Lecture 11: Linear Classifiers

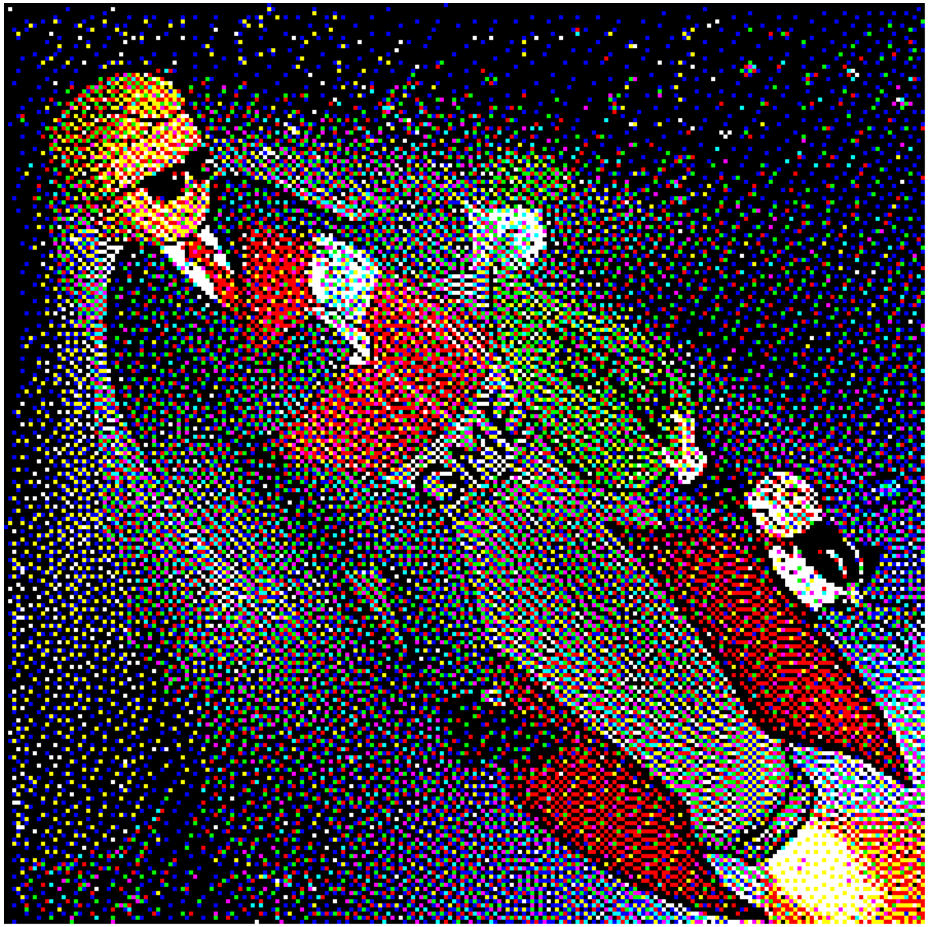
Administrative: HW2

- HW2 due Friday 2/26

Administrative: Well-Being Break

- Wednesday 2/24 is an official Well-Being Day
- No lecture on Thursday 2/25
- Regular office hours and discussion sections this week

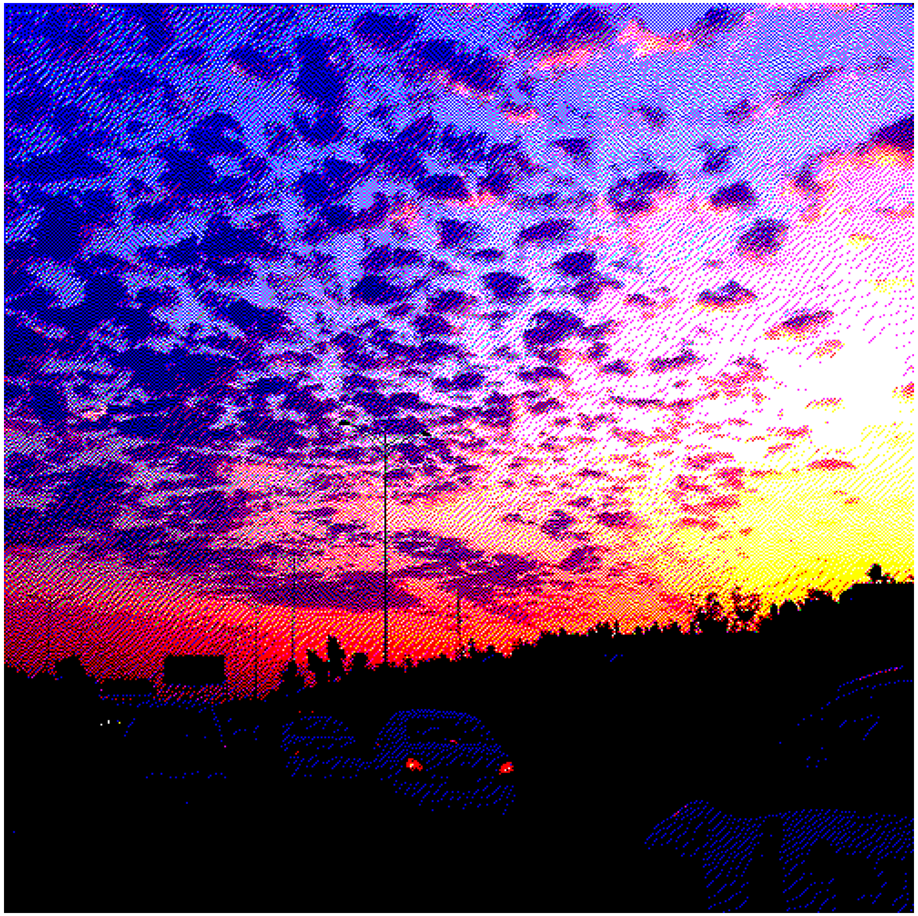
Dithering Winners! 4th Place



Dithering Winners! 4th Place

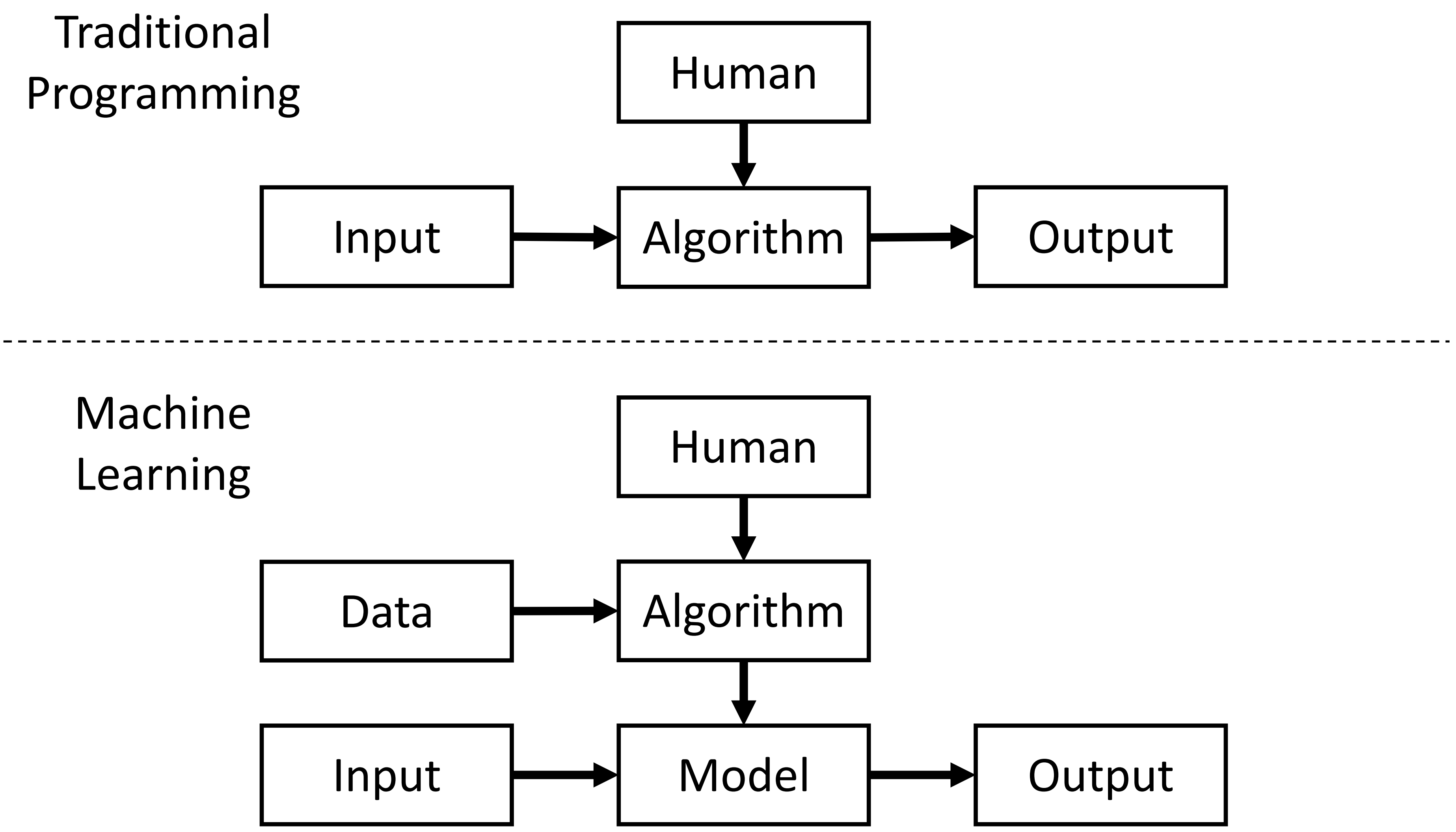
Dithering Winners! 3rd Place

Dithering Winners! 2nd Place



Dithering Winners! 1st Place

Last Time: Machine Learning



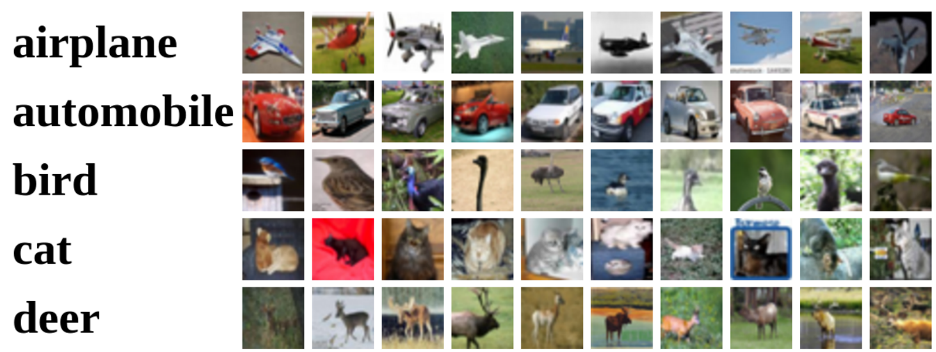
Last Time: Supervised Learning

1. Collect a dataset of images and labels
2. Use Machine Learning to train a classifier
3. Evaluate the classifier on new images

```python
def train(images, labels):
  # Machine learning!
  return model
```

```python
def predict(model, test_images):
  # Use model to predict labels
  return test_labels
```

Example training set



Last Time: Types of ML

Supervised Learning

Data: (x, y)

x is input / feature

y is label / target

Goal: Learn a *function* to map x -> y

Unsupervised Learning

Data: x

Just data, no labels!

Goal: Learn underlying *structure* in the data

Last Time: Least Squares

"Least squares" = Find the line that minimizes squared error

Data:

$$(x_1, y_1), (x_2, y_2), \dots (x_N, y_N)$$

$$x_i, y_i \in \mathbb{R}$$

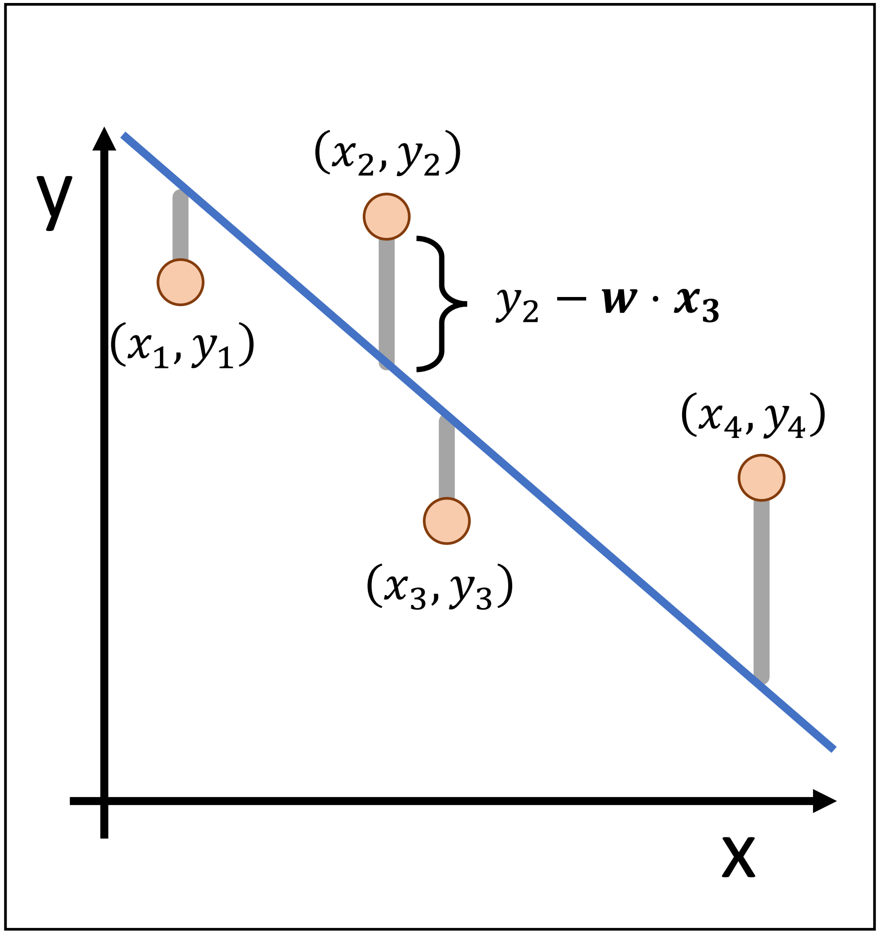
Model: $y = mx + b$

Or: $\boldsymbol{x} = (x, 1); \boldsymbol{w} = (m, b)$

$y = \boldsymbol{w} \cdot \boldsymbol{x}$

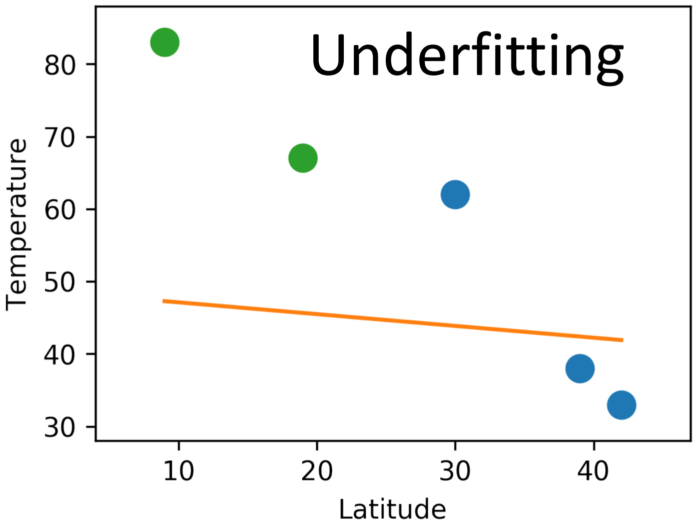
Training:

$$\boldsymbol{w}^* = \arg\min_{\boldsymbol{w}} \sum_{i=1}^{N} (y_i - \boldsymbol{w} \cdot \boldsymbol{x}_i)^2$$

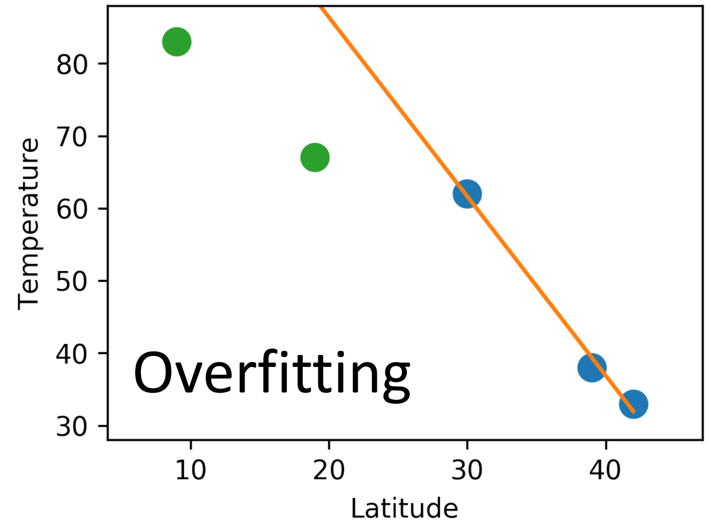


Last Time: Over/Under Fitting, Regularization

Test data **Training data**



Test data **Training data**



L2-Regularized Least Squares

$$\arg\min_{\boldsymbol{w}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|^2 + \lambda \|\boldsymbol{w}\|^2$$

Fit training data **Regularization Strength** **Penalize complexity**

Last Time: Choosing Hyperparameters

Idea #1: Choose hyperparameters that work best on the data

BAD: λ =0 always works best on training data

Your Dataset

Idea #2: Split data into **train** and **test**, choose hyperparameters that work best on test data

BAD: No idea how we will perform on new data

train	test

Idea #3: Split data into **train**, **val**, and **test**; choose hyperparameters on val and evaluate on test

Better!

train	validation	test

Today: Linear Classifiers

Image Classification: Core Vision Task

Input: image

Output: Assign image to one of a fixed set of categories

cat
bird
deer
dog
truck

Classification with Least Squares

$\boldsymbol{x}_i \in \mathbb{R}^D$ is image feature

$\boldsymbol{y}_i \in \mathbb{R}^C$ is **one-hot** label

$y_{i,c} = 1$ if $\boldsymbol{x}_i$ has category c, 0 otherwise

Training (**x**$_i$,**y**$_i$): $$\arg\min_{\boldsymbol{W}} \sum_{i=1}^{n} \|\boldsymbol{W}\boldsymbol{x}_i - \boldsymbol{y}_i\|^2$$

Inference (x): $$\boldsymbol{W}\boldsymbol{x} > t$$

Unprincipled in theory, but often effective in practice

The reverse (regression via discrete bins) is also common

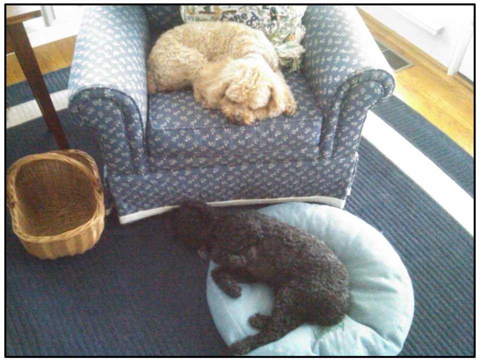
Rifkin, Yeo, Poggio. *Regularized Least Squares Classification* (http://cbcl.mit.edu/publications/ps/rlsc.pdf). 2003

Redmon, Divvala, Girshick, Farhadi. *You Only Look Once: Unified, Real-Time Object Detection.* CVPR 2016.

Classification via Memorization

Just **memorize** (as in a Python dictionary)
Consider cat/dog/hippo classification.

If this:
cat.



If this:
dog.

If this:
hippo.

Classification via Memorization

Where does this go wrong?

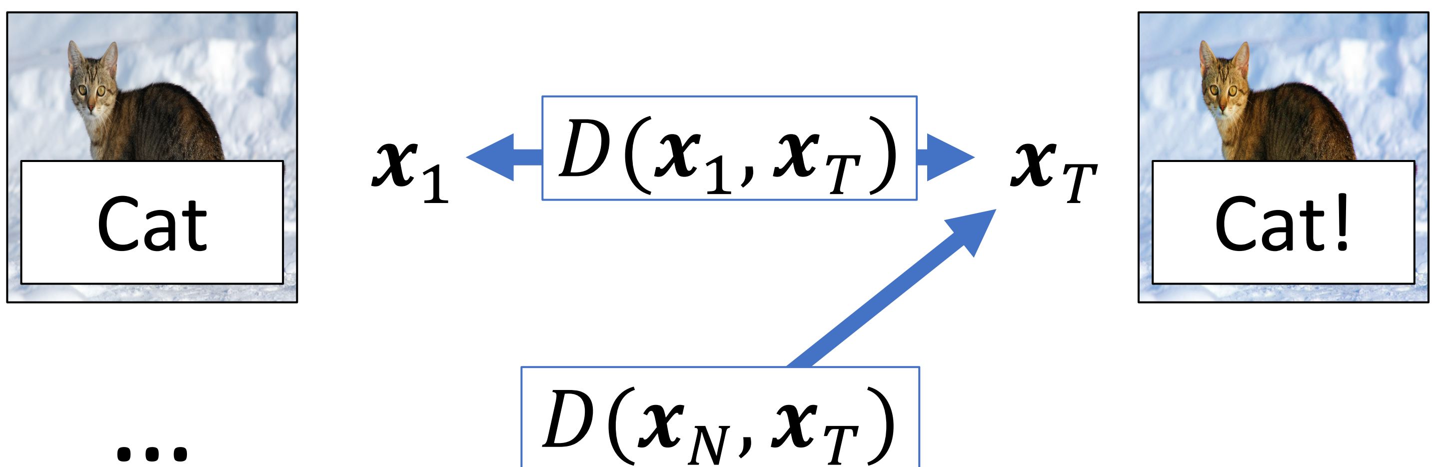
Rule: if this, then cat

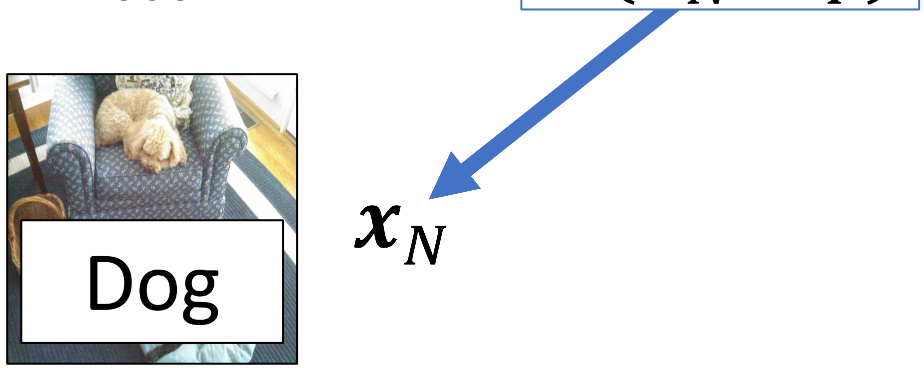
Hmmm. Not quite the same.

Classification via Memorization

Known Images
Labels

Test
Image





(1) Compute distance between feature vectors
(2) find nearest
(3) use label.

Nearest Neighbor

"Algorithm"

Training ($\mathbf{x}_i$,y_i): Memorize training set

Inference (x):

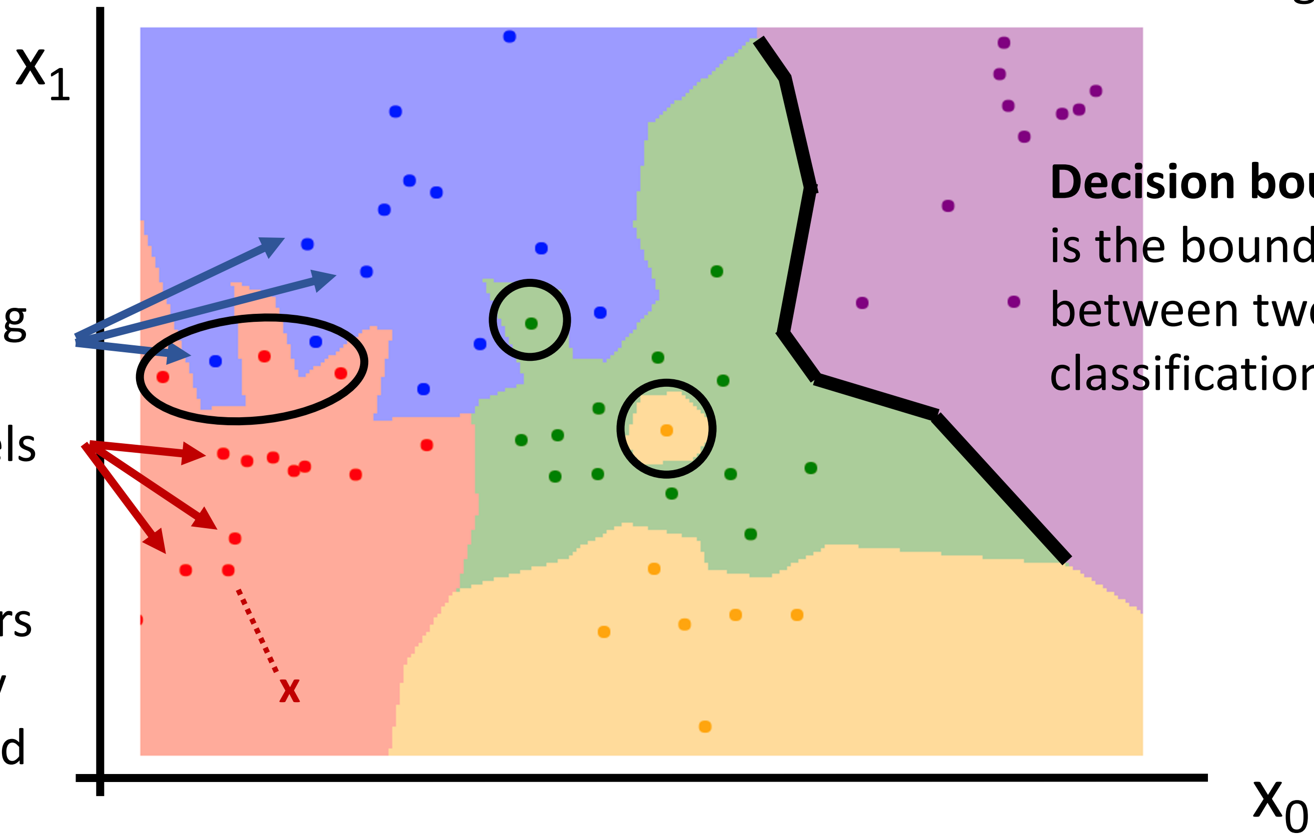
```
bestDist, prediction = Inf, None
for i in range(N):
    if dist(x_i,x) < bestDist:
        bestDist = dist(x_i,x)
        prediction = y_i
```

Nearest Neighbor

Nearest neighbors in two dimensions

Decision boundaries can be noisy; affected by outliers

How to smooth out decision boundaries? Use more neighbors!



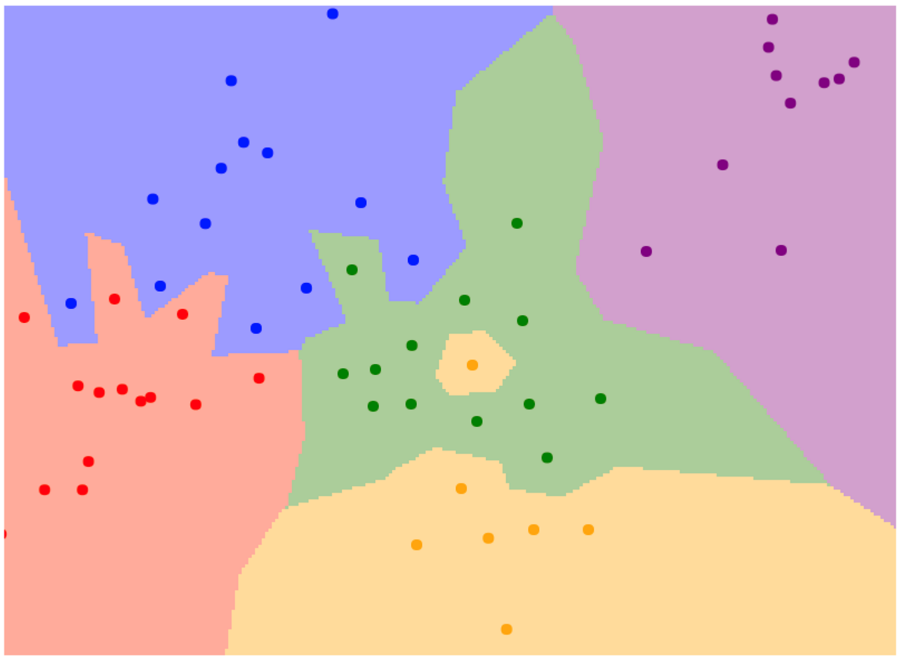
Decision boundary is the boundary between two classification regions

Points are training examples; colors give training labels

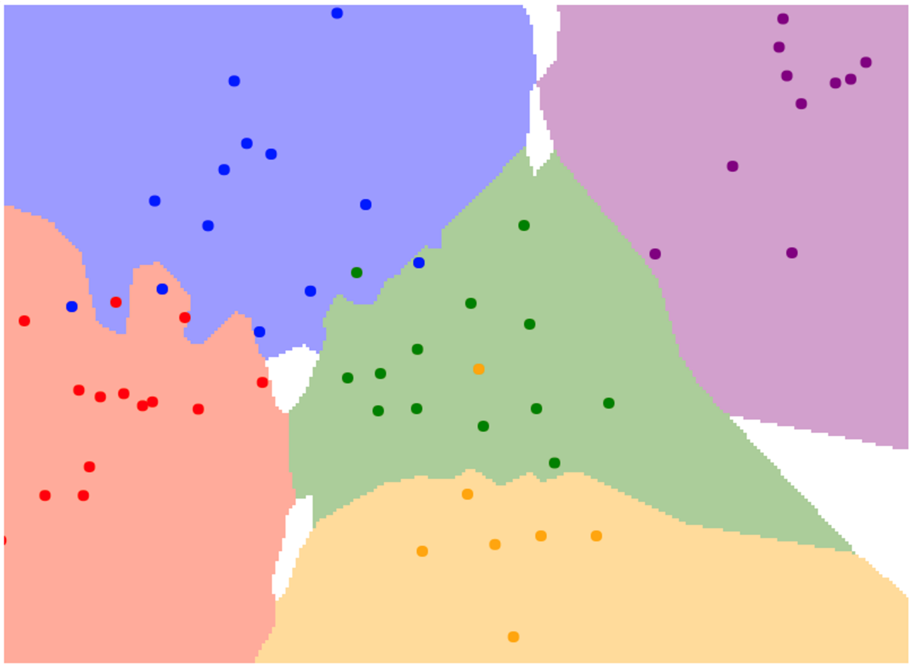
Background colors give the category a test point would be assigned

K-Nearest Neighbors

K = 1



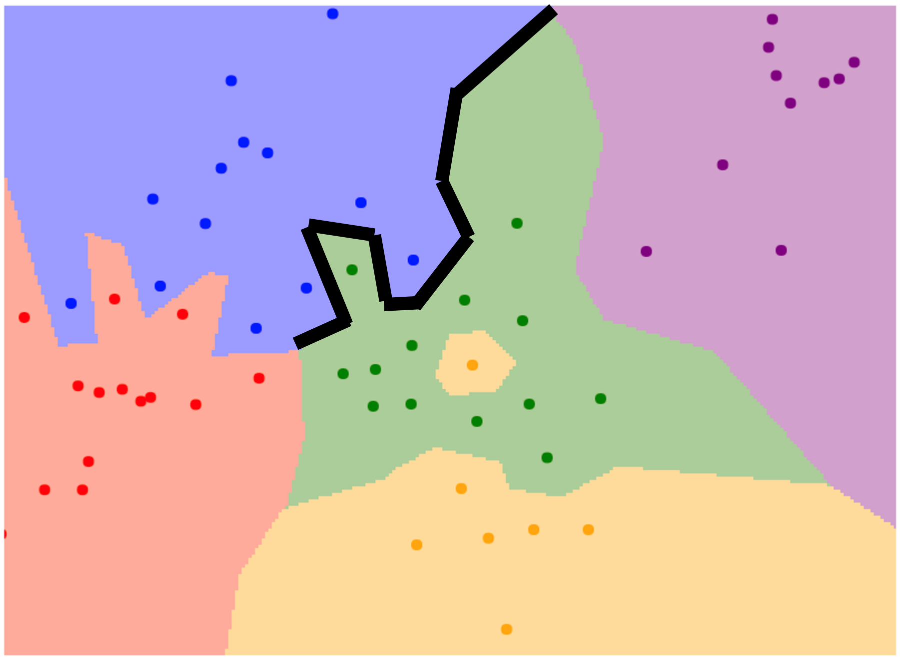
K = 3



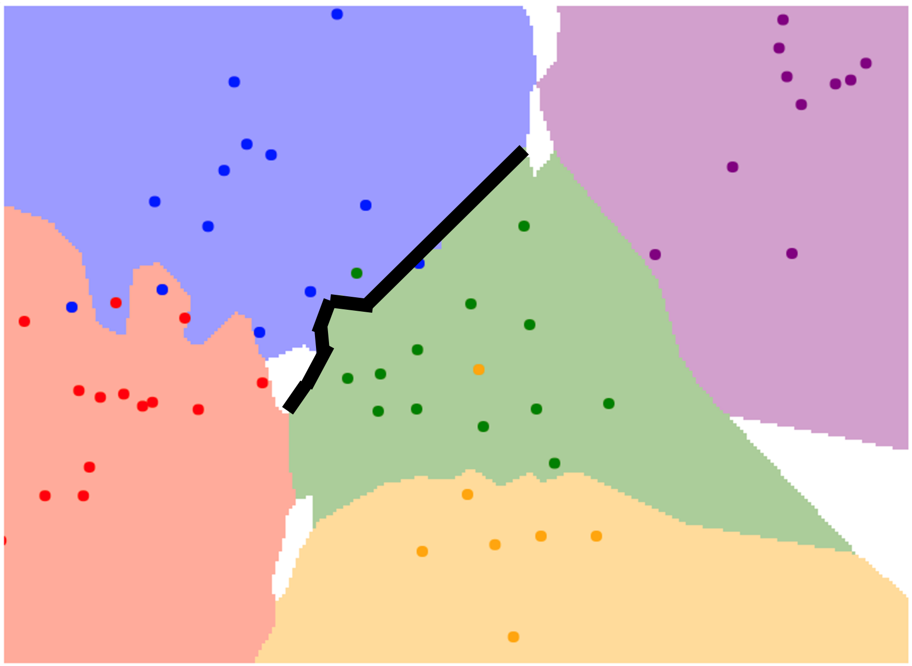
Instead of copying label from nearest neighbor, take **majority vote** from K closest points

K-Nearest Neighbors

K = 1



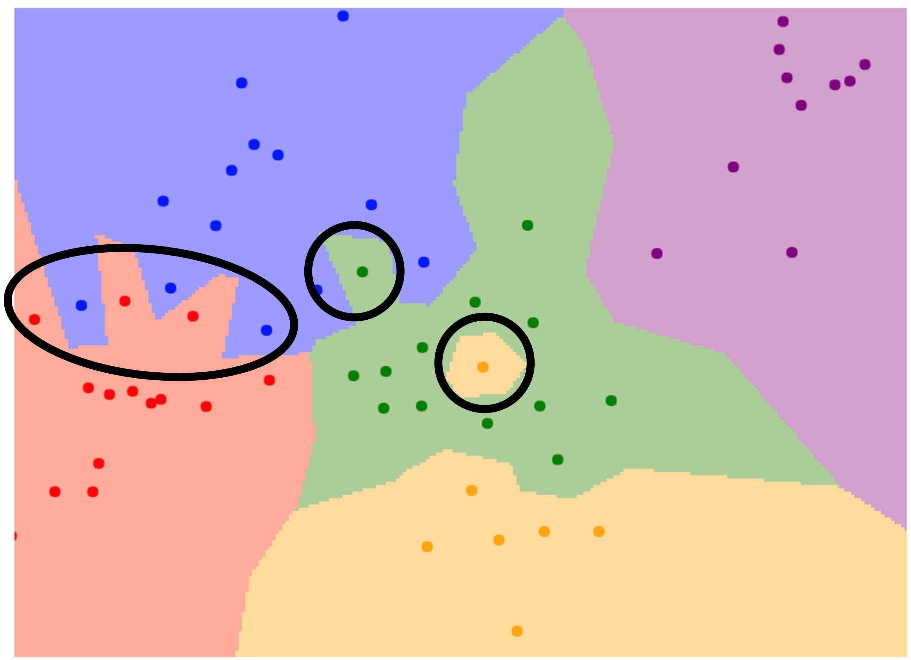
K = 3



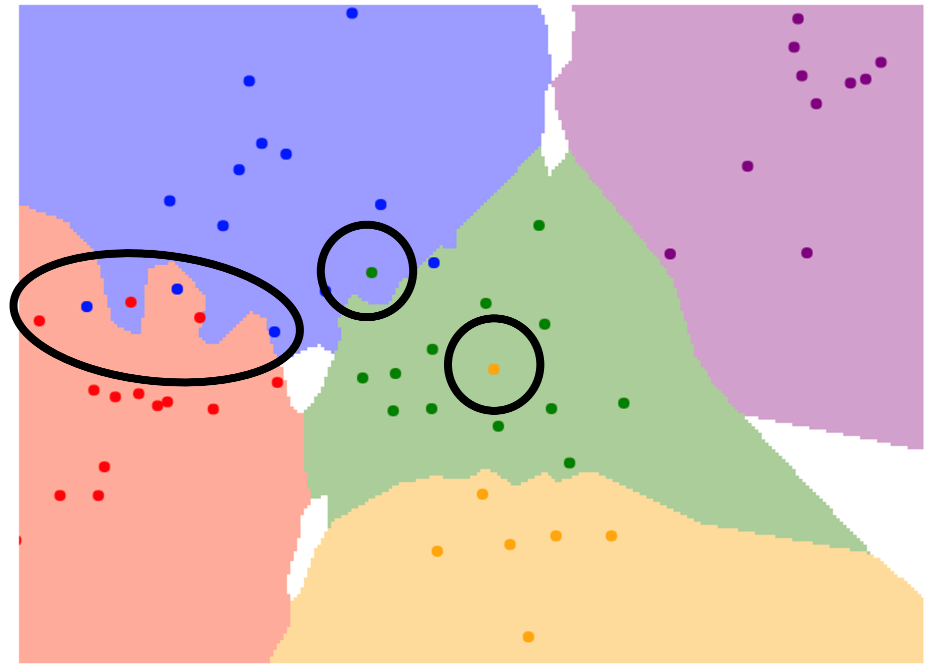
Using more neighbors helps smooth out rough decision boundaries

K-Nearest Neighbors

K = 1



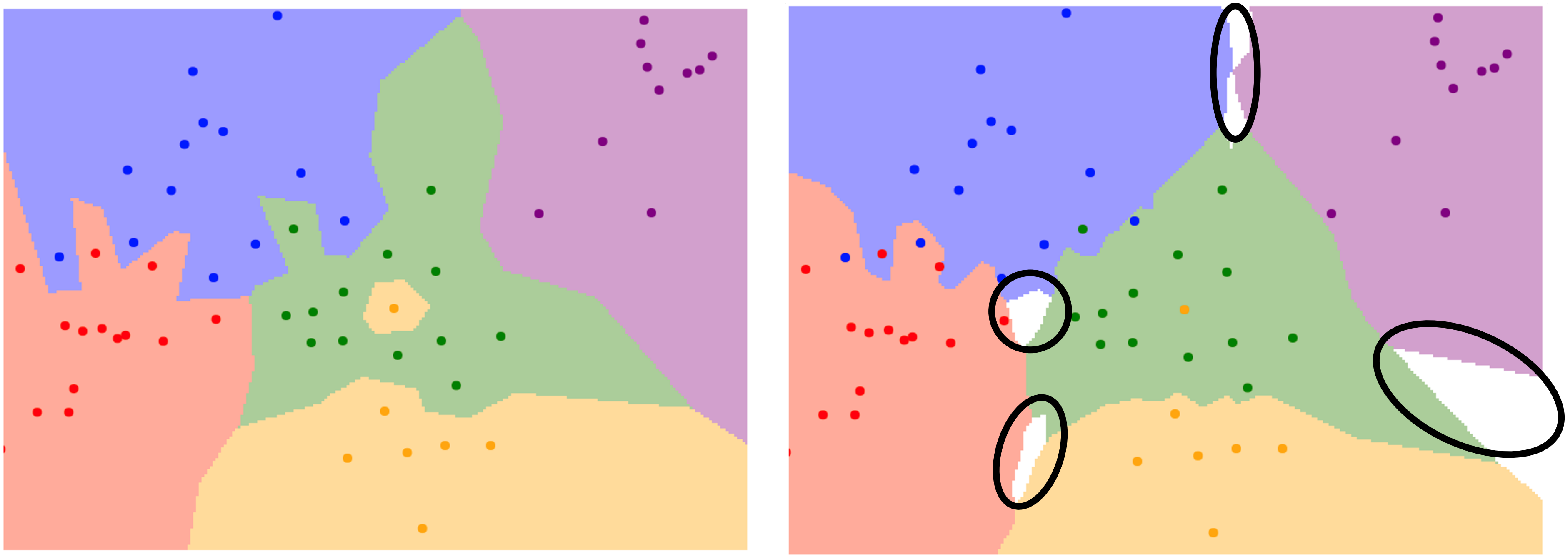
K = 3



Using more neighbors helps reduce the effect of outliers

K-Nearest Neighbors

K = 1 K = 3

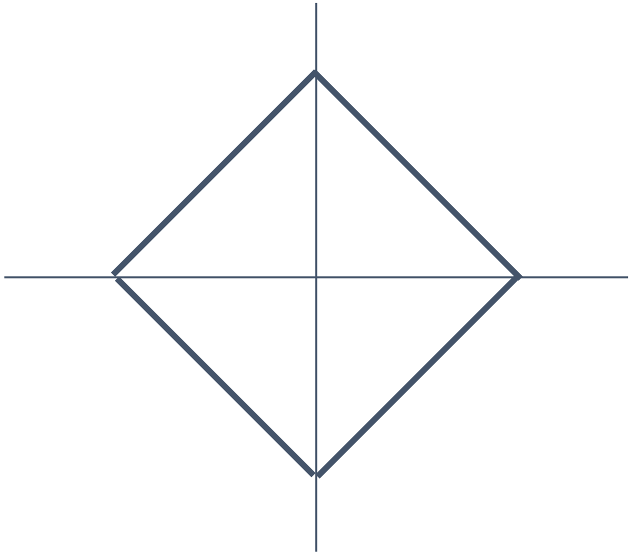


When K > 1 there can be ties!
Need to break them somehow

K-Nearest Neighbors: Distance Metric

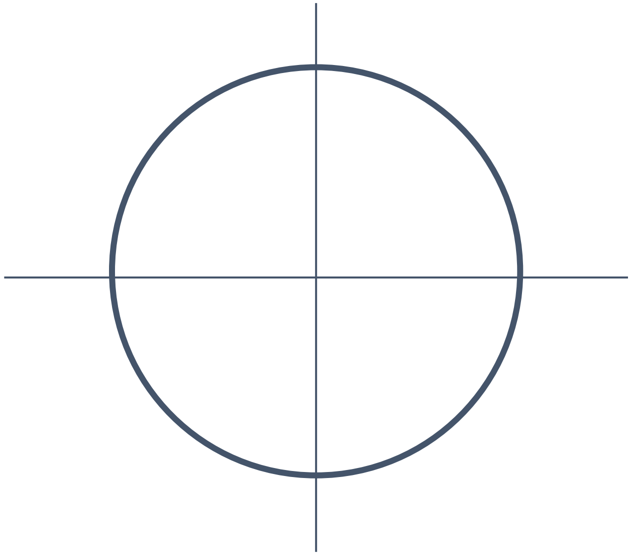
L1 (Manhattan) Distance

$$d(x, y) = \sum_i |x_i - y_i|$$



L2 (Euclidean) Distance

$$d(x, y) = \left(\sum_i (x_i - y_i)^2\right)^{1/2}$$



K-Nearest Neighbors: Distance Metric

L1 (Manhattan) Distance

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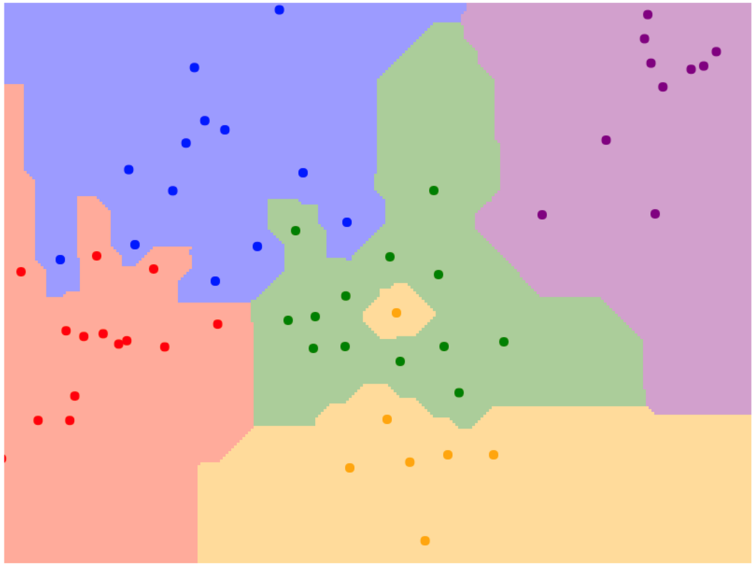
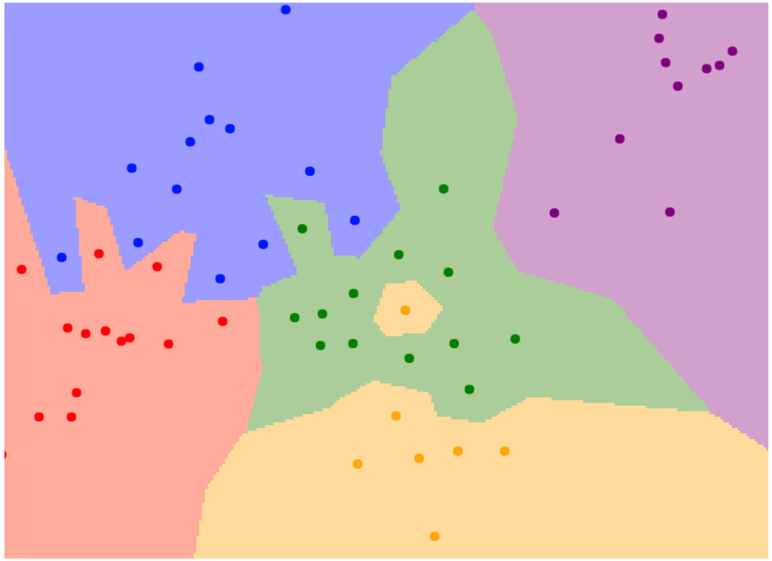


K = 1

L2 (Euclidean) Distance

$$d(x,y) = \left(\sum_i (x_i - y_i)^2\right)^{1/2}$$

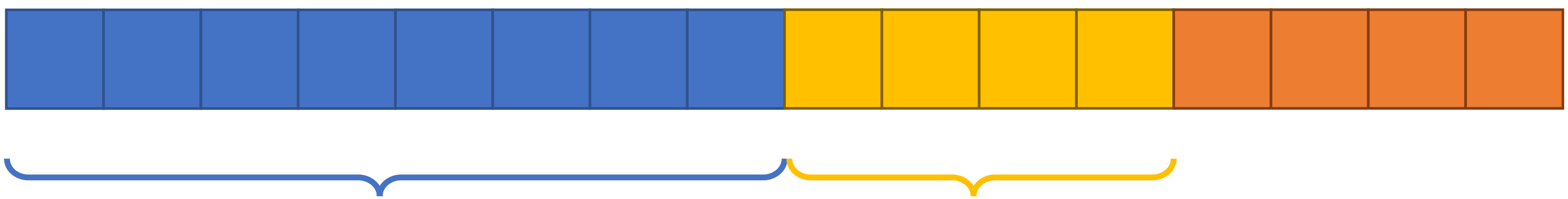


K = 1

K-Nearest Neighbors

What distance? What value for K?

Training Validation Test



Use these data points for lookup

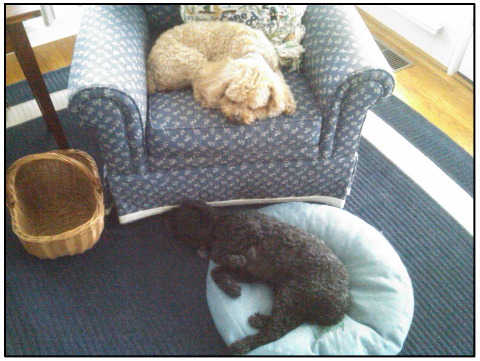
Evaluate on these points for different k, distances

K-Nearest Neighbors

- No learning going on but usually effective
- Same algorithm for every task
- As number of datapoints $\rightarrow \infty$, error rate is guaranteed to be at most 2x worse than optimal you could do on data
- Training is fast, but inference is slow. Opposite of what we want!

Linear Classifiers

Example Setup: 3 classes


Model – one weight per class: $\boldsymbol{w}_0, \boldsymbol{w}_1, \boldsymbol{w}_2$

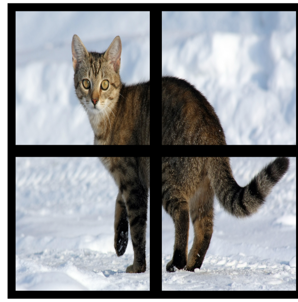
$\boldsymbol{w}_0^T\boldsymbol{x}$ big if cat

$\boldsymbol{w}_1^T\boldsymbol{x}$ big if dog

$\boldsymbol{w}_2^T\boldsymbol{x}$ big if hippo

Stack together: $\boldsymbol{W}_{3xF}$ where **x** is in R^F

Linear Classifiers



Cat weight vector	0.2	-0.5	0.1	2.0	1.1
Dog weight vector	1.5	1.3	2.1	0.0	3.2
Hippo weight vector	0.0	0.3	0.2	-0.3	-1.2

$\boldsymbol{W}$

56
231
24
2
1

$\boldsymbol{x_i}$

→

-96.8	Cat score
437.9	Dog score
61.95	Hippo score

$\boldsymbol{Wx_i}$

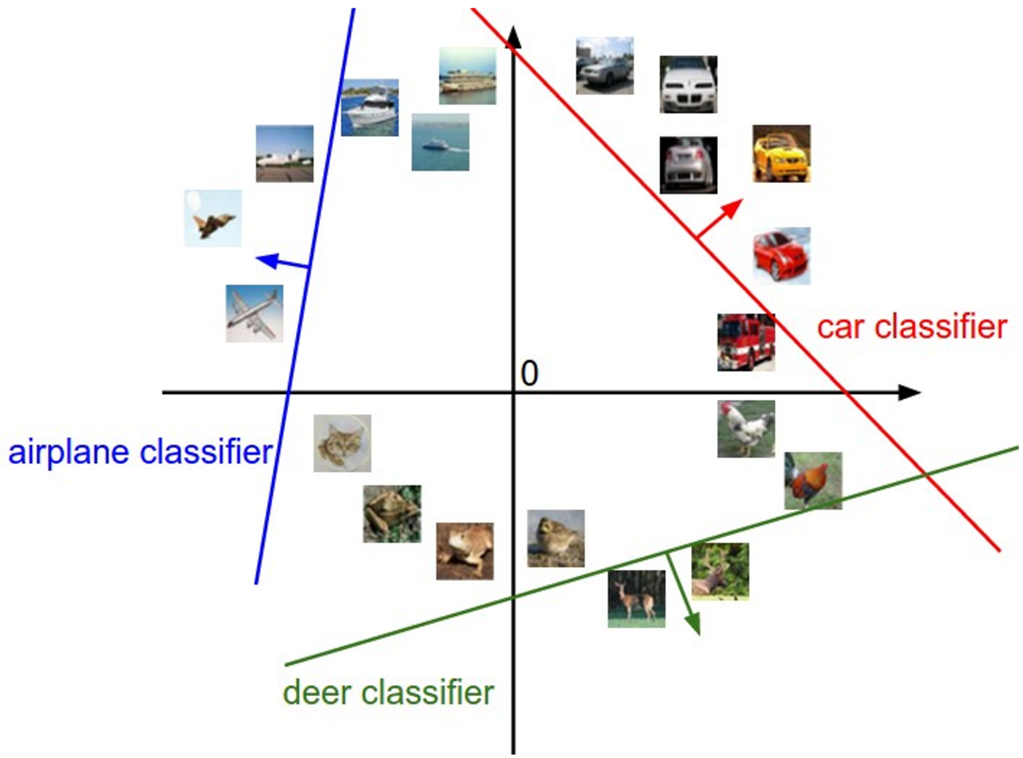
Weight matrix a collection of scoring functions, one per class

Prediction is vector where jth component is "score" for jth class.

Diagram by: Karpathy, Fei-Fei

Linear Classifiers: Geometric Intuition

What does a linear classifier look like in 2D?

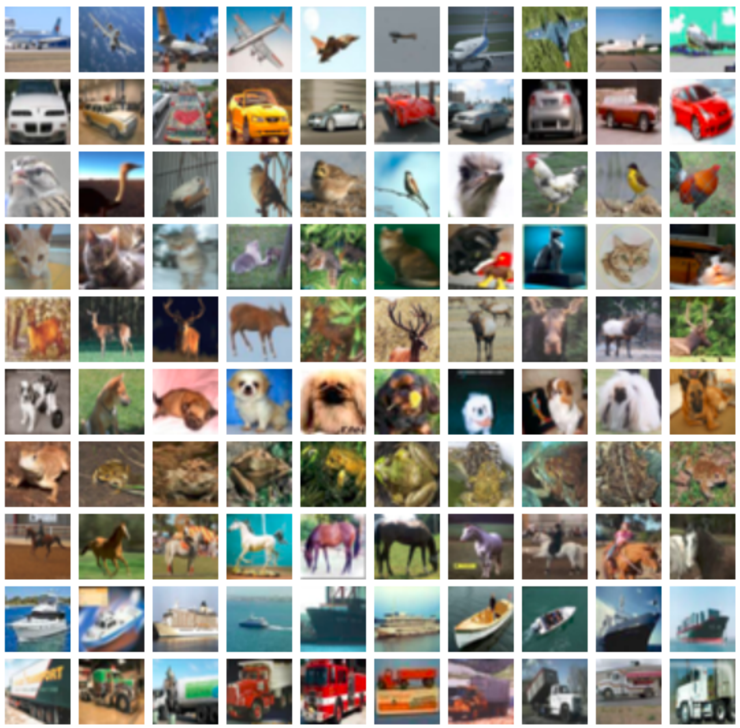


Be aware: Intuition from 2D doesn't always carry over into high-dimensional spaces. See: *On the Surprising Behavior of Distance Metrics in High Dimensional Space.* Charu, Hinneburg, Keim. ICDT 2001

Diagram credit: Karpathy & Fei-Fei

Linear Classifiers: Visual Intuition

CIFAR 10:
32x32x3 Images, 10 Classes



- Turn each image into feature by unrolling all pixels
- Train a linear model to recognize 10 classes

Linear Classifiers: Visual Intuition

Decision rule is $\mathbf{w}^T\mathbf{x}$. If $\mathbf{w}_i$ is big, then big values of x_i are indicative of the class.

Deer or Plane?

Linear Classifiers: Visual Intuition

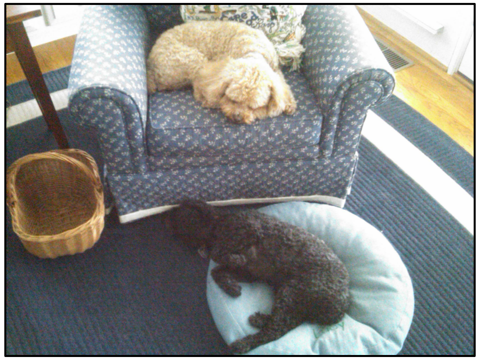
Decision rule is $\mathbf{w}^T\mathbf{x}$. If $\mathbf{w}_i$ is big, then big values of x_i are indicative of the class.

Ship or Dog?

Linear Classifiers: Visual Intuition

Decision rule is $\mathbf{w}^T\mathbf{x}$. If $\mathbf{w}_i$ is big, then big values of x_i are indicative of the class.

So Far: Linear Score Function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$\boldsymbol{w}_0^T\boldsymbol{x}$ big if cat

$\boldsymbol{w}_1^T\boldsymbol{x}$ big if dog

$\boldsymbol{w}_2^T\boldsymbol{x}$ big if hippo

Stack together: $\boldsymbol{W}_{\boldsymbol{3xF}}$ where **x** is in R^F

How do we know which W is best?

Choosing W: Loss Function

A **loss function** tells how good our current classifier is

Low loss = good classifier
High loss = bad classifier

(Also called: **objective function**; **cost function**)

Negative loss function sometimes called **reward function**, **profit function**, **utility function**, **fitness function**, etc

Given a dataset

$$\{(x_i, y_i)\}_{i=1}^N$$

of images x_i and labels y_i,

Loss for a single example is:

$$L_i(f(x_i, W), y_i)$$

Loss for the dataset is

$$L = \frac{1}{N}\sum_{i=1}^{N} L_i(f(x_i, W), y_i)$$

Cross-Entropy Loss

Want to interpret raw classifier scores as **probabilities**

Classifier scores
$s = f(x_i, W)$

cat	**3.2**
car	5.1
frog	-1.7

Cross-Entropy Loss

Want to interpret raw classifier scores as **probabilities**

Classifier scores
$s = f(x_i, W)$

Softmax function
$$p_i = \frac{\exp(s_i)}{\sum_j \exp(s_j)}$$

cat	**3.2**
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frog	-1.7

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Unnormalized log-probabilities / logits

Cross-Entropy Loss

Want to interpret raw classifier scores as **probabilities**

Softmax function

$$p_i = \frac{\exp(s_i)}{\sum_j \exp(s_j)}$$

Classifier scores

$$s = f(x_i, W)$$

Probabilities must be >= 0

		exp →	
cat	**3.2**		**24.5**
car	5.1		164
frog	-1.7		0.18

Unnormalized log-probabilities / logits

unnormalized probabilities

Cross-Entropy Loss

Want to interpret raw classifier scores as **probabilities**

Classifier scores
$s = f(x_i, W)$

Softmax function
$$p_i = \frac{\exp(s_i)}{\sum_j \exp(s_j)}$$

Probabilities must be >= 0

Probabilities must sum to 1

		exp		normalize	
cat	**3.2**	→	**24.5**	→	**0.13**
car	5.1		164		0.87
frog	-1.7		0.18		0.00

Unnormalized log-probabilities / logits

unnormalized probabilities

probabilities

Cross-Entropy Loss

Want to interpret raw classifier scores as **probabilities**

Classifier scores
$s = f(x_i, W)$

Softmax function
$$p_i = \frac{\exp(s_i)}{\sum_j \exp(s_j)}$$

Loss
$$L_i = -\log(p_{y_i})$$

	Unnormalized log-probabilities / logits		unnormalized probabilities		probabilities
cat	**3.2**	exp →	**24.5**	normalize →	**0.13**
car	5.1		164		0.87
frog	-1.7		0.18		0.00

Probabilities must be >= 0

Probabilities must sum to 1

$L_i = -\log(0.13)$
$= \mathbf{2.04}$

Cross-Entropy Loss

Want to interpret raw classifier scores as **probabilities**

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$s = f(x_i, W)$

Softmax function
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Probabilities must be >= 0

Probabilities must sum to 1

	Unnormalized log-probabilities / logits	$\xrightarrow{\text{exp}}$	unnormalized probabilities	$\xrightarrow{\text{normalize}}$	probabilities
cat	**3.2**		**24.5**		**0.13**
car	5.1		164		0.87
frog	-1.7		0.18		0.00

$L_i = -\log(0.13)$
$= \mathbf{2.04}$

Maximum Likelihood Estimation
Choose weights to maximize the likelihood of the observed data
(See EECS 445 or EECS 545)

Cross-Entropy Loss

Want to interpret raw classifier scores as **probabilities**

Classifier scores
$s = f(x_i, W)$

Softmax function
$$p_i = \frac{\exp(s_i)}{\sum_j \exp(s_j)}$$

Loss
$$L_i = -\log(p_{y_i})$$

	Unnormalized log-probabilities / logits	exp →	Probabilities must be >= 0 — unnormalized probabilities	normalize →	Probabilities must sum to 1 — probabilities	Correct probs
cat	**3.2**		**24.5**		**0.13**	**1.00**
car	5.1		164		0.87	0.00
frog	-1.7		0.18		0.00	0.00

Probabilities → Compare ← Correct probs

Cross-Entropy Loss

Want to interpret raw classifier scores as **probabilities**

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$s = f(x_i, W)$

Softmax function
$$p_i = \frac{\exp(s_i)}{\sum_j \exp(s_j)}$$

Loss
$$L_i = -\log(p_{y_i})$$

Probabilities must be >= 0

Probabilities must sum to 1

	Unnormalized log-probabilities / logits		unnormalized probabilities		probabilities		Correct probs
cat	**3.2**	exp →	**24.5**	normalize →	**0.13**	→ Compare ←	**1.00**
car	5.1		164		0.87		0.00
frog	-1.7		0.18		0.00		0.00

Kullback-Leibler Divergence
$$D_{KL}(\boldsymbol{P} \,||\, \boldsymbol{Q}) = \sum_y \boldsymbol{P}(y) \log \frac{\boldsymbol{P}(y)}{\boldsymbol{Q}(y)}$$

Cross-Entropy Loss

Want to interpret raw classifier scores as **probabilities**

Softmax function

Classifier scores

$$s = f(x_i, W)$$

$$p_i = \frac{\exp(s_i)}{\sum_j \exp(s_j)}$$

Loss

$$L_i = -\log(p_{y_i})$$

Probabilities must be >= 0

Probabilities must sum to 1

cat	**3.2**	exp →	**24.5**	normalize →	**0.13**	→ Compare ←	**1.00**
car	5.1		164		0.87		0.00
frog	-1.7		0.18		0.00		0.00

Cross-Entropy:

$$H(P,Q) = H(P) + D_{KL}(P \,||\, Q)$$

Unnormalized log-probabilities / logits

unnormalized probabilities

probabilities

Correct probs

Cross-Entropy Loss

Want to interpret raw classifier scores as **probabilities**

cat	**3.2**
car	5.1
frog	-1.7

Classifier scores

$s = f(x_i, W)$

Softmax function

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Loss

$$L_i = -\log(p_{y_i})$$

Putting it all together:

$$L_i = -\log \frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}$$

Cross-Entropy Loss

Want to interpret raw classifier scores as **probabilities**

cat	**3.2**
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Classifier scores

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Softmax function

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Loss

$$L_i = -\log(p_{y_i})$$

Putting it all together:

$$L_i = -\log \frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}$$

Q: What is the min / max possible loss L_i?

Cross-Entropy Loss

Want to interpret raw classifier scores as **probabilities**

cat	**3.2**
car	5.1
frog	-1.7

Classifier scores
$s = f(x_i, W)$

Softmax function
$$p_i = \frac{\exp(s_i)}{\sum_j \exp(s_j)}$$

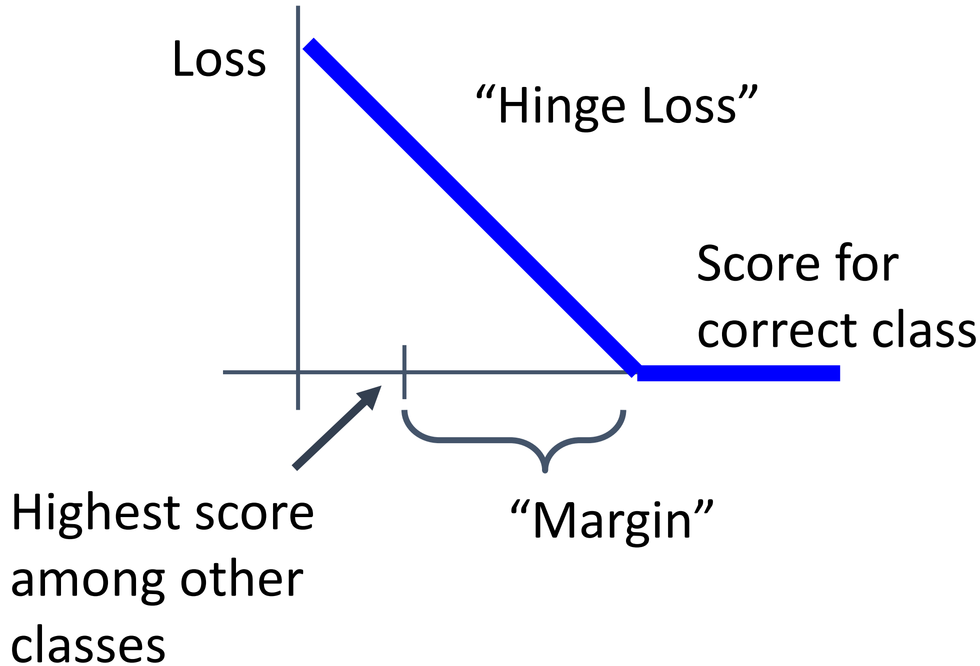
Loss
$$L_i = -\log(p_{y_i})$$

Putting it all together:
$$L_i = -\log \frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}$$

Q: If all scores are small random values, what is the loss?

Multiclass SVM Loss

"The score of the correct class should be higher than all the other scores"



Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Multiclass SVM Loss

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Multiclass SVM Loss



cat	**3.2**	1.3	2.2
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Multiclass SVM Loss



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frog	-1.7	2.0	**-3.1**
Loss	2.9		

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$= \max(0, 5.1 - 3.2 + 1)$
$\quad + \max(0, -1.7 - 3.2 + 1)$
$= \max(0, 2.9) + \max(0, -3.9)$
$= 2.9 + 0$
$= 2.9$

Multiclass SVM Loss



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9	0	

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

Multiclass SVM Loss

  

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 2.2 - (-3.1) + 1)
+max(0, 2.5 - (-3.1) + 1)
= max(0, 6.3) + max(0, 6.6)
= 6.3 + 6.6
= 12.9

Multiclass SVM Loss



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Loss over the dataset is:

L = (2.9 + 0.0 + 12.9) / 3
= 5.27

Multiclass SVM Loss



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Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q: What happens to the loss if the scores for the car image change a bit?

Multiclass SVM Loss



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Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

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Q: What are the min and max possible loss?

Multiclass SVM Loss



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Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q: If all scores were random, what loss would we expect?

Multiclass SVM Loss



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Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q: What would happen if sum were over all classes? (including $j = y_i$)

Multiclass SVM Loss

  

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Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q: What if the loss used mean instead of sum?

Multiclass SVM Loss

			
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Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q: What if we used this loss instead?

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)^2$$

Cross-Entropy vs SVM Loss

$$L_i = -\log \frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Assume scores:
[10, -2, 3]
[10, 9, 9]
[10, -100, -100]
and $y_i = 0$

Q: What is cross-entropy loss? What is SVM loss?

A: Cross-entropy loss > 0
SVM loss = 0

Cross-Entropy vs SVM Loss

$$L_i = -\log \frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Assume scores:
[10, -2, 3]
[10, 9, 9]
[10, -100, -100]
and $y_i = 0$

Q: What happens to each loss if I slightly change the scores of the last datapoint?

A: Cross-entropy loss will change; SVM loss will stay the same

Cross-Entropy vs SVM Loss

$$L_i = -\log \frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Assume scores:

[10, -2, 3]

[10, 9, 9]

[10, -100, -100]

and $y_i = 0$

Q: What happens to each loss if I double the score of the correct class from 10 to 20?

A: Cross-entropy loss will decrease, SVM loss still 0

Recap

- **Image Classification** is a core computer vision task
- **K-Nearest Neighbors** is classification via memorization
- **Linear classifiers** learn one template per category to match with the input
- A **loss function** specifies your preference over different settings of weights
- **Cross-Entropy loss** maximizes probability of correct class
- **SVM Loss** wants correct score larger than other scores

Next Time:
How to choose W?
Optimization!