Lecture 13: Neural Networks

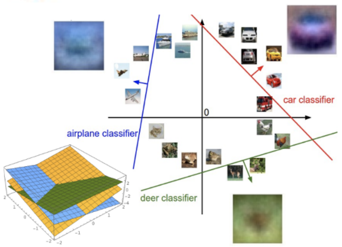
Administrative

- HW3 due **Wednesday 3/10**

Where we are:

1. Use **Linear Models** for image classification problems

$$s = f(x; W) = Wx$$



2. Use **Loss Functions** to express preferences over different choices of weights

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right) \quad \text{Softmax}$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \quad \text{SVM}$$

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i + R(W)$$

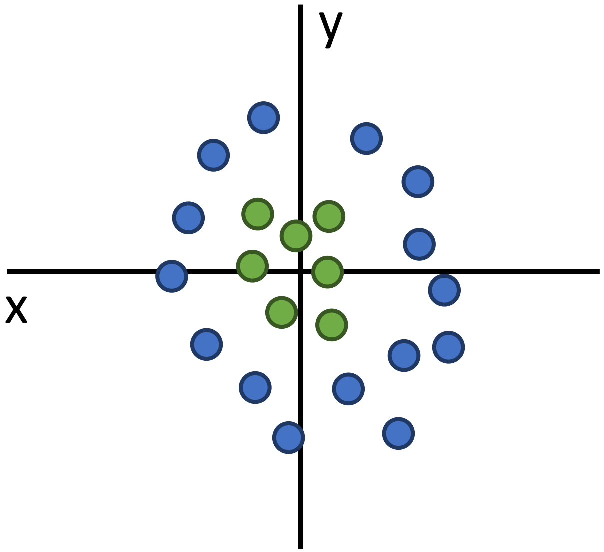
3. Use **Stochastic Gradient Descent** to minimize our loss functions and train the model

```
v = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  v = rho * v + dw
  w -= learning_rate * v
```

4. Add **Regularization** to control overfitting

Problem: Linear Classifiers not enough

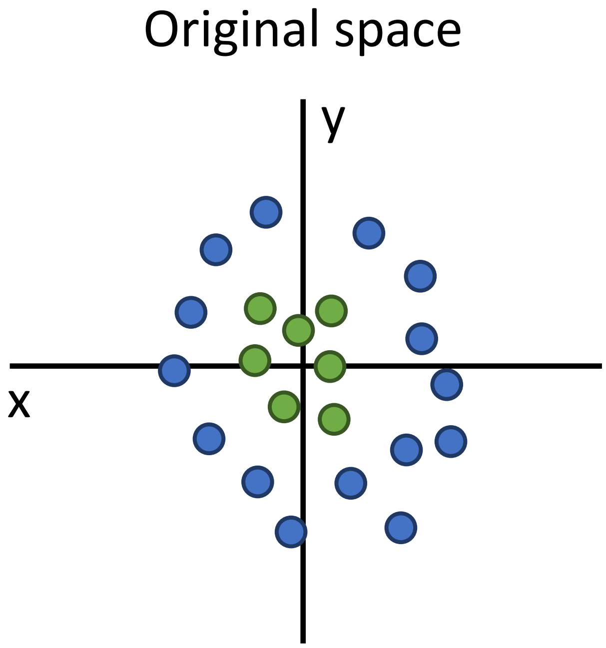
<u>Geometric Viewpoint</u>



<u>Visual Viewpoint</u>

One template per class:
Can't recognize different
modes of a class

One solution: **Feature Transforms**

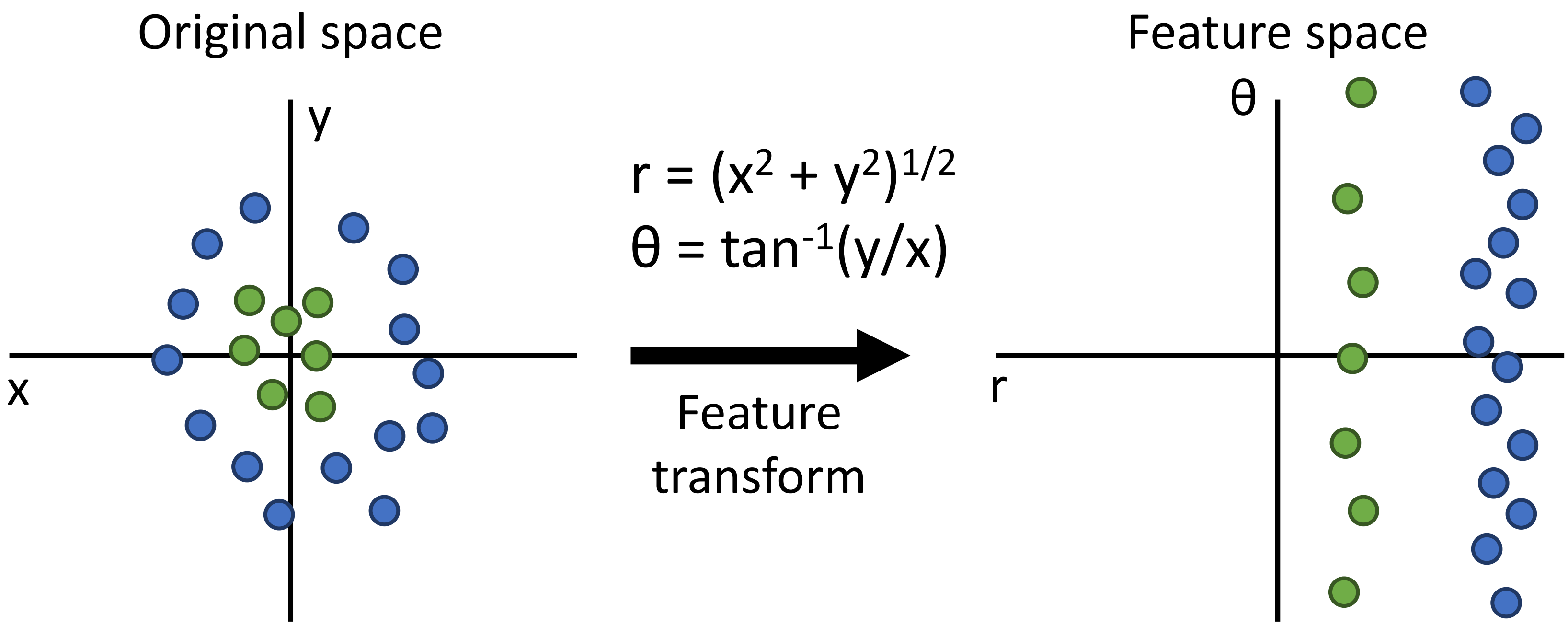


$r = (x^2 + y^2)^{1/2}$

$\theta = \tan^{-1}(y/x)$

Feature transform

One solution: **Feature Transforms**



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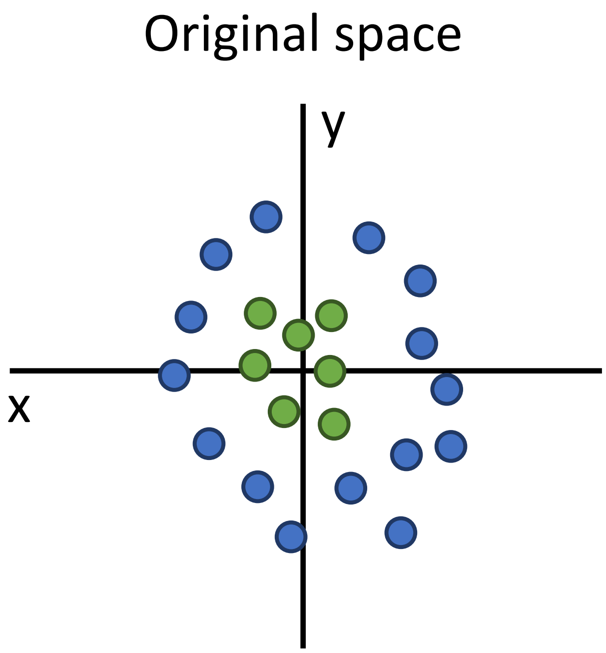

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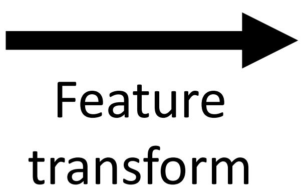
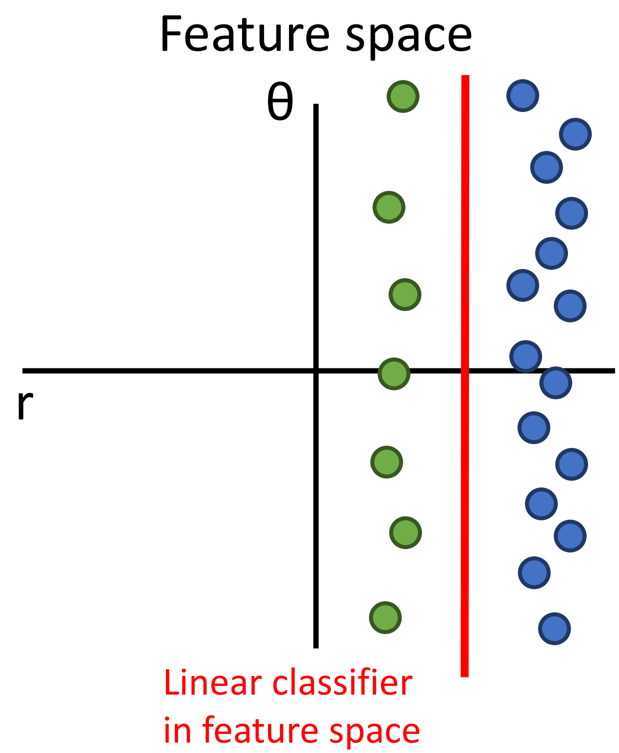
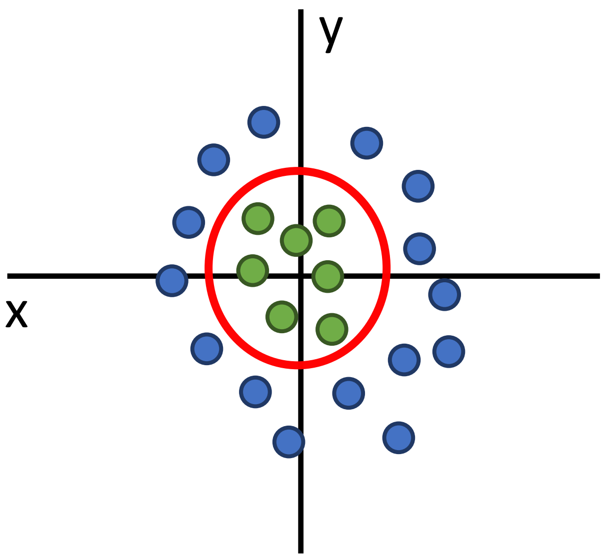




One solution: **Feature Transforms**

Original space



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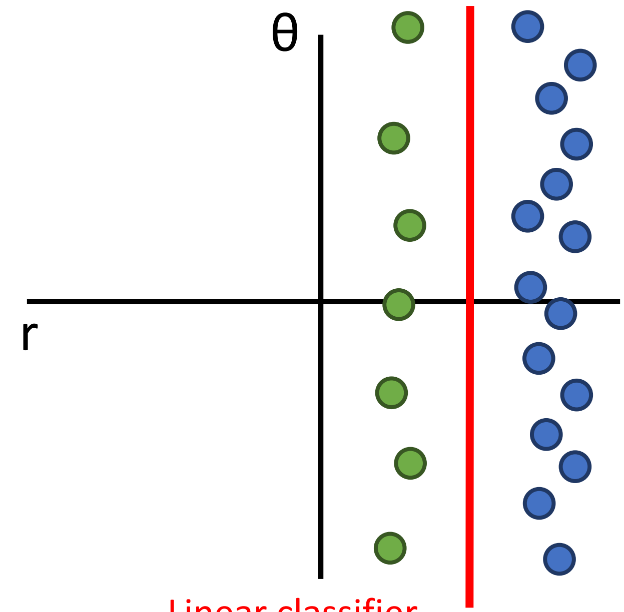
Feature transform

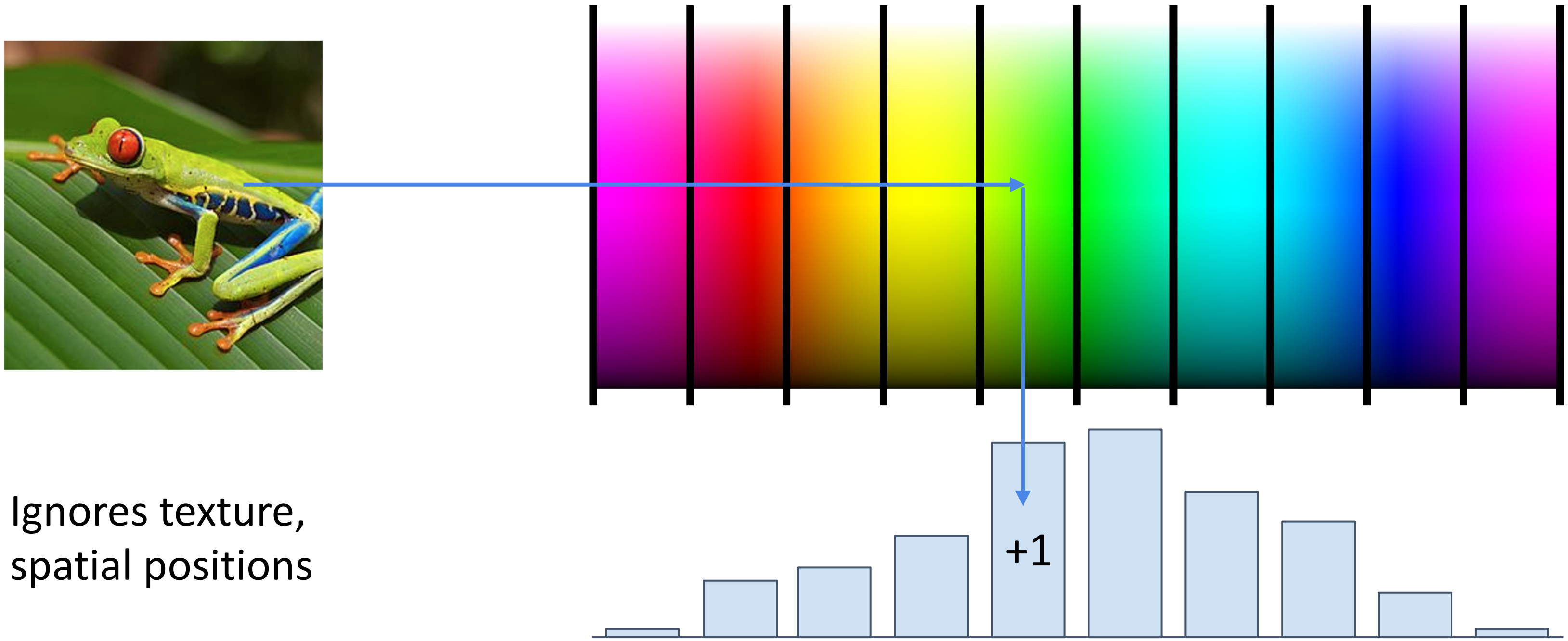
Feature space



Nonlinear classifier in original space!

Linear classifier in feature space

Image Features: Color Histogram



Ignores texture,
spatial positions

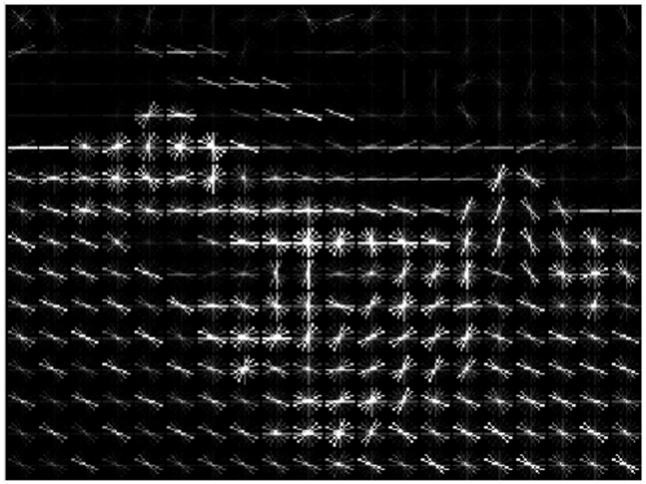
Frog image is in the public domain

Image Features: Histogram of Oriented Gradients (HoG)

1. Compute edge direction / strength at each pixel
2. Divide image into 8x8 regions
3. Within each region compute a histogram of edge directions weighted by edge strength

Lowe, "Object recognition from local scale-invariant features", ICCV 1999
Dalal and Triggs, "Histograms of oriented gradients for human detection," CVPR 2005

Image Features: Histogram of Oriented Gradients (HoG)



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Example: 320x240 image gets divided into 40x30 bins; 8 directions per bin; feature vector has 30*40*9 = 10,800 numbers

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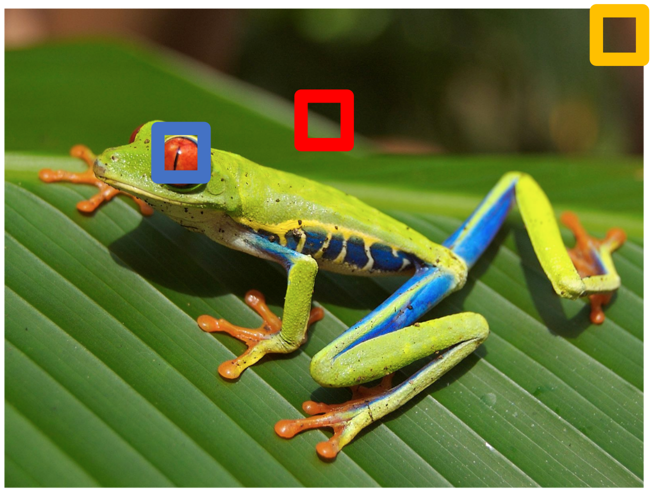

Weak edges

Strong diagonal edges

Edges in all directions

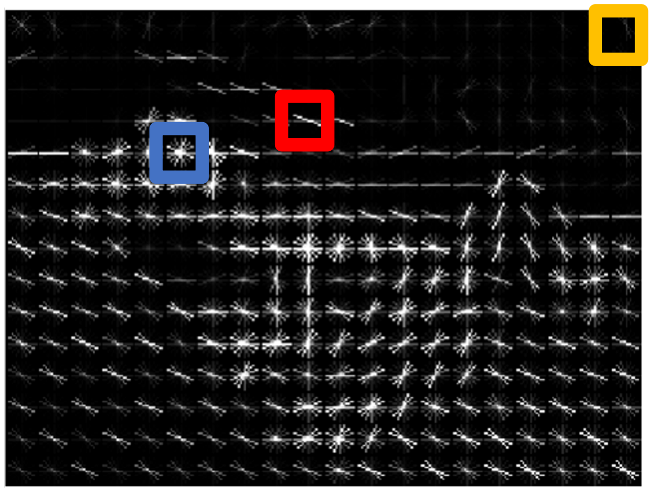

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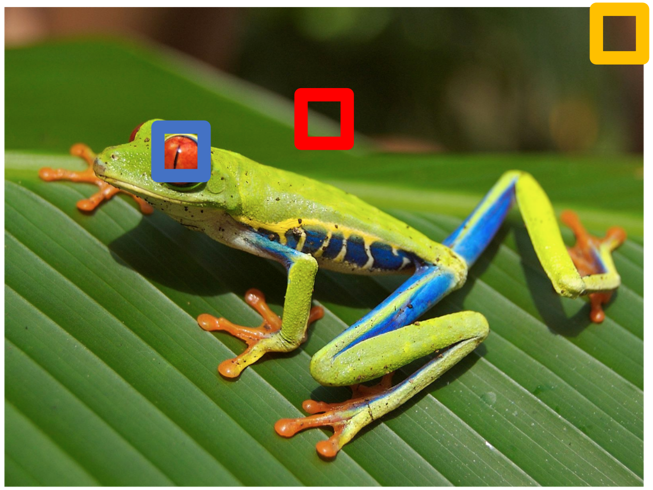
Image Features: Histogram of Oriented Gradients (HoG)

Captures texture and position, robust to small image changes



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Edges in all directions

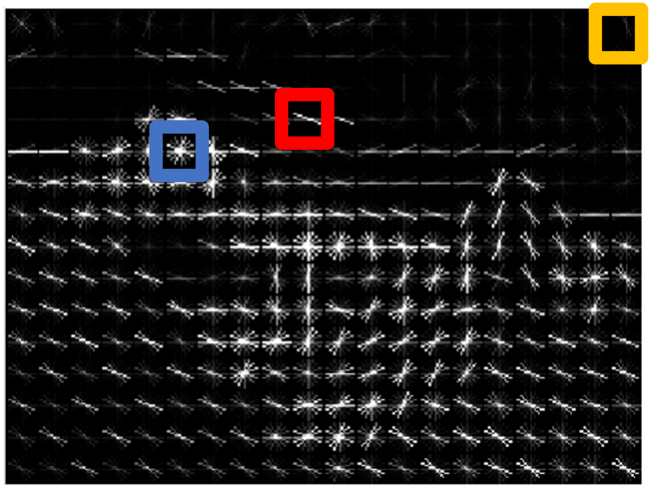

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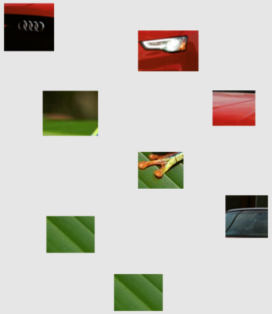
Image Features: Bag of Words

Learn a feature transform from data!

Step 1: Build codebook

Extract random patches



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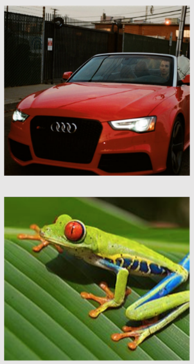

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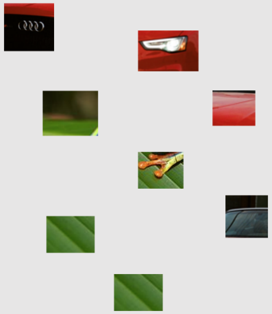

Cluster patches to form “codebook” of “visual words”

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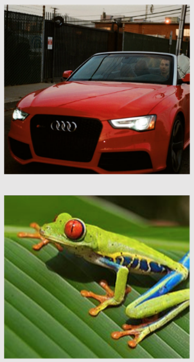
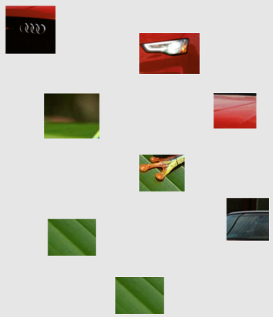
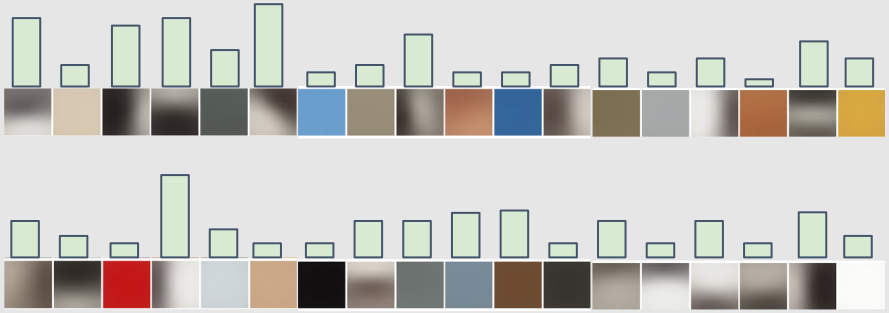


Extract random patches



Cluster patches to form “codebook” of “visual words”

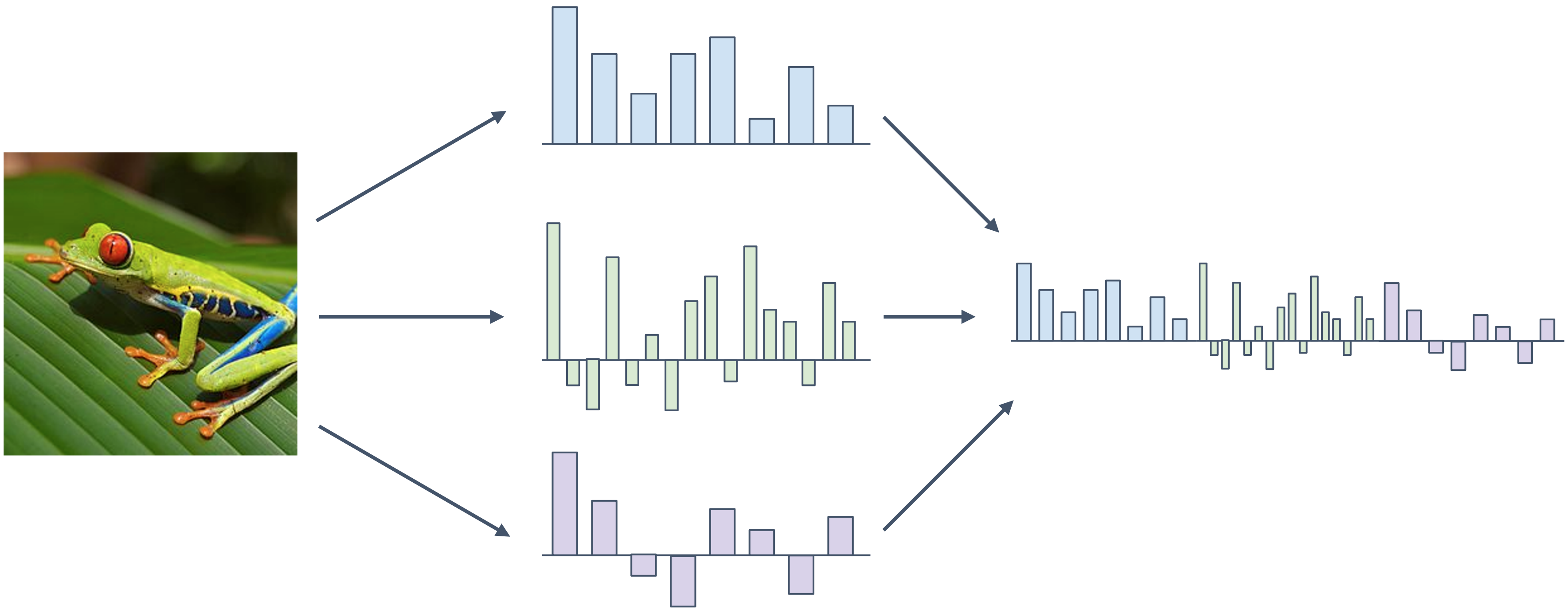
Step 2: Encode images



Fei-Fei and Perona, “A bayesian hierarchical model for learning natural scene categories”, CVPR 2005

Image Features

Common trick: Combine multiple feature transforms



Winner of 2011 ImageNet Challenge

Low-level feature extraction ≈ 10k patches per image

- SIFT: 128-dim
- color: 96-dim

$\Big\}$ reduced to 64-dim with PCA

FV extraction and compression:

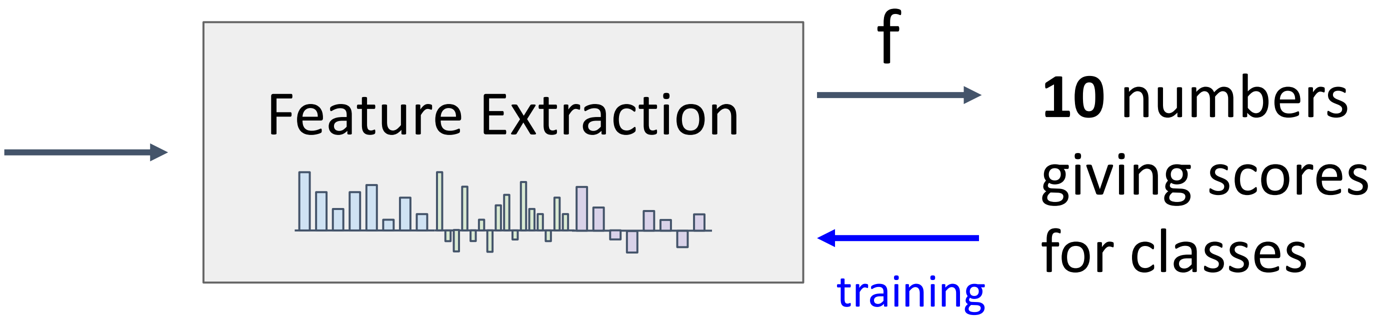
- N=1,024 Gaussians, R=4 regions ⇨ 520K dim x 2
- compression: G=8, b=1 bit per dimension

One-vs-all SVM learning with SGD

Late fusion of SIFT and color systems

F. Perronnin, J. Sánchez, "Compressed Fisher vectors for LSVRC", PASCAL VOC / ImageNet workshop, ICCV, 2011.

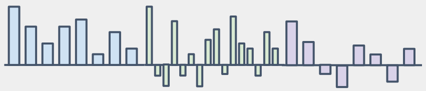
Image Features vs Neural Networks



Krizhevsky, Sutskever, and Hinton, "Imagenet classification with deep convolutional neural networks", NIPS 2012

Image Features vs Neural Networks

Feature Extraction



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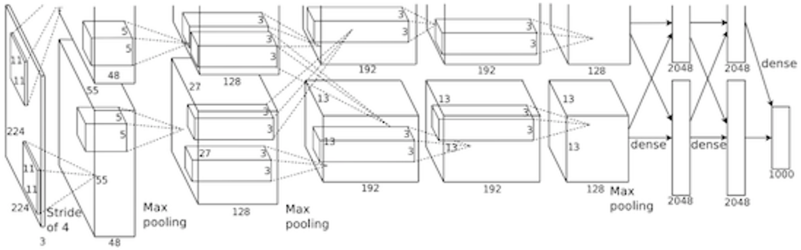
10 numbers giving scores for classes

training

Deep Neural Network



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Neural Networks

Input image: $x \in \mathbb{R}^D$
Category scores: $s \in \mathbb{R}^C$

Linear Classifier:

$$s = Wx$$

$$W \in \mathbb{R}^{C \times D}$$

In practice we add a learnable bias
+b after each matrix multiply

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2-layer Neural Net:

$$s = \mathrm{W}_2 \max(0, W_1 x)$$
$$W_1 \in \mathbb{R}^{H \times D}$$
$$W_2 \in \mathbb{R}^{C \times H}$$

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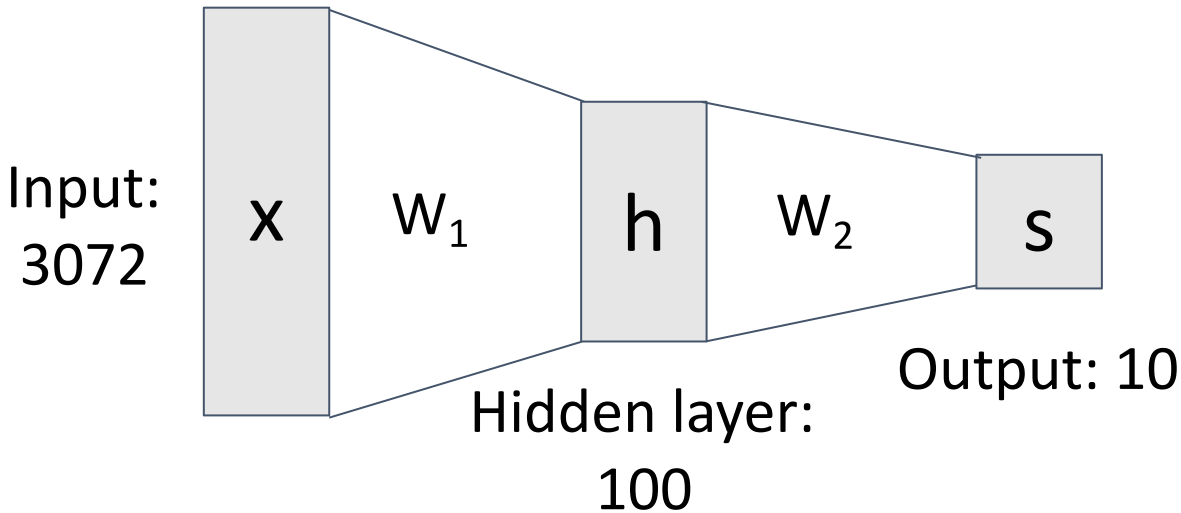
$$W_2 \in \mathbb{R}^{C \times H}$$

3-layer Neural Net:

$$s = \mathrm{W}_3 \max(0, W_2 \max(0, W_1 x))$$

Neural Networks

Two-Layer Neural Network: $s = \mathrm{W}_2 \max(0, W_1 x)$



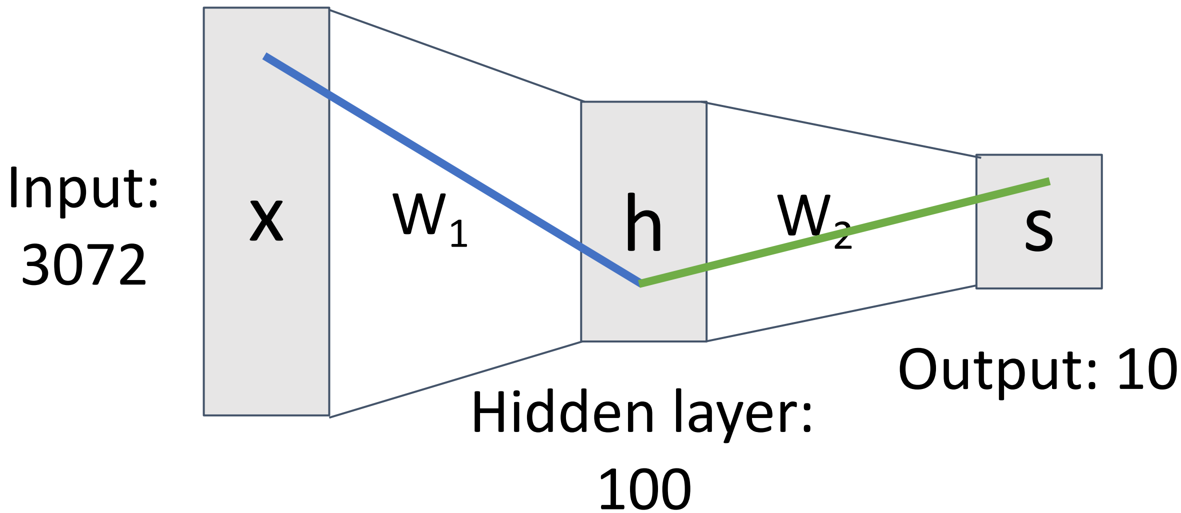
$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H\times D}, W_2 \in \mathbb{R}^{C\times H}$$

Neural Networks

Two-Layer Neural Network: $s = \mathrm{W}_2 \max(0, W_1 x)$

Element (i, j) of W_1 gives the effect on h_i from x_j

Element (i, j) of W_2 gives the effect on s_i from h_j



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Neural Networks

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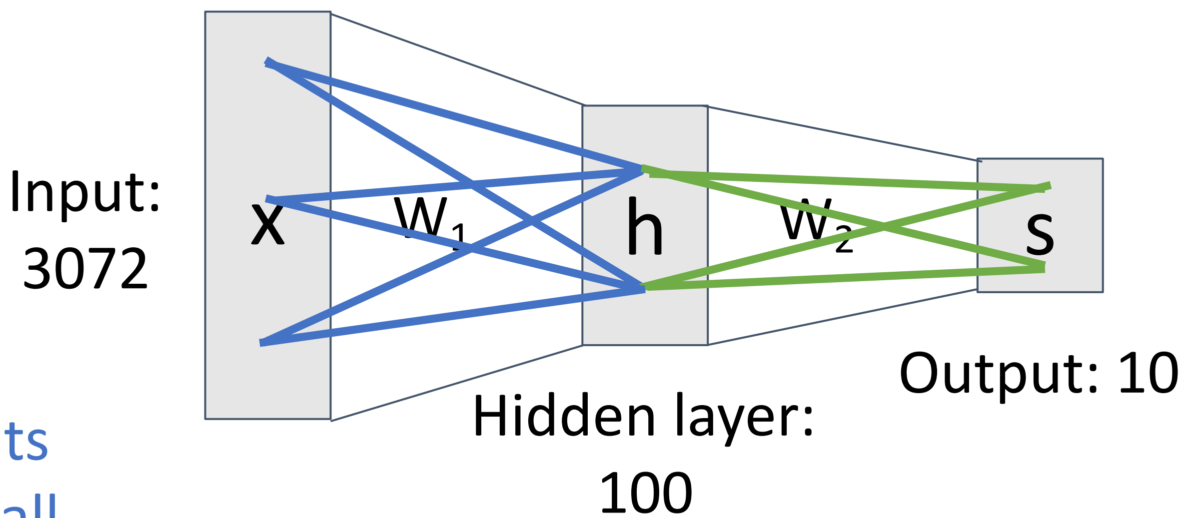


All elements of x affect all elements of h

All elements of h affect all elements of s

"Fully-Connected" neural network
Also "Multi-Layer Perceptron" (MLP)

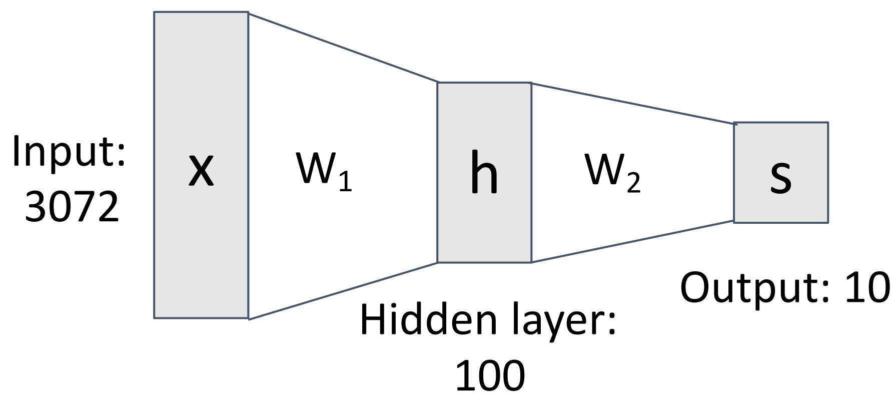
Neural Networks

Linear classifier: $s = Wx$

One template per class

Two-Layer Neural Network:

$s = \mathrm{W}_2 \max(0, W_1 x)$

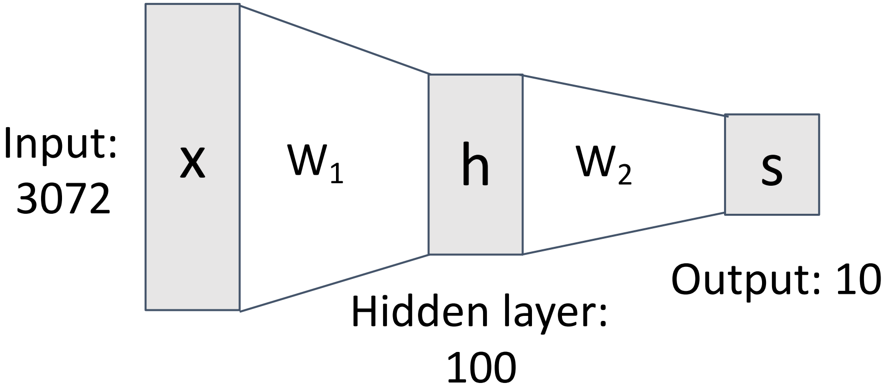


$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$

Neural Networks

Neural Network:
First layer is a bank of templates
Second layer recombines templates

Two-Layer Neural Network:
$s = \mathrm{W}_2 \max(0, W_1 x)$



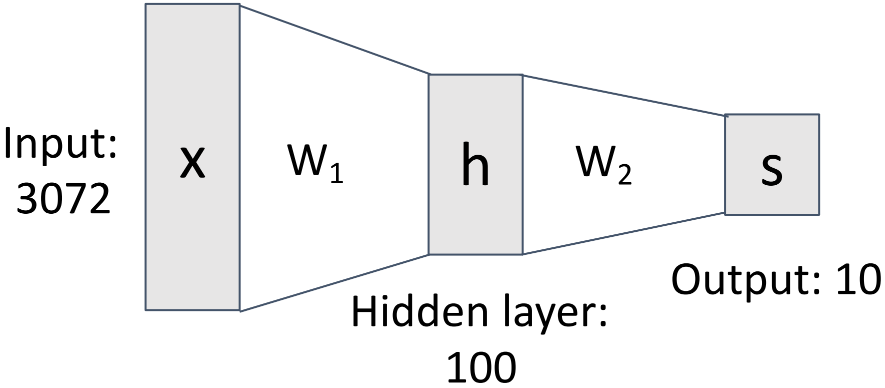
$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$

Neural Networks

Different templates can cover different modes of a class!

Two-Layer Neural Network:

$s = W_2 \max(0, W_1 x)$



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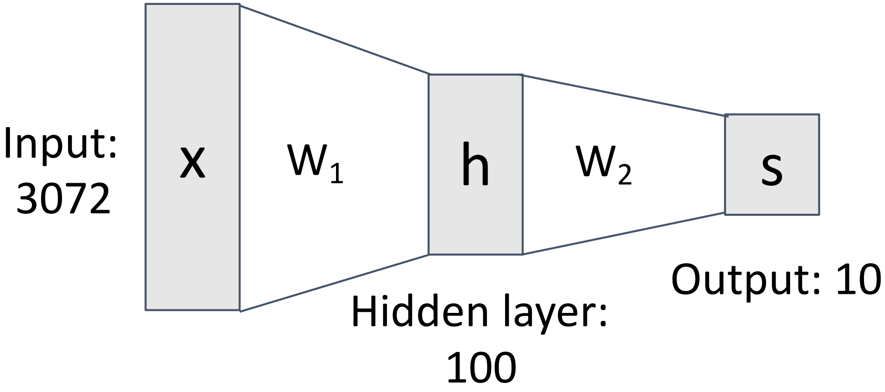
Neural Networks

Many templates not interpretable:
"Distributed representation"

Two-Layer Neural Network:

$$s = W_2 \max(0, W_1 x)$$



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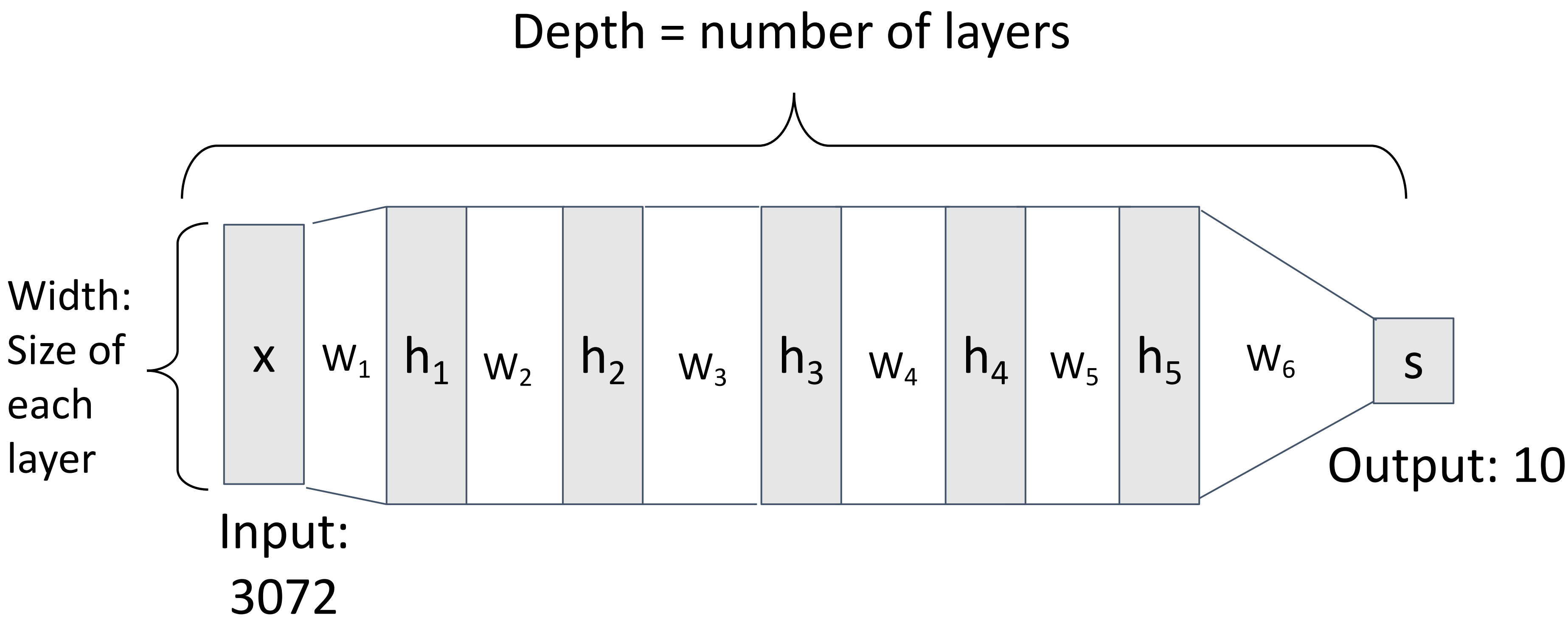
Deep Neural Networks



$$s = W_6 \max(0, W_5 \max(0, W_4 \max(0, W_3 \max(0, W_2 \max(0, W_1 x)))))$$

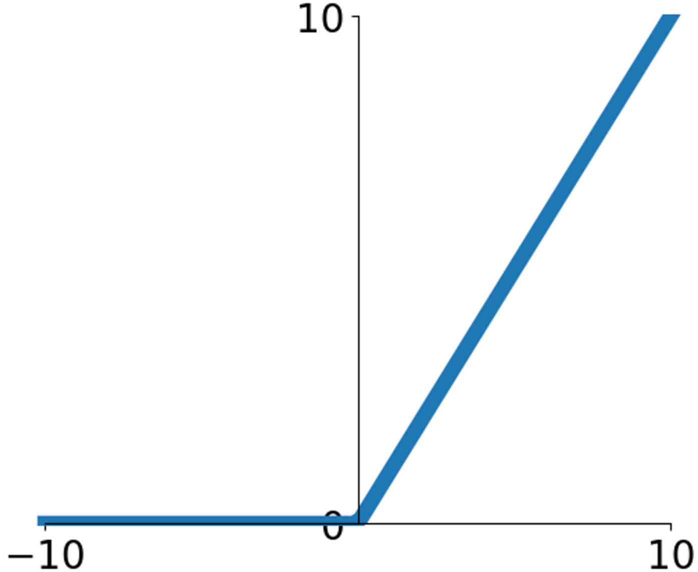
Activation Functions

2-layer Neural Network

$$s = W_2 \max(0, W_1 x)$$

The function $ReLU(z) = \max(0, z)$ is called "Rectified Linear Unit"

This is called the **activation function** of the neural network



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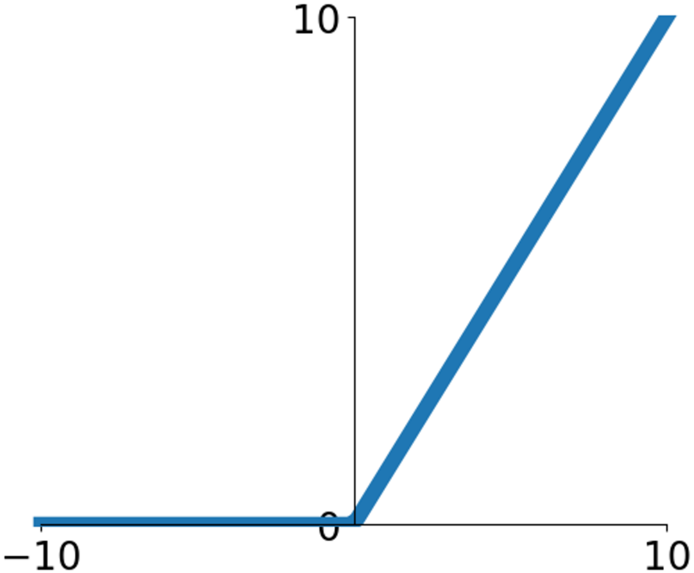

Q: What happens if we build a neural network with no activation function?

$$s = W_2 W_1 x$$

Activation Functions

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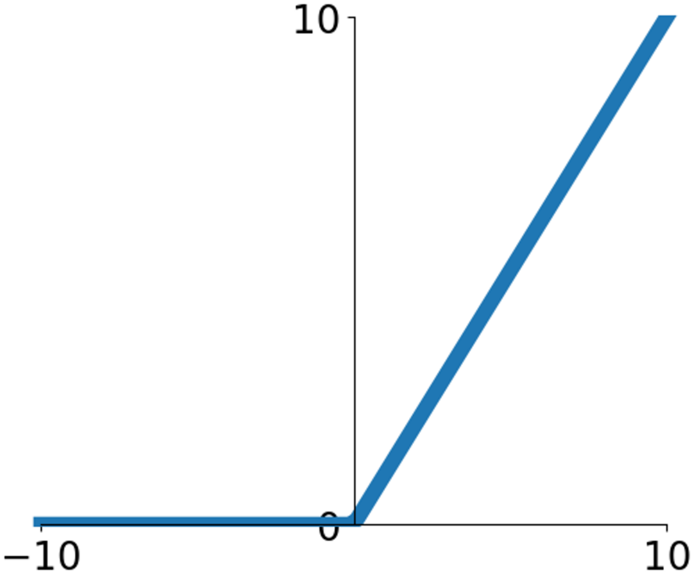


Q: What happens if we build a neural network with no activation function?

$$s = W_2 W_1 x$$

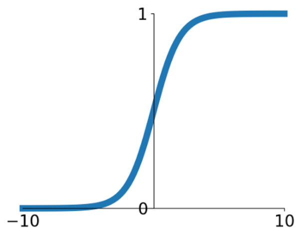
A: We get a linear classifier!

$$W_3 = W_2 W_1 \in \mathbb{R}^{C \times D}$$
$$s = W_3 x$$

Activation Functions

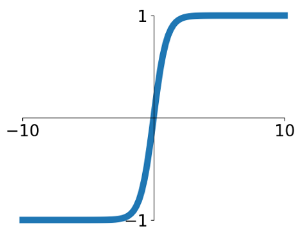
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



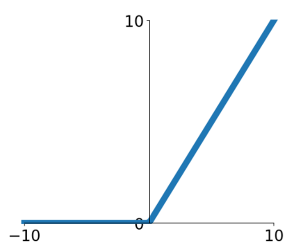
tanh

$\tanh(x)$



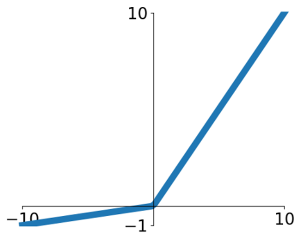
ReLU

$\max(0, x)$



Leaky ReLU

$\max(0.1x, x)$

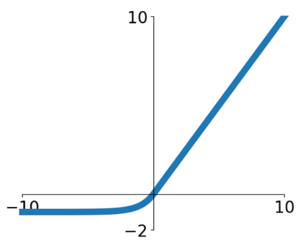


Maxout

$\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$

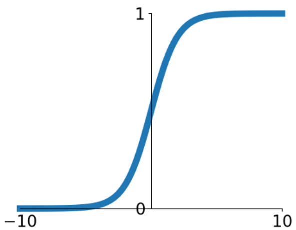


Activation Functions

ReLU is a good default choice

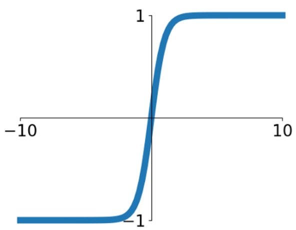
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



tanh

$\tanh(x)$



ReLU

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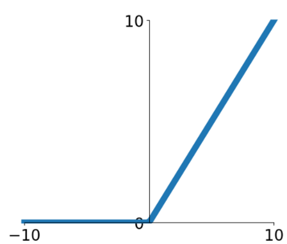

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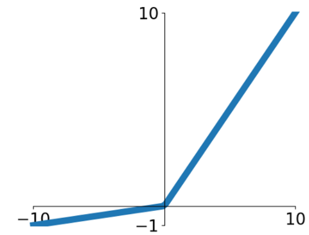

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ELU

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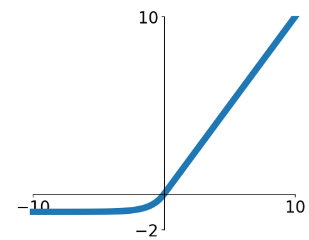

Neural Net in <20 lines!

```python
import numpy as np
from numpy.random import randn

N, Din, H, Dout = 64, 1000, 100, 10
x, y = randn(N, Din), randn(N, Dout)
w1, w2 = randn(Din, H), randn(H, Dout)
for t in range(10000):
  h = 1.0 / (1.0 + np.exp(-x.dot(w1)))
  y_pred = h.dot(w2)
  loss = np.square(y_pred - y).sum()
  dy_pred = 2.0 * (y_pred - y)
  dw2 = h.T.dot(dy_pred)
  dh = dy_pred.dot(w2.T)
  dw1 = x.T.dot(dh * h * (1 - h))
  w1 -= 1e-4 * dw1
  w2 -= 1e-4 * dw2
```

Neural Net in <20 lines!

Initialize weights and data

```python
import numpy as np
from numpy.random import randn

N, Din, H, Dout = 64, 1000, 100, 10
x, y = randn(N, Din), randn(N, Dout)
w1, w2 = randn(Din, H), randn(H, Dout)
for t in range(10000):
  h = 1.0 / (1.0 + np.exp(-x.dot(w1)))
  y_pred = h.dot(w2)
  loss = np.square(y_pred - y).sum()
  dy_pred = 2.0 * (y_pred - y)
  dw2 = h.T.dot(dy_pred)
  dh = dy_pred.dot(w2.T)
  dw1 = x.T.dot(dh * h * (1 - h))
  w1 -= 1e-4 * dw1
  w2 -= 1e-4 * dw2
```

Neural Net in <20 lines!

Initialize weights and data

Compute loss (sigmoid activation, L2 loss)

```
import numpy as np
from numpy.random import randn

N, Din, H, Dout = 64, 1000, 100, 10
x, y = randn(N, Din), randn(N, Dout)
w1, w2 = randn(Din, H), randn(H, Dout)
for t in range(10000):
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  dw1 = x.T.dot(dh * h * (1 - h))
  w1 -= 1e-4 * dw1
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```

Neural Net in <20 lines!

Initialize weights and data

Compute loss (sigmoid activation, L2 loss)

Compute gradients

```
import numpy as np
from numpy.random import randn

N, Din, H, Dout = 64, 1000, 100, 10
x, y = randn(N, Din), randn(N, Dout)
w1, w2 = randn(Din, H), randn(H, Dout)
for t in range(10000):
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  dy_pred = 2.0 * (y_pred - y)
  dw2 = h.T.dot(dy_pred)
  dh = dy_pred.dot(w2.T)
  dw1 = x.T.dot(dh * h * (1 - h))
  w1 -= 1e-4 * dw1
  w2 -= 1e-4 * dw2
```

Neural Net in <20 lines!

Initialize weights and data

Compute loss (sigmoid activation, L2 loss)

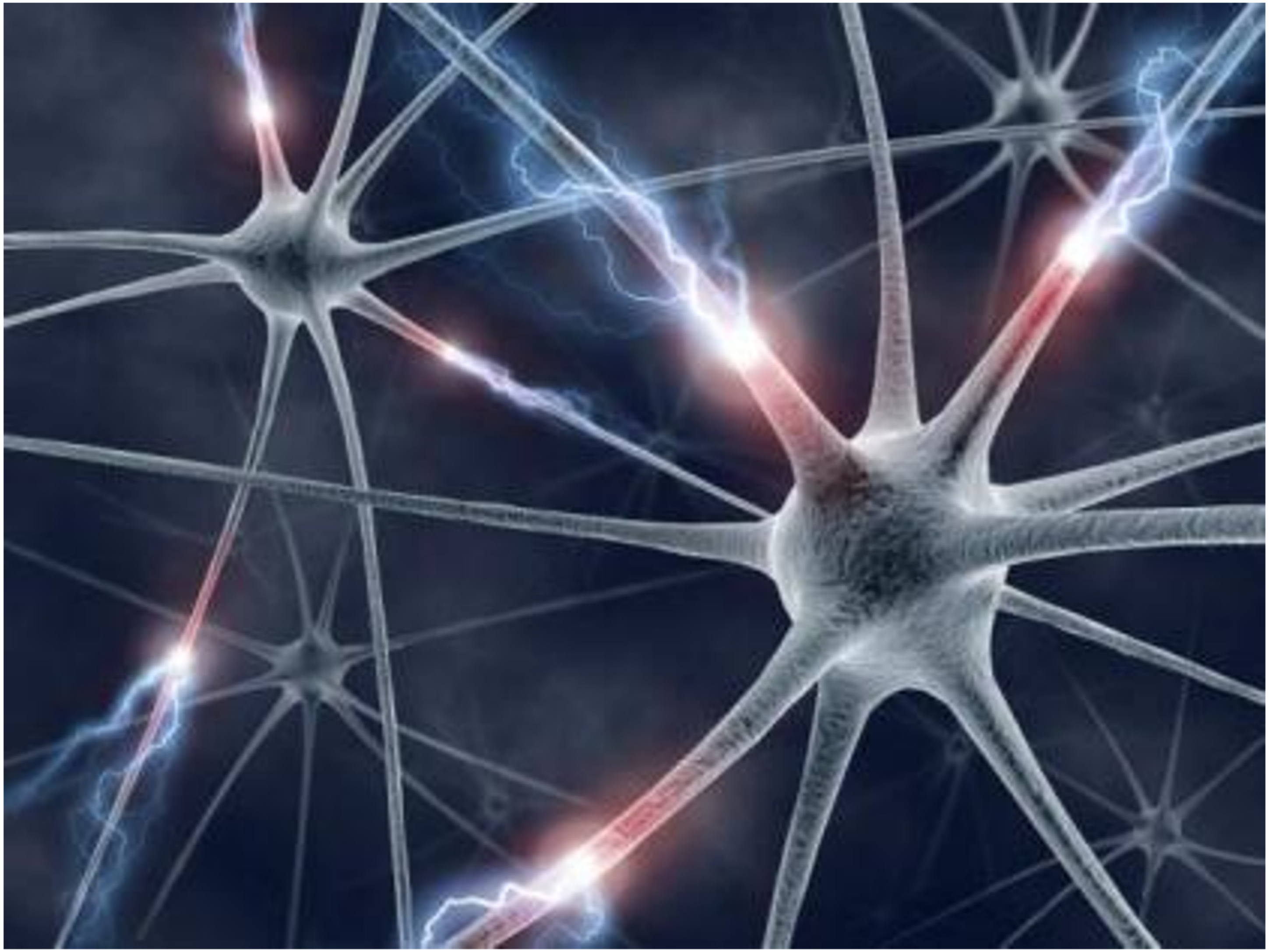
Compute gradients

SGD step

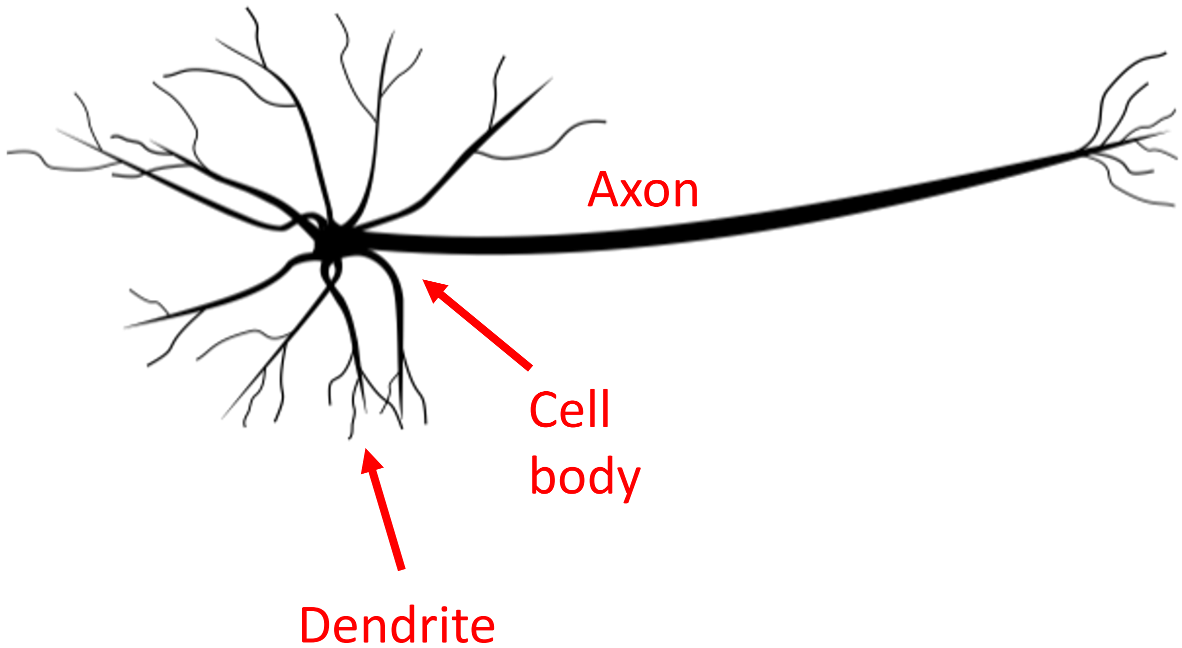
```python
import numpy as np
from numpy.random import randn

N, Din, H, Dout = 64, 1000, 100, 10
x, y = randn(N, Din), randn(N, Dout)
w1, w2 = randn(Din, H), randn(H, Dout)
for t in range(10000):
  h = 1.0 / (1.0 + np.exp(-x.dot(w1)))
  y_pred = h.dot(w2)
  loss = np.square(y_pred - y).sum()
  dy_pred = 2.0 * (y_pred - y)
  dw2 = h.T.dot(dy_pred)
  dh = dy_pred.dot(w2.T)
  dw1 = x.T.dot(dh * h * (1 - h))
  w1 -= 1e-4 * dw1
  w2 -= 1e-4 * dw2
```

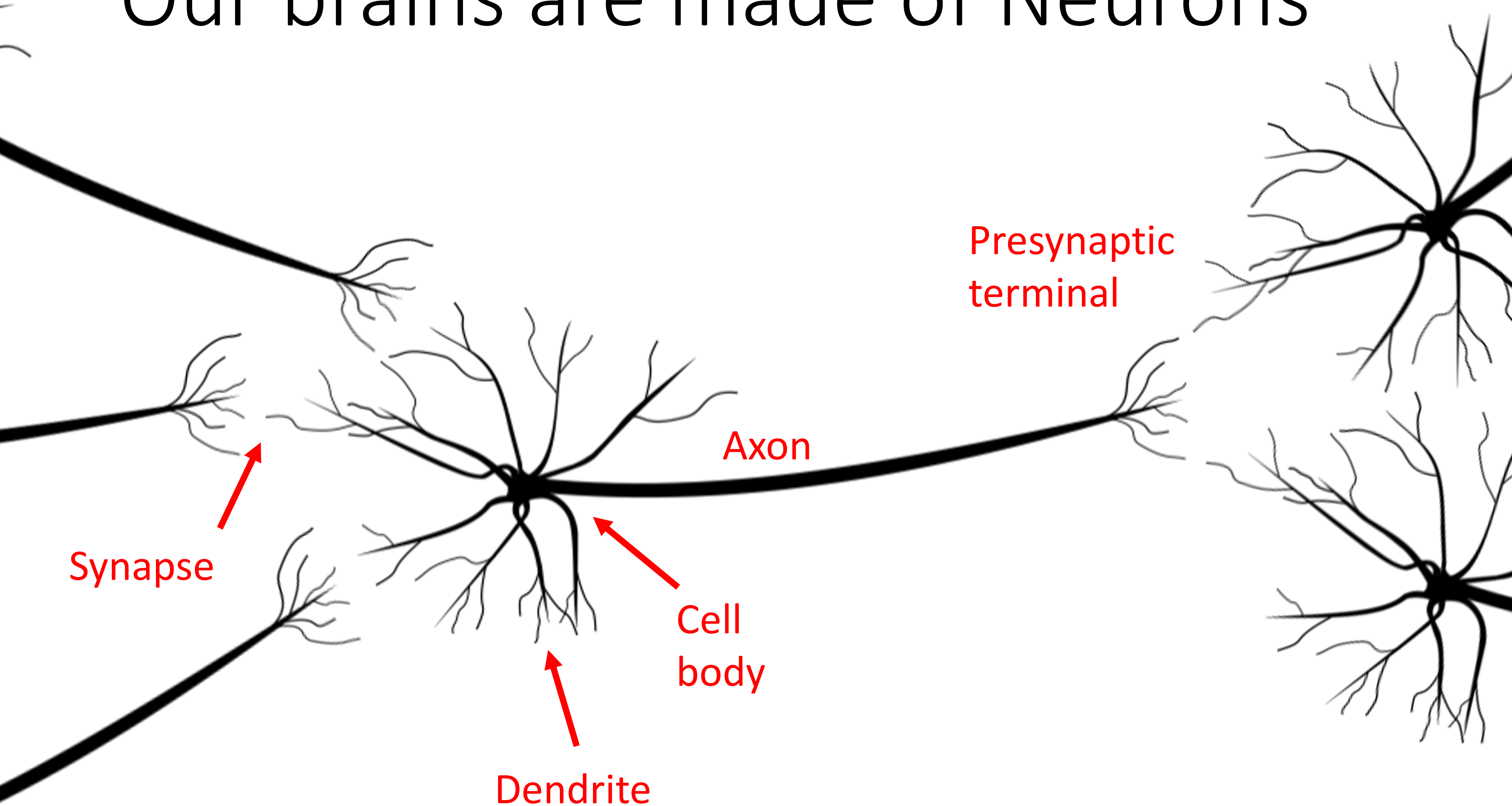
“Neural” Networks



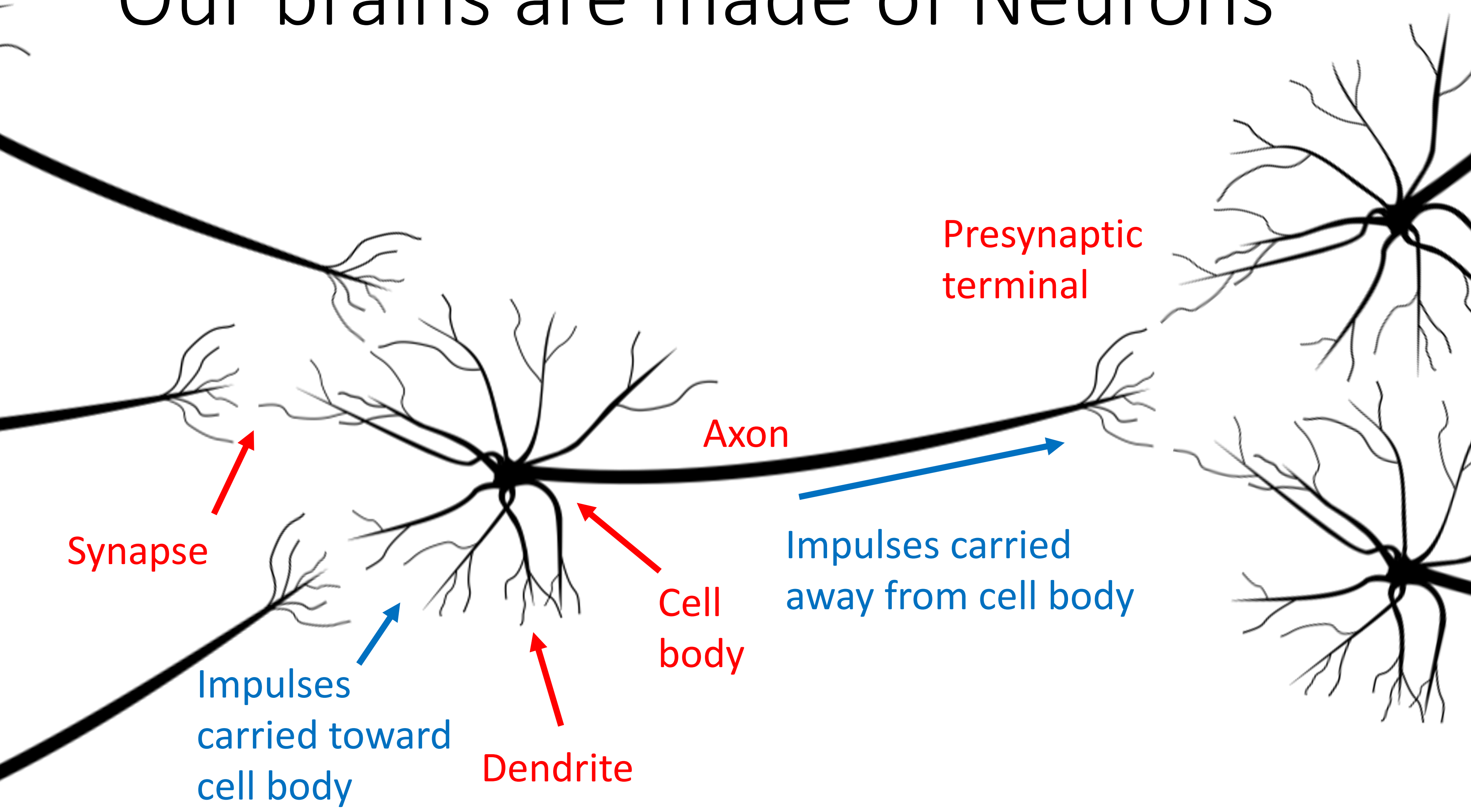
Our brains are made of Neurons



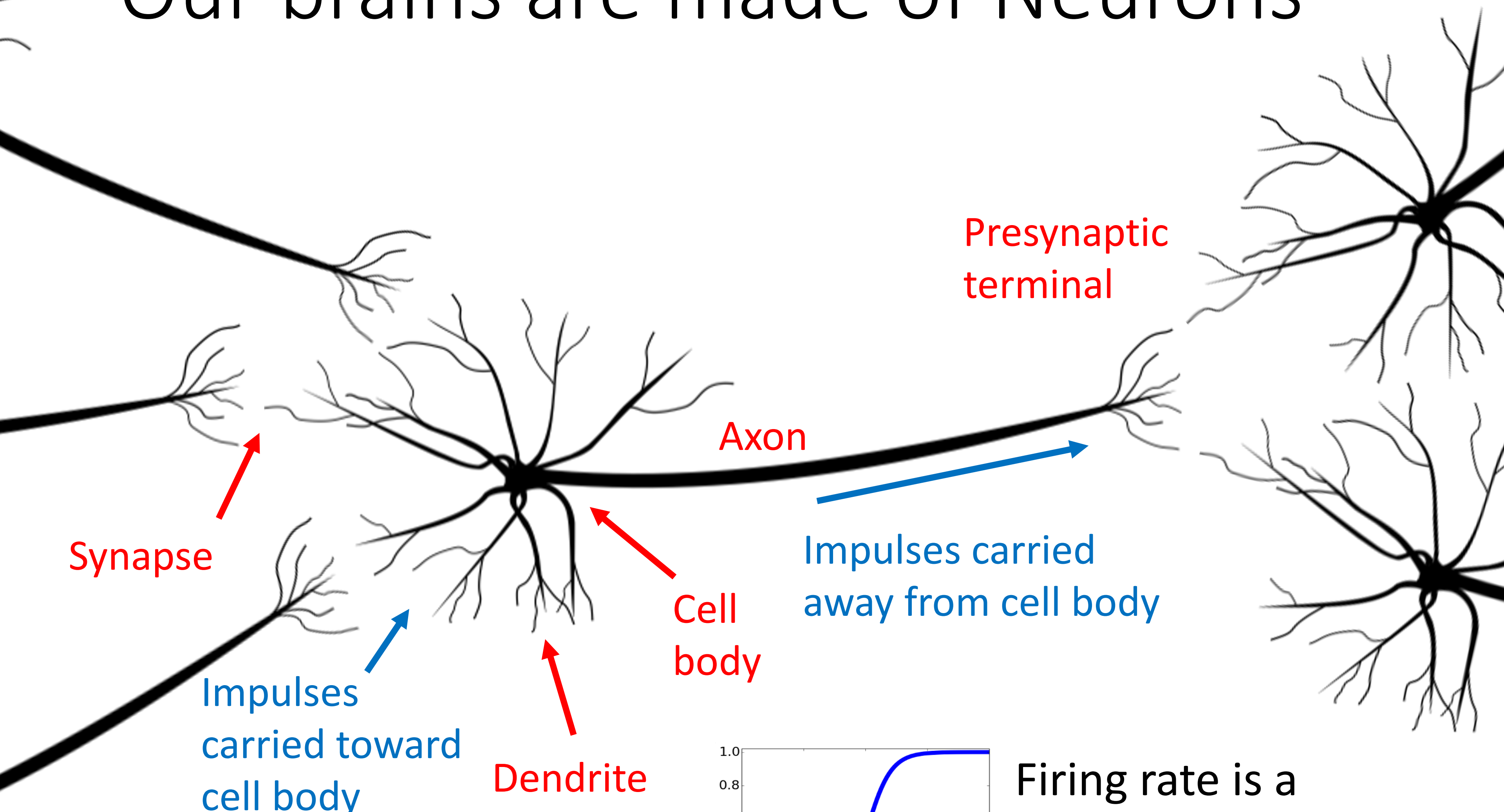
Our brains are made of Neurons

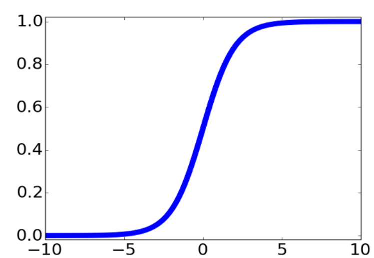


Our brains are made of Neurons

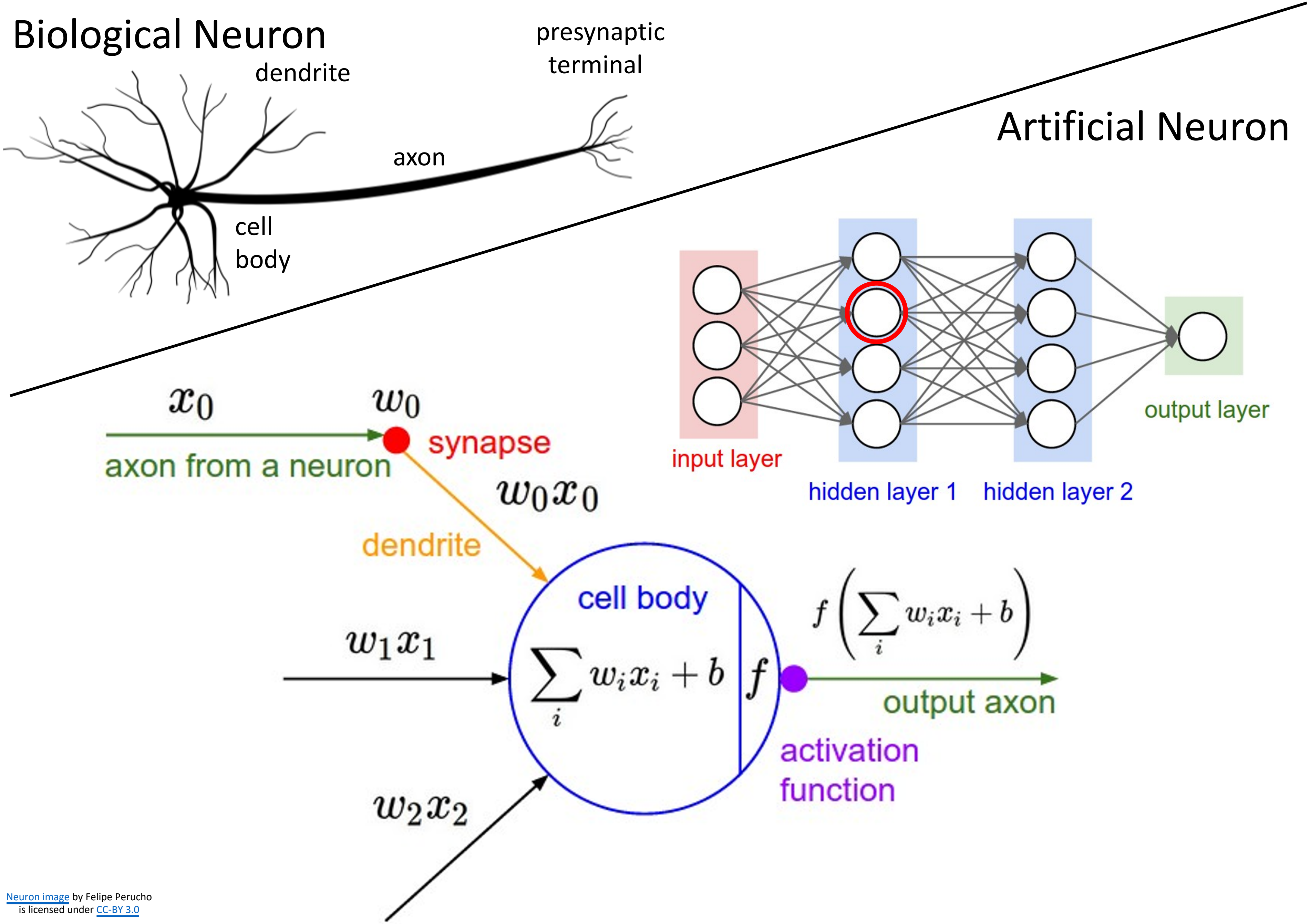


Our brains are made of Neurons

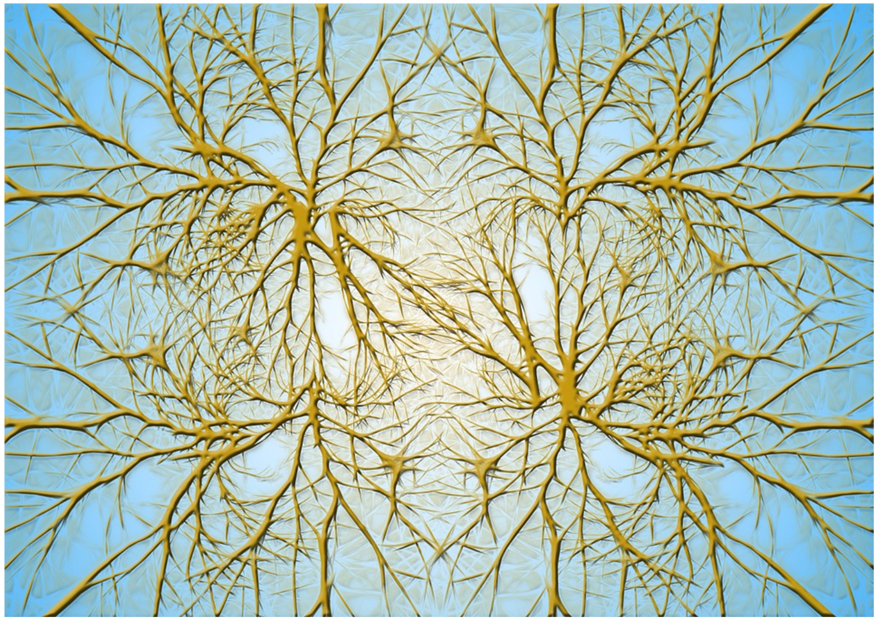




Firing rate is a nonlinear function of inputs

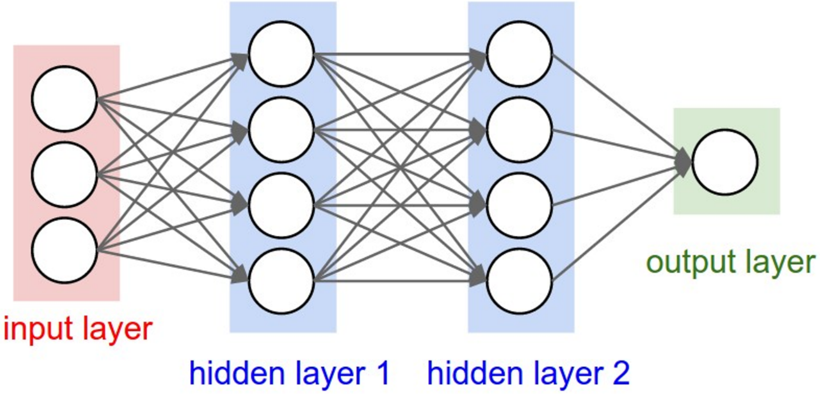

Biological Neuron
dendrite
presynaptic
terminal
axon
cell
body
Artificial Neuron
input layer
hidden layer 1
hidden layer 2
output layer
x_0
w_0
synapse
axon from a neuron
$w_0 x_0$
dendrite
cell body
$\sum_i w_i x_i + b$
f
$f\left(\sum_i w_i x_i + b\right)$
output axon
activation
function
$w_1 x_1$
$w_2 x_2$

Biological Neurons:
Complex connectivity patterns



This image is CC0 Public Domain

Neurons in a neural network:
Organized into regular layers for computational efficiency



Be very careful with brain analogies!

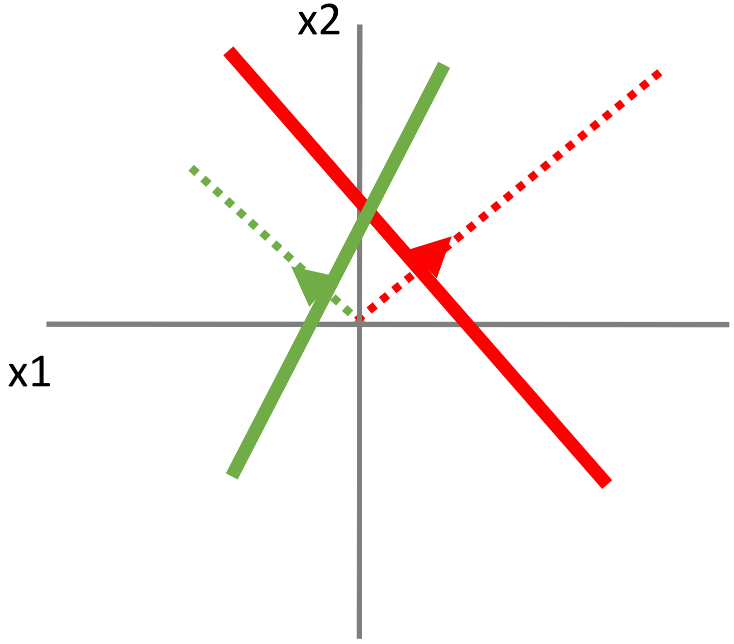
Biological Neurons:

- Many different types
- Can perform complex non-linear computations
- Synapses are not a single weight but a complex non-linear dynamical system
- Can have feedback, time-dependent
- Probably don't learn via gradient descent

[Dendritic Computation. London and Hausser]

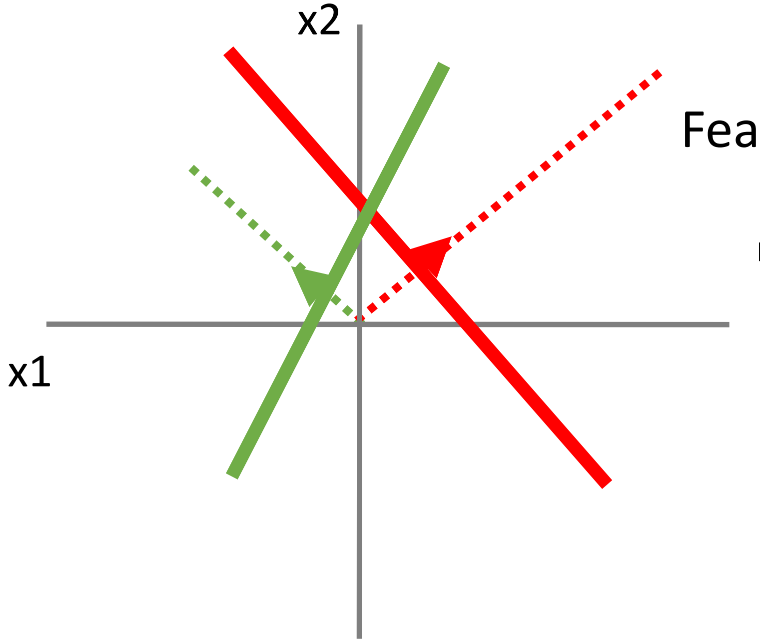
Space Warping

Consider a linear transform: h = Wx
Where x, h are both 2-dimensional

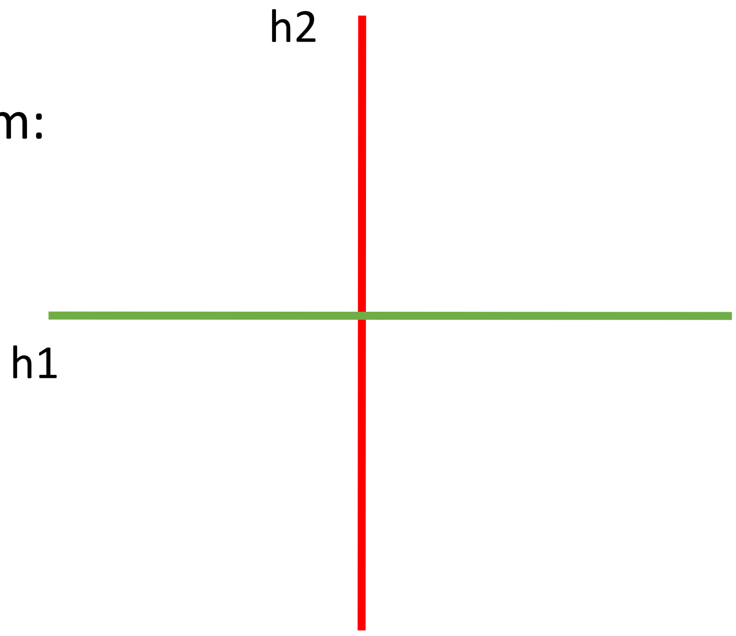


Space Warping

Consider a linear transform: h = Wx
Where x, h are both 2-dimensional



Feature transform:
h = Wx



Space Warping

Consider a linear transform: h = Wx
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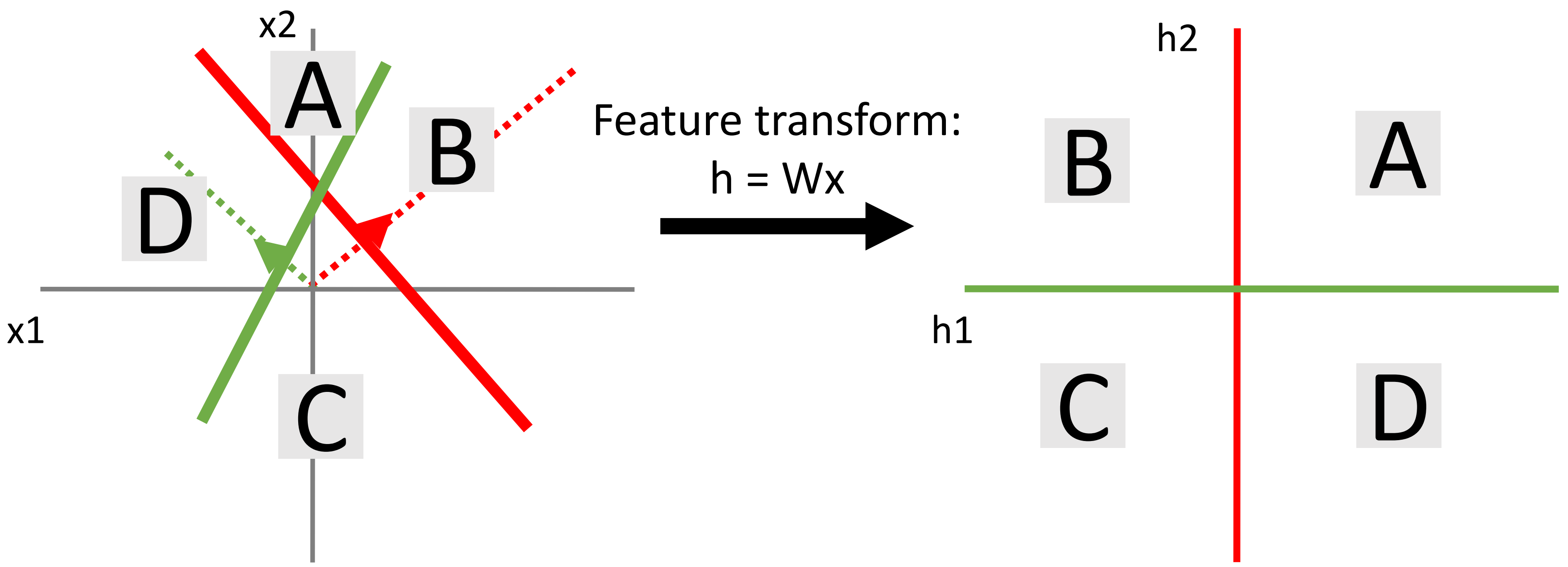
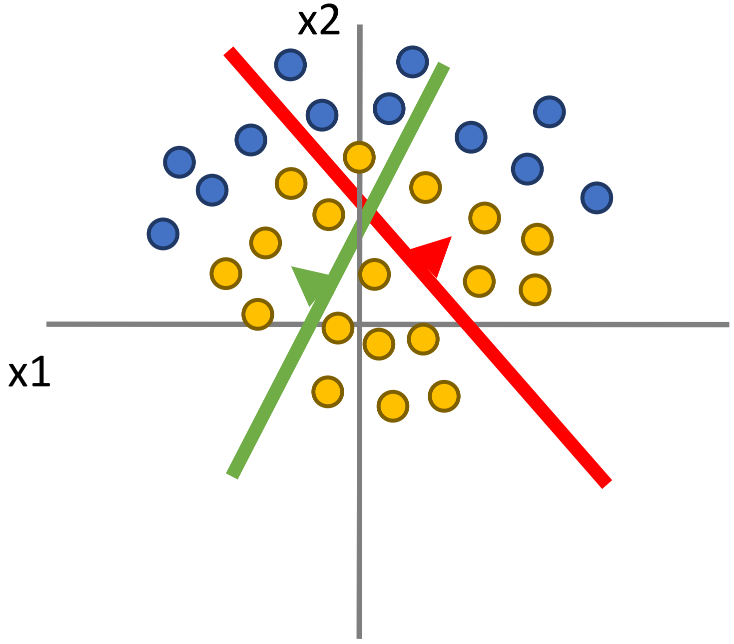


Space Warping

Consider a linear transform: h = Wx
Where x, h are both 2-dimensional

Points not linearly separable in original space



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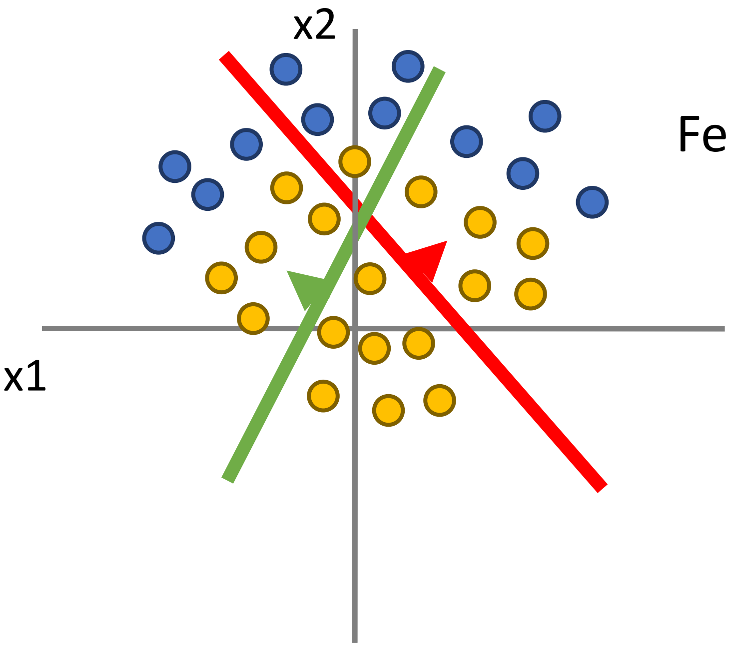
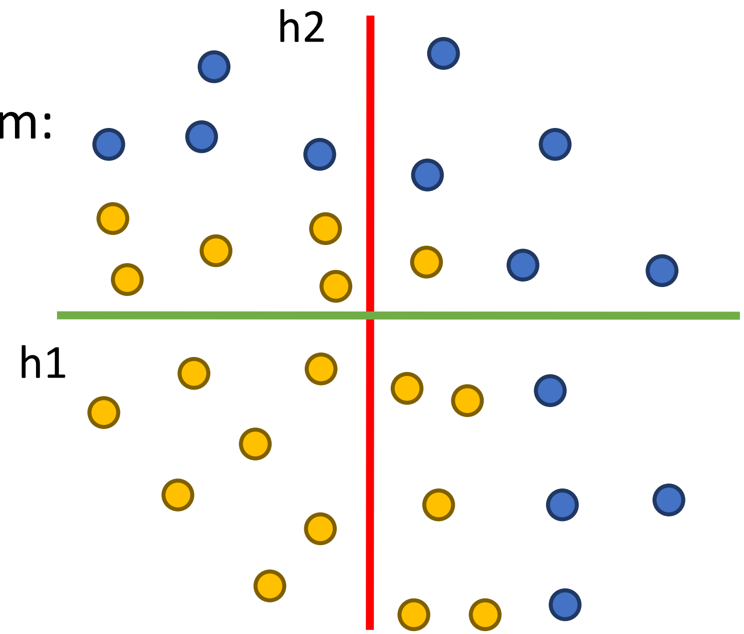
Points not linearly separable in original space



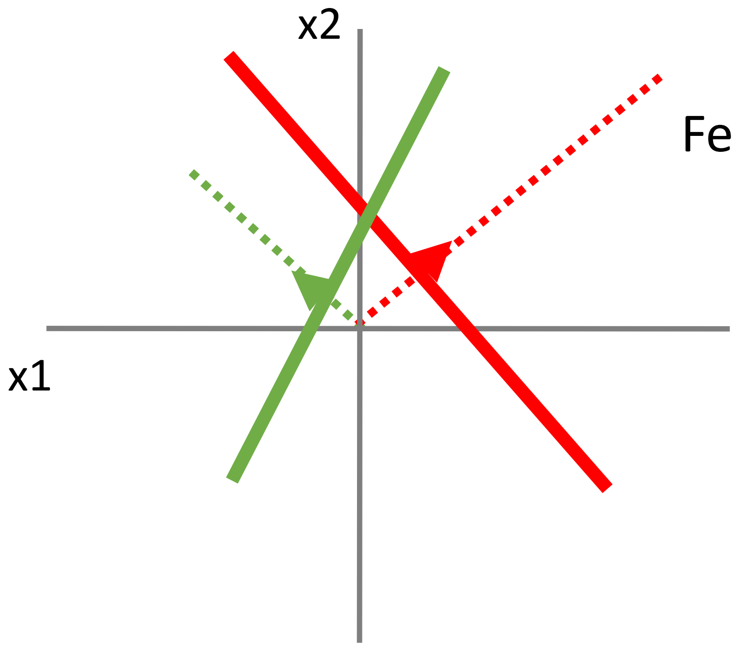
Feature transform:
h = Wx

Points not linearly separable in feature space

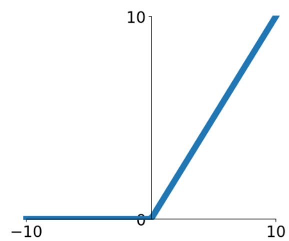


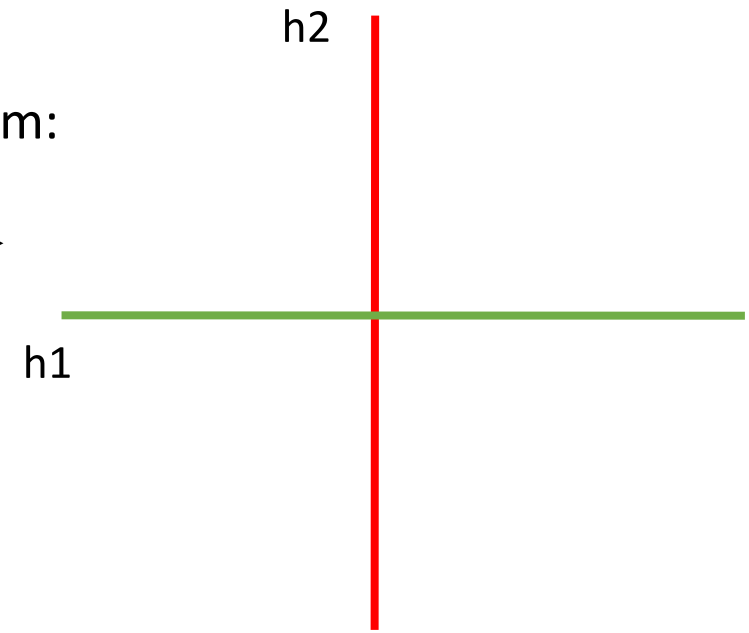
Space Warping

Consider a neural net hidden layer:
h = ReLU(Wx) = max(0, Wx)
Where x, h are both 2-dimensional



Feature transform:
h = ReLU(Wx)





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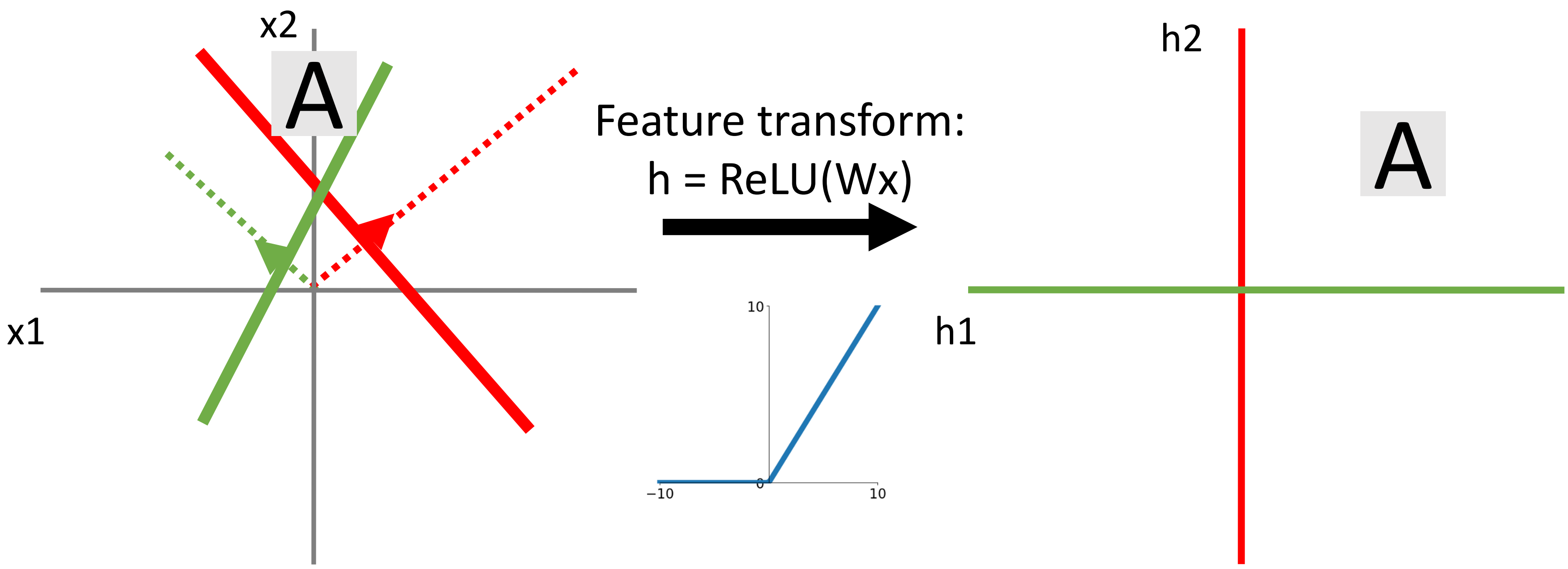

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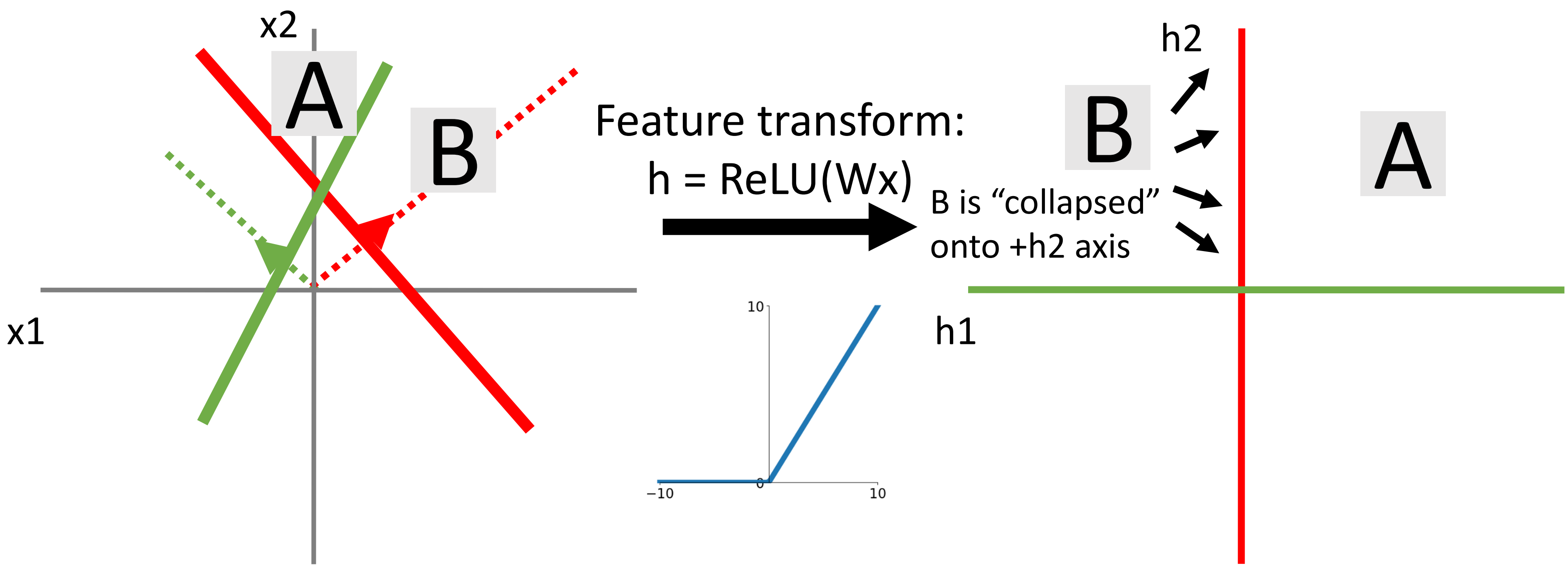

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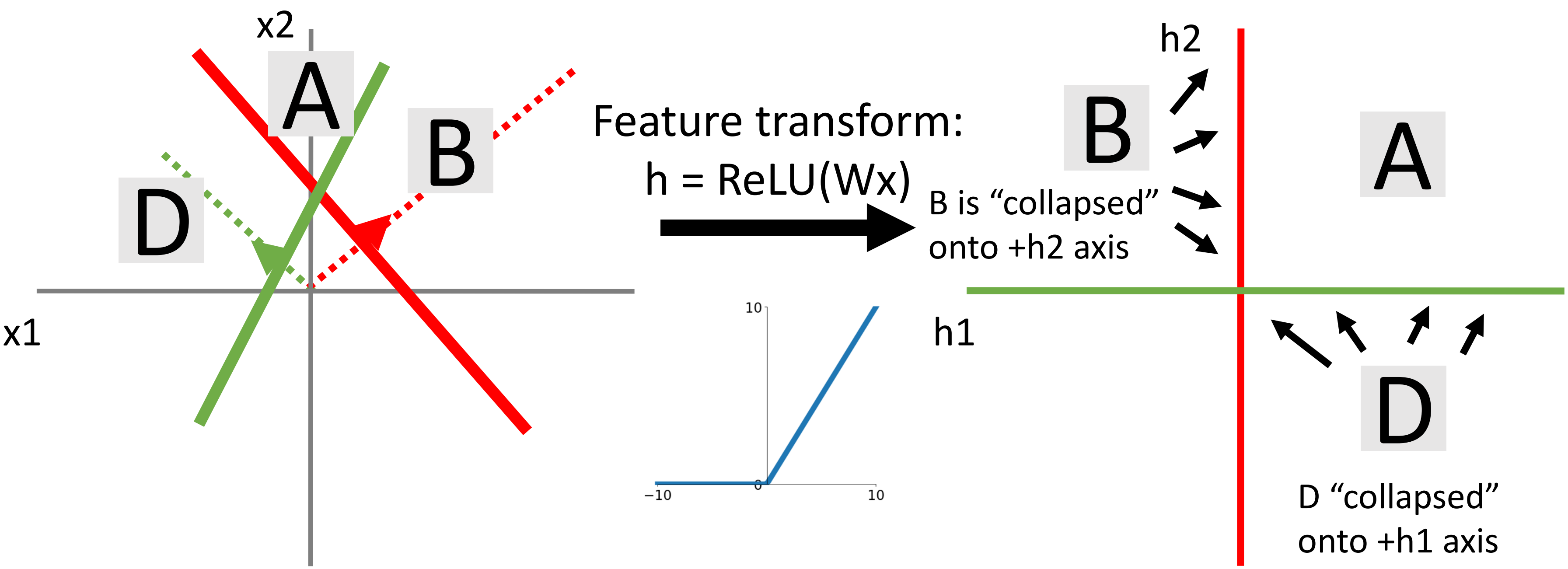

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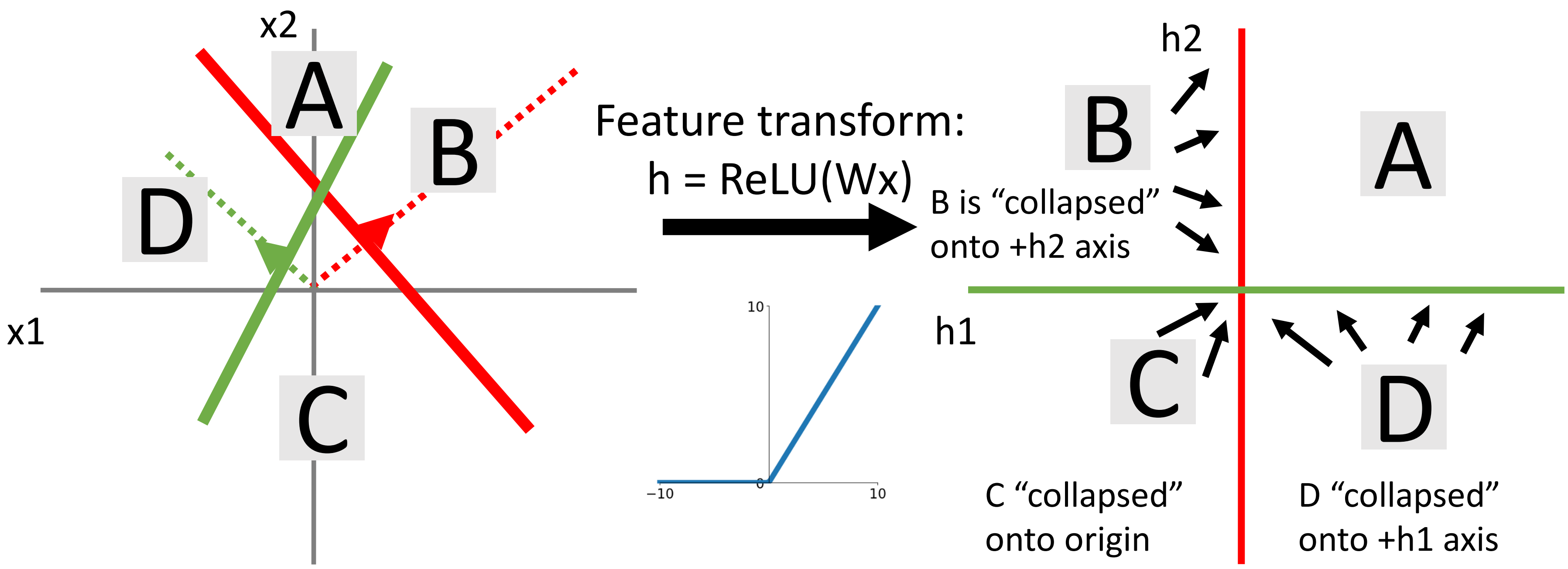

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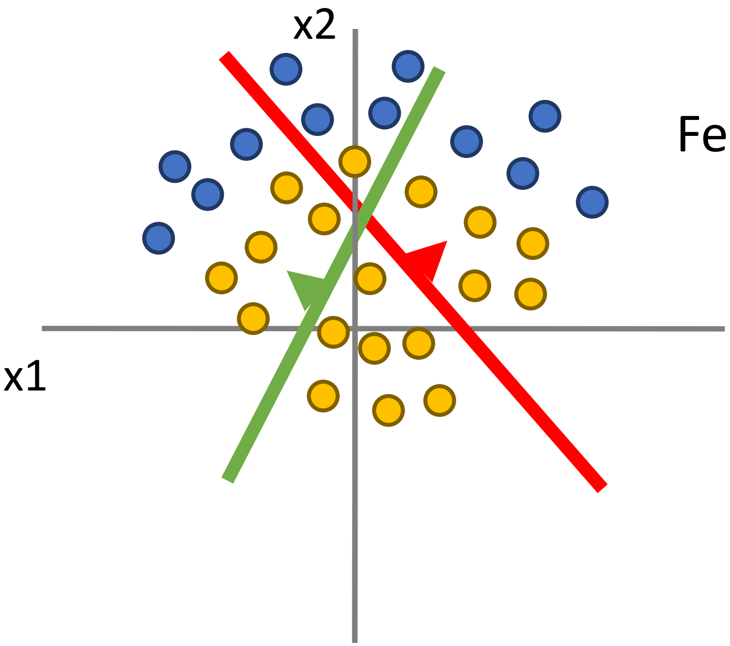
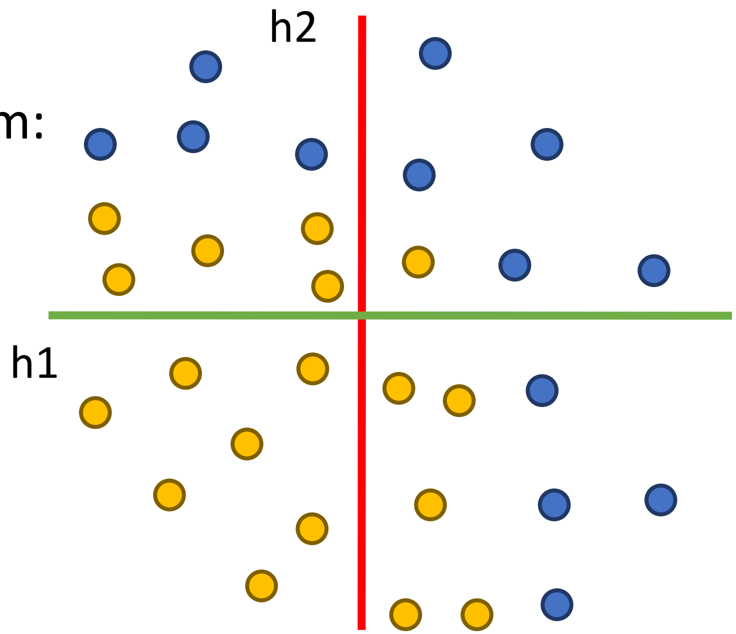


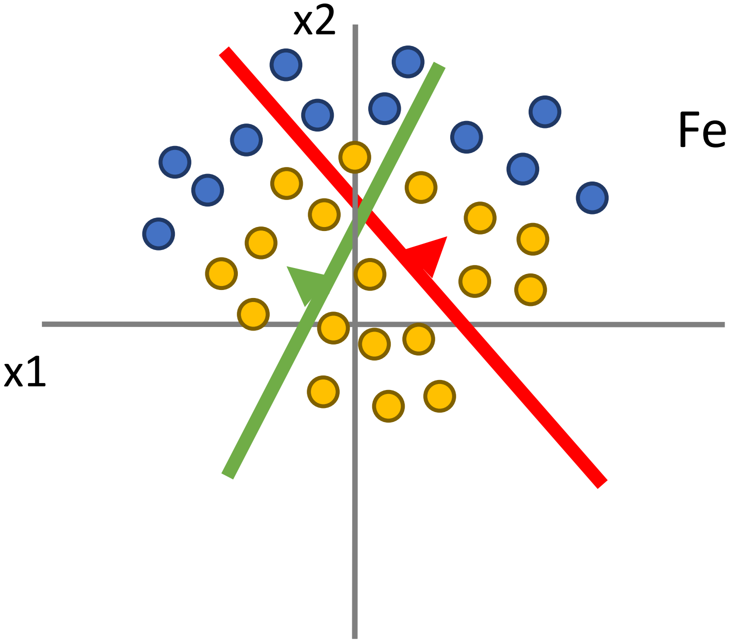
Feature transform:
h = Wx



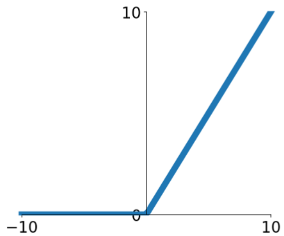
Points not linearly separable in original space

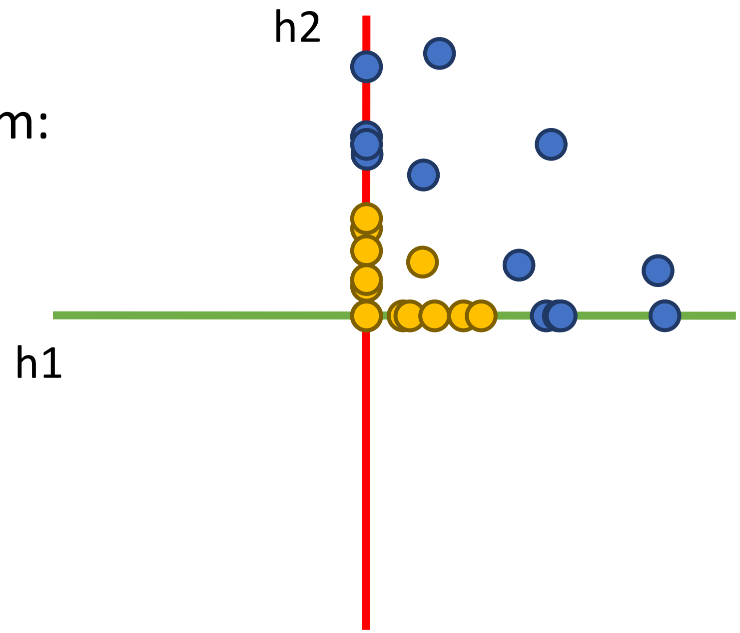
Space Warping

Consider a neural net hidden layer:
h = ReLU(Wx) = max(0, Wx)
Where x, h are both 2-dimensional



Feature transform:
h = ReLU(Wx)





Points not linearly separable in original space

Space Warping

Consider a neural net hidden layer:
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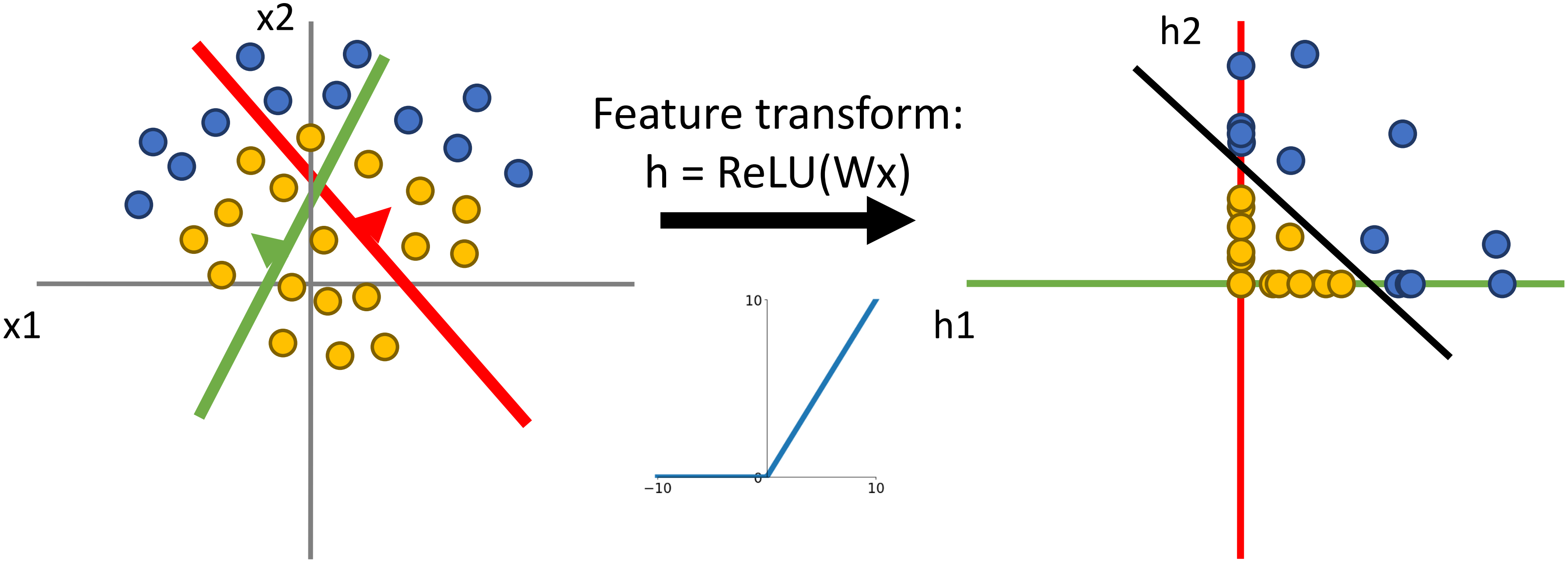
Where x, h are both 2-dimensional



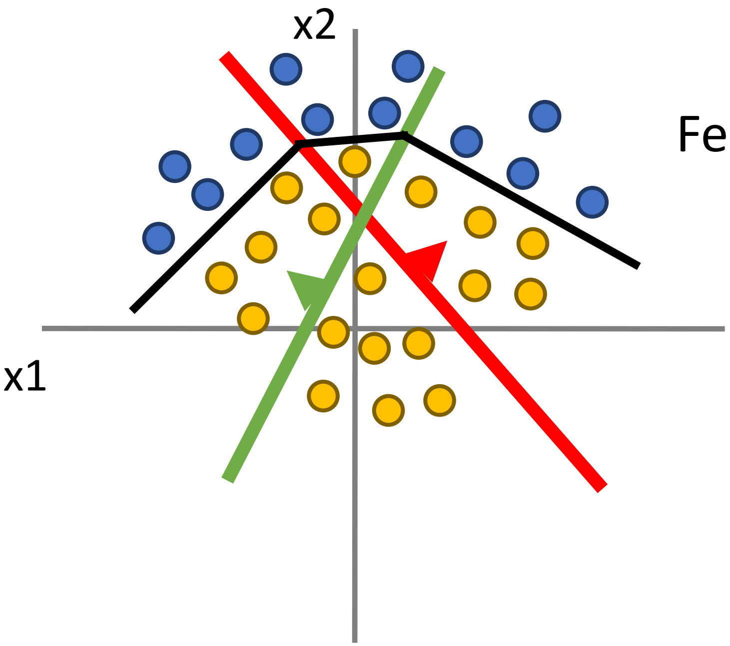
Points not linearly separable in original space

Points are linearly separable in features space!

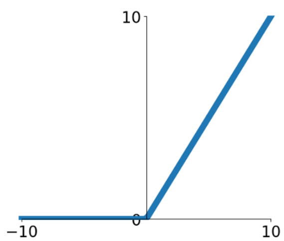
Space Warping

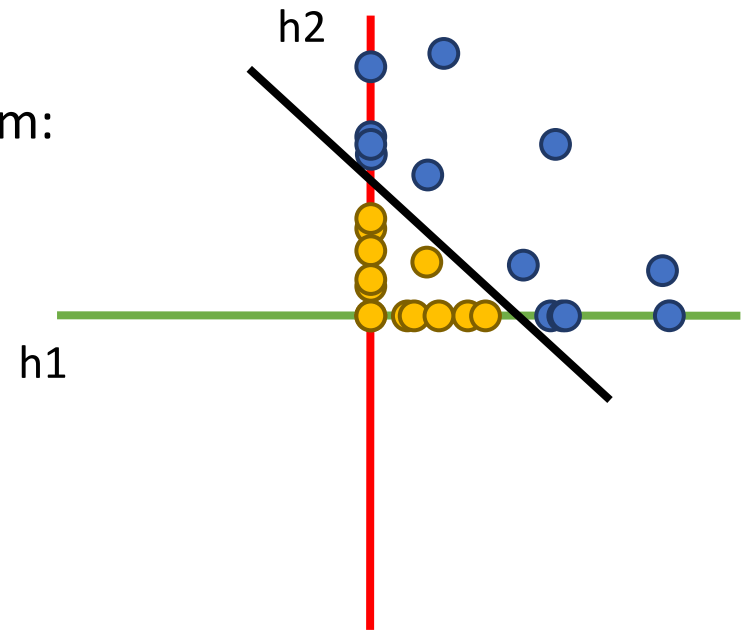
Linear classifier in feature space gives nonlinear classifier in original space

Consider a neural net hidden layer:
h = ReLU(Wx) = max(0, Wx)
Where x, h are both 2-dimensional



Feature transform:
h = ReLU(Wx)

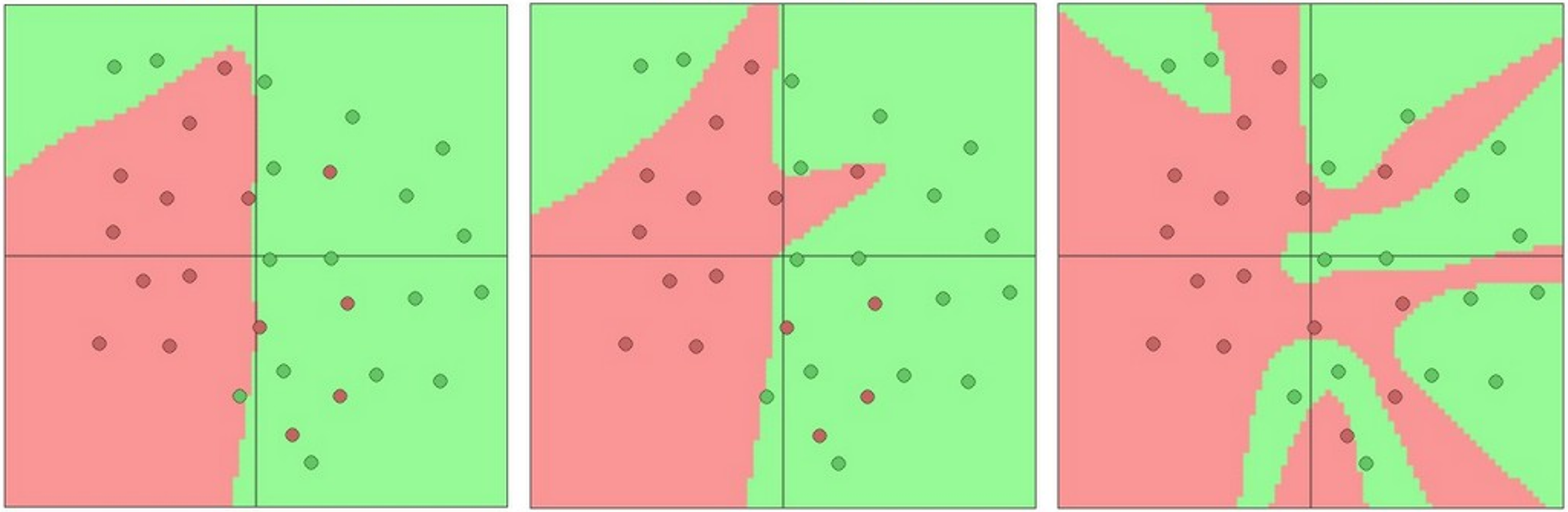




Points not linearly separable in original space

Points are linearly separable in features space!

Neural Networks Web Demo



(Web demo with ConvNetJS:
http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html)

Next Time: How to compute gradients? Backpropagation