Lecture 2: Cameras I

Administrative

HW0 is released, will be due Friday 1/24 at 11:59pm

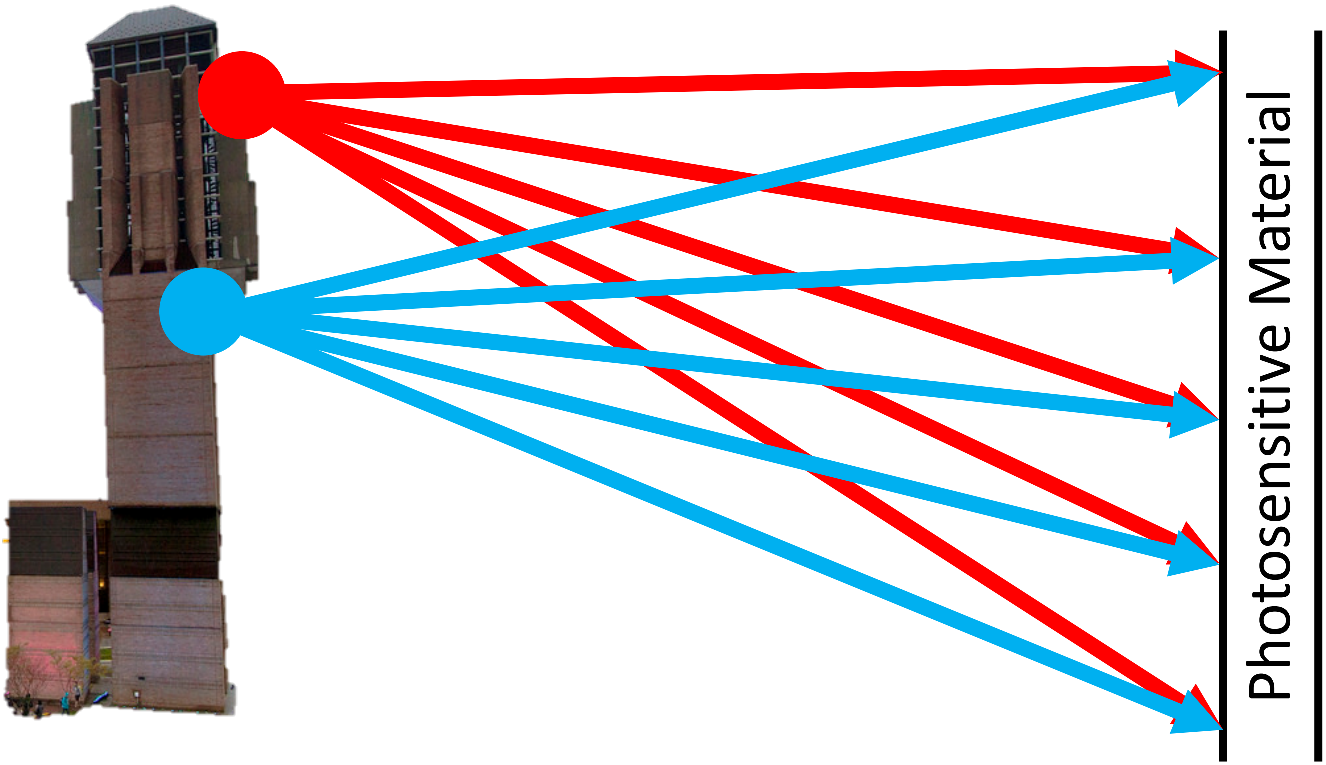
This week: How do images form?

Computer Vision: Writing software to understand images

Photography: Capturing images of the real world

Computer Graphics: Writing software to generate images

Let's take a picture!

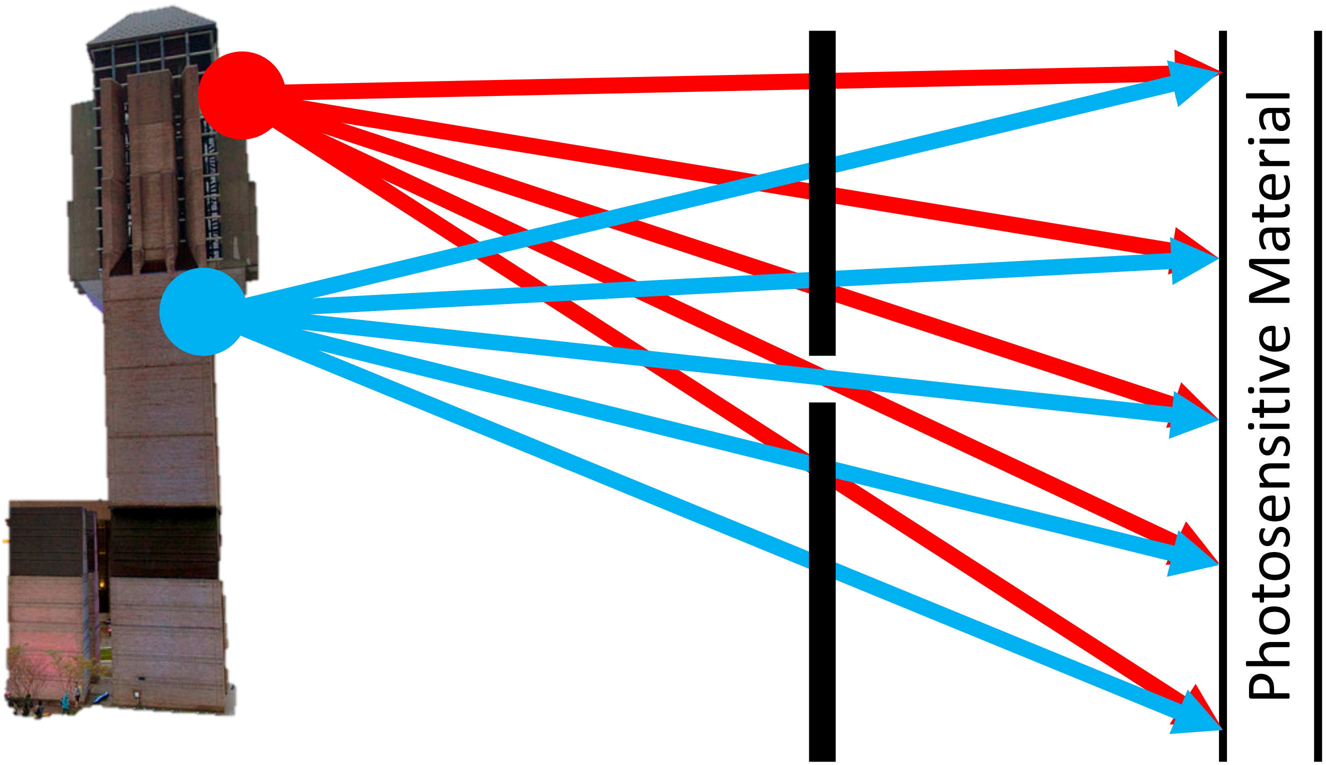


Idea 1: Just use film

Result: **Junk**

Slide inspired by S. Seitz; image from Michigan Engineering

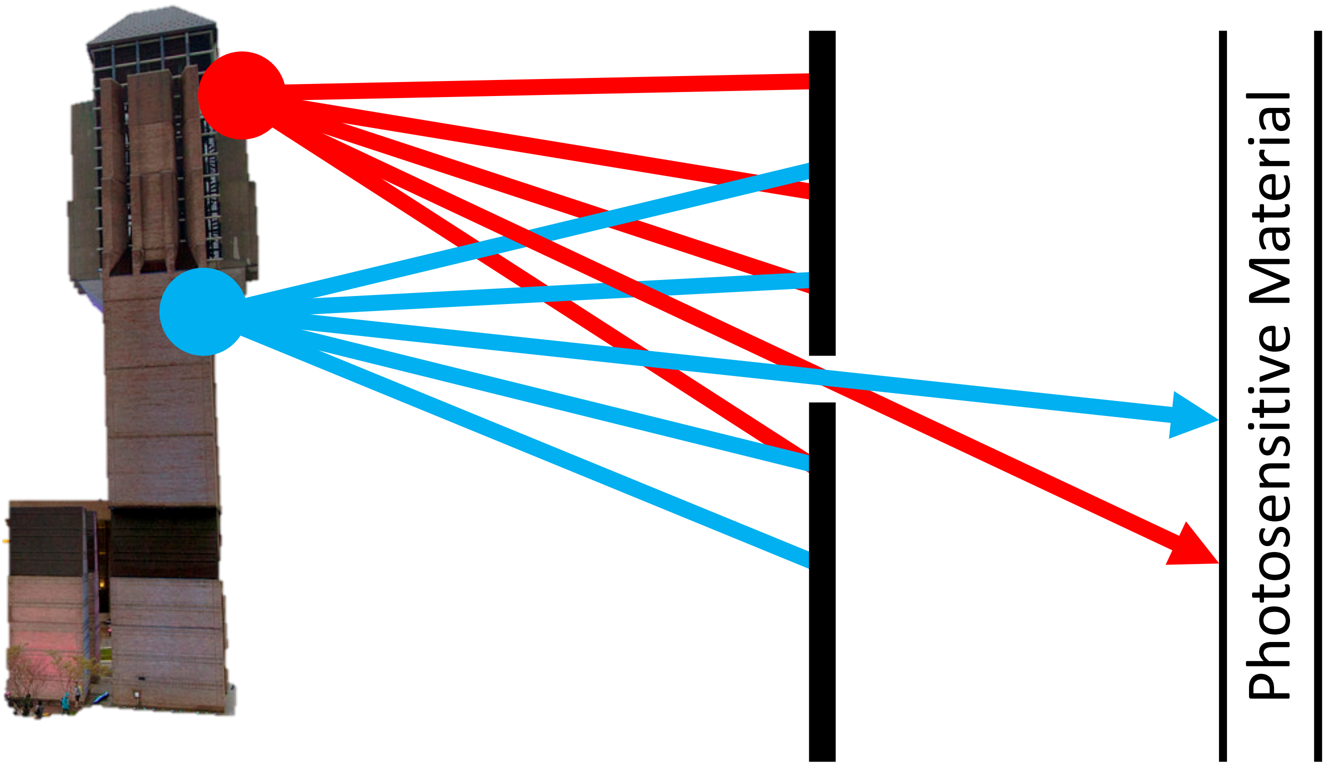
Let's take a picture!



Idea 2: add a barrier

Slide inspired by S. Seitz; image from Michigan Engineering

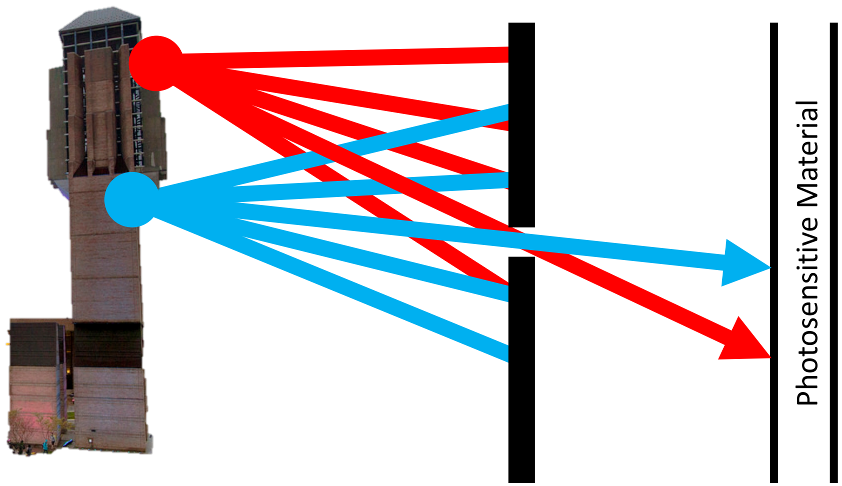
Let’s take a picture!



Idea 2: add a barrier

Slide inspired by S. Seitz; image from Michigan Engineering

Let's take a picture!

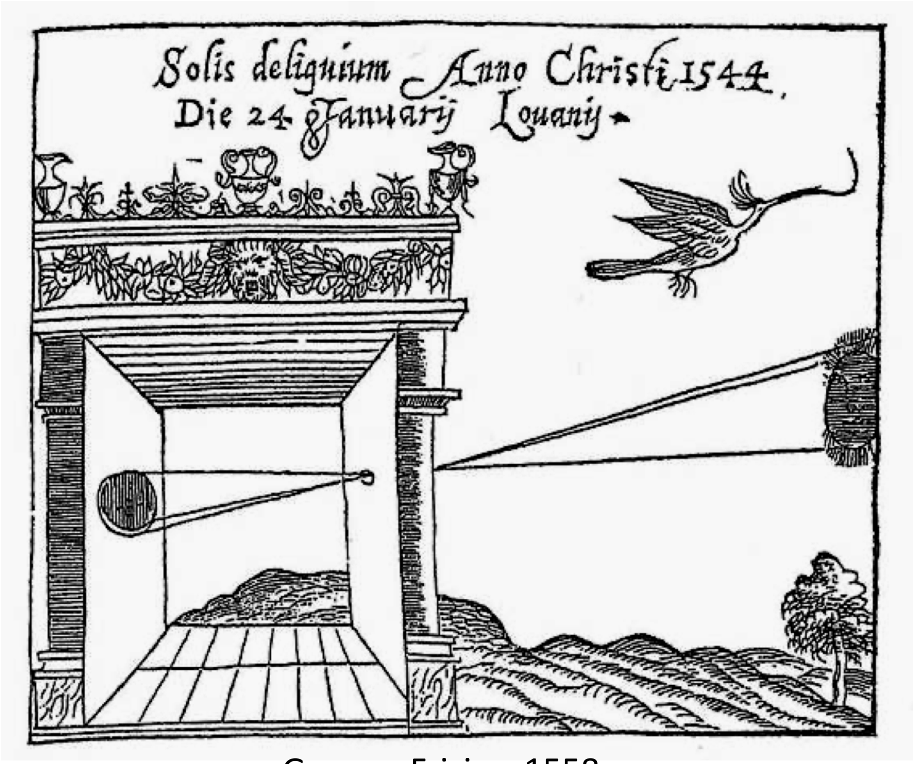


Film captures all the rays going through a point (a p*encil of rays).*

Result: good in theory!

Slide inspired by S. Seitz; image from Michigan Engineering

Camera Obscura



Gemma Frisius, 1558

- Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)

Source: A. Efros

Camera Obscura

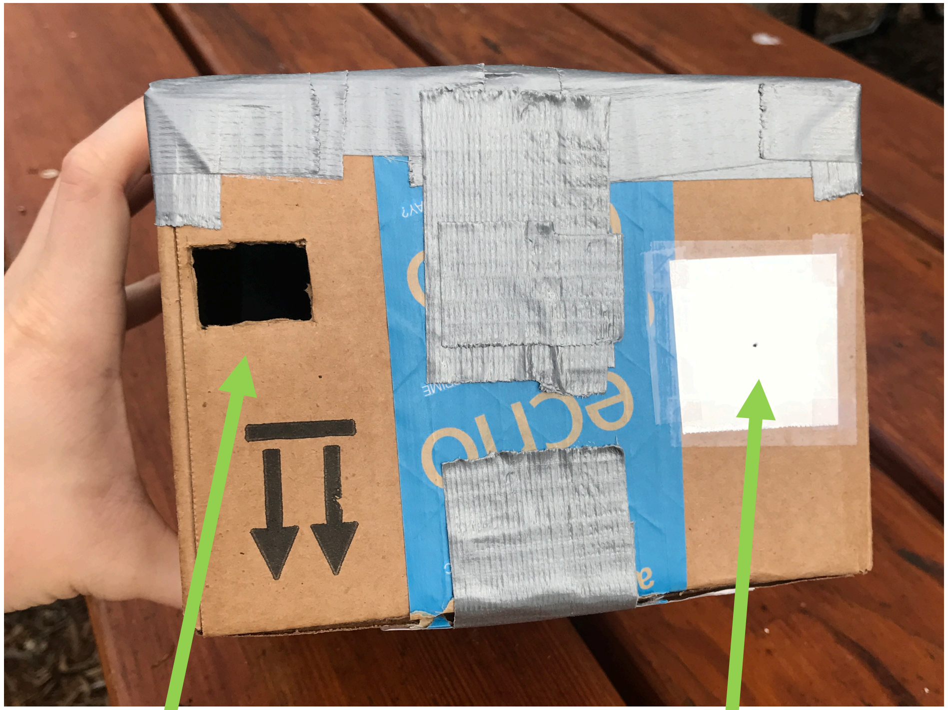
My bedroom with blackout curtains

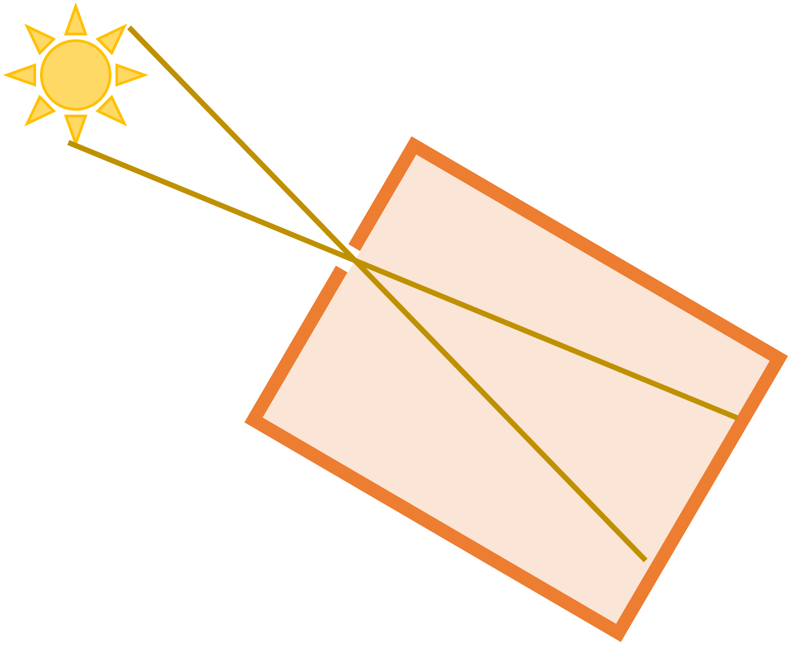
The view out the window

Image credit: Justin Johnson, 9/29/2018

Camera Obscura

Useful for viewing solar eclipses!



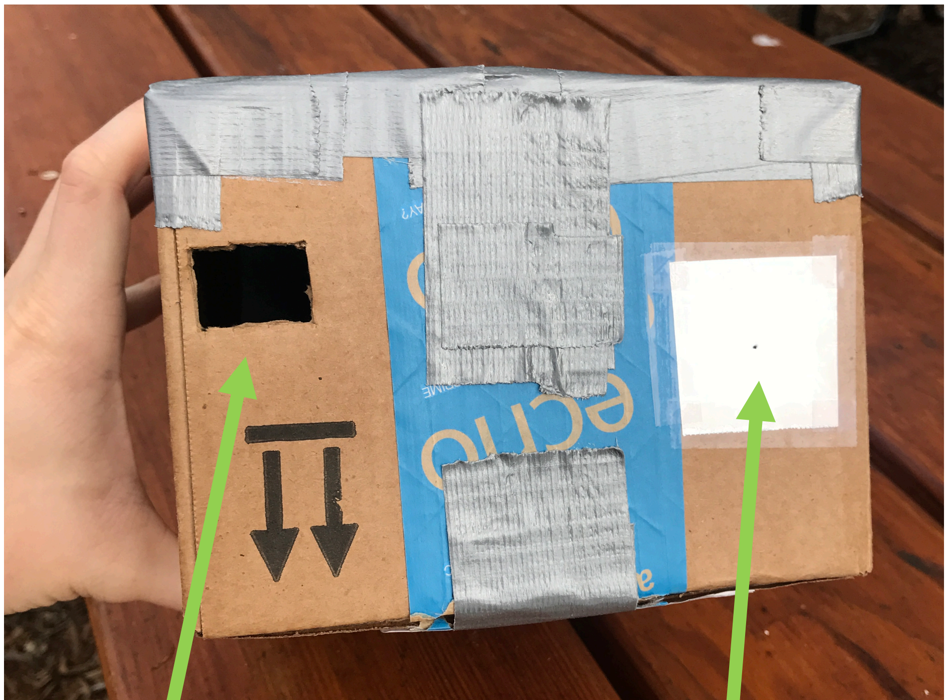


Put your eye here

Pinhole: aluminum foil with a tiny hole

Camera Obscura

Useful for viewing solar eclipses!



Put your eye here

Pinhole: aluminum foil with a tiny hole

Me on 8/21/2017

Camera Obscura

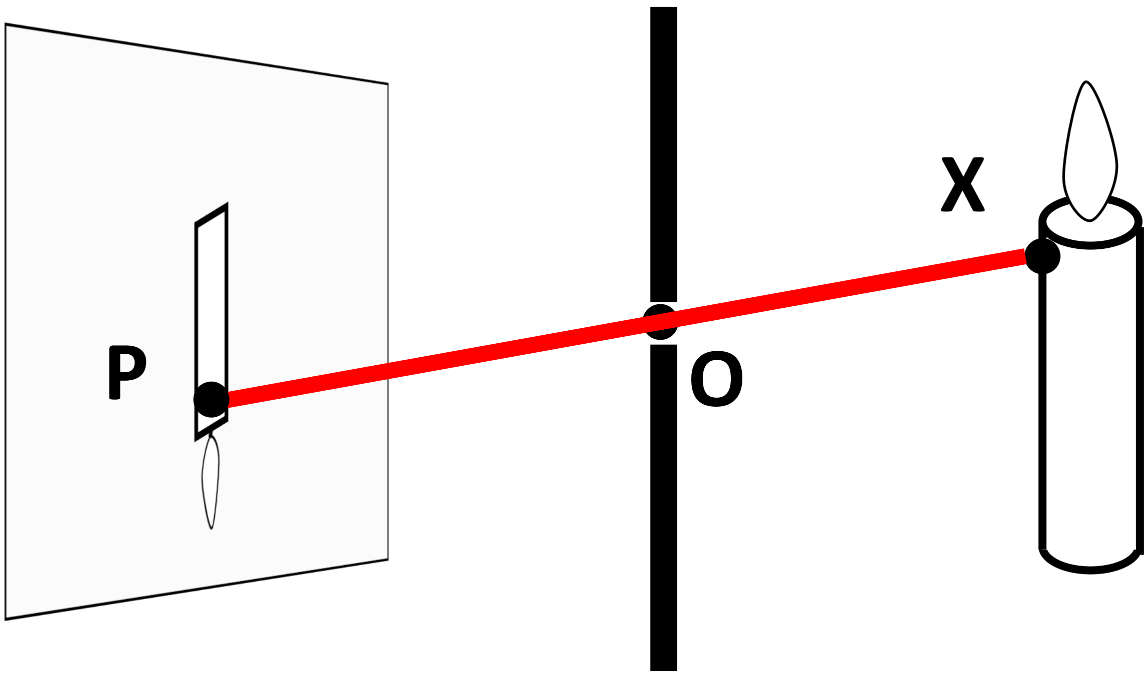
Useful for viewing solar eclipses!

Photo of the sun

View in the box

Me on 8/21/2017

Projection

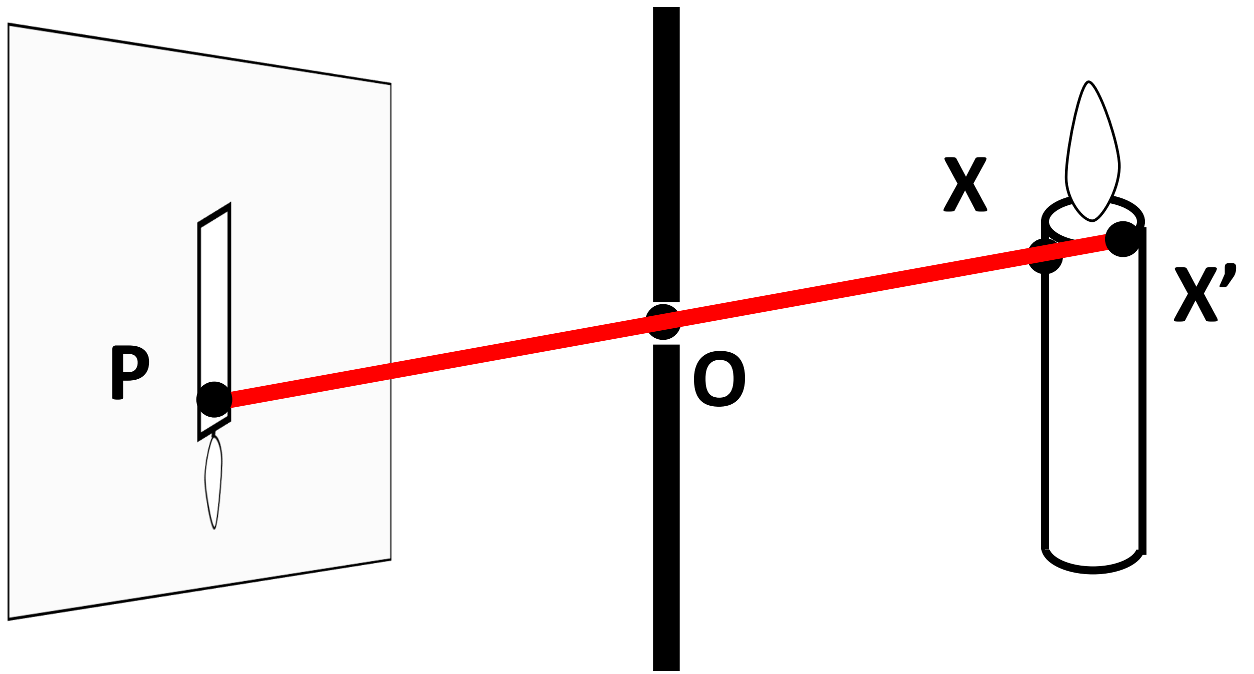


How do we find the projection P of a point X?

Form visual ray from X to camera center and intersect it with camera plane

Source: L Lazebnik

Projection

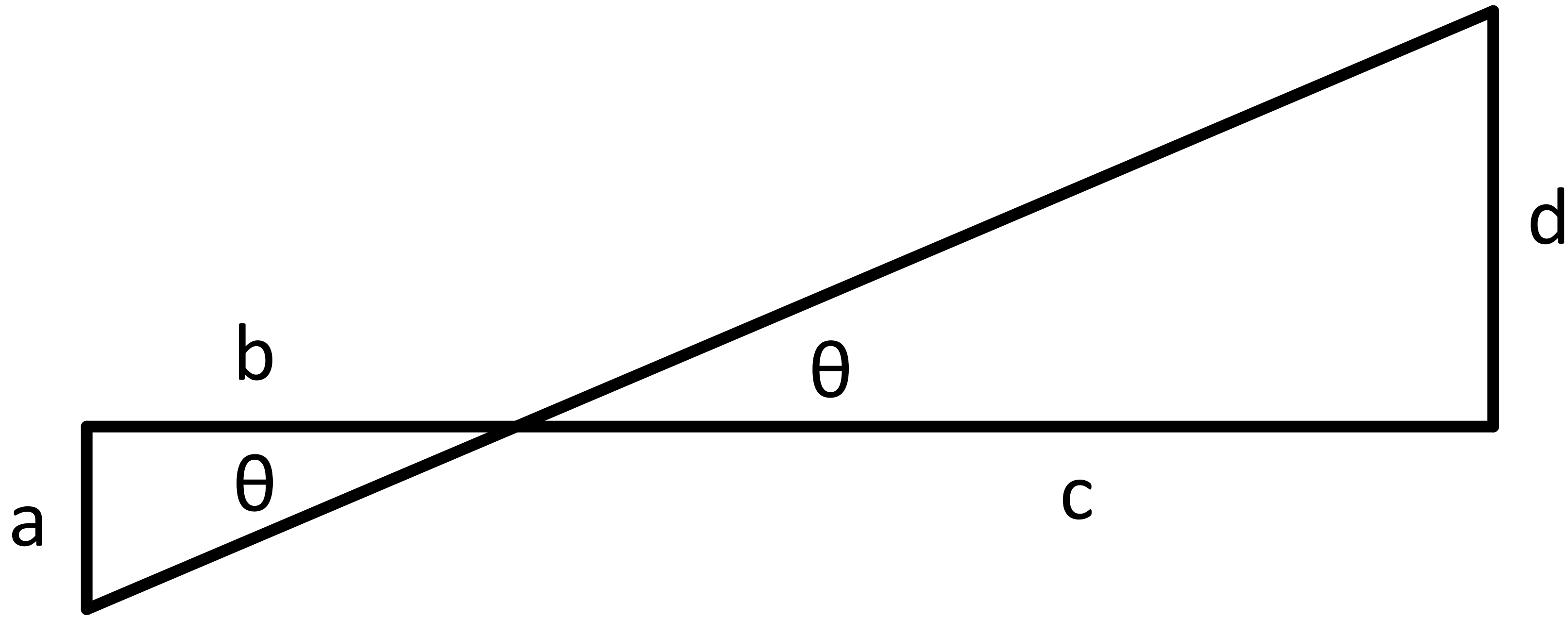


Both X and X' project to P. Which appears in the image?

Are there points for which projection is undefined?

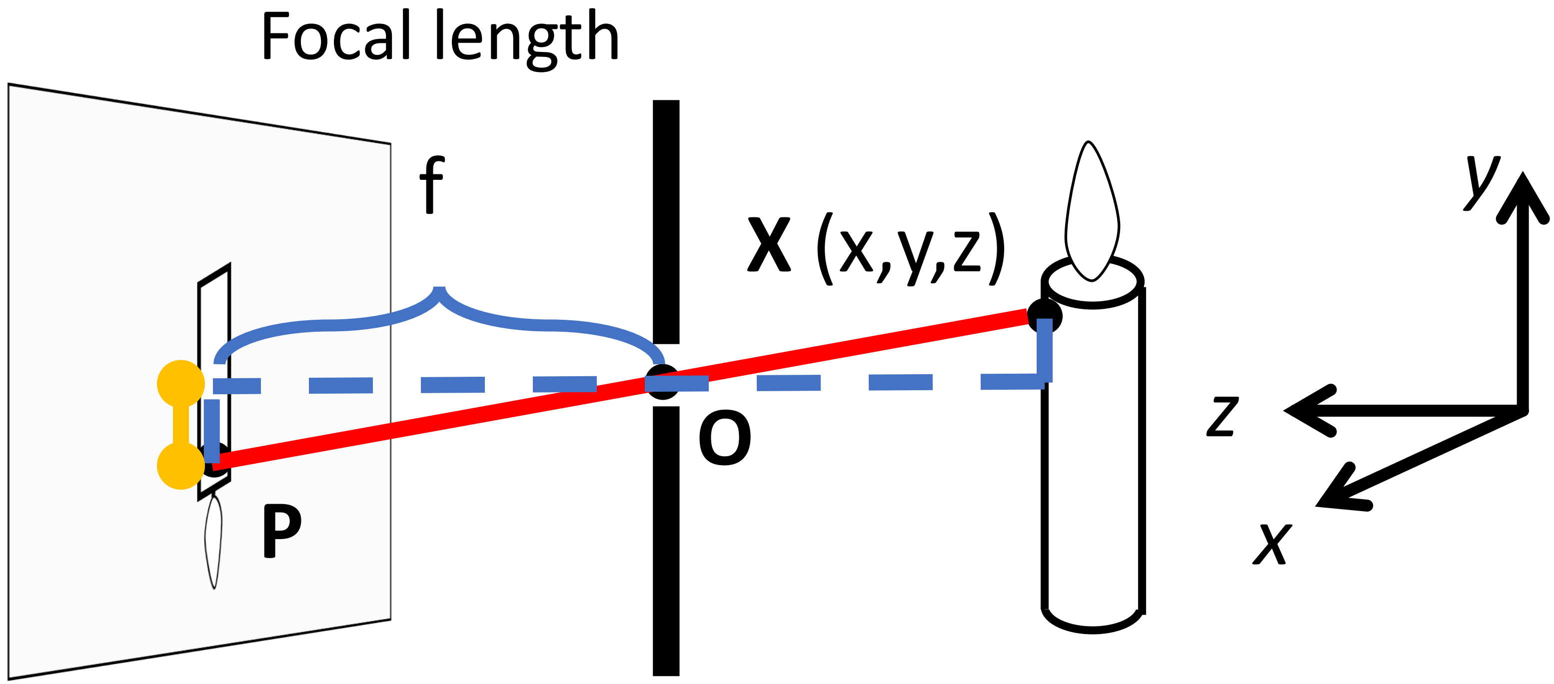
Source: L Lazebnik

Aside: Remember Trigonometry?



$$\frac{a}{b} = \frac{d}{c} \longrightarrow a = \frac{bd}{c}$$

Projection



Coordinate system: **O** is origin, XY in image, Z sticks out.
XY is image plane, Z is optical axis.

(x,y,z) projects to (fx/z,fy/z) via similar triangles

Source: L Lazebnik

Source: L Lazebnik

Facts about Projection

3D lines project to 2D lines

The projection of any 3D parallel lines converge at a vanishing point

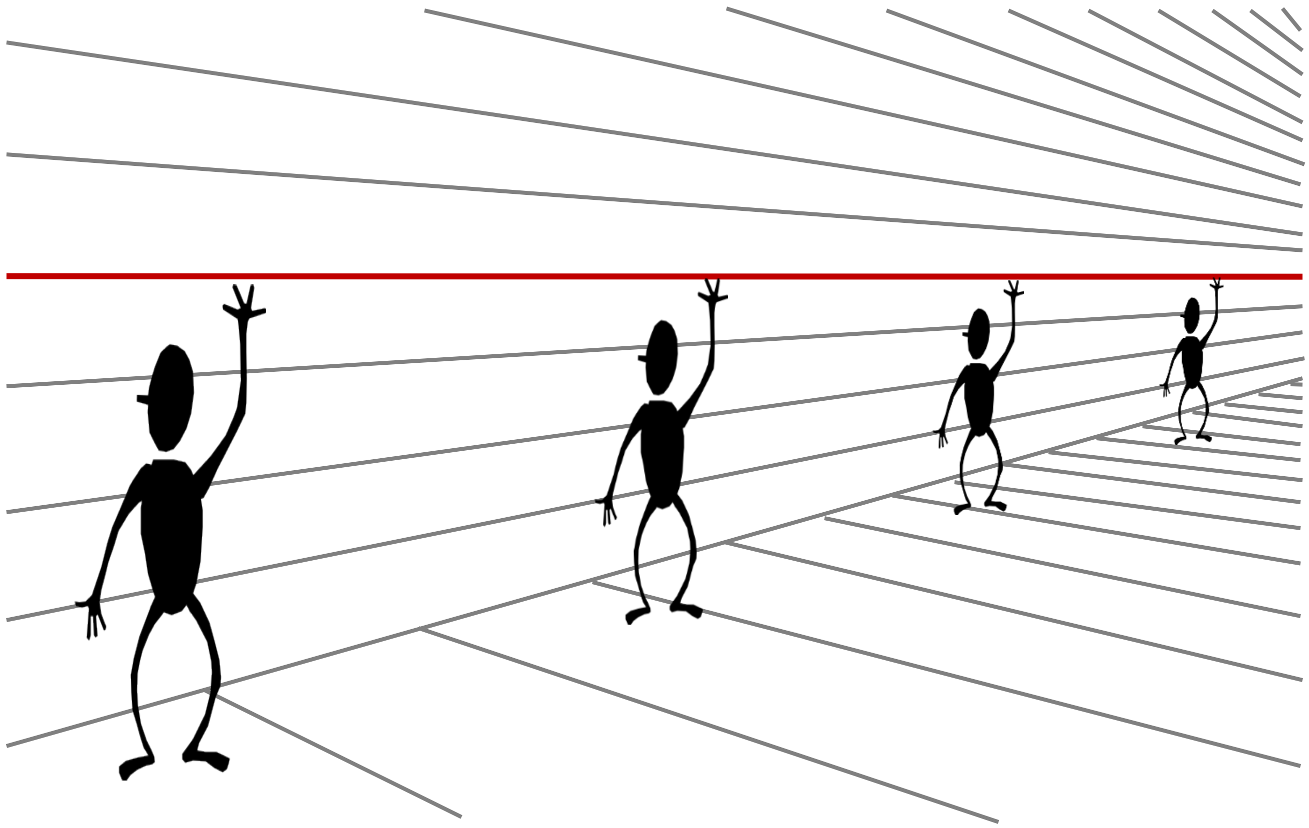
Distant objects are smaller

List of properties from M. Hebert

Facts about Projection

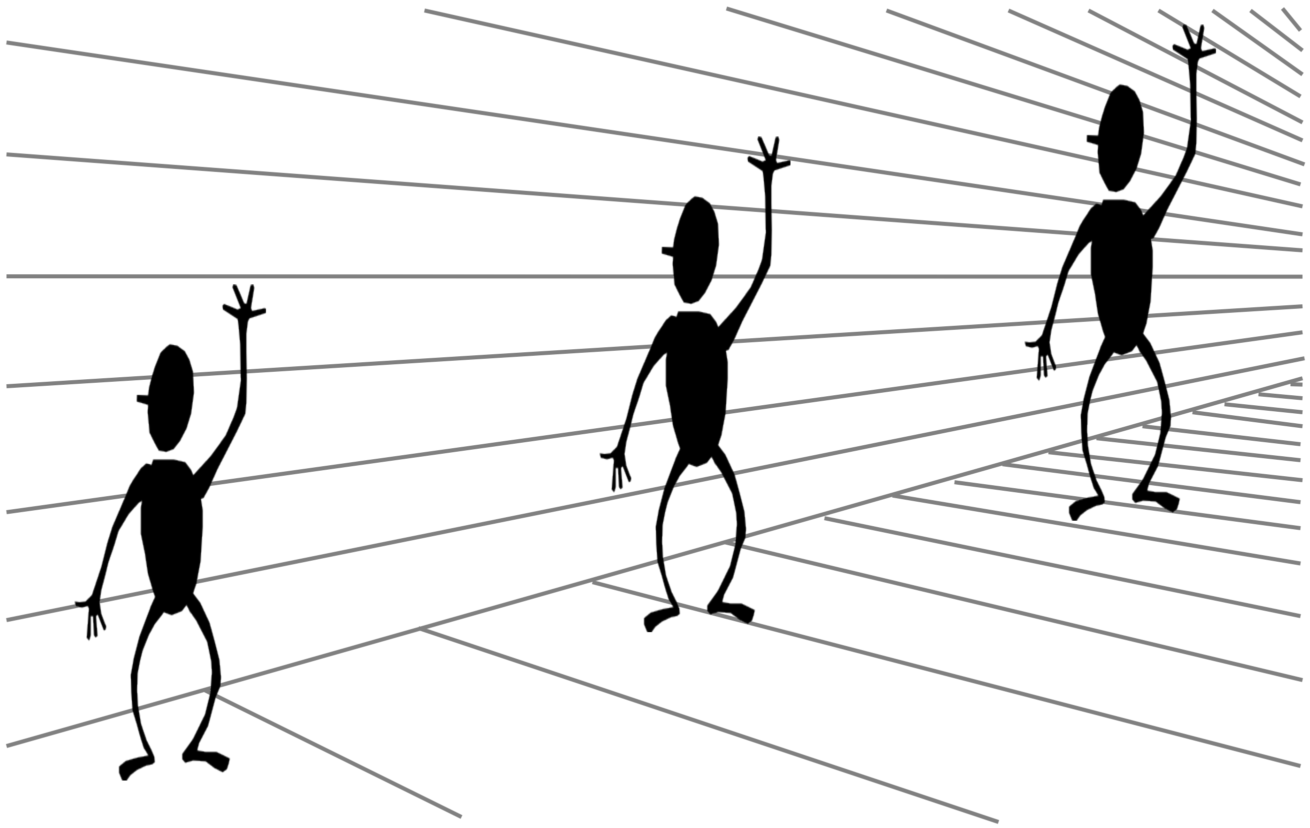
Let's try some fake images

Facts about Projection



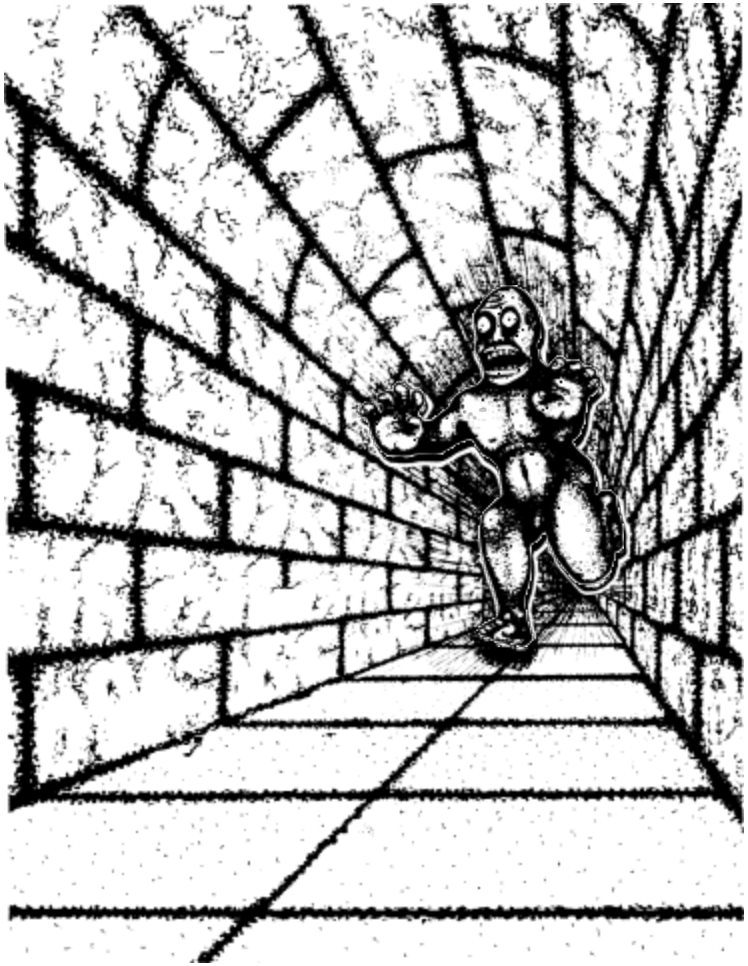
Slide by Steve Seitz

Facts about Projection



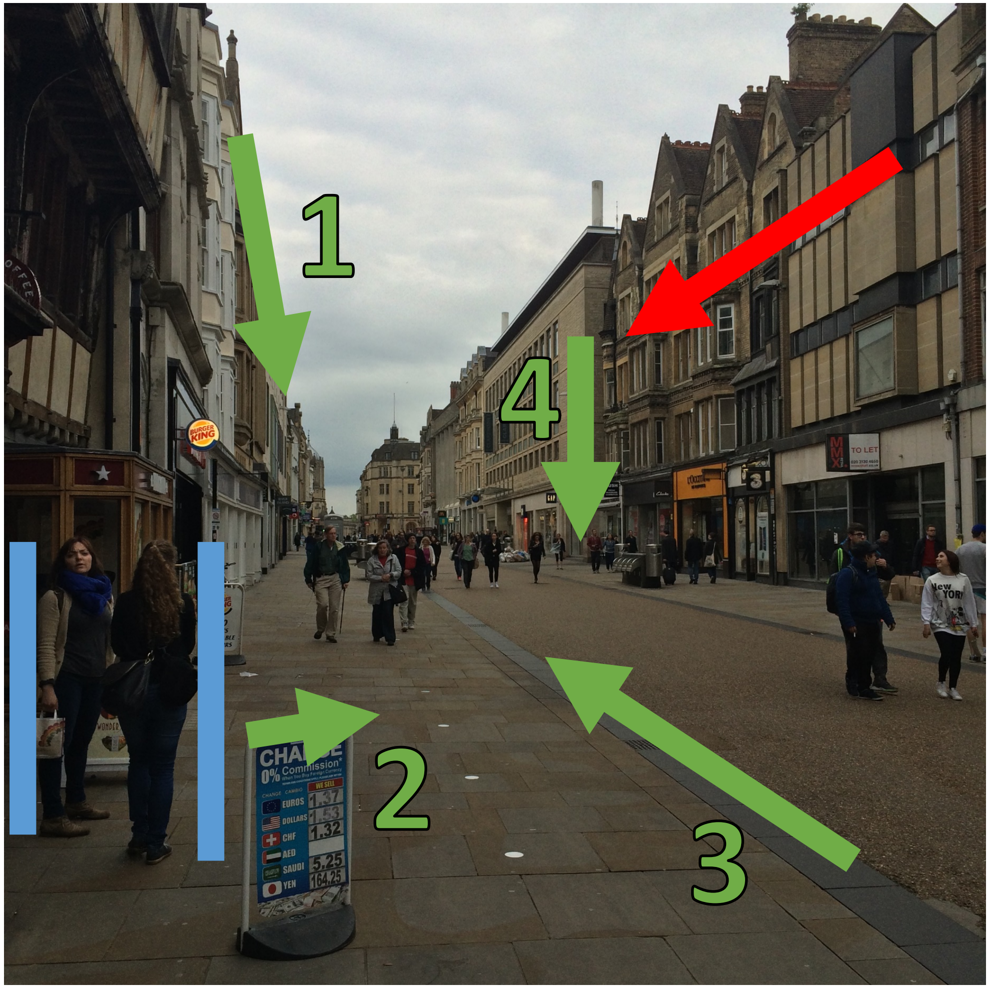
Slide by Steve Seitz

Facts about Projection



Illusion Credit: RN Shepard, Mind Sights: Original Visual Illusions, Ambiguities, and other Anomalies

What is lost under projection?

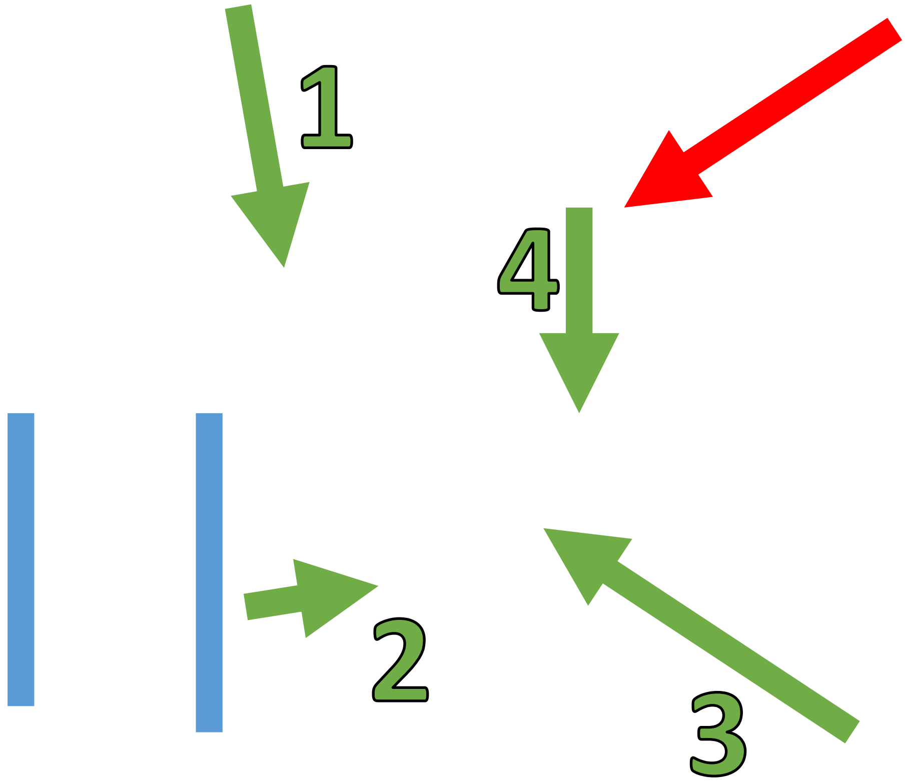


Is she shorter or further away?

Are the green lines we see parallel / perpendicular / neither to the red line?

Inspired by D. Hoiem slide

What is lost under projection?



Is she shorter or further away?

Are the green lines we see parallel / perpendicular / neither to the red line?

Inspired by D. Hoiem slide

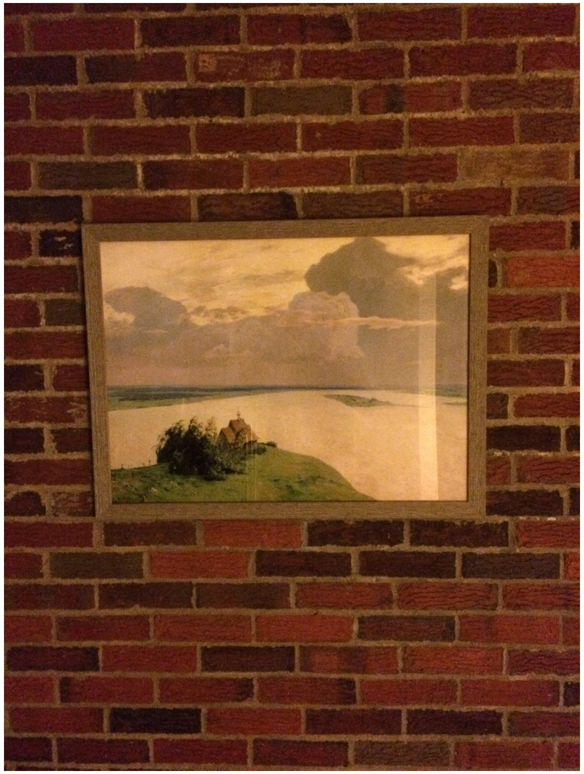
What is lost under projection?

Be careful of drawing conclusions:

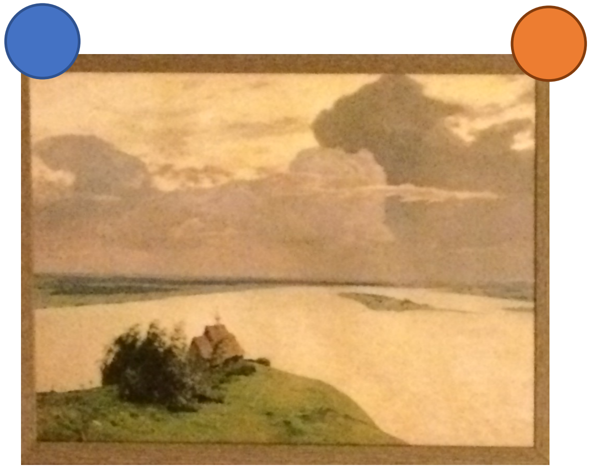
- Projection of 3D line is 2D line; NOT 2D line is 3D line.
- **Can you think of a counter-example (a 2D line that is not a 3D line)?**

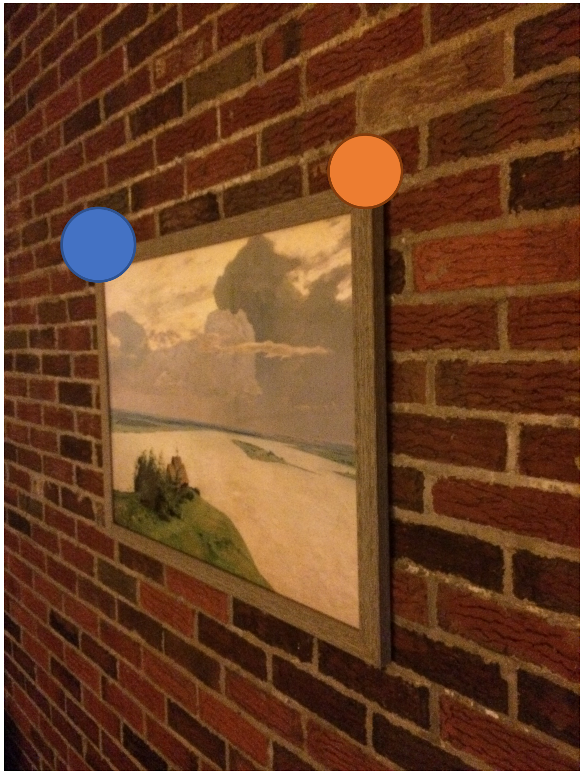
- Projections of parallel 3D lines converge at VP; NOT any pair of lines that converge are parallel in 3D.
- **Can you think of a counter-example?**

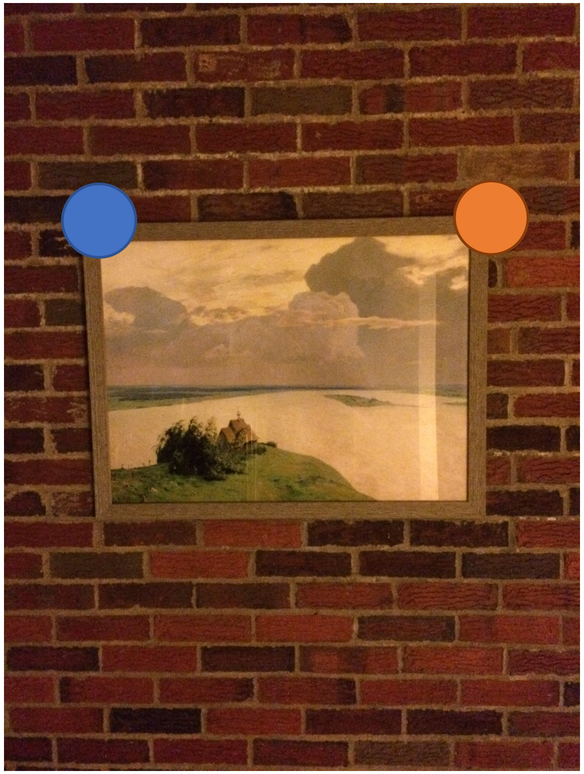
Do we always get perspective?



Do we always get perspective?



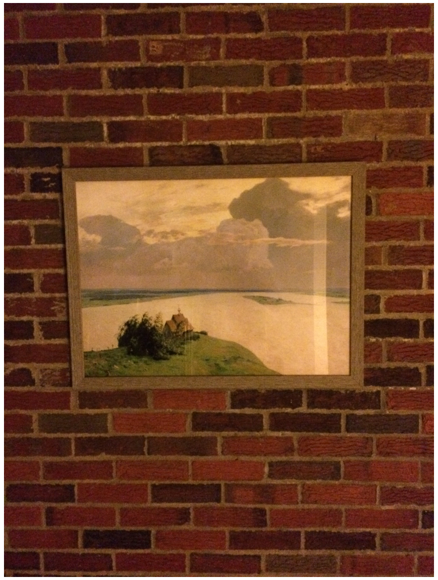




Y location of blue and orange dots in image:

$$\frac{fy}{z_2} \qquad \frac{fy}{z_1} \qquad\qquad \frac{fy}{z} \qquad \frac{fy}{z}$$

Do we always get perspective?

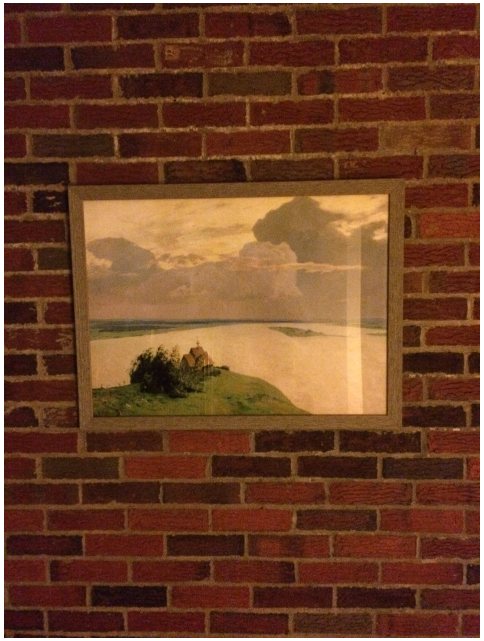


When plane is fronto-parallel (parallel to camera plane), everything is:

- scaled by f/z
- otherwise is preserved.

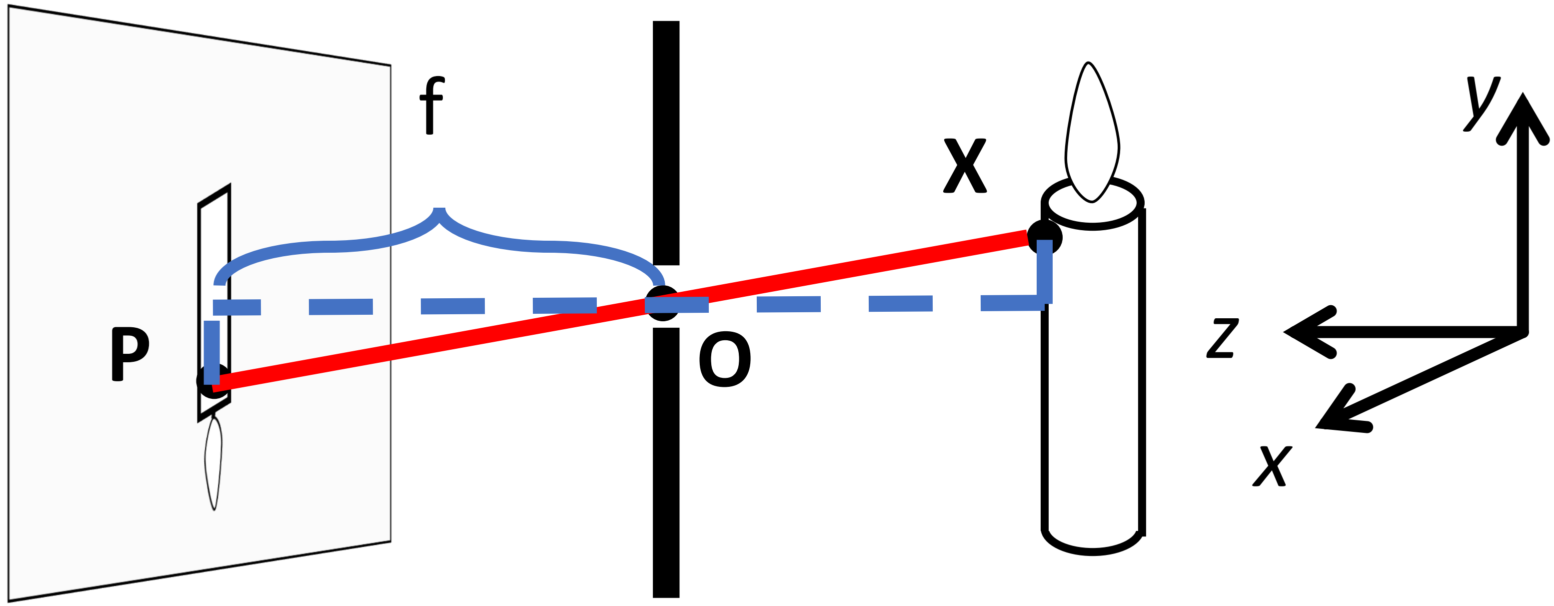
Why is this useful?





Things looking different when viewed from different angles seems like a nuisance. It's also a cue. **Why?**

Projection



$(x,y,z) \rightarrow (fx/z, fy/z)$

I promised you linear algebra: is this linear?

Nope: division by z is non-linear
(and risks division by 0)

Adapted from S. Seitz slide

Homogeneous Coordinates (2D)

Trick: add a dimension!

This also clears up lots of nasty special cases

Physical Point		Homogeneous Point		Physical Point
$\begin{bmatrix} x \\ y \end{bmatrix}$	$\longrightarrow$ Concat w=1	$\begin{bmatrix} u \\ v \\ w \end{bmatrix}$	$\longrightarrow$ Divide by w	$\begin{bmatrix} u/w \\ v/w \end{bmatrix}$

What if w = 0?

Adapted from M. Hebert slide

Homogenous Coordinates

Triple / Equivalent

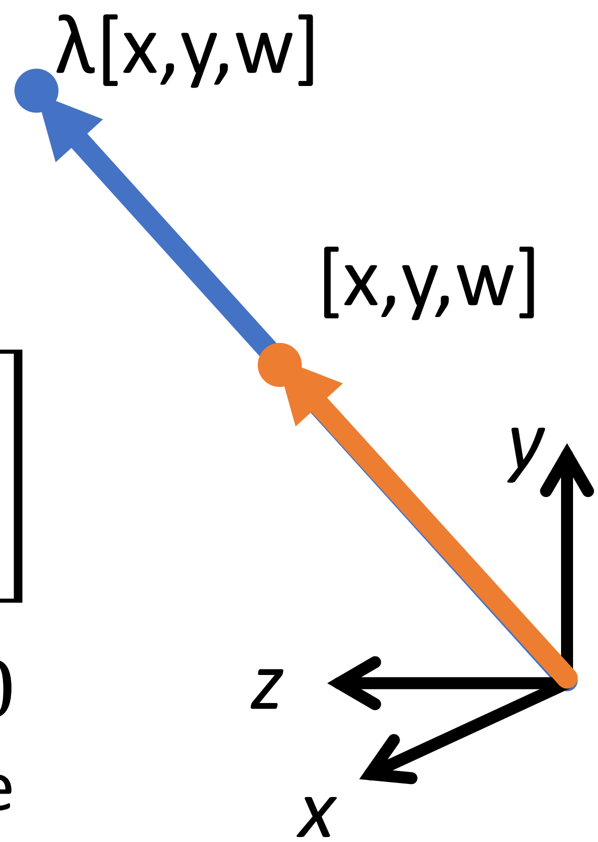
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \boxed{\equiv} \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix}$$

$\leftrightarrow$

Double / Equals

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \boxed{=} \lambda \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix}$$

$$\lambda \neq 0$$



Two homogeneous coordinates are **equivalent** if they are proportional to each other. **Not = !**

Benefits of Homogenous Coords

General equation of 2D line:

$$ax + by + c = 0$$

Homogeneous Coordinates

$$\boldsymbol{l}^T\boldsymbol{p} = 0, \qquad \boldsymbol{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \boldsymbol{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

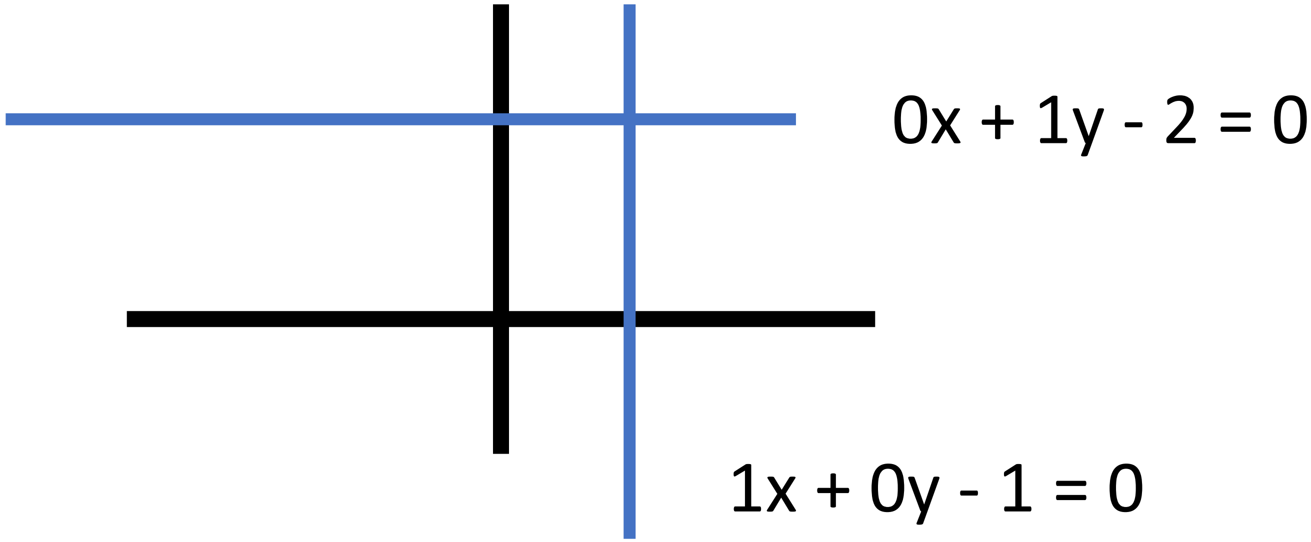
Slide from M. Hebert

Benefits of Homogenous Coords

- Lines (3D) and points (2D → 3D) are now the same dimension.
- Use the *cross* (x) and *dot product* for:
  - Point **p** on line **l**: $\mathbf{l}^T\mathbf{p} = 0$
  - Intersection of lines **l** and **m**: **l** x **m**
  - Line through two points **p** and **q**: **p** x **q**
- Parallel lines, vertical lines become easy (compared to y=mx+b)

Benefits of Homogenous Coords

What's the intersection?



$$[0,1,-2] \times [1,0,-1] = [-1,-2,-1]$$

Converting back (divide by -1)

(1,2)

Benefits of Homogenous Coords

Intersection of y=2, y=1

$[0,1,-2] \times [0,1,-1] = [1,0,0]$

Does it lie on y=3? Intuitively?

$[0,1,-3]^T[1,0,0] = 0$

Benefits of Homogenous Coords

Translation is now linear / matrix-multiply

If w = 1 $$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} u + t_x \\ v + t_y \\ 1 \end{bmatrix}$$

Generically $$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} u + wt_x \\ v + wt_y \\ w \end{bmatrix}$$

Rigid body transforms (rot + trans) now linear

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

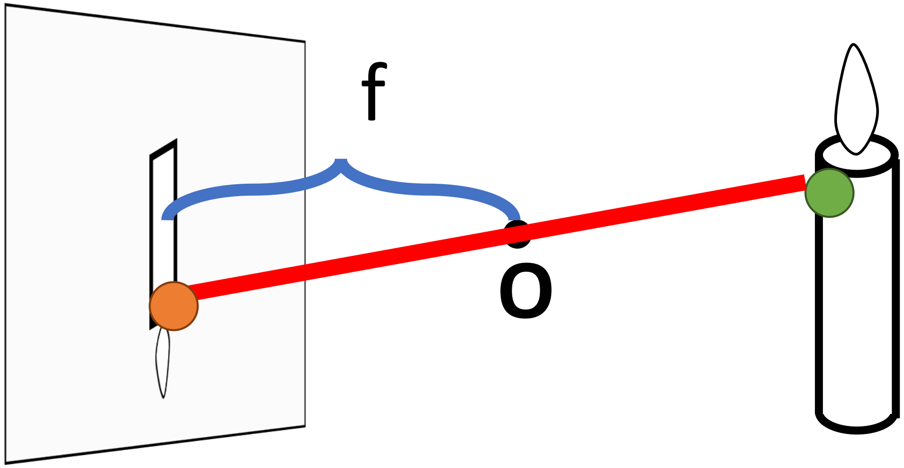
3D Homogenous Coordinates

Same story: add a coordinate, things are equivalent if they're proportional

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} u \\ v \\ w \\ t \end{bmatrix} \rightarrow \begin{bmatrix} u/t \\ v/t \\ w/t \end{bmatrix}$$

Projection Matrix

Projection (x, y, z) -> (fx/z, fy/z) is matrix multiplication



Slide inspired from L. Lazebnik

Projection Matrix

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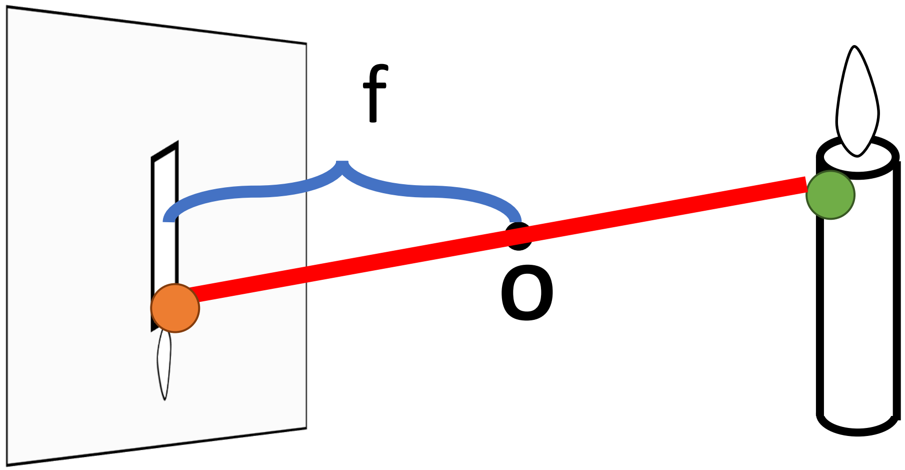


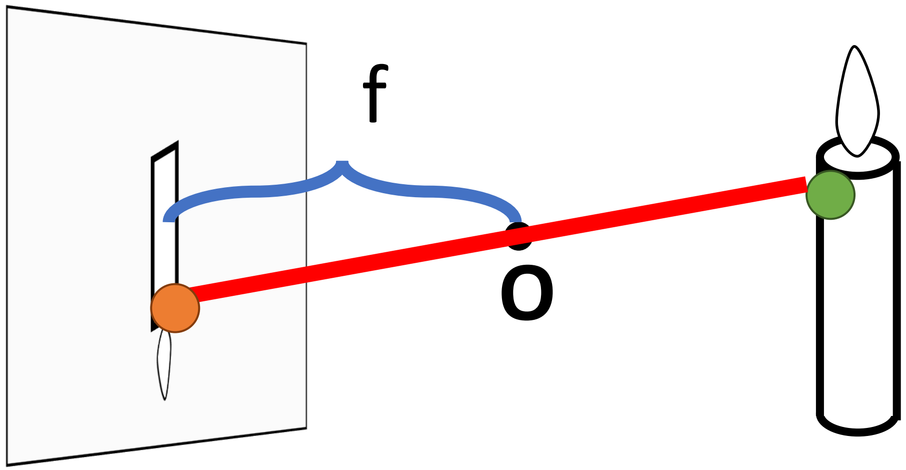
$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D homogenous point

Slide inspired from L. Lazebnik

Projection Matrix

Projection (x, y, z) -> (fx/z, fy/z) is matrix multiplication



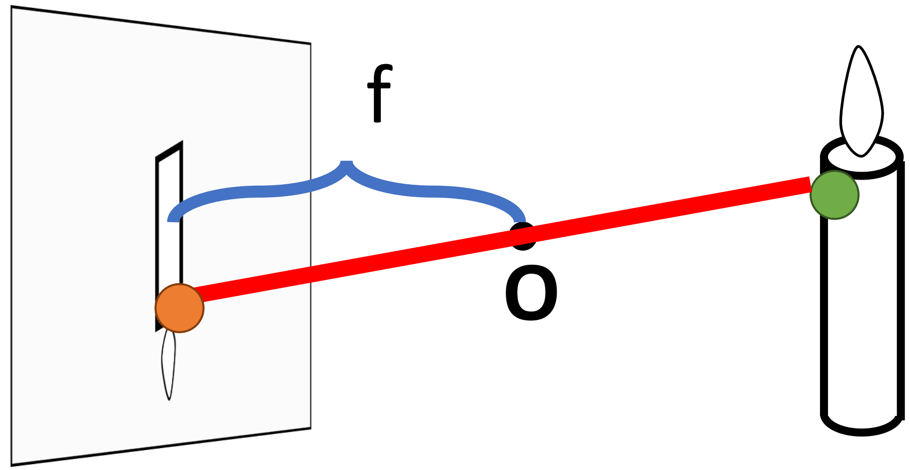
$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \\ \boldsymbol{1} \end{bmatrix}$$

3D homogenous point

Slide inspired from L. Lazebnik

Projection Matrix

Projection (x, y, z) -> (fx/z, fy/z) is matrix multiplication



$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \\ \boldsymbol{1} \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix}$$

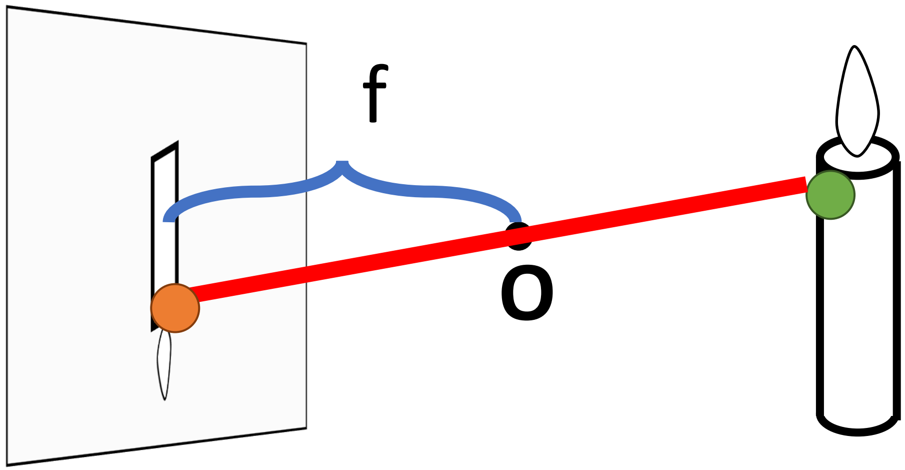
3D homogenous point

2D homogenous point

Slide inspired from L. Lazebnik

Projection Matrix

Projection (x, y, z) -> (fx/z, fy/z) is matrix multiplication



$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \\ \boldsymbol{1} \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \equiv \begin{bmatrix} fx/z \\ fy/z \\ 1 \end{bmatrix}$$

3D homogenous point

2D homogenous point

Slide inspired from L. Lazebnik

Typical Perspective Model

P: 2D homogeneous point (3D)

X: 3d homogeneous point (4D)

$$\boldsymbol{P} \equiv \qquad\qquad \boldsymbol{X}_{4x1}$$

Typical Perspective Model

R: rotation between world system and camera

t: translation between world system and camera

$$\boldsymbol{P} \equiv \qquad [\boldsymbol{R}_{3x3} \quad \boldsymbol{t}_{3x1}] \quad \boldsymbol{X}_{4x1}$$

Typical Perspective Model

f focal length

u0,v0: principal point (image coords of camera origin on retina)

$$\boldsymbol{P} \equiv \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{R}_{3x3} & \boldsymbol{t}_{3x1} \end{bmatrix} \boldsymbol{X}_{4x1}$$

Typical Perspective Model

Intrinsic Matrix K | **Extrinsic Matrix [R,t]**

$$\boldsymbol{P} \equiv \boxed{\begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}} \quad \boxed{\begin{bmatrix} \boldsymbol{R}_{3x3} & \boldsymbol{t}_{3x1} \end{bmatrix}} \quad \boldsymbol{X}_{4x1}$$

$$\boldsymbol{P} \equiv \boldsymbol{K}[\boldsymbol{R} \mid \boldsymbol{t}]\boldsymbol{X} \equiv \boldsymbol{M}_{3x4}\boldsymbol{X}_{4x1}$$

Nice interactive demo: http://ksimek.github.io/2012/08/22/extrinsic/

Other Cameras: Orthographic

Orthographic Camera (z infinite)

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \boldsymbol{X}_{3x1}$$

Image Credit: Wikipedia

Other Cameras: Orthographic

Why does this make things easy and why is this popular in old games?

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Next Time:
More Cameras