Detectors and Descriptors

EECS 442 – David Fouhey and Justin Johnson
Winter 2021, University of Michigan

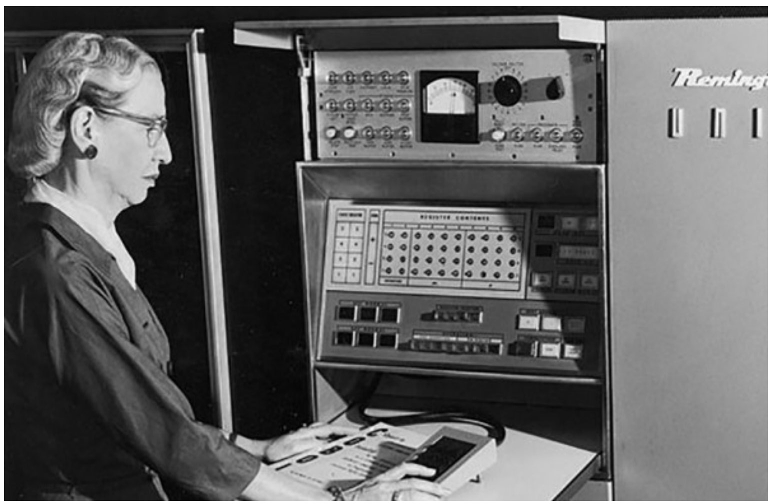
https://web.eecs.umich.edu/~justincj/teaching/eecs442/WI2021/

Goal

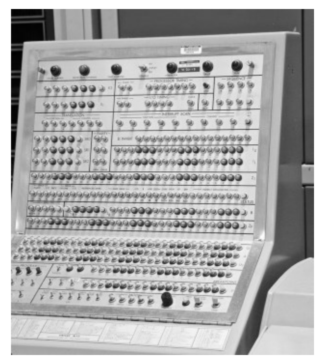
How big is this image as a vector?

389x600 = 233,400 dimensions **(big)**

Applications To Have In Mind



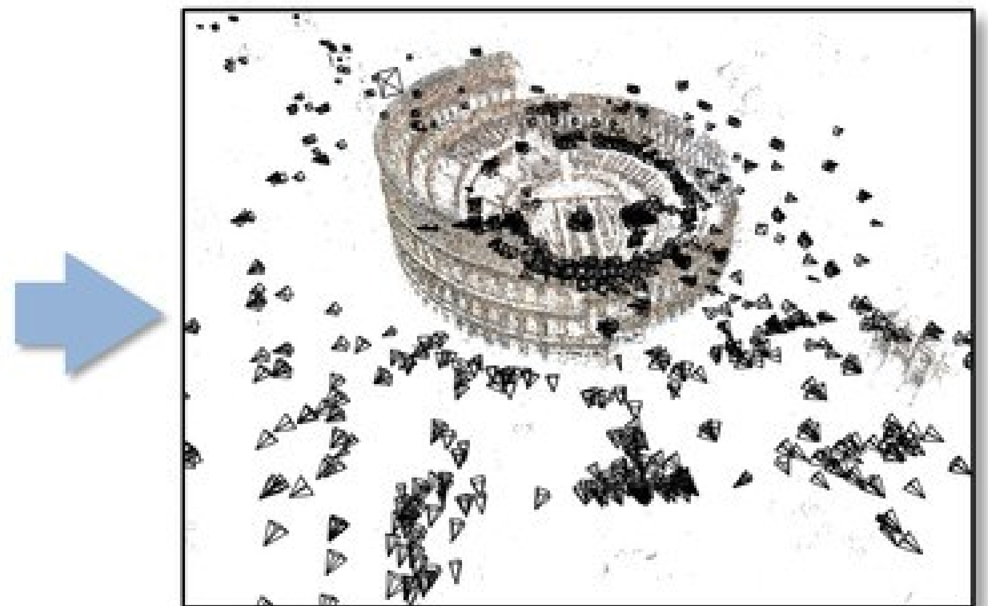
Part of the same photo?



Same computer from another angle?

Applications To Have In Mind

Building a 3D Reconstruction Out Of Images



Slide Credit: N. Seitz

Applications To Have In Mind

Stitching photos taken at different angles

One Possibly Familiar Example

Given two images: how do you align them?

One (Soon To Be Familiar) Solution

```
for y in range(-ySearch,ySearch+1):
    for x in range(-xSearch,xSearch+1):
        #Touches all HxW pixels!
        check_alignment_with_images()
```

One Motivating Example

Given these images: how do you align them?

These aren’t off by a small 2D translation but instead by a 3D rotation + translation of the camera.

Photo credit: M. Brown, D. Lowe

One (Soon To Be Familiar) Solution

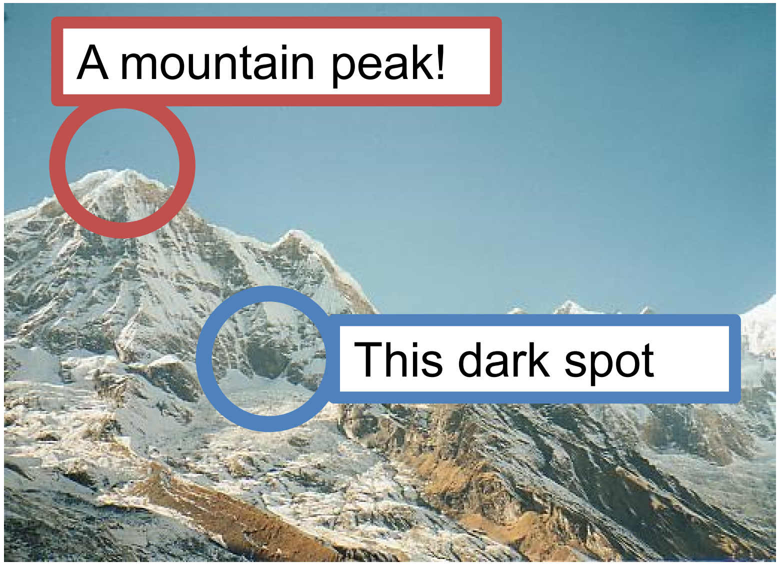
```
for y in yRange:
    for x in xRange:
        for z in zRange:
            for xRot in xRotVals:
                for yRot in yRotVals:
                    for zRot in zRotVals:
                        #touches all HxW pixels!
                        check_alignment_with_images()
```

This code should make you really <u>unhappy</u>

Note: this actually isn't even the full number of parameters; it's actually 8 for loops.

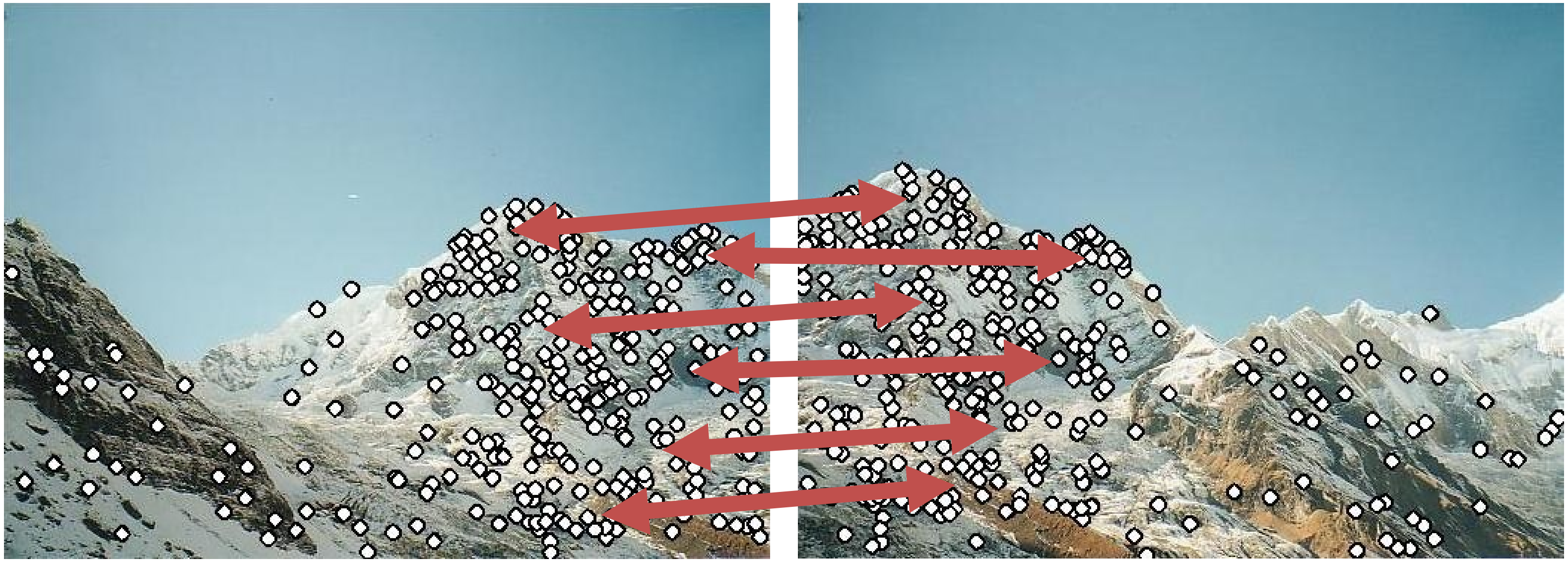
An Alternate Approach

Given these images: how would **<u>you</u>** align them?



An Alternate Approach

Finding and Matching



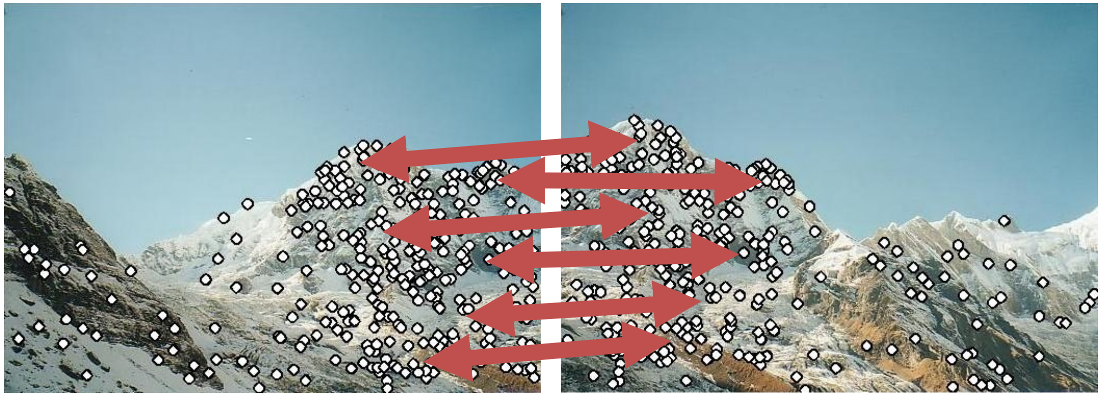
1: find corners+features

2: match based on local image data

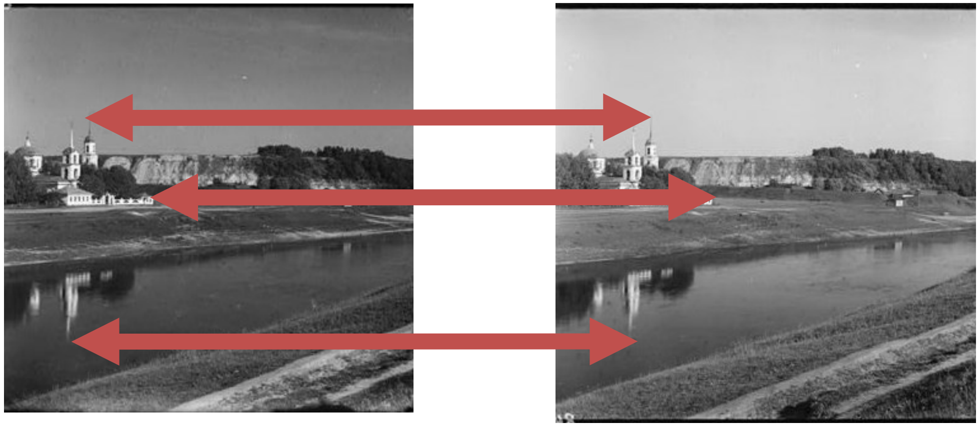
Slide Credit: S. Lazebnik, original figure: M. Brown, D. Lowe

What Now?

Given pairs **p1**,**p2** of correspondence, **how do I align?**

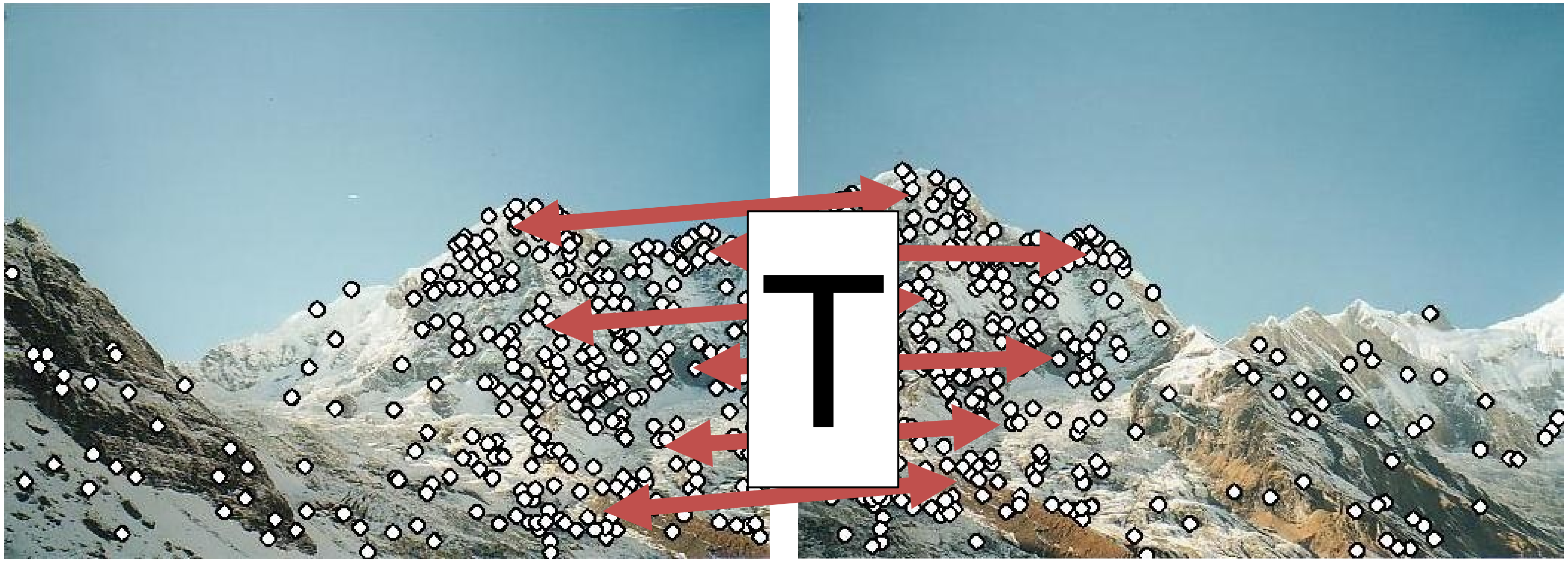


Consider translation-only case from HW1.



An Alternate Approach

Solving for a Transformation



3: Solve for transformation T (e.g. such that **p1 ≡ T p2**) that fits the matches well

Note the homogeneous coordinates, you'll see them again.

Slide Credit: S. Lazebnik, original figure: M. Brown, D. Lowe

An Alternate Approach

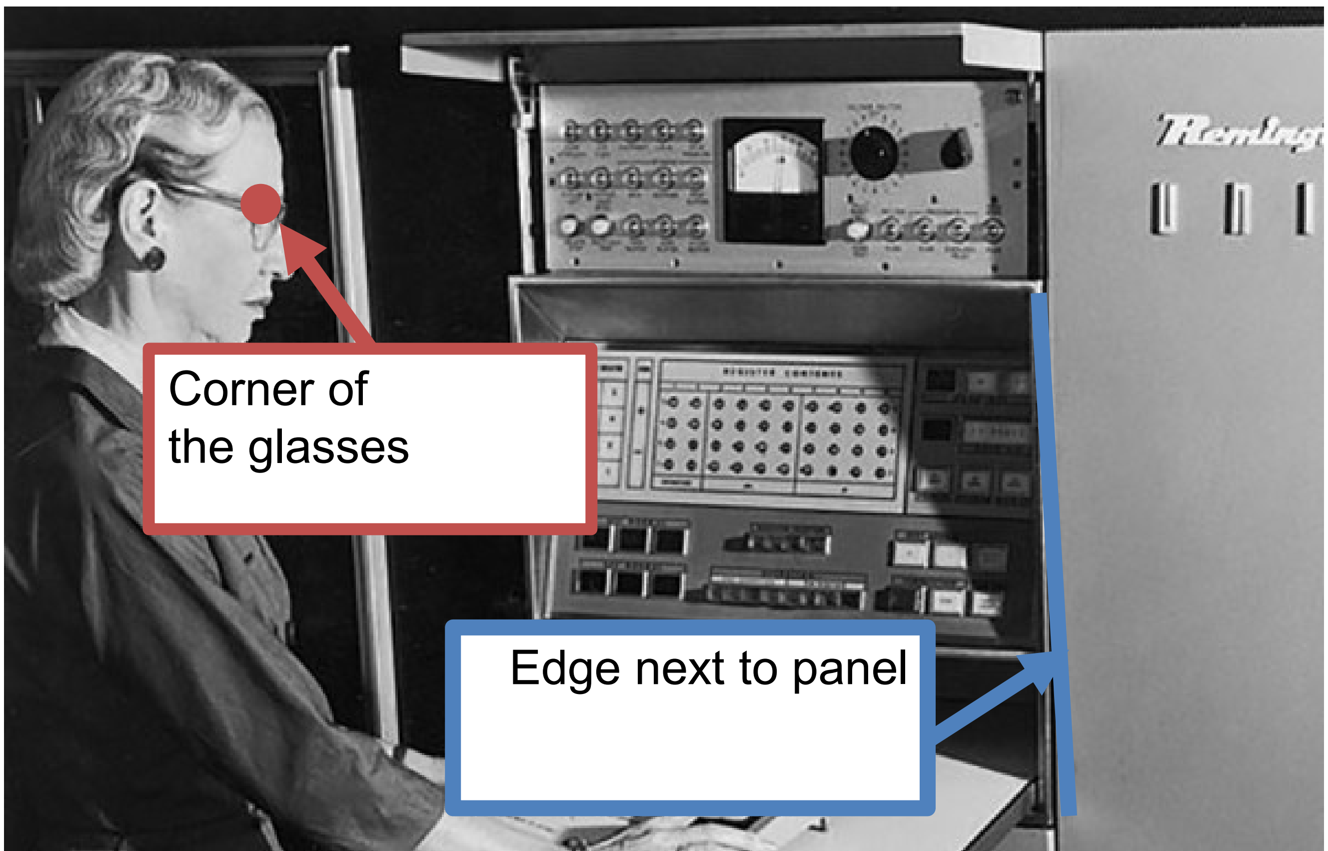
Blend Them Together

Key insight: we don't work with full image. We work with only parts of the image.

Photo Credit: M. Brown, D. Lowe

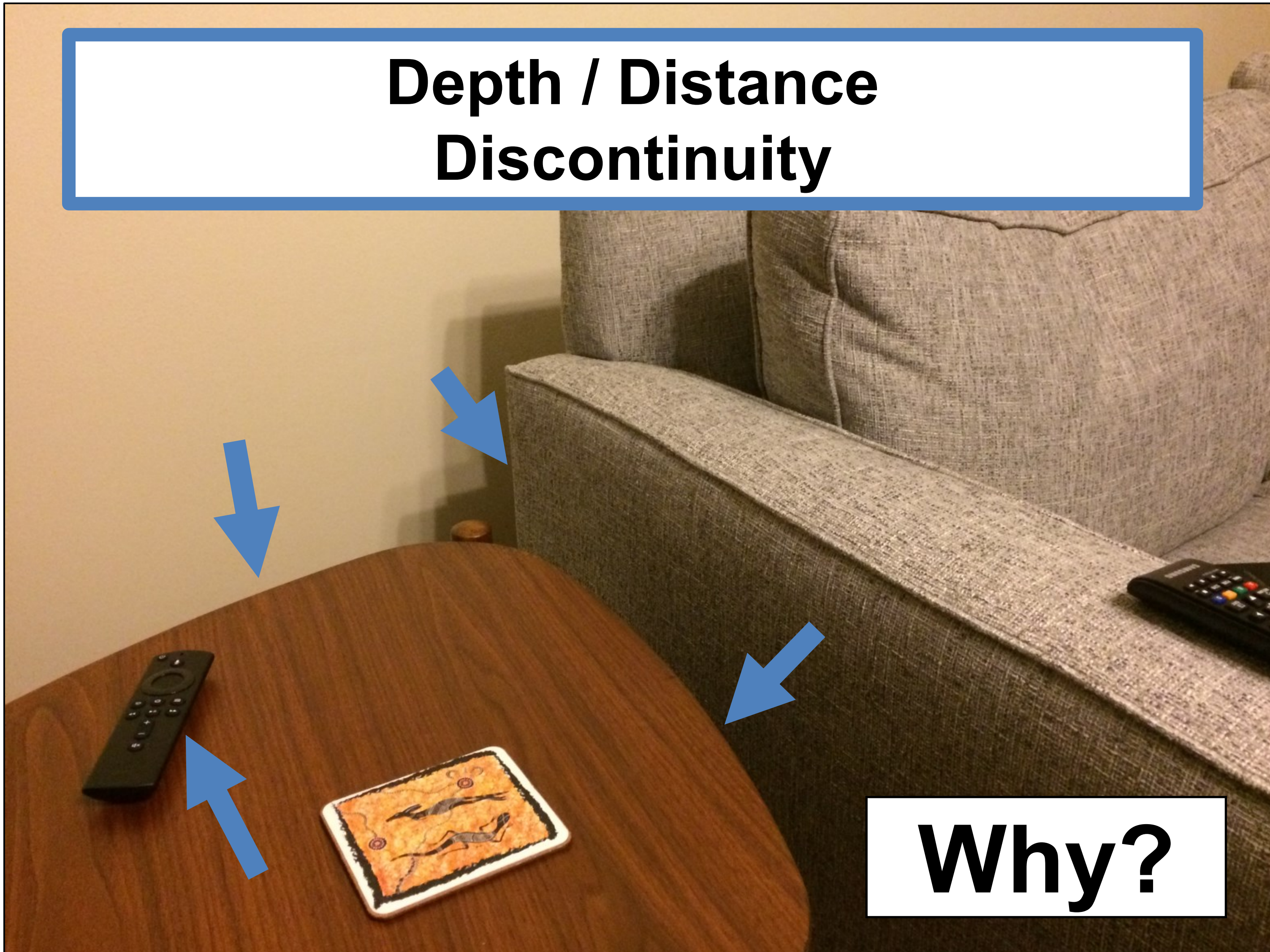
Today

Finding edges (part 1) and corners (part 2) in images.



Where do Edges Come From?

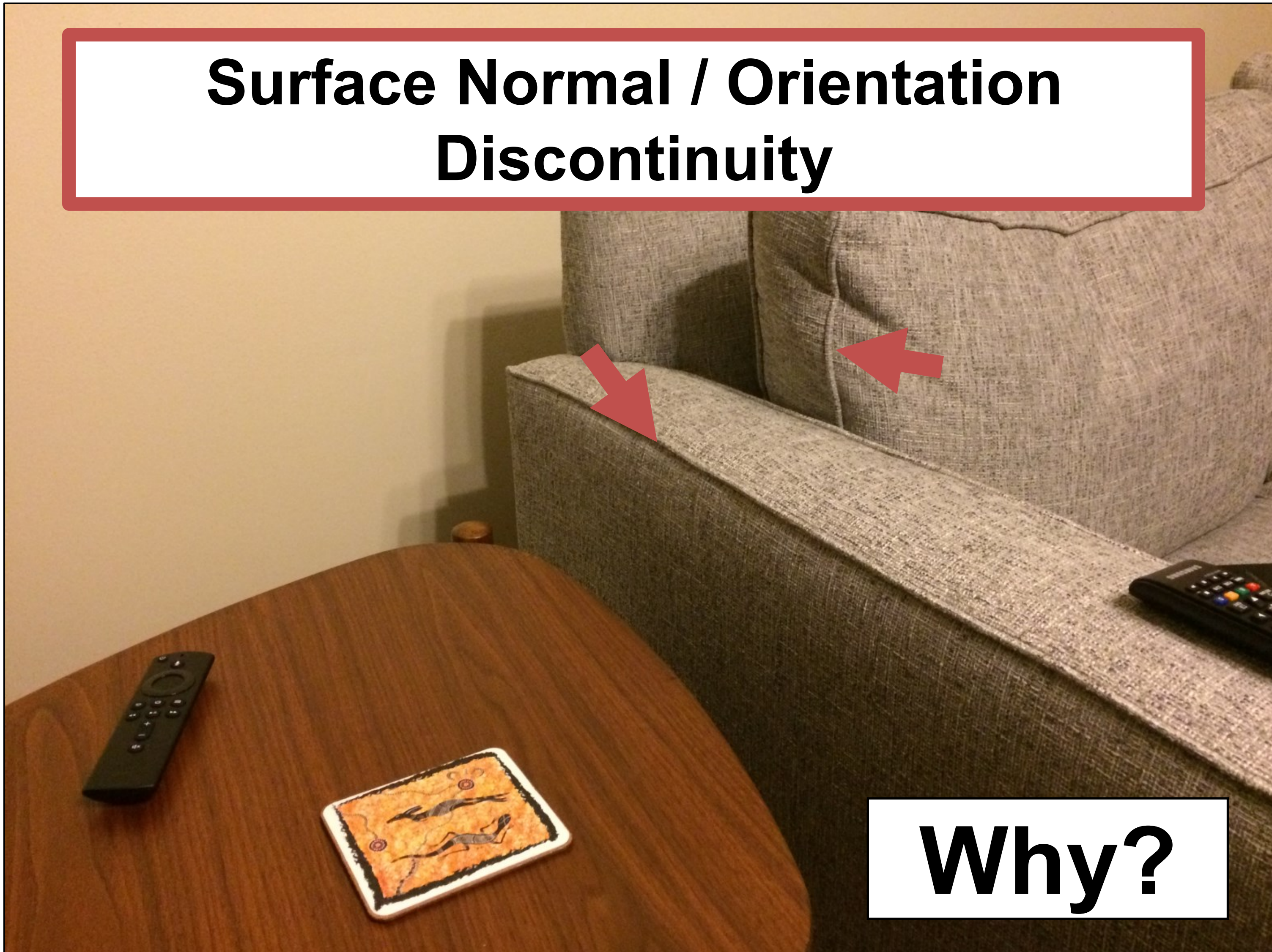
Where do Edges Come From?



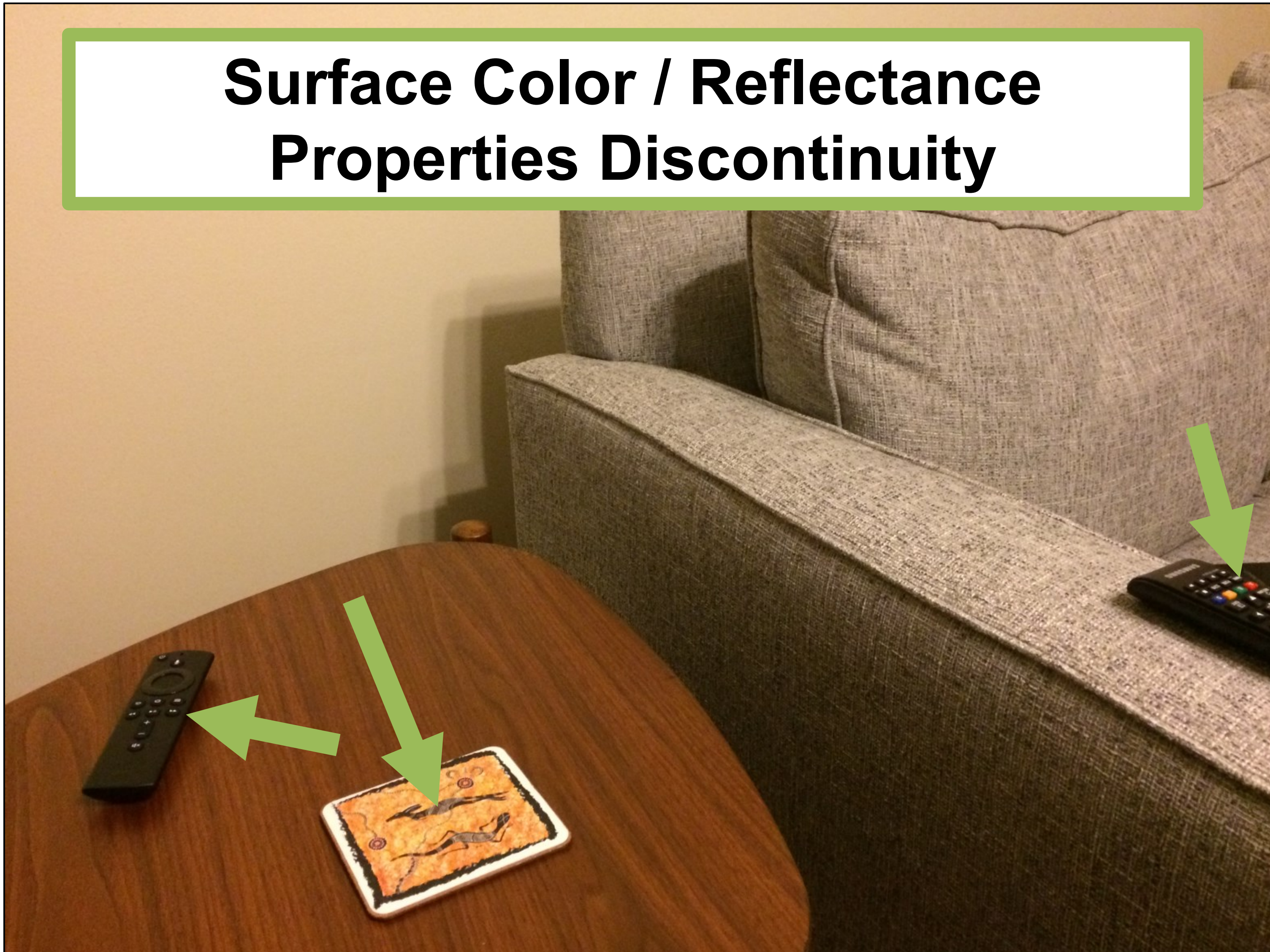
Depth / Distance Discontinuity

Why?

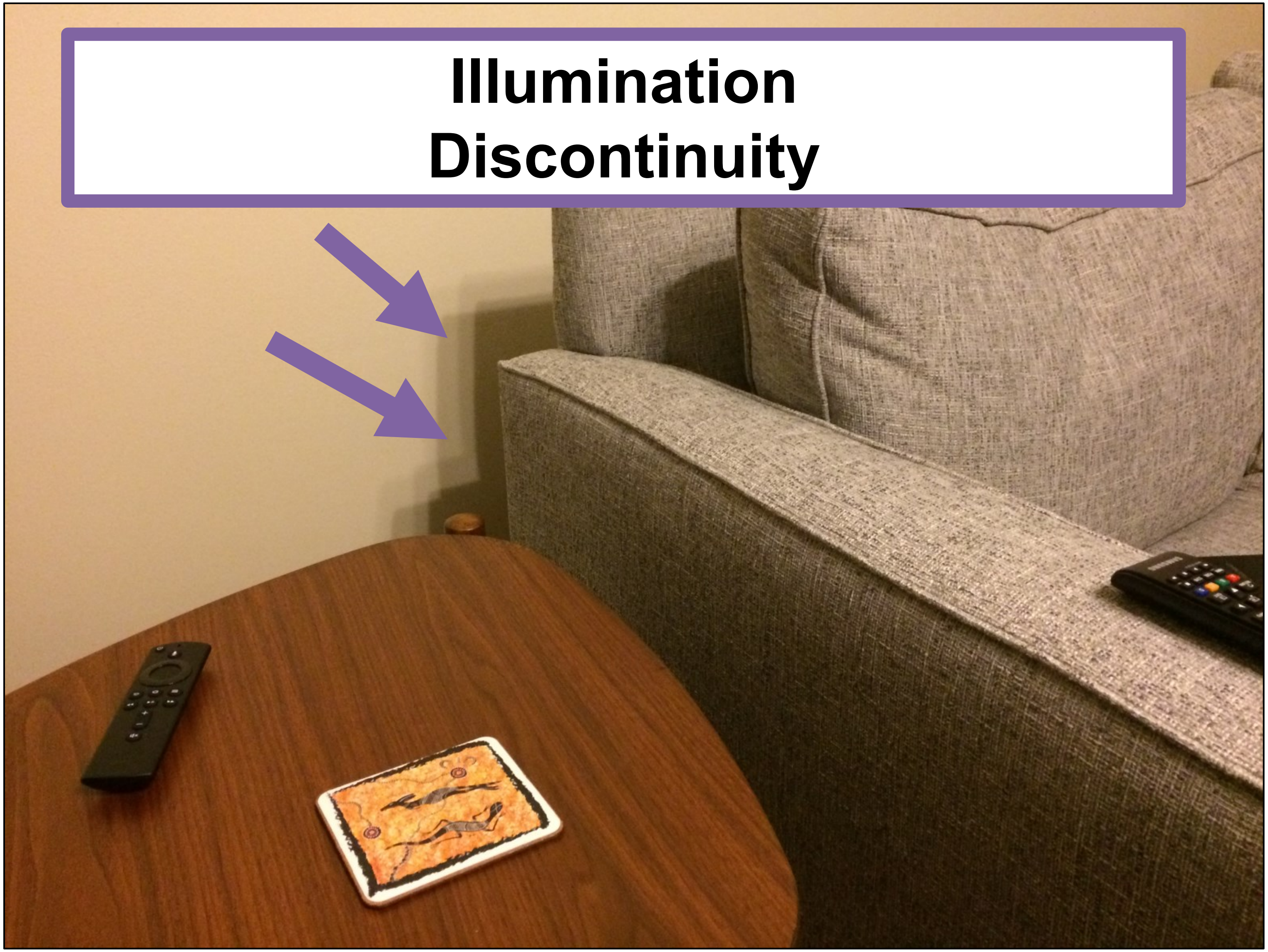
Where do Edges Come From?



Where do Edges Come From?



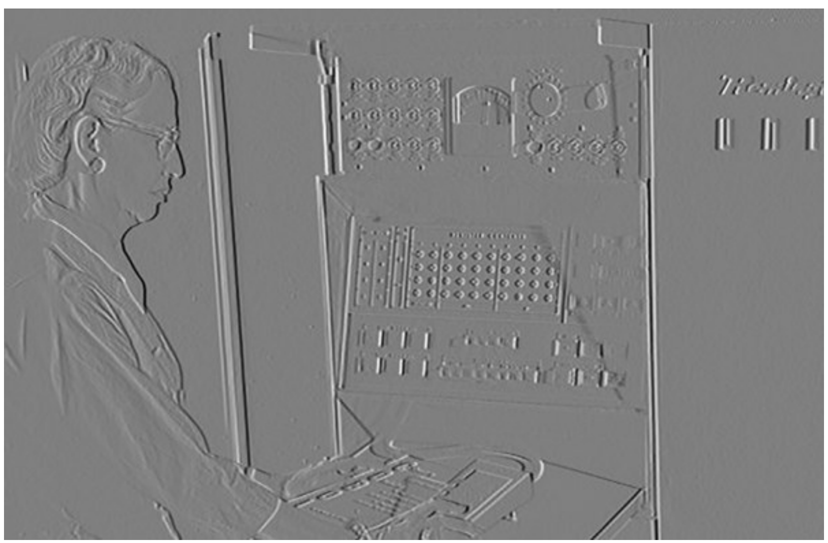
Where do Edges Come From?



Last Time

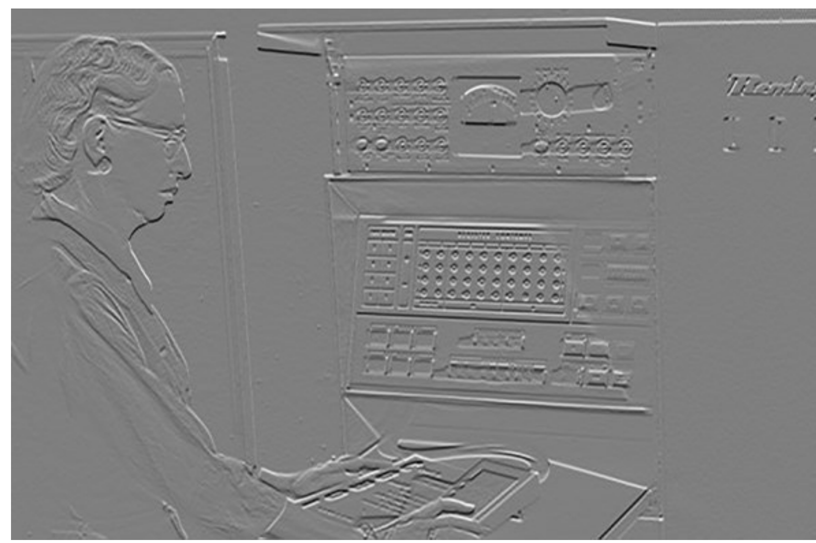
$$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

Ix



$$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}^T$$

Iy



Derivatives

Remember derivatives?

Derivative: rate at which a function f(x) changes at a point as well as the direction that increases the function

Gradient: all of the partial derivatives (derivatives in only one direction) stacked together.

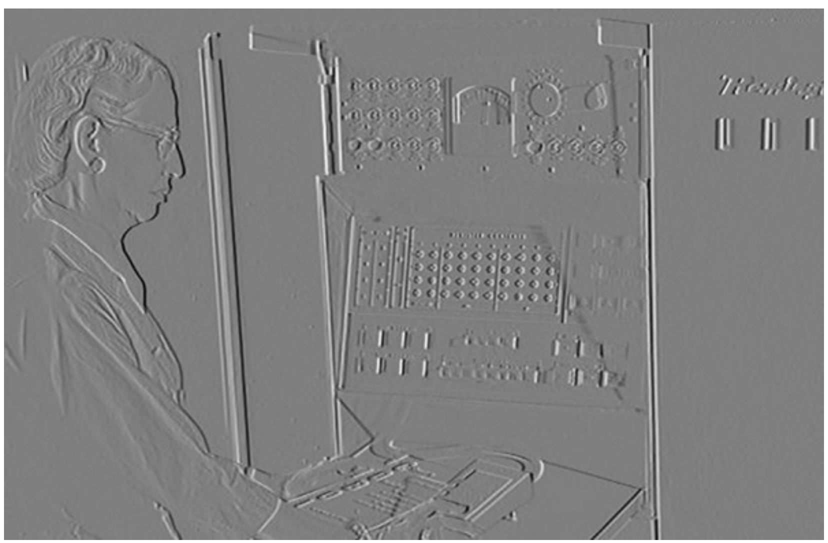
What Should I Know?

- Gradients are simply partial derivatives per-dimension: if $\boldsymbol{x}$ in $f(\boldsymbol{x})$ has n dimensions, $\nabla_f(x)$ has n dimensions
- Gradients point in direction of ascent and tell the rate of ascent
- If a is minimum of $f(\boldsymbol{x}) \rightarrow \nabla_{\mathrm{f}}(\mathrm{a}) = \mathbf{0}$
- Reverse is not true, especially in high-dimensional spaces

Last Time

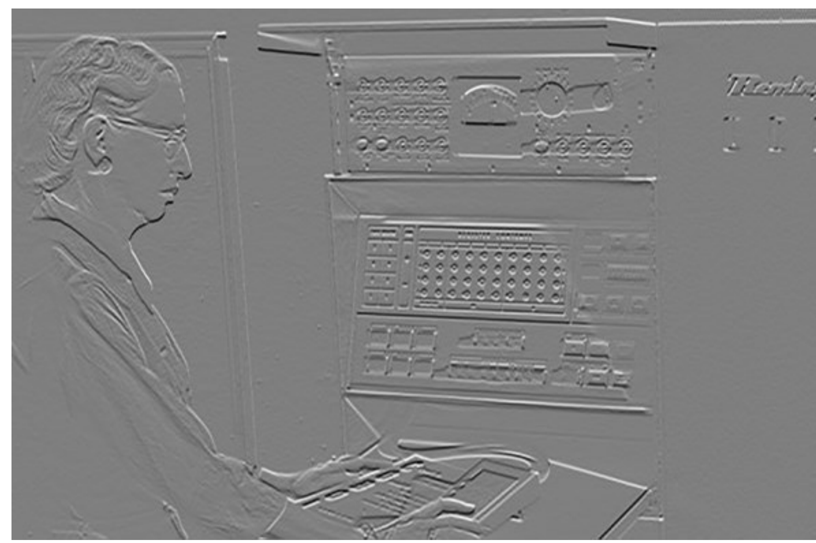
$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$

Ix



$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}^T$

Iy



Why Does This Work?

Image is function f(x,y)

Remember:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon, y) - f(x,y)}{\epsilon}$$

Approximate:

-1	1

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

Another one:

-1	0	1

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x-1,y)}{2}$$

Other Differentiation Operations

	Horizontal	Vertical
Prewitt	$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$
Sobel	$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

Why might people use these compared to [-1,0,1]?

Images as Functions or Points

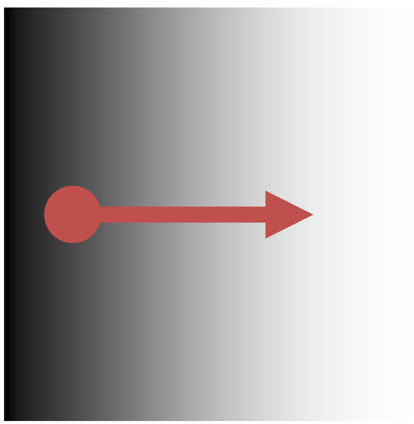
Key idea: can treat image as a point in $R^{(H \times W)}$ or as a function of x,y.

$$\nabla I(x,y) = \begin{bmatrix} \frac{\partial I}{\partial x}(x,y) \\ \\ \frac{\partial I}{\partial y}(x,y) \end{bmatrix}$$

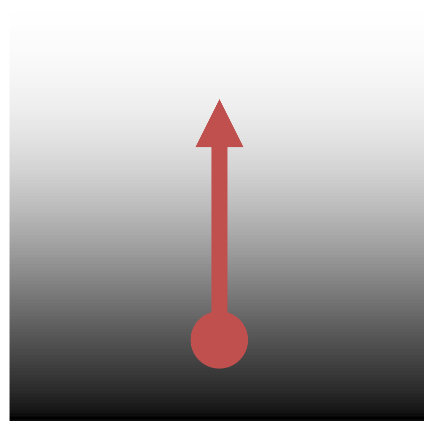
How much the intensity of the image changes as you go horizontally at (x,y)
(Often called Ix)

Image Gradient Direction

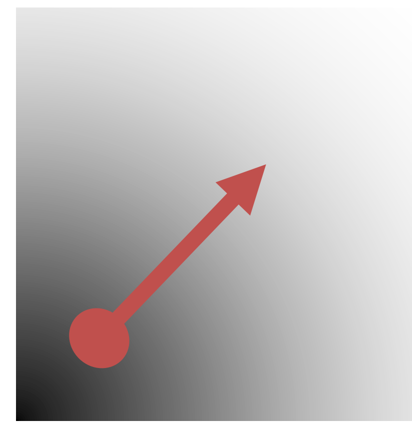
Some gradients



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



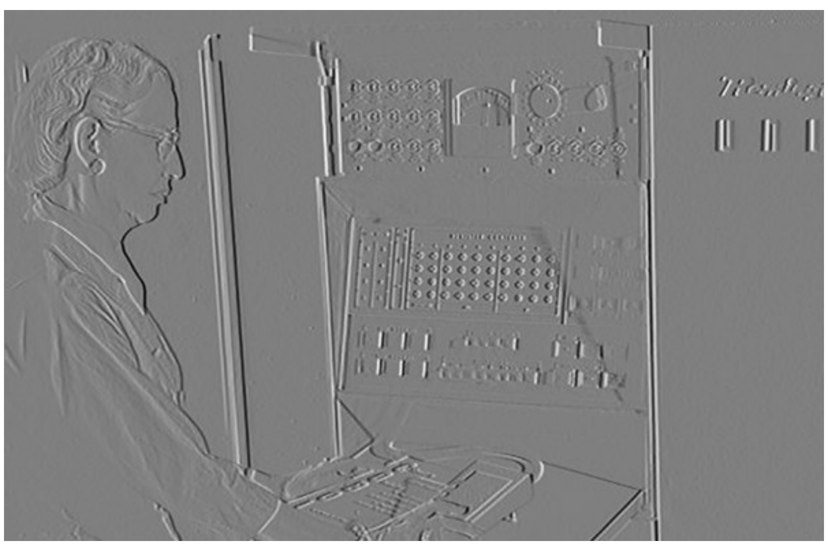
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

Figure Credit: S. Seitz

Image Gradient

Gradient: direction of maximum change.
What's the relationship to edge direction?

Ix



Iy

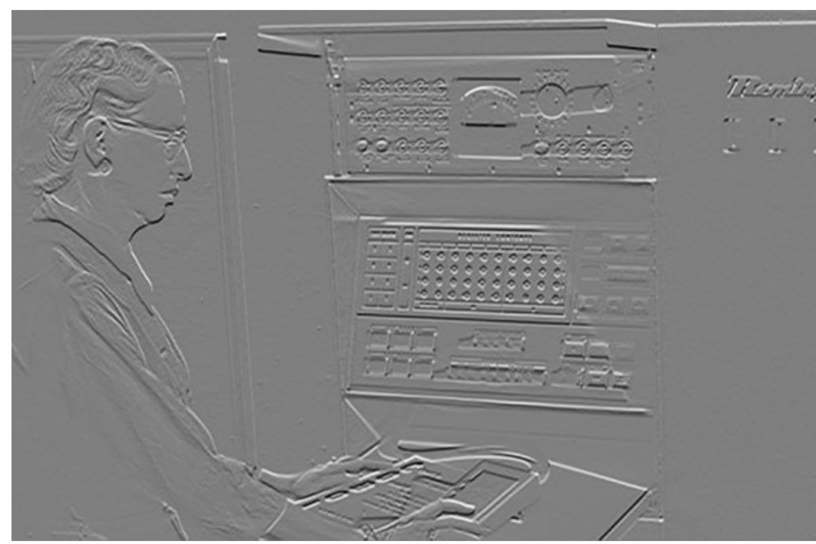
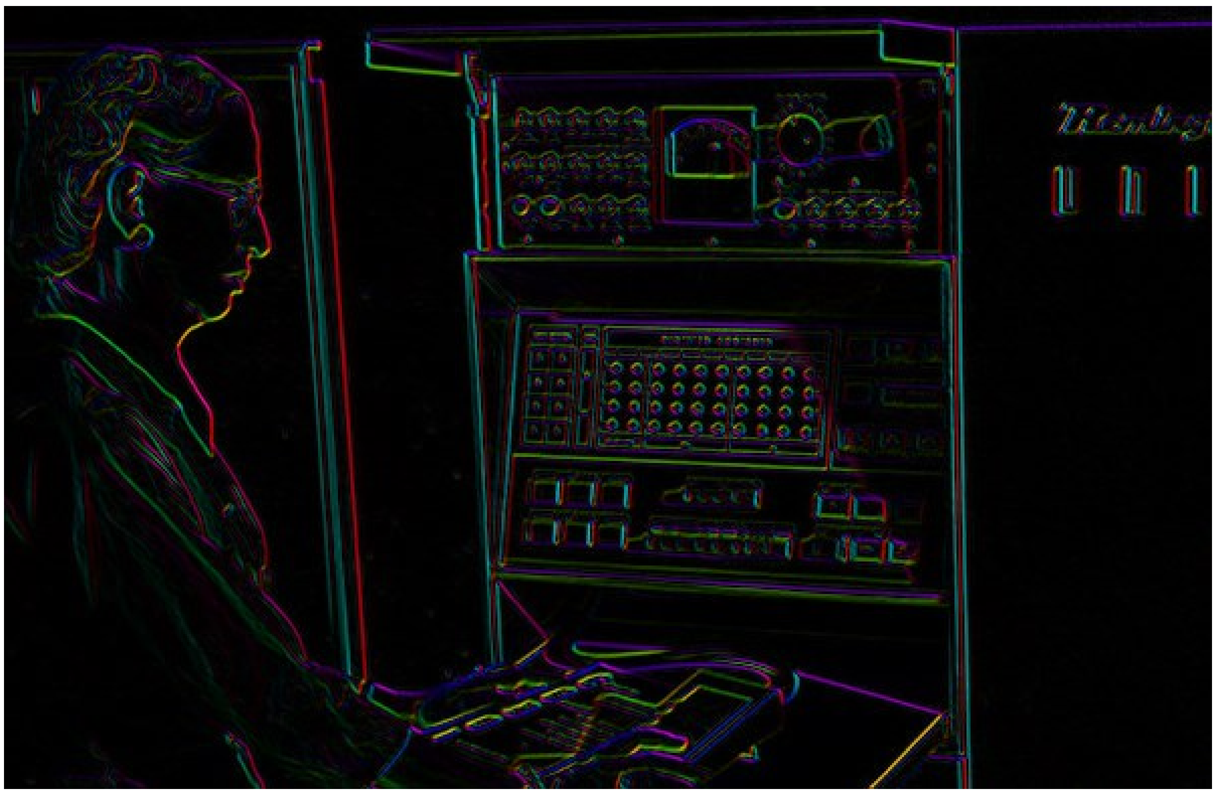


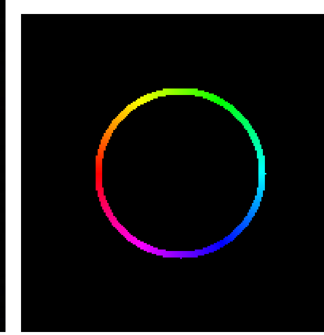
Image Gradient

$(Ix^2 + Iy^2)^{1/2}$: magnitude

Image Gradient

atan2(Iy,Ix): orientation

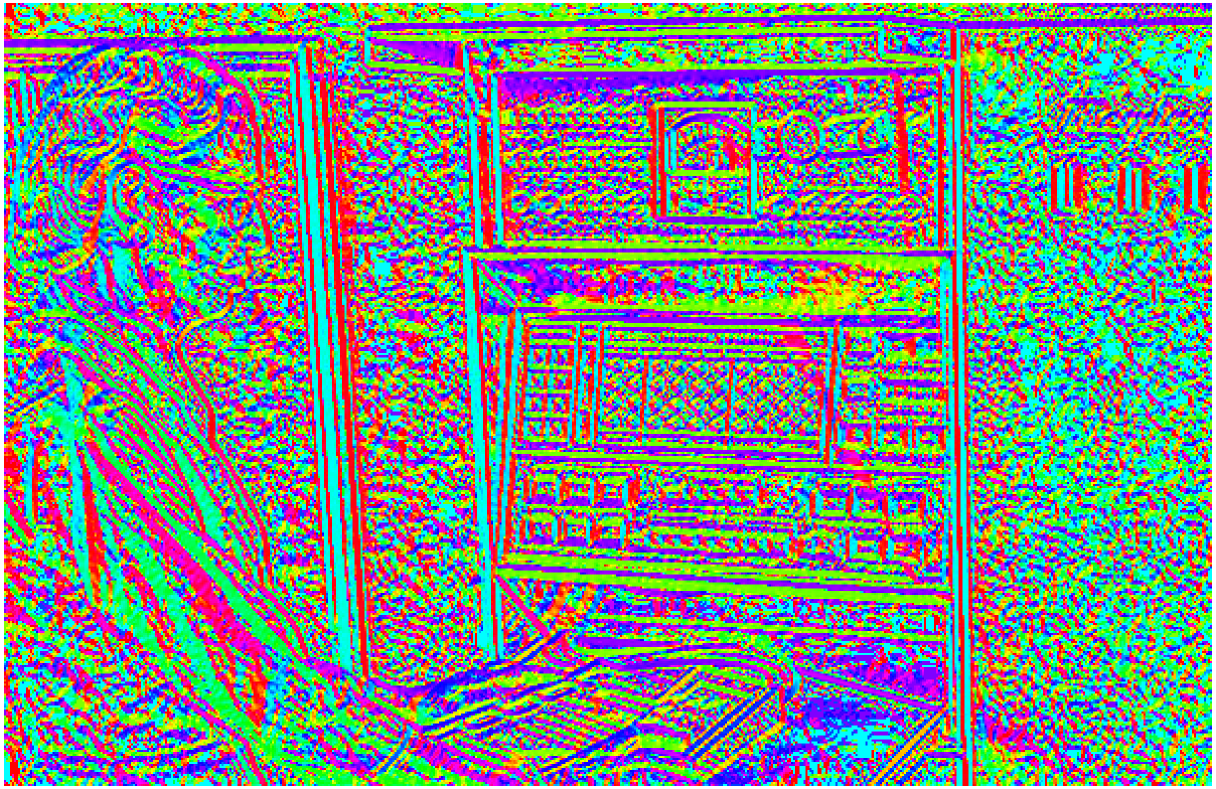


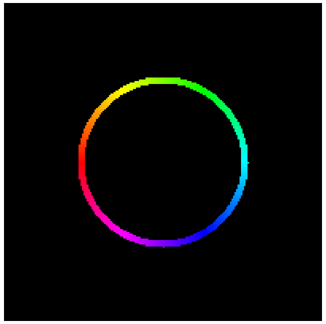


I’m making the lightness equal to gradient magnitude

Image Gradient

atan2(Iy,Ix): orientation



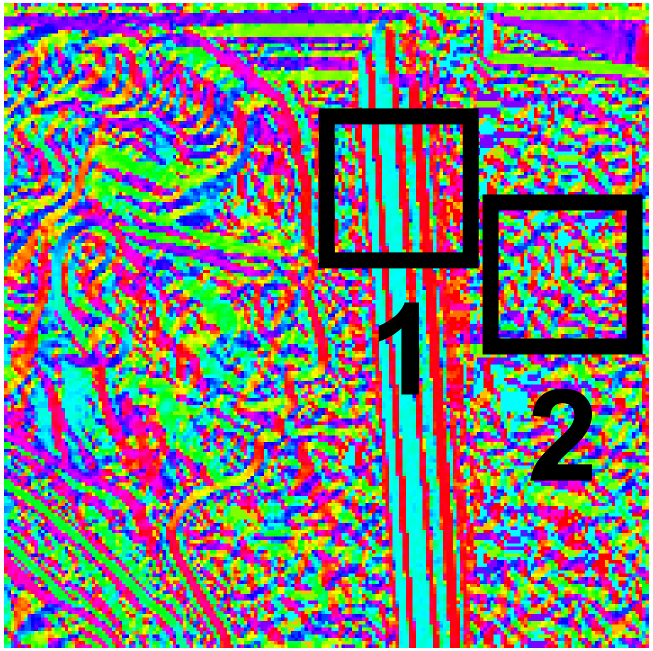


Now I'm showing *all* the gradients

Image Gradient

atan2(Iy,Ix): orientation

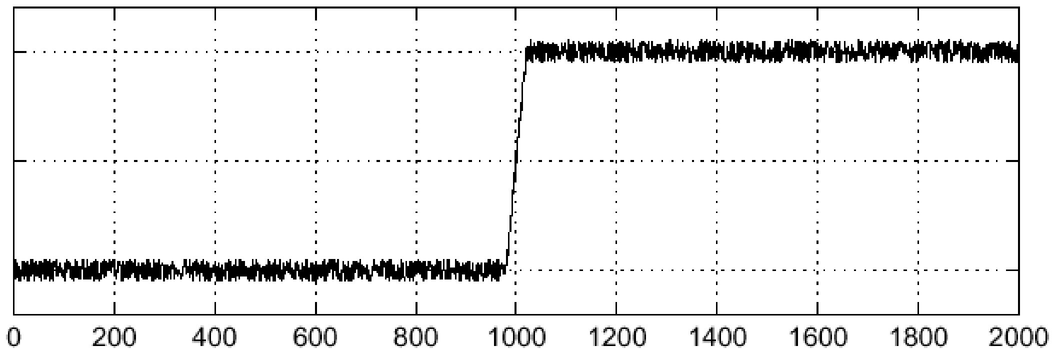
Why is there structure at 1 and not at 2?



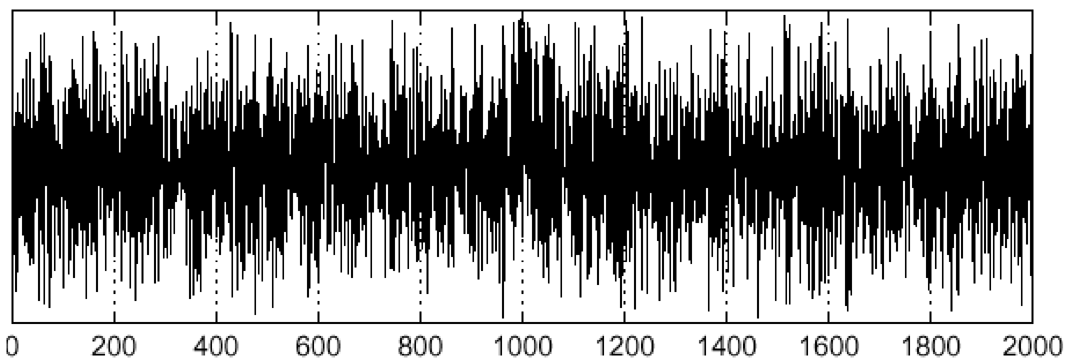
Noise

Consider a row of f(x,y) (i.e., fix y)

$f(x)$



$\frac{d}{dx}f(x)$



Slide Credit: S. Seitz

Noise

Conv. image + per-pixel noise with

-1	0	1

$I_{i,j} =$ True image $\quad \epsilon_{i,j} \sim N(0, \sigma^2)$

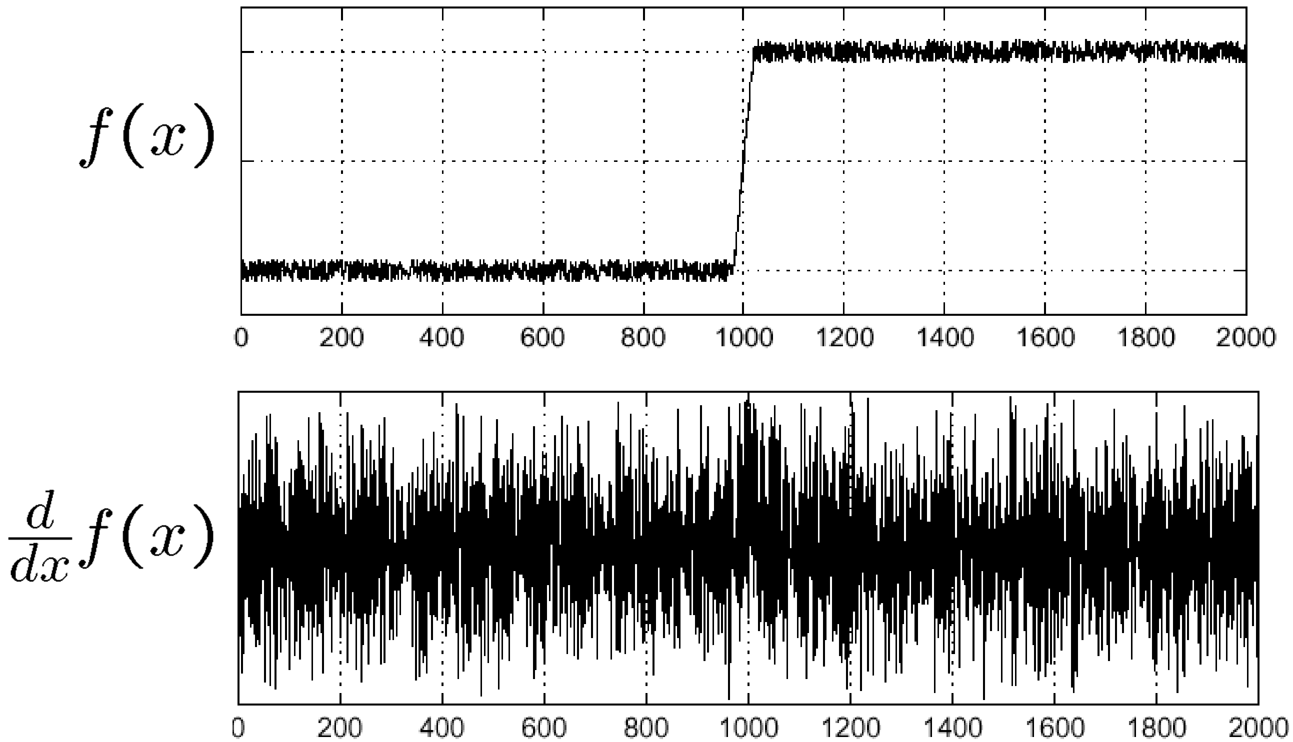
$$D_{i,j} = (I_{i,j+1} + \epsilon_{i,j+1}) - (I_{i,j-1} + \epsilon_{i,j-1})$$

$$D_{i,j} = \underbrace{(I_{i,j+1} - I_{i,j-1})}_{\text{True difference}} + \underbrace{\epsilon_{i,j+1} - \epsilon_{i,j-1}}_{\text{Sum of 2 Gaussians}}$$

$\epsilon_{i,j} - \epsilon_{k,l} \sim N\left(0, 2\sigma^2\right) \rightarrow$ Variance doubles!

Noise

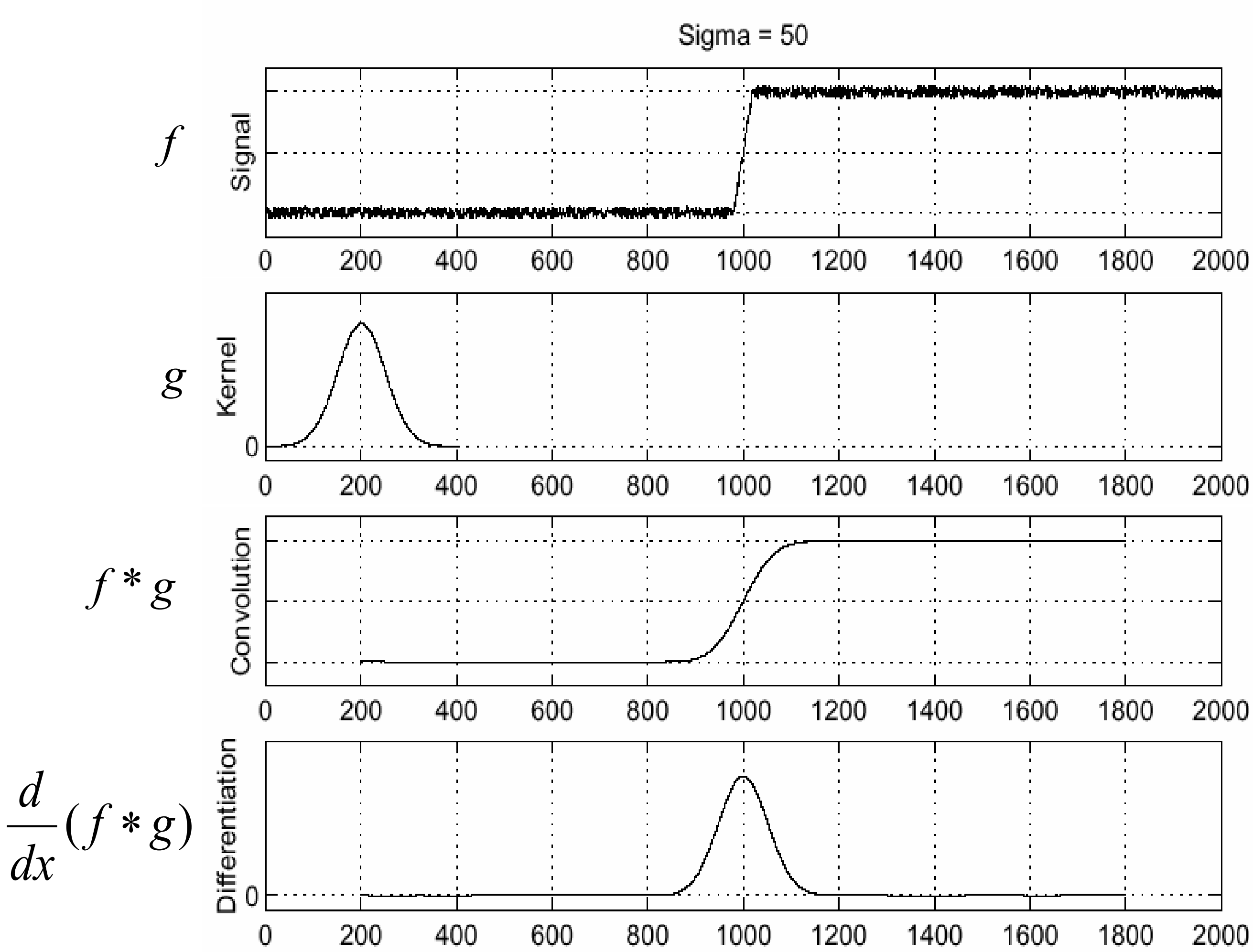
Consider a row of f(x,y) (i.e., make y constant)



How can we use the last class to fix this?

Slide Credit: S. Seitz

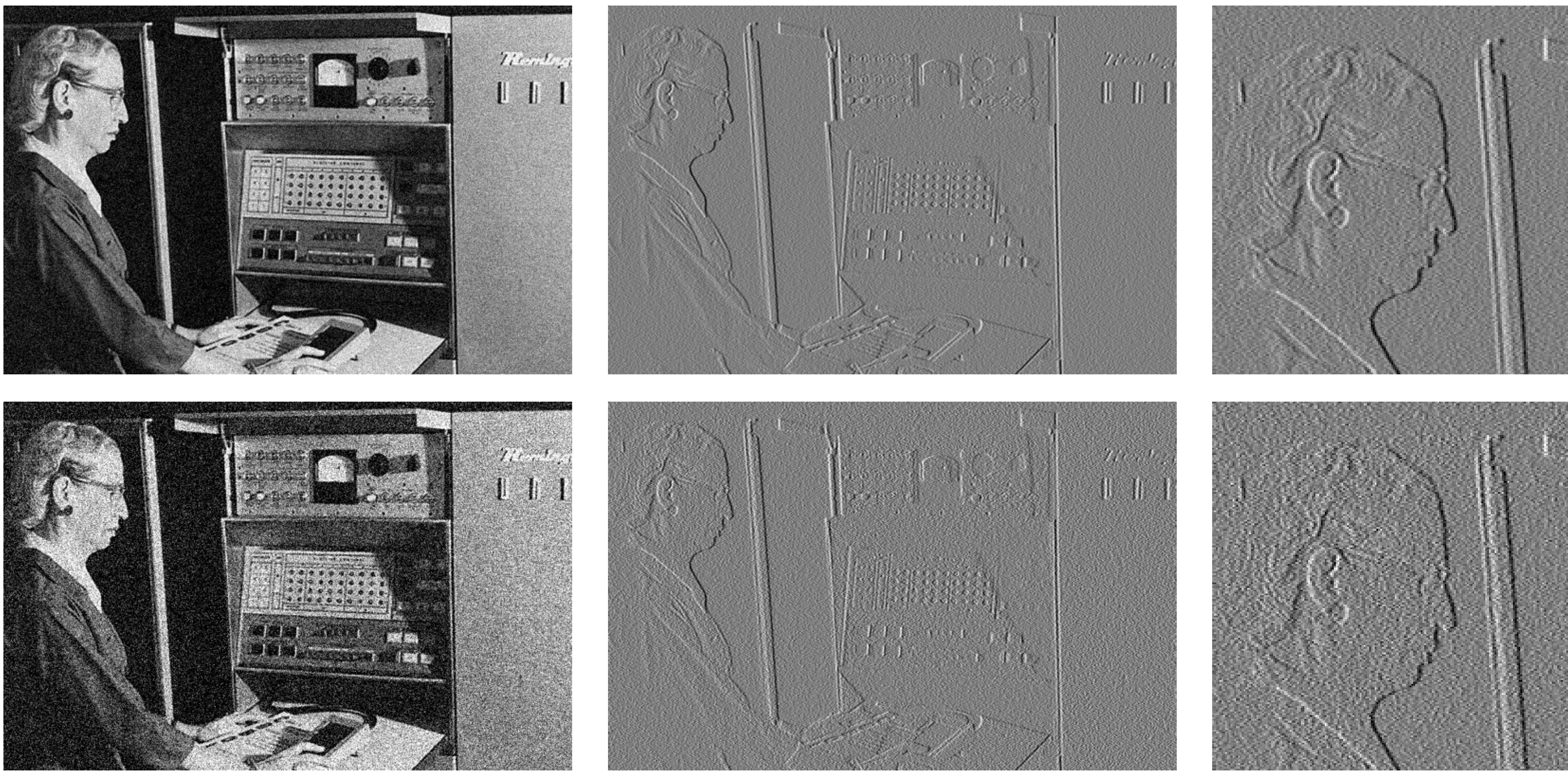
Handling Noise



Slide Credit: S. Seitz

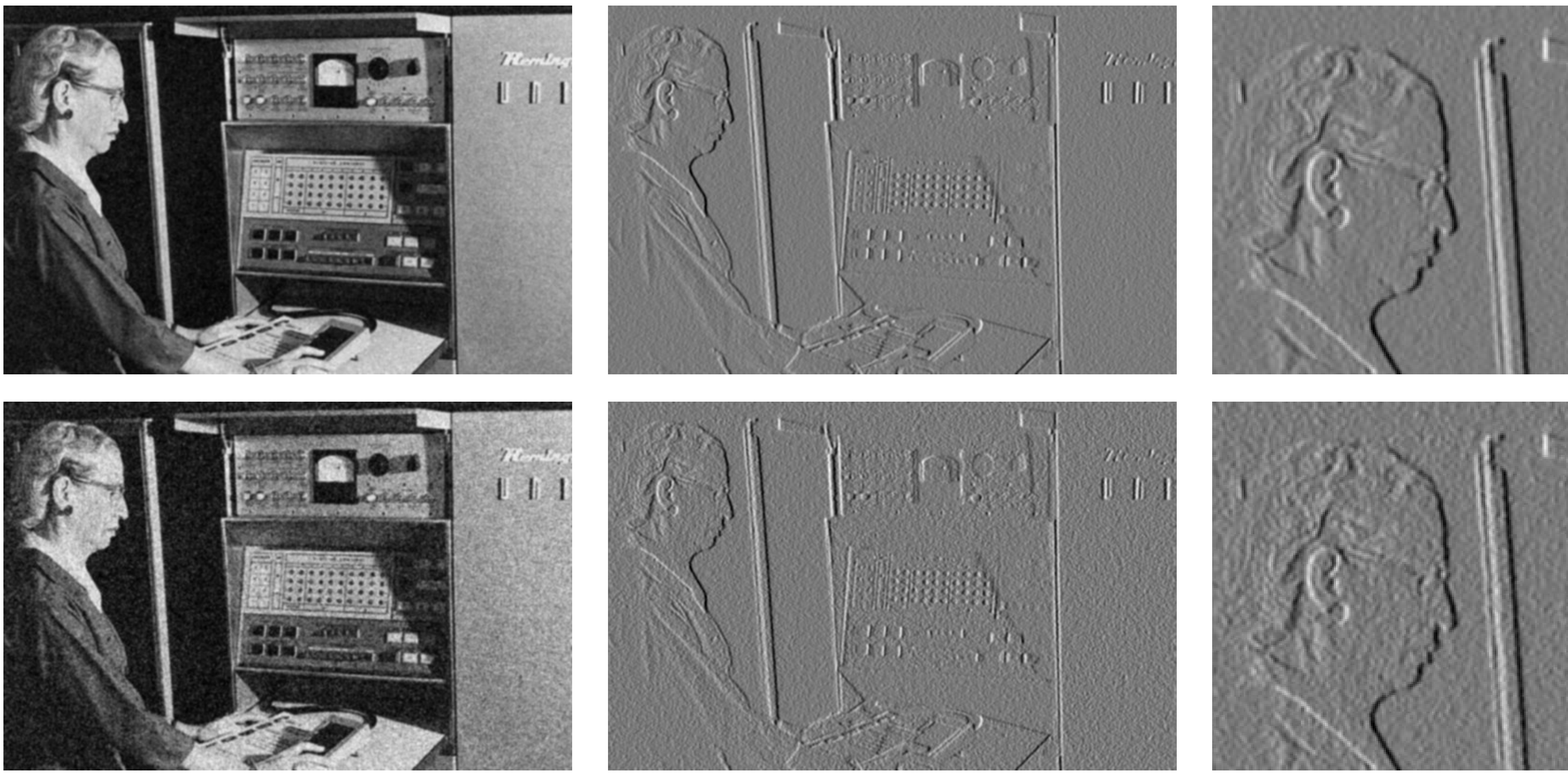
Noise in 2D

Noisy Input | Ix via [-1,01] | Zoom



Noise + Smoothing

Smoothed Input | Ix via [-1,01] | Zoom



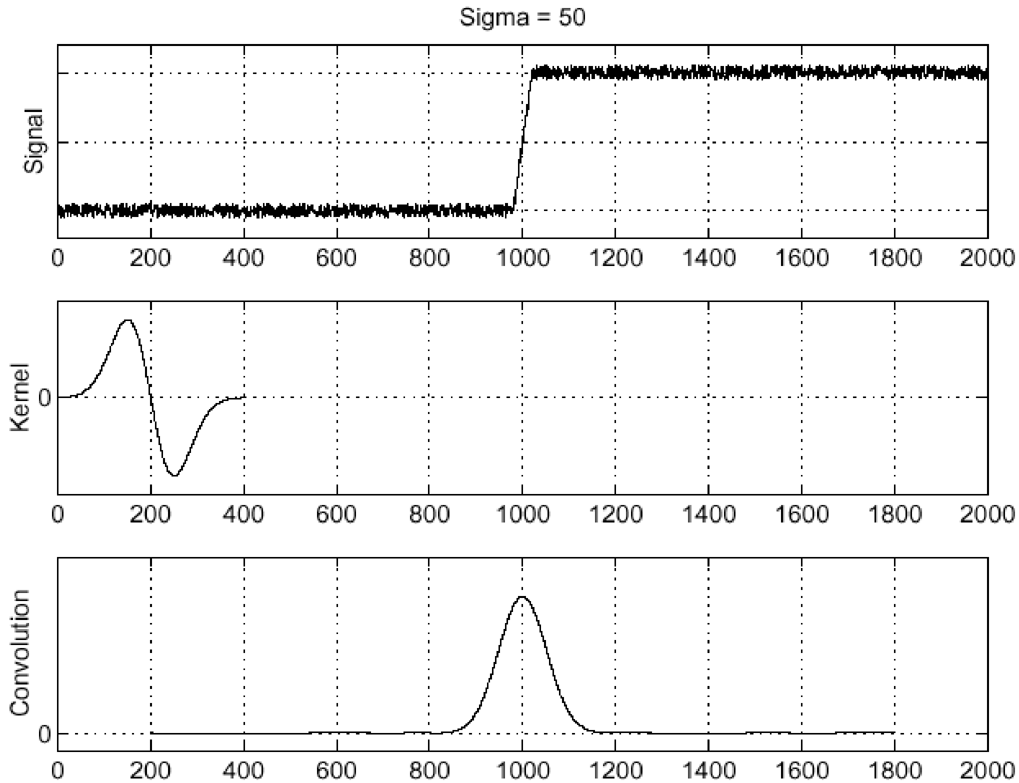
Let's Make It One Pass (1D)

$$\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$$

f

$\frac{d}{dx} g$

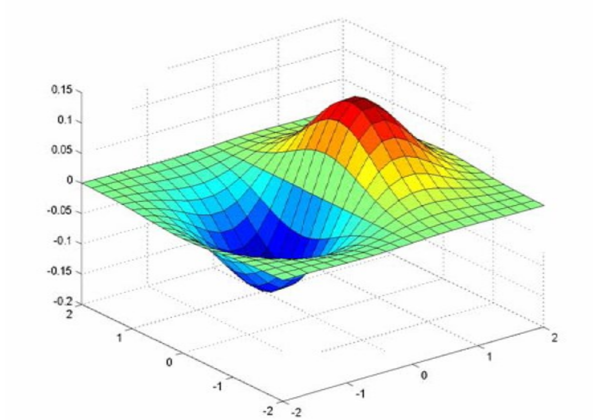
$f * \frac{d}{dx} g$

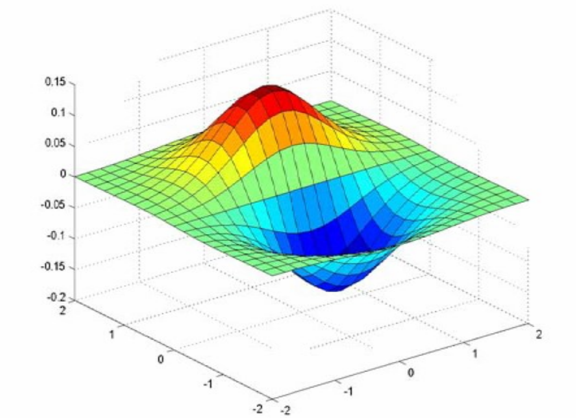


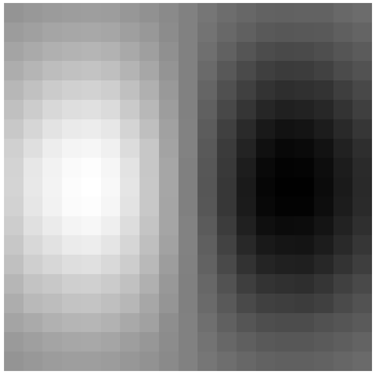
Slide Credit: S. Seitz

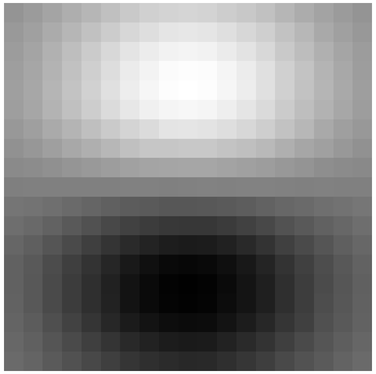
Let’s Make It One Pass (2D)

Gaussian Derivative Filter







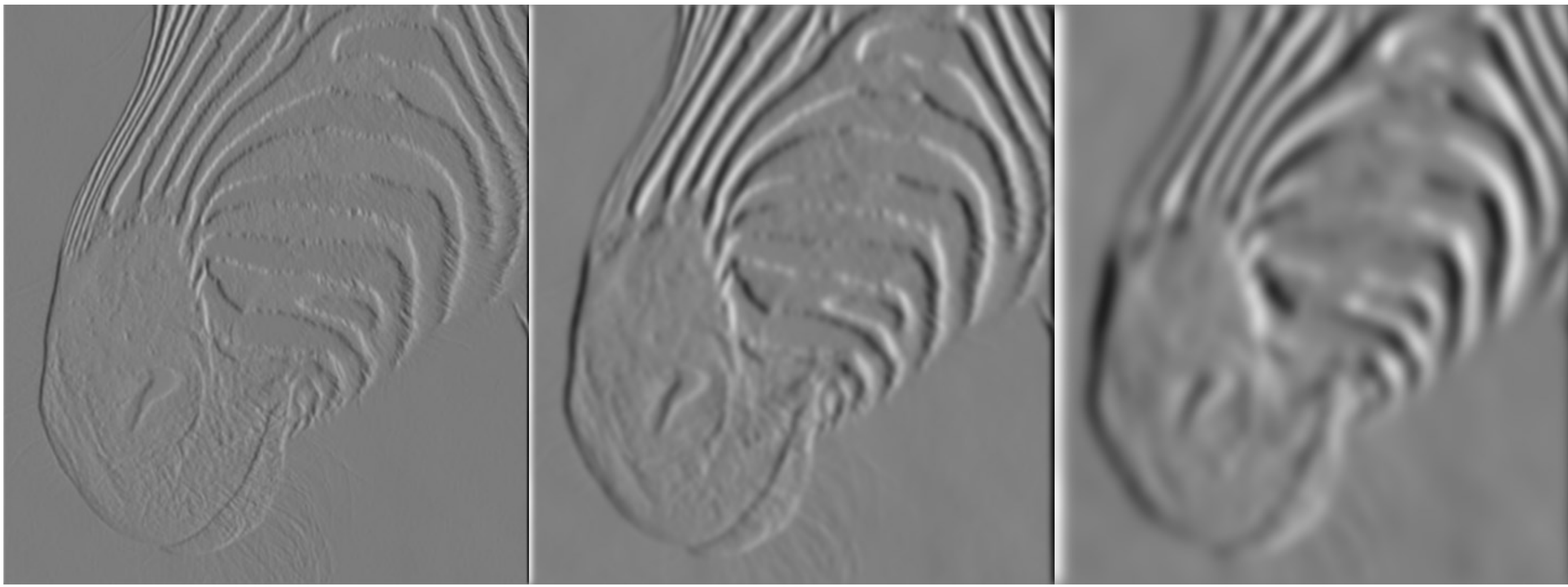


Which one finds the X direction?

Slide Credit: L. Lazebnik

Applying the Gaussian Derivative

1 pixel 3 pixels 7 pixels

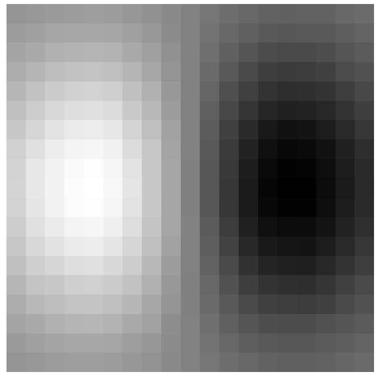


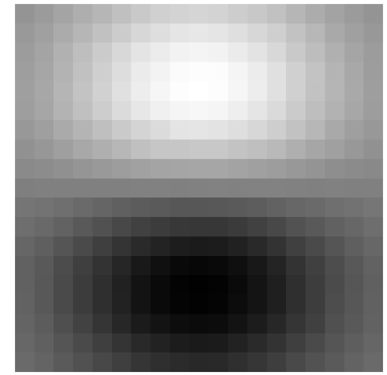
Removes noise, but blurs edge

Slide Credit: D. Forsyth

Compared with the Past

Gaussian Derivative



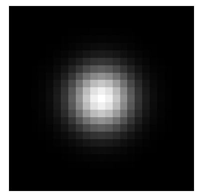
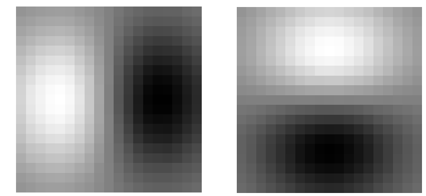


Sobel Filter

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Why would anybody use the bottom filter?

Filters We've Seen

	Smoothing	Derivative
		
Example	Gaussian	Deriv. of gauss
Goal	Remove noise	Find edges
Only +?	**Yes**	**No**
Sums to	1	0

Why sum to 1 or 0, intuitively?

Slide Credit: J. Deng

Problems

Image | human segmentation | gradient magnitude

Still an unsolved problem

Localizing Reliably

- Suppose you need to meet someone but you can’t use your cell phone to coordinate
- Where do you agree to meet?

A: Along the Huron river

B: Along State Street

C: At Liberty and State Street

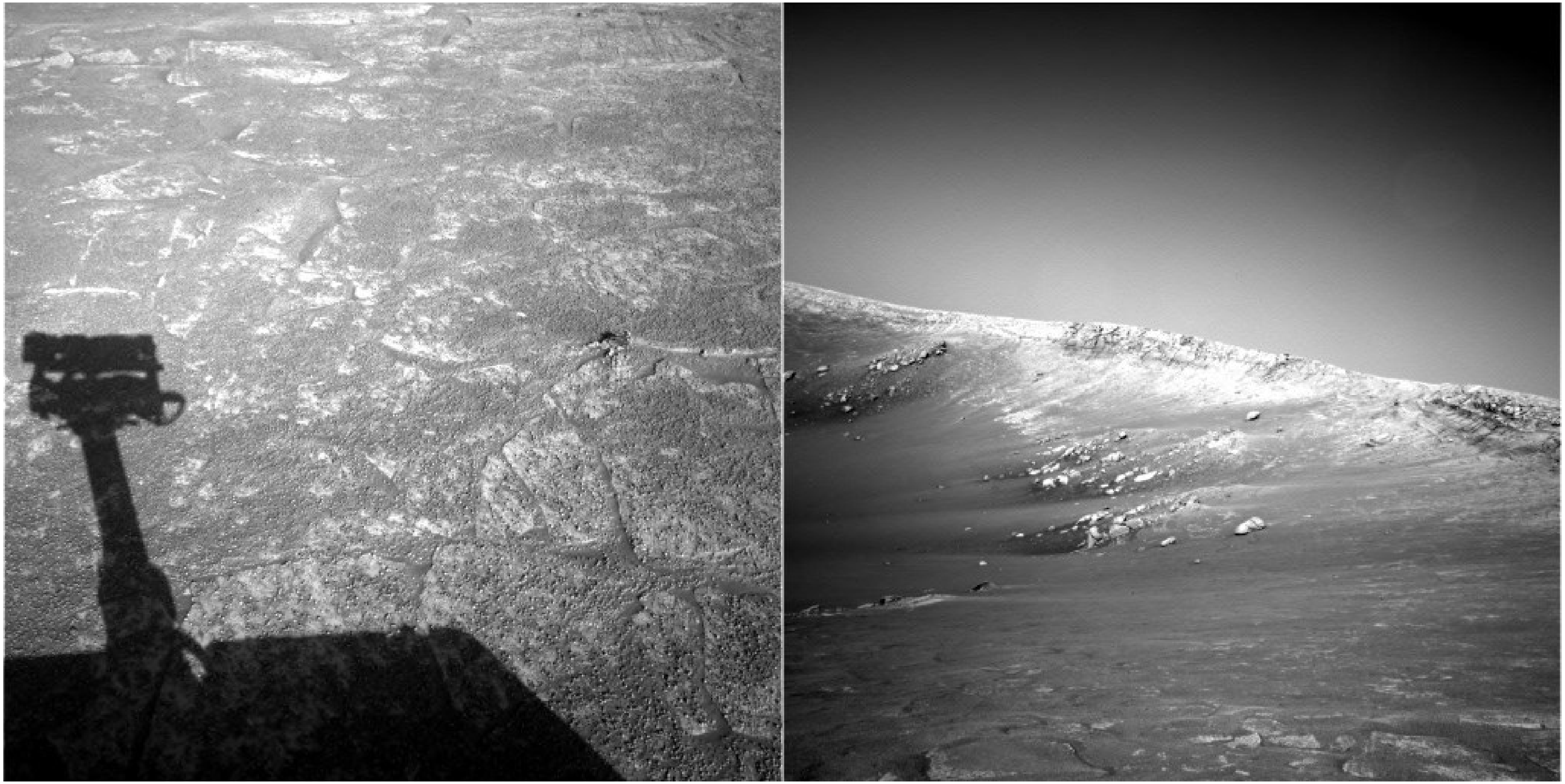
D: On North Campus

Desirables

- Repeatable: should find same things even with distortion
- Saliency: each feature should be distinctive
- Compactness: shouldn’t just be all the pixels
- Locality: should only depend on local image data

Property list: S. Lazebnik

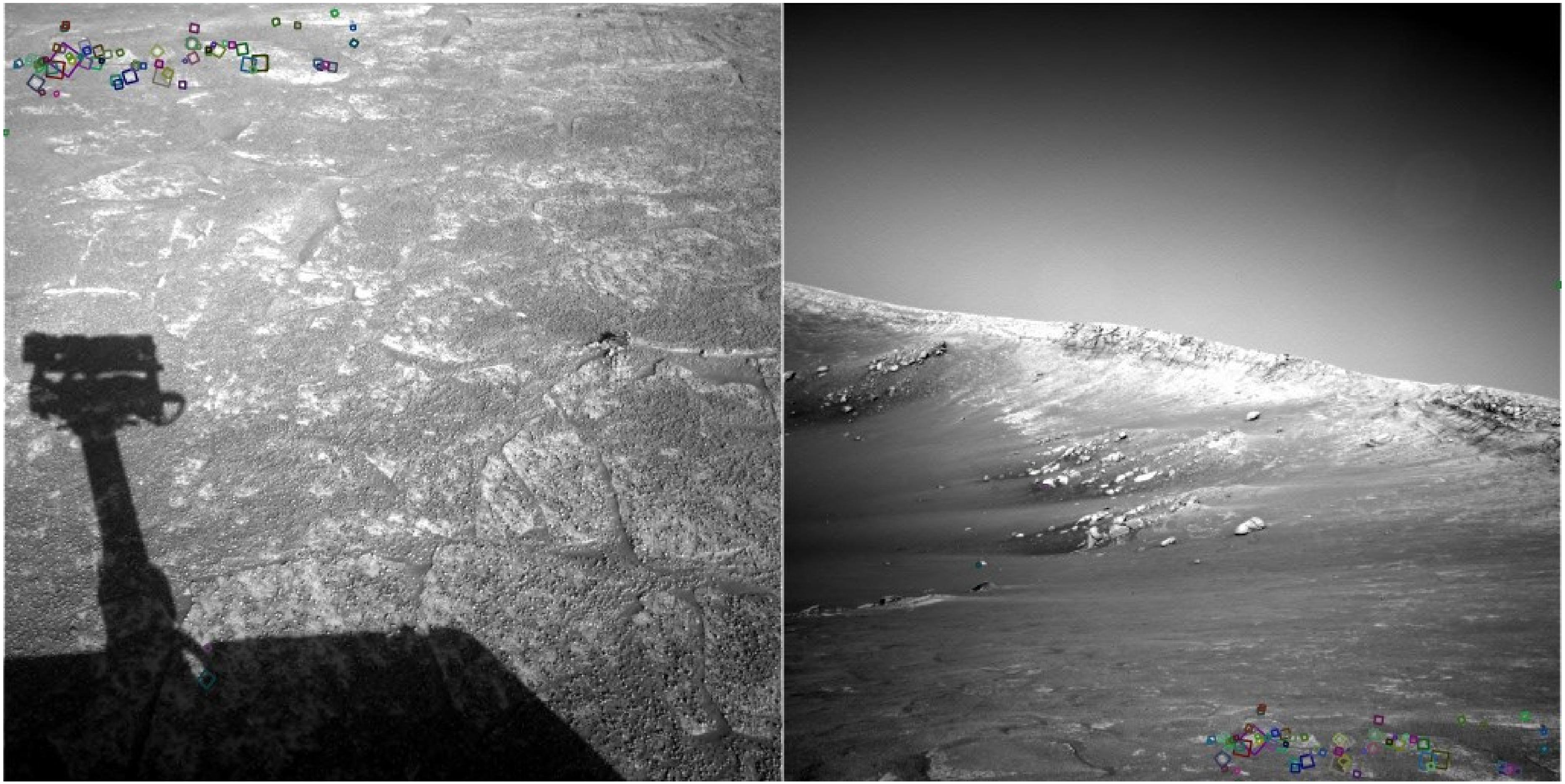
Example



Can you find the correspondences?

Slide credit: N. Snavely

Example Matches

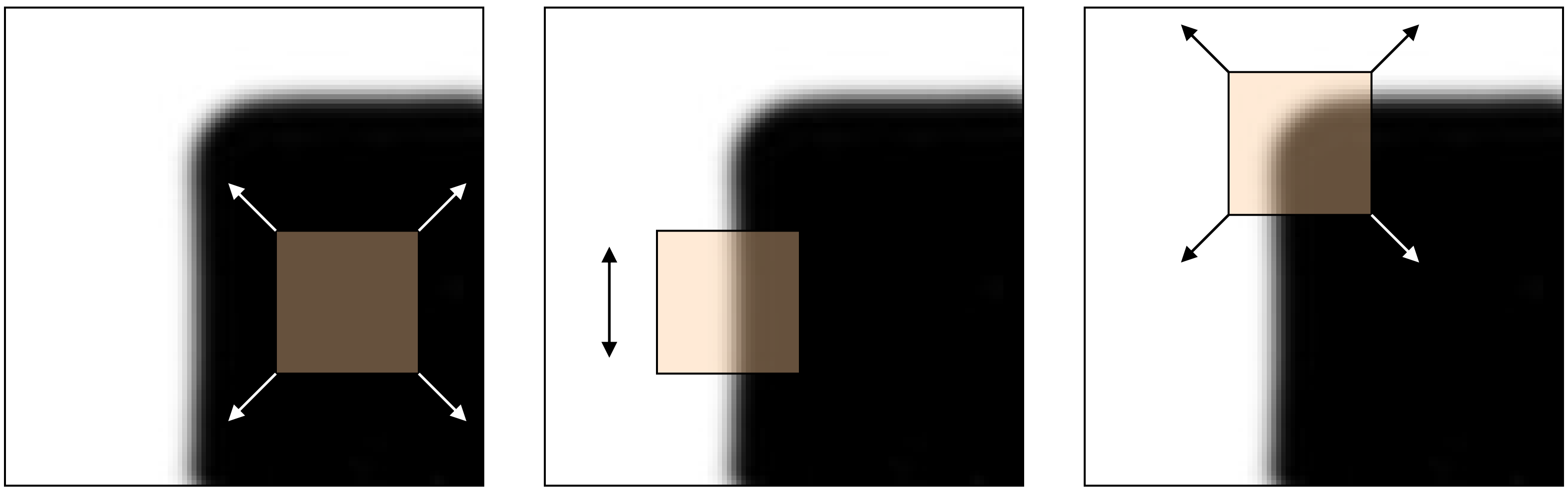


Look for the colored squares

Slide credit: N. Snavely

Basic Idea

Should see where we are based on small window, or any shift → big intensity change.



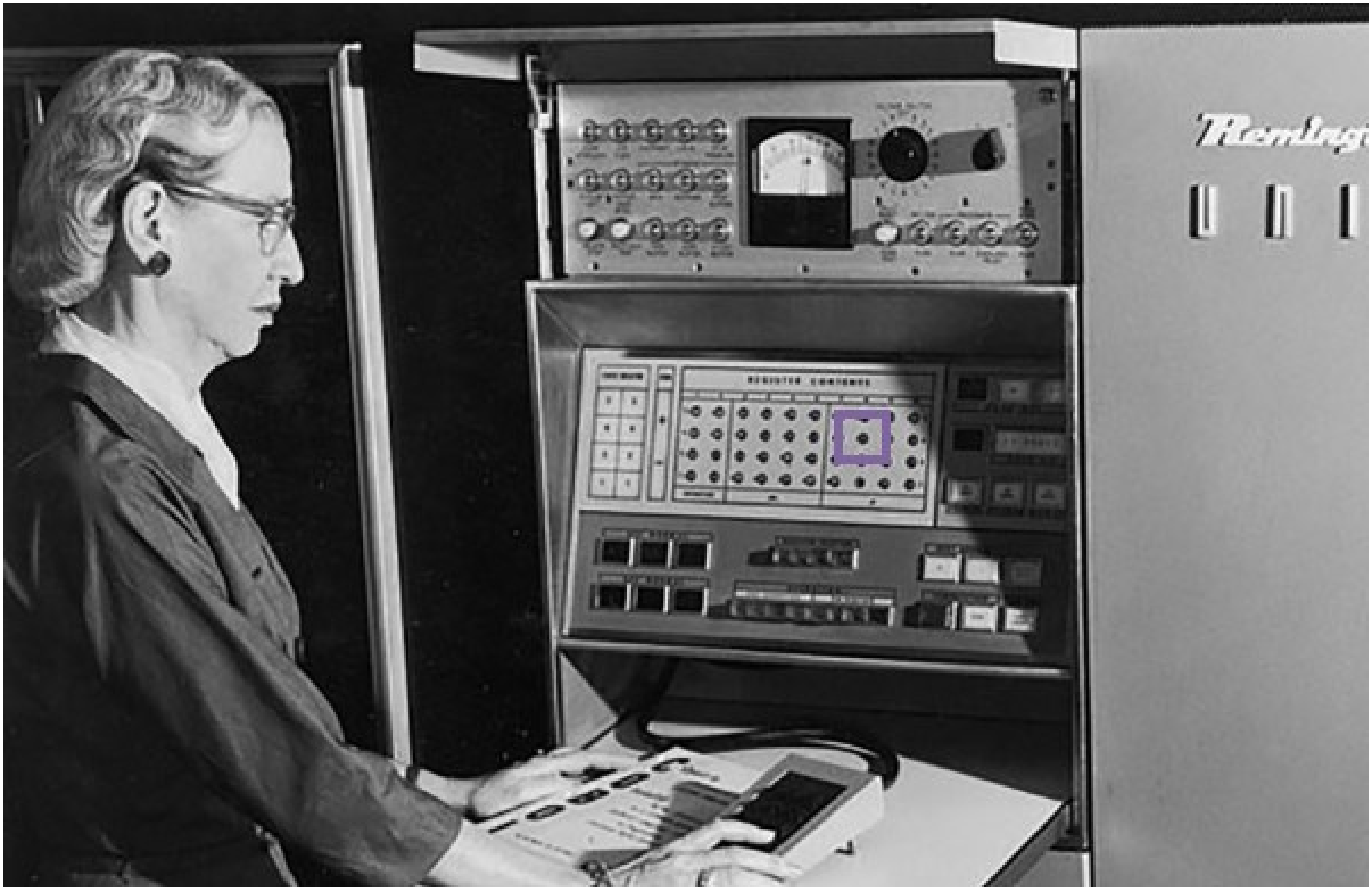
"flat" region: no change in all directions

"edge": no change along the edge direction

"corner": significant change in all directions

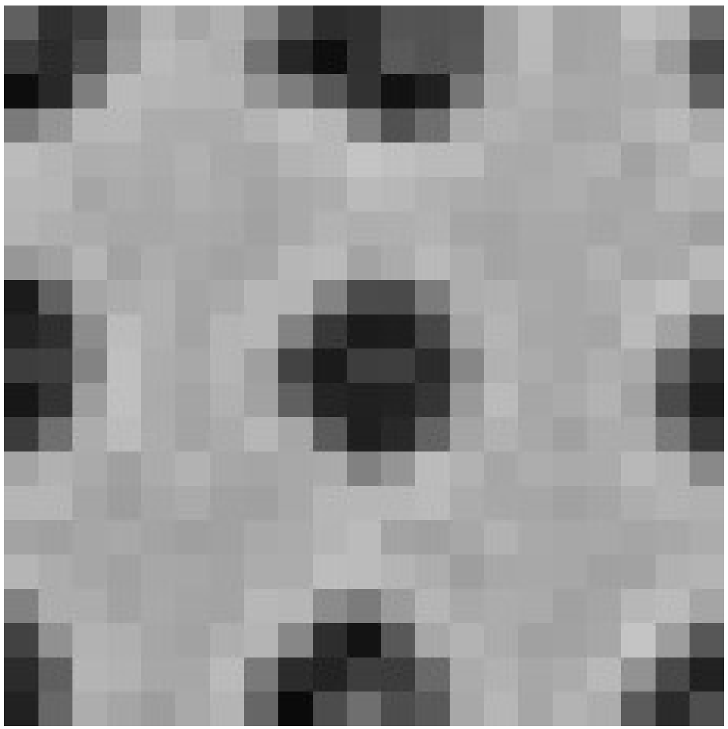
Slide Credit: S. Lazebnik

Formalizing Corner Detection



Formalizing Corner Detection

Zoom-In at x,y

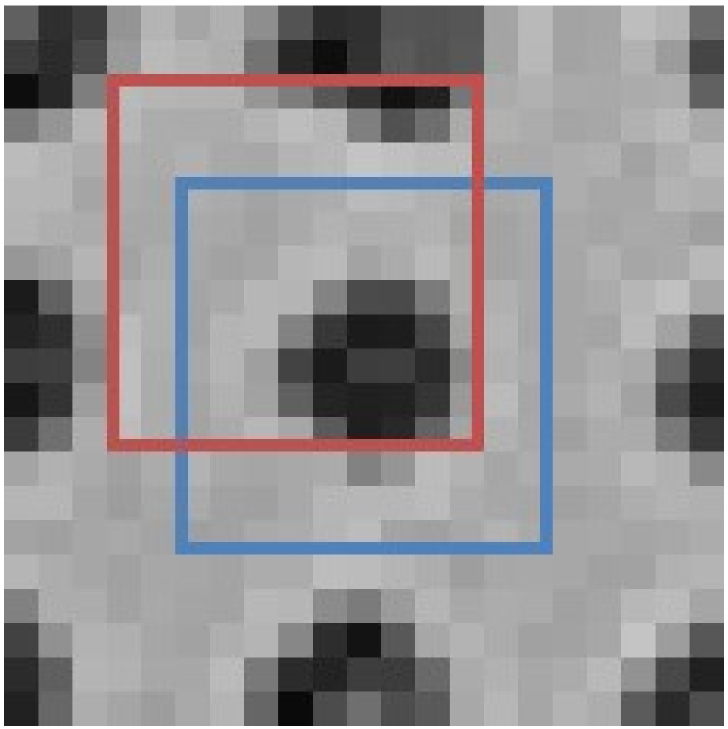


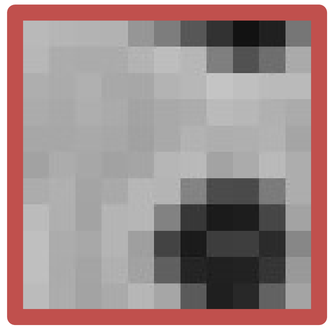
Original Image

Formalizing Corner Detection

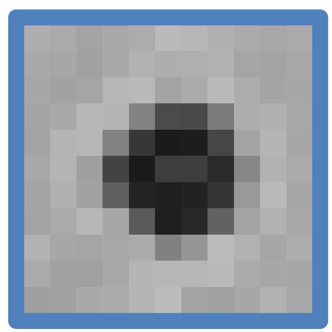
Zoom-In at x,y

Window with and w/o Offset





“Window”
At x+u, y+v
Here: u=-2,v=-3

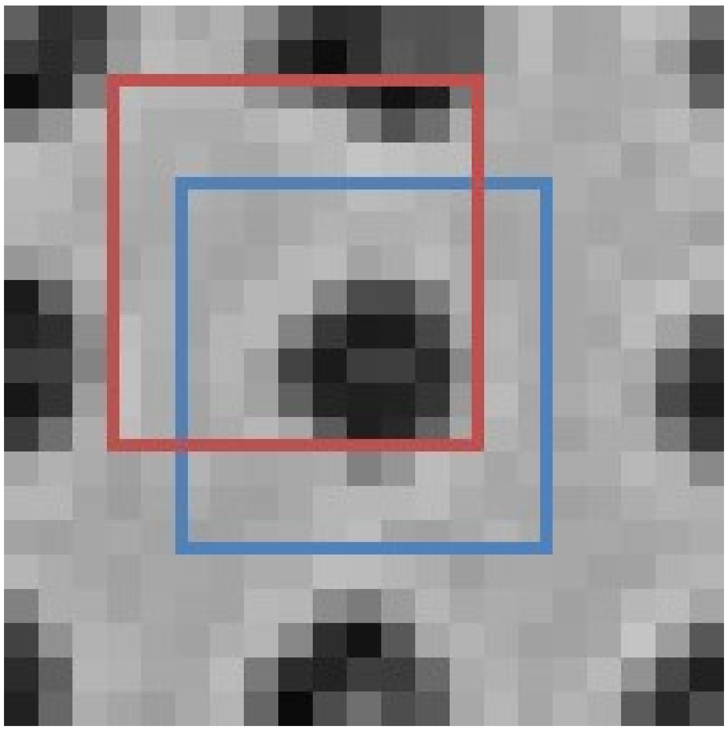


“Window”
At x, y

How might we measure similarity?

Formalizing Corner Detection

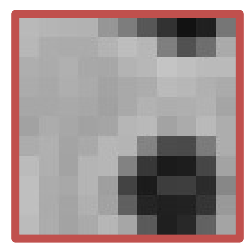
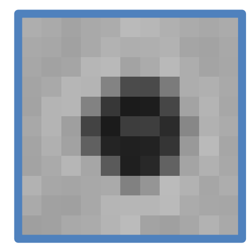
Zoom-In at x,y



Error (Sum Sqs) for u,v offset

$$E(u,v) =$$

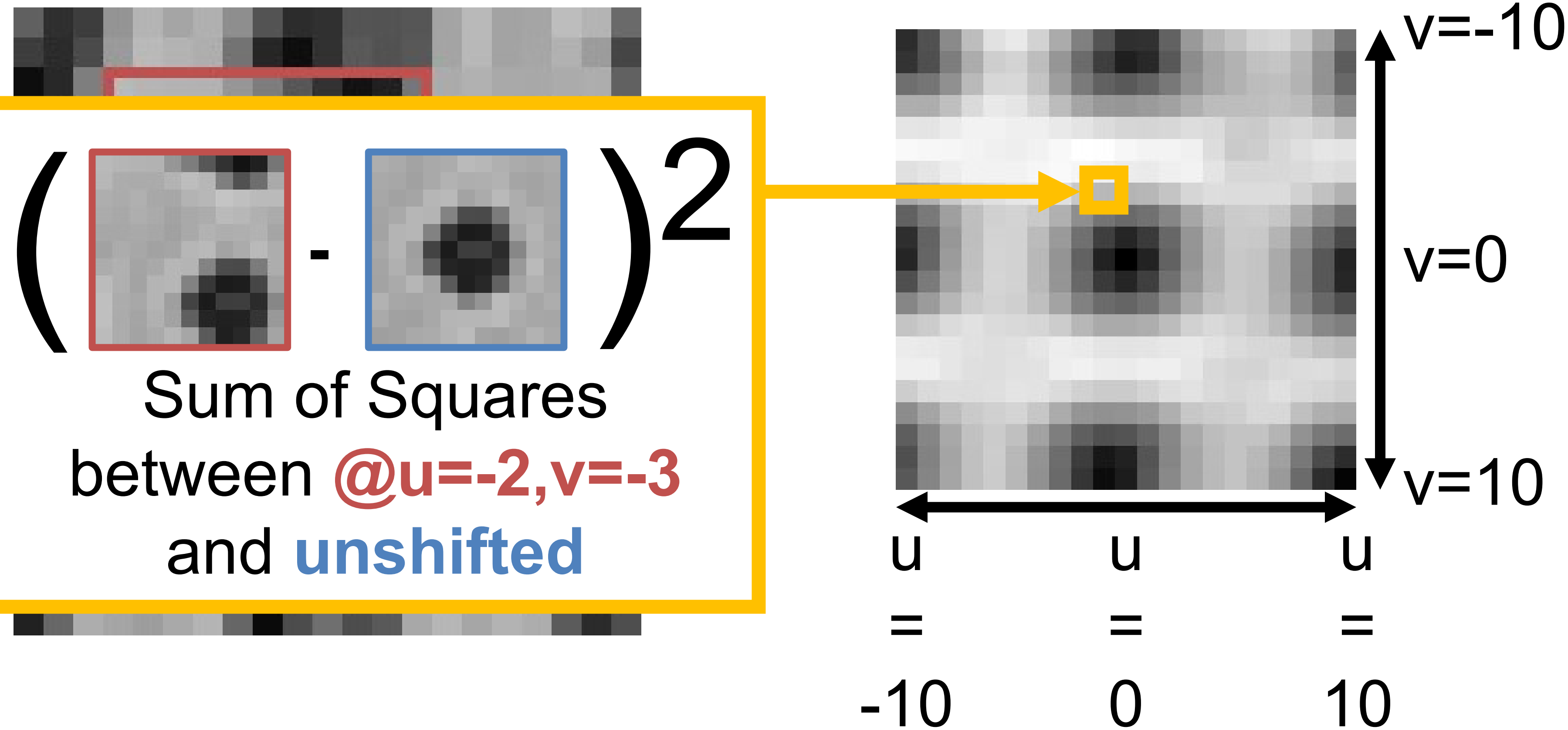
$$\sum_{(x,y)\in W} (I[x+u,y+v] - I[x,y])^2$$

( - )^2

Formalizing Corner Detection

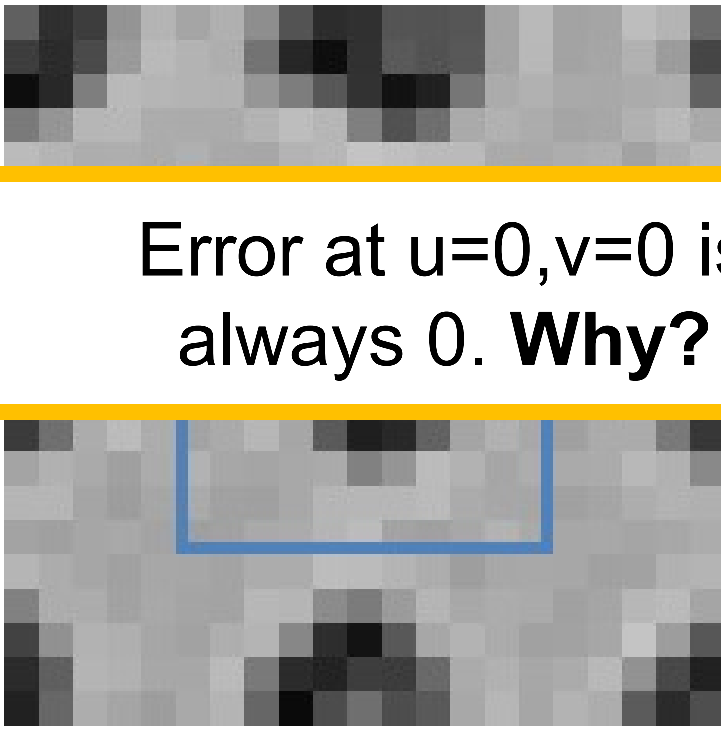
Zoom-In at x,y

Error (Sum Sqs) for u,v offset



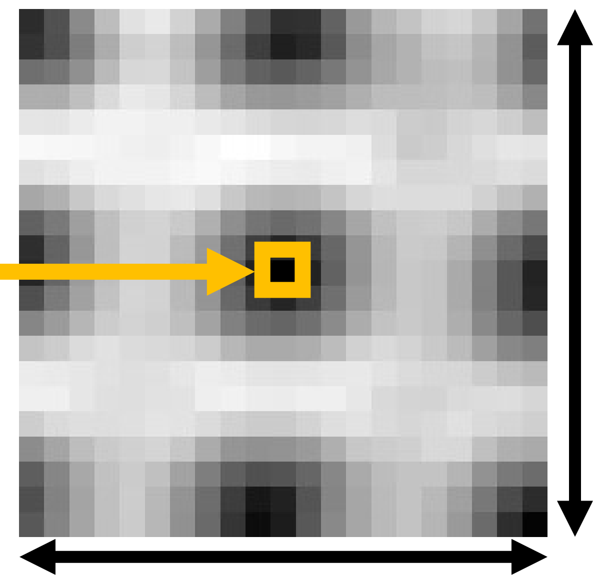
Formalizing Corner Detection

Zoom-In at x,y



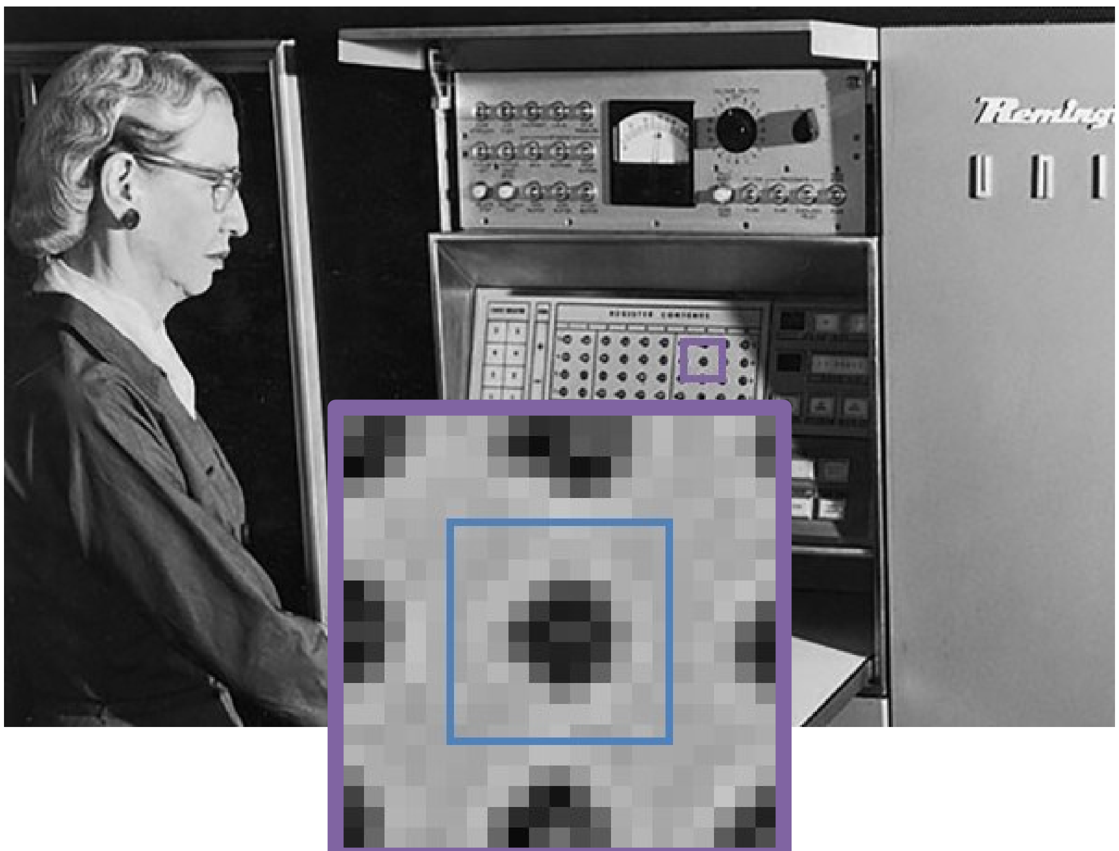
Error at u=0,v=0 is always 0. **Why?**

Error (Sum Sqs) for u,v offset



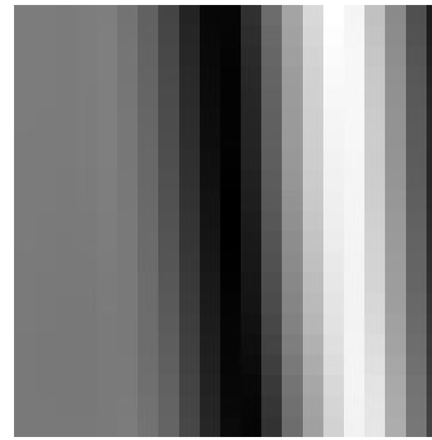
Match The Location and Plot

Original Image and Zoom-In

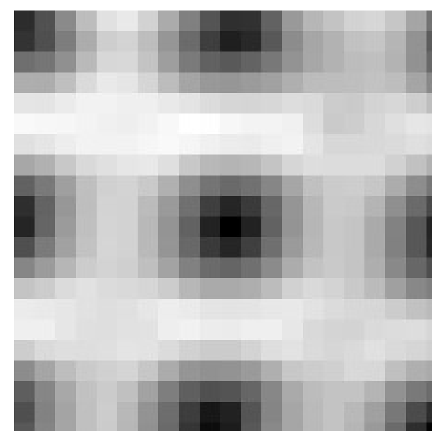


Error Options

A

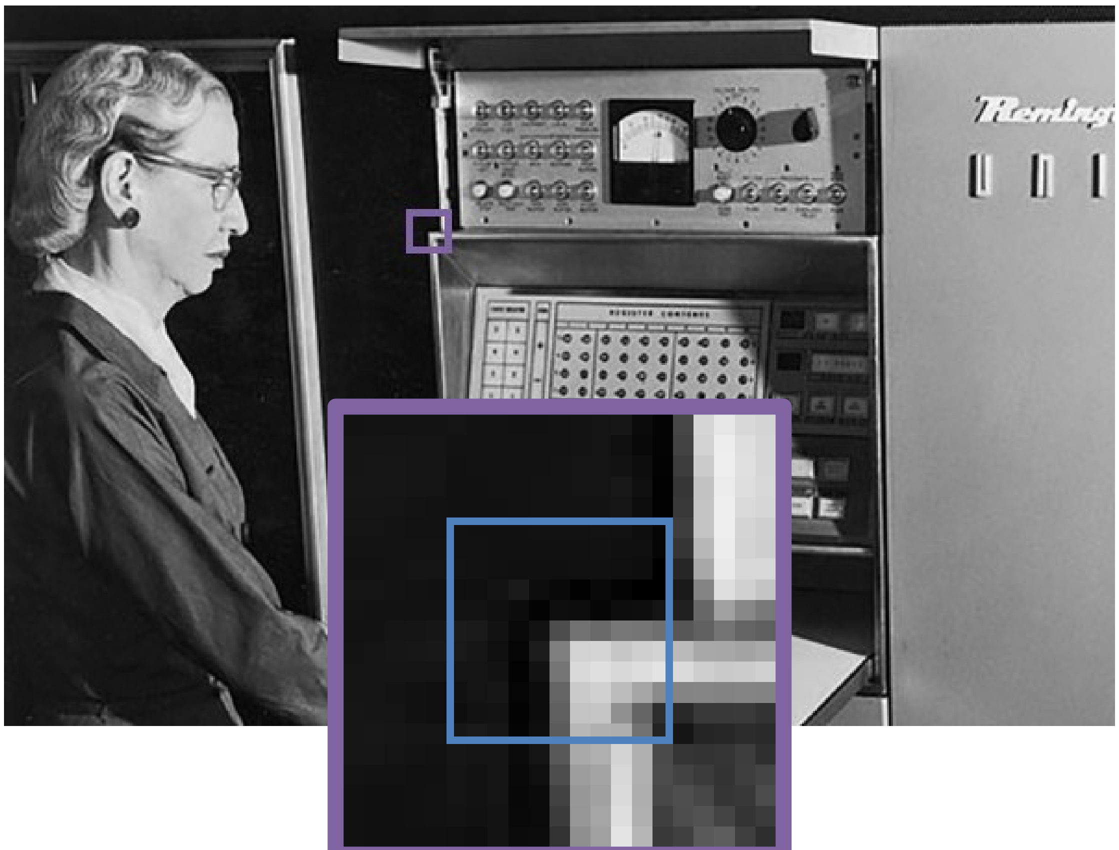


B



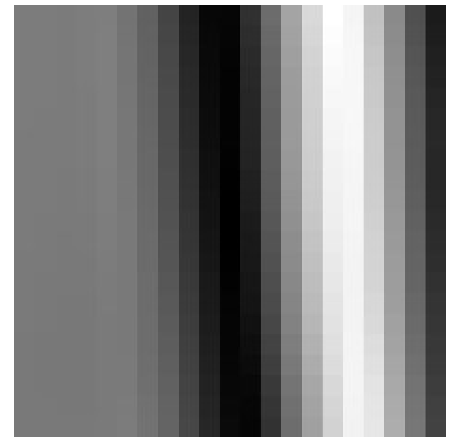
Match The Location and Plot

Original Image and Zoom-In

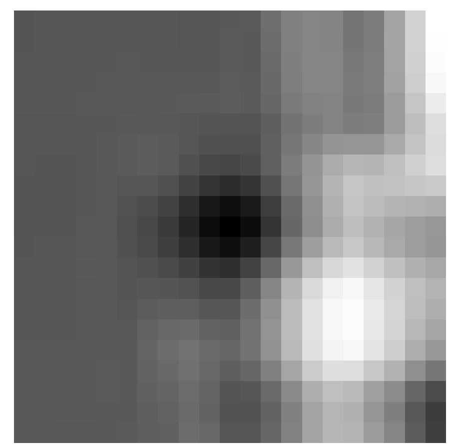


Error Options

A

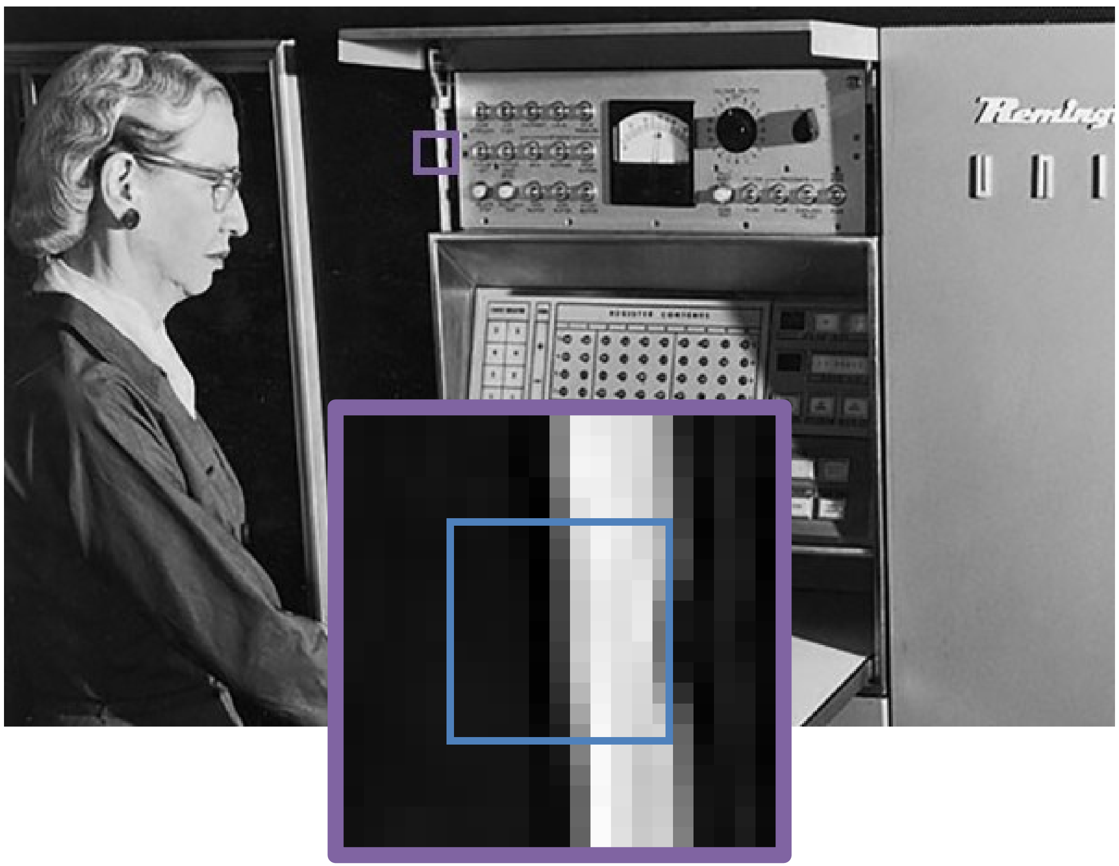


B



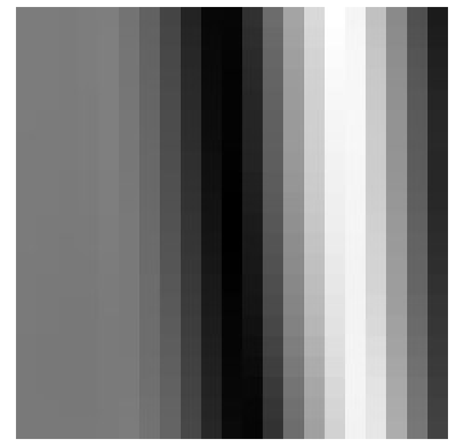
Match The Location and Plot

Original Image and Zoom-In



Error Options

A



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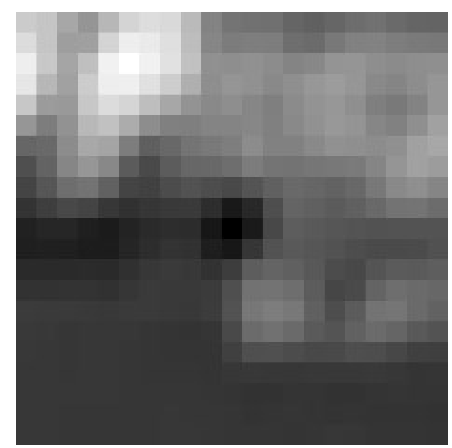
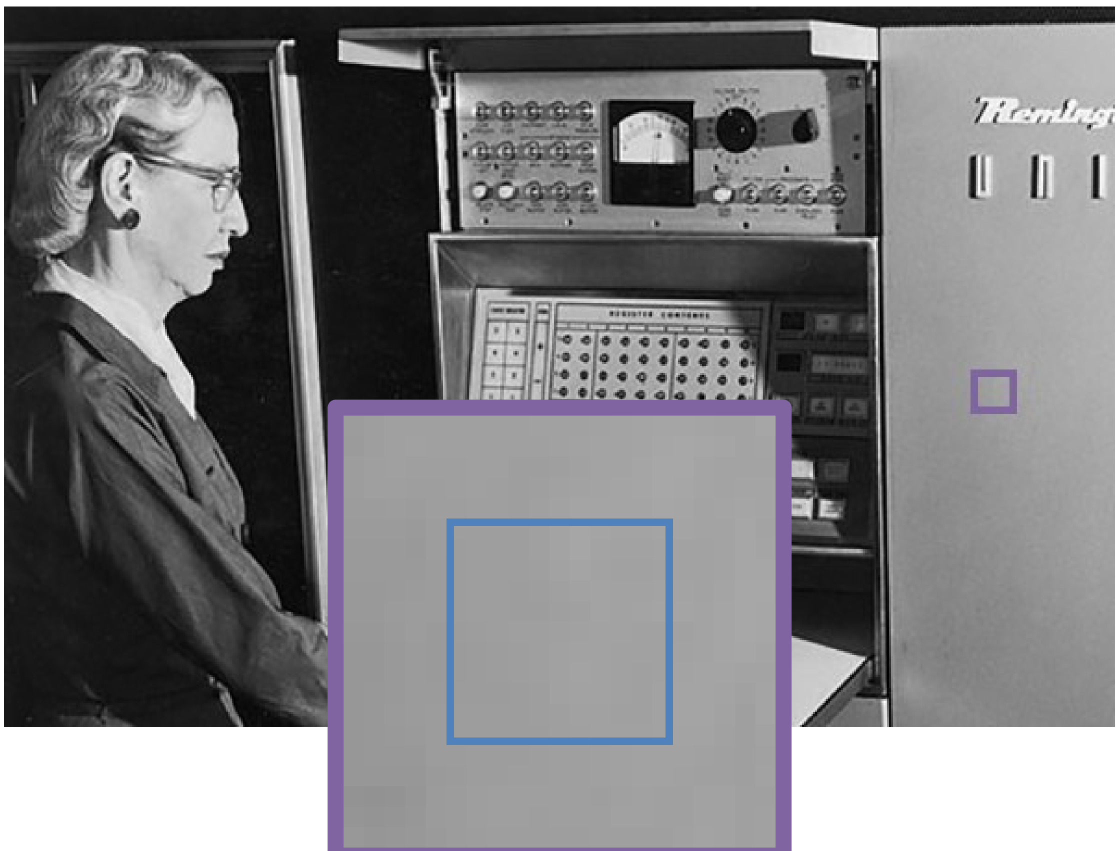


Match The Location and Plot

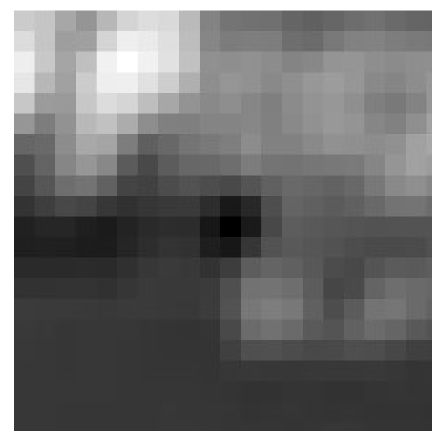
Original Image and Zoom-In



Error Options

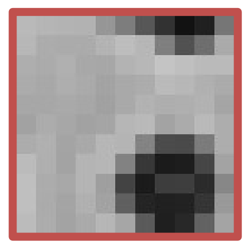
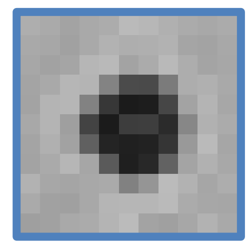
A

B



Ok But Back To Math

$$E(u,v) = \sum_{(x,y)\in W} (I[x+u,y+v] - I[x,y])^2$$

( - )^2

Shifting windows around is expensive!
We'll find a trick to approximate this.

Note: only need to get the gist

Aside: Taylor Series for Images

Recall Taylor Series – way of *linearizing* a function:

$$f(x+d) \approx f(x) + \frac{\partial f}{\partial x} d$$

Do the same with images, treating them as function of x, y

$$I(x+u, y+v) \approx I(x,y) + I_x u + I_y v$$

For brevity: Ix = Ix at point (x,y), Iy = Iy at point (x,y)

Formalizing Corner Detection

$$E(u,v) = \sum_{(x,y)\in W} (I[x+u, y+v] - I[x,y])^2$$

Taylor series expansion for I at every single point in window

$$\approx \sum_{(x,y)\in W} \left(\underline{I[x,y]} + I_x u + I_y v - \underline{I[x,y]}\right)^2$$

Cancel

$$= \sum_{(x,y)\in W} \left(I_x u + I_y v\right)^2$$

Expand

$$= \sum_{(x,y)\in W} I_x^2 u^2 + 2 I_x I_y u v + I_y^2 v^2$$

For brevity: Ix = Ix at point (x,y), Iy = Iy at point (x,y)

Formalizing corner Detection

By linearizing image, we can approximate E(u,v) with quadratic function of u and v

$$E(u,v) \approx \sum_{(x,y)\in W} \left(I_x^2 u^2 + 2 I_x I_y uv + I_y^2 v^2 \right)$$
$$= [u,v] \boldsymbol{M} [u,v]^T$$

$$\boldsymbol{M} = \begin{bmatrix} \sum\limits_{x,y\in W} I_x^2 & \sum\limits_{x,y\in W} I_x I_y \\ \sum\limits_{x,y\in W} I_x I_y & \sum\limits_{x,y\in W} I_y^2 \end{bmatrix}$$

M is called the second moment matrix

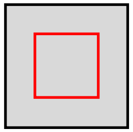
Intuitively what is M?

Pretend for now gradients are either vertical or horizontal at a pixel (so Ix Iy = 0)

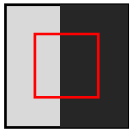
Obviously Wrong!

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

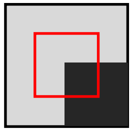
If a,b are both small: flat



If one is big, one is small: edge



If a,b both big: corner



Review: Quadratic Forms

Suppose have symmetric matrix **M,** scalar a, vector [u,v]:

$$E([u,v]) = [u,v]\boldsymbol{M}[u,v]^T$$

Then the isocontour / slice-through of F, i.e.

$$E([u,v]) = a$$

is an ellipse.

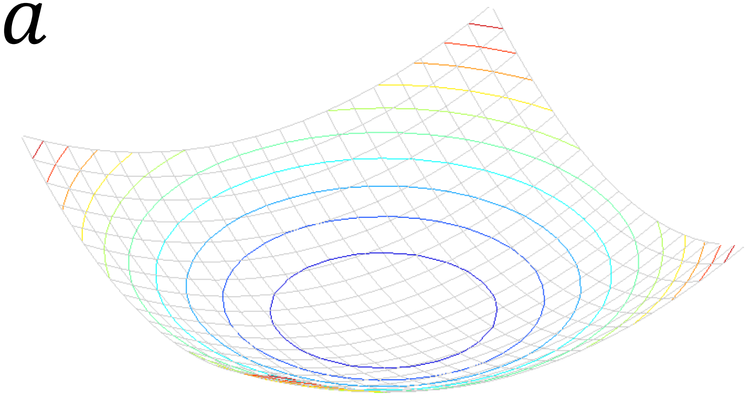


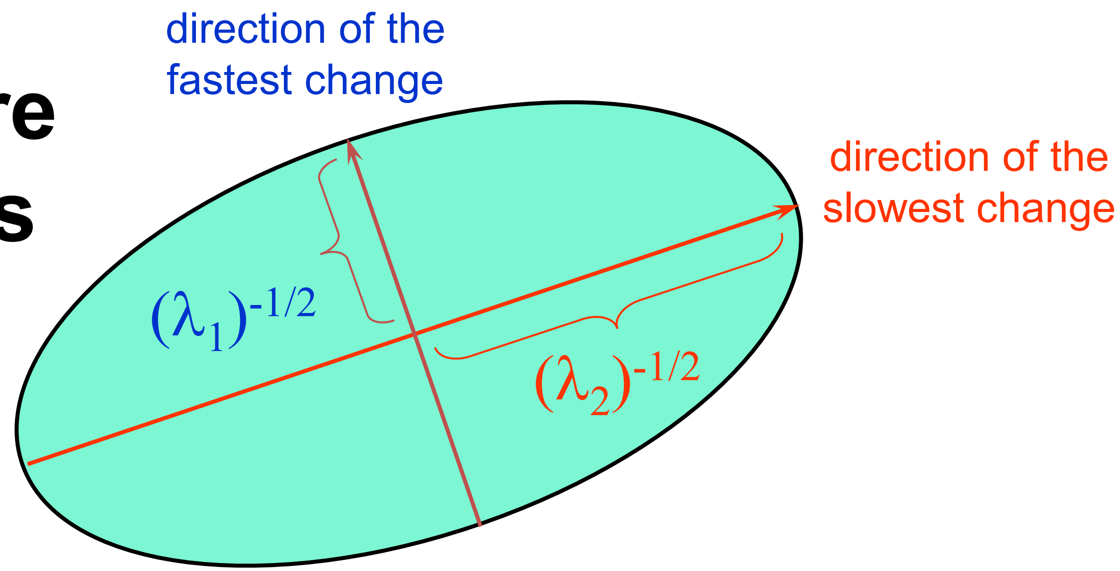
Diagram credit: S. Lazebnik

Review: Quadratic Forms

We can look at the shape of this ellipse by decomposing M into a rotation + scaling

$$\boldsymbol{M} = \boldsymbol{R}^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \boldsymbol{R}$$

λ_1 and λ_2 are eigenvalues



Slide credit: S. Lazebnik

Interpreting The Matrix M

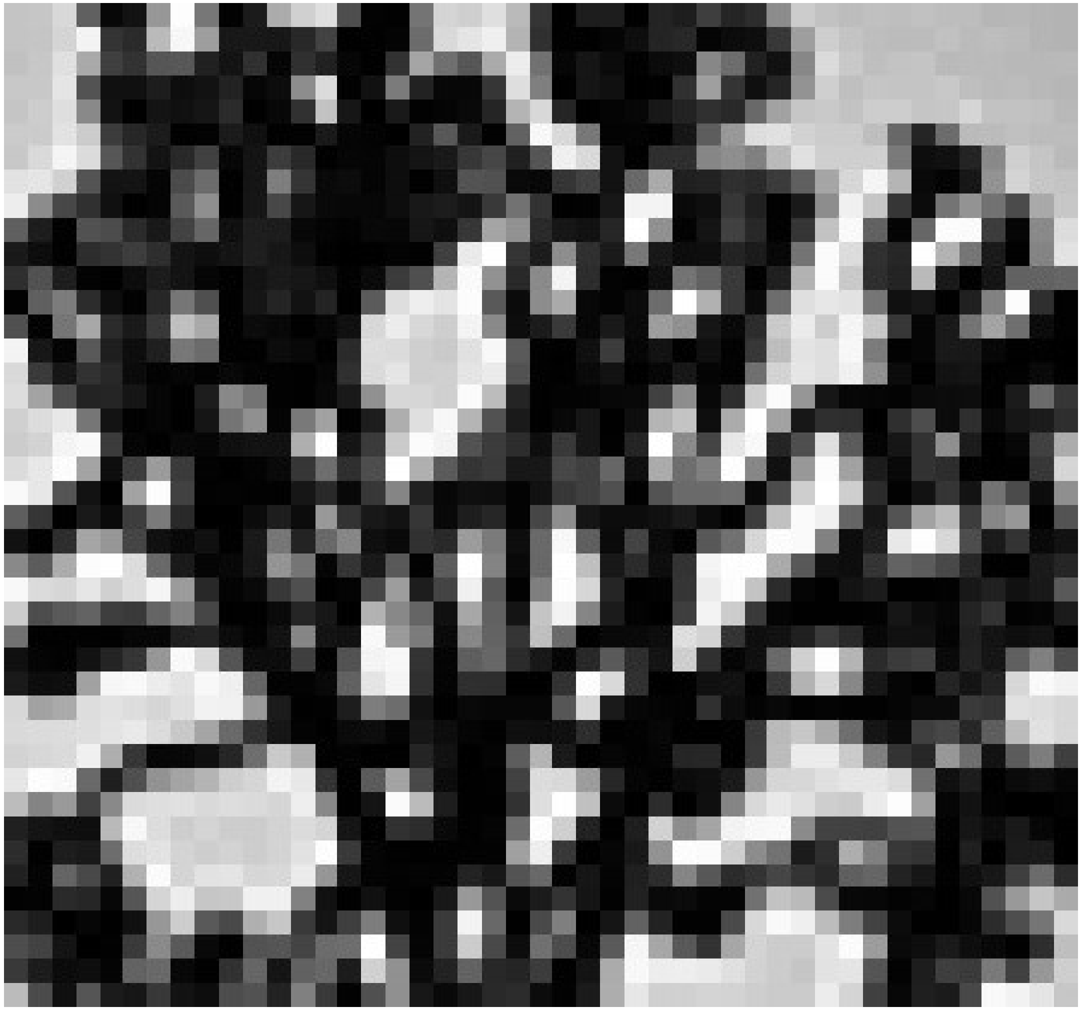
The second moment matrix tells us how quickly the image changes and in which directions.

Directions

$$\boldsymbol{M} = \overbrace{\begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix}}^{\text{Can compute at each pixel}} = \boldsymbol{R}^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \boldsymbol{R}$$

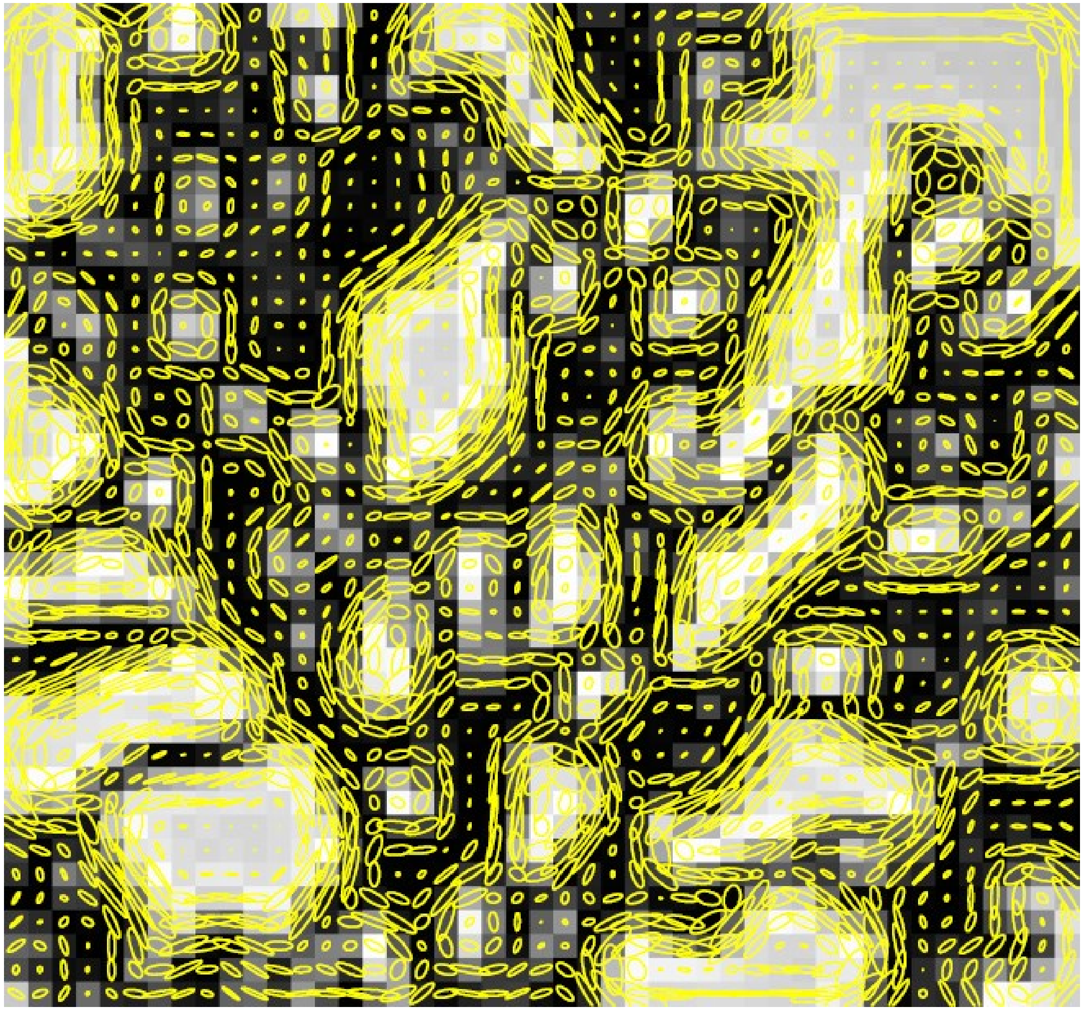
Amounts

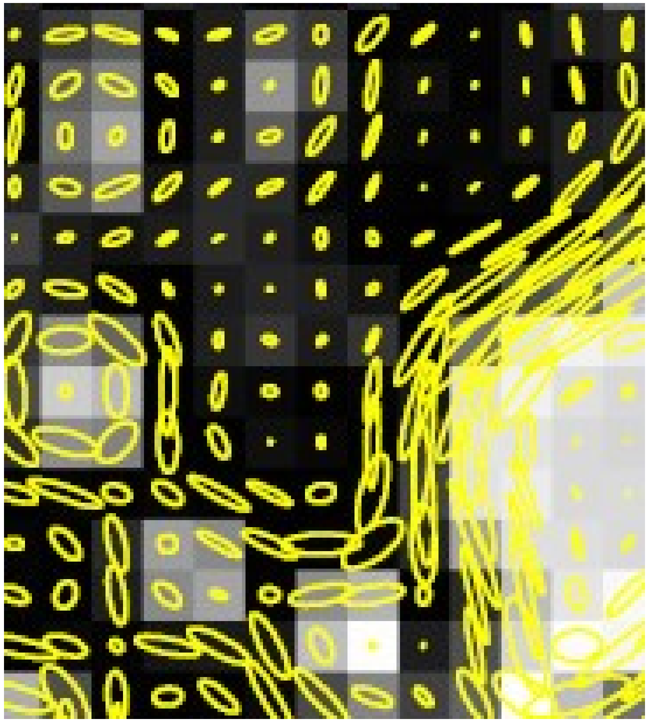
Visualizing M



Slide credit: S. Lazebnik

Visualizing M



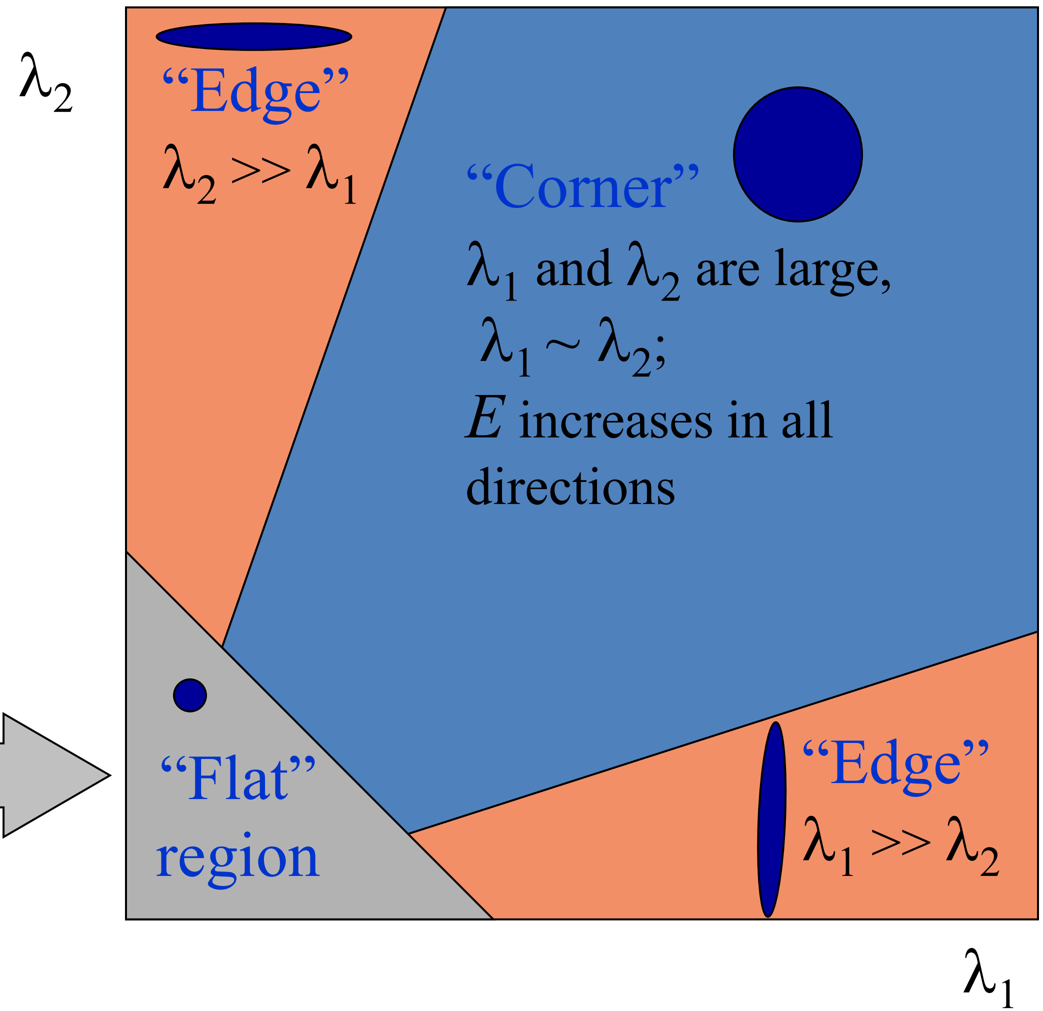


Technical note: M is often best *visualized* by first taking inverse, so long edge of ellipse goes along edge

Slide credit: S. Lazebnik

Interpreting Eigenvalues of M

λ_1 and λ_2 are small;
E is almost constant
in all directions

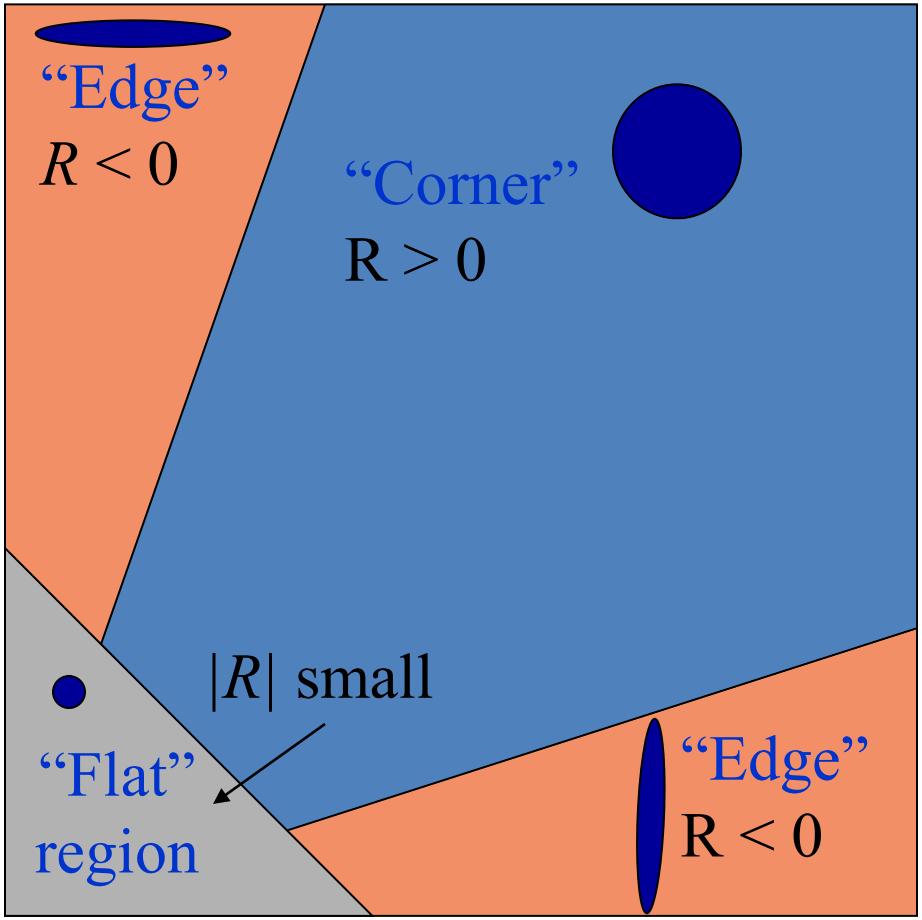


Slide credit: S. Lazebnik; Note: this refers to previous ellipses, not original M ellipse. Other slides on the internet may vary

Putting Together The Eigenvalues

$$R = \det(\boldsymbol{M}) - \alpha\ trace(\boldsymbol{M})^2$$
$$= \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)



Slide credit: S. Lazebnik; Note: this refers to previous ellipses, not original M ellipse. Other slides on the internet may vary

What Do I Need To Know?

- Need to be able to take derivatives of image
- Need to be able to compute the entries of **M** at every pixel.
- Should know that some properties of **M** indicate whether a pixel is a corner or not.

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix}$$

In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} w(x,y) I_x^2 & \sum_{x,y \in W} w(x,y) I_x I_y \\ \sum_{x,y \in W} w(x,y) I_x I_y & \sum_{x,y \in W} w(x,y) I_y^2 \end{bmatrix}$$

C.Harris and M.Stephens. [“A Combined Corner and Edge Detector.”] *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Slide credit: S. Lazebnik

In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w
3. Compute response function R

$$R = \det(\boldsymbol{M}) - \alpha\, trace(\boldsymbol{M})^2$$
$$= \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

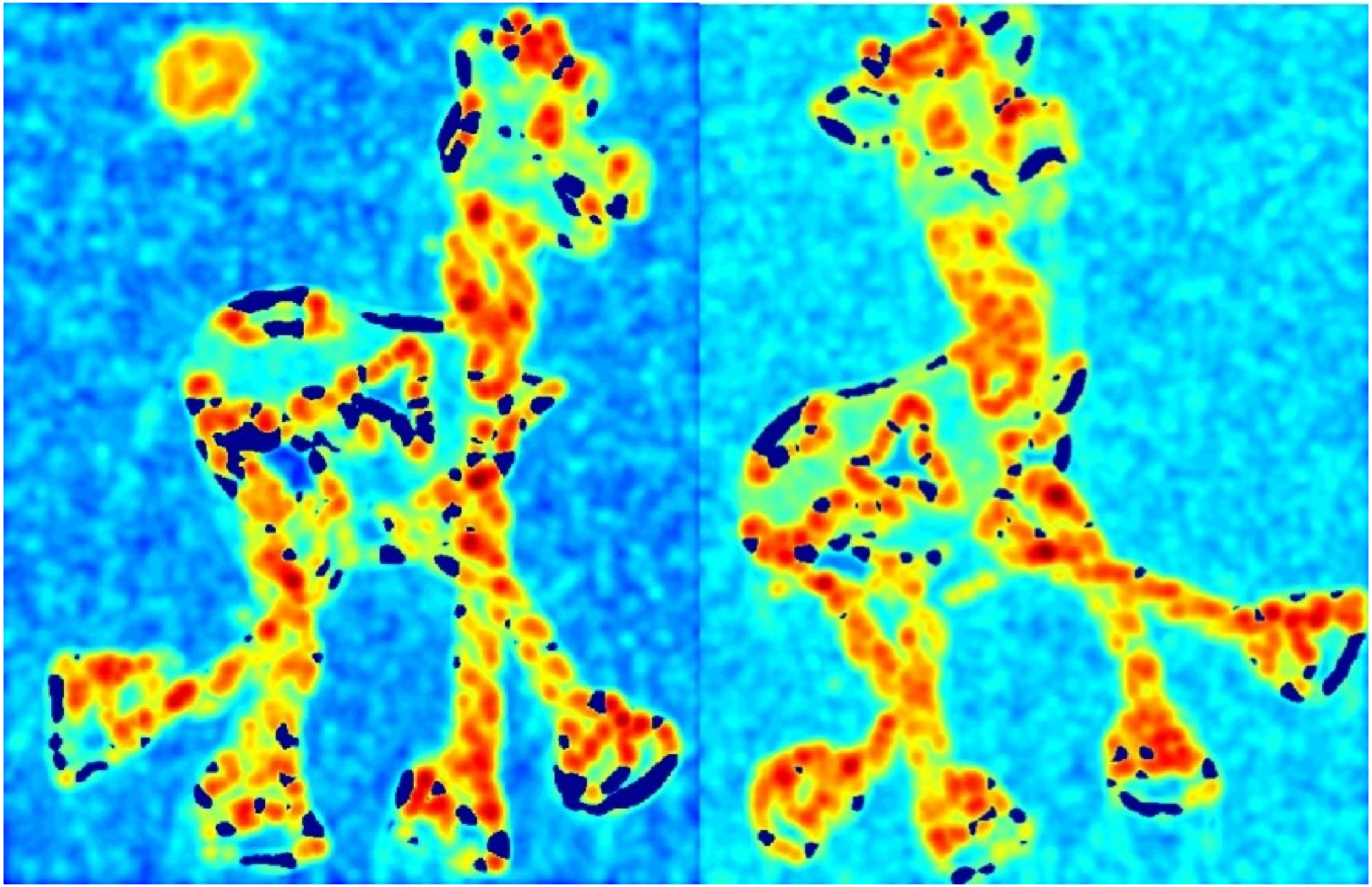
C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Slide credit: S. Lazebnik

Computing R

Slide credit: S. Lazebnik

Computing R



Slide credit: S. Lazebnik

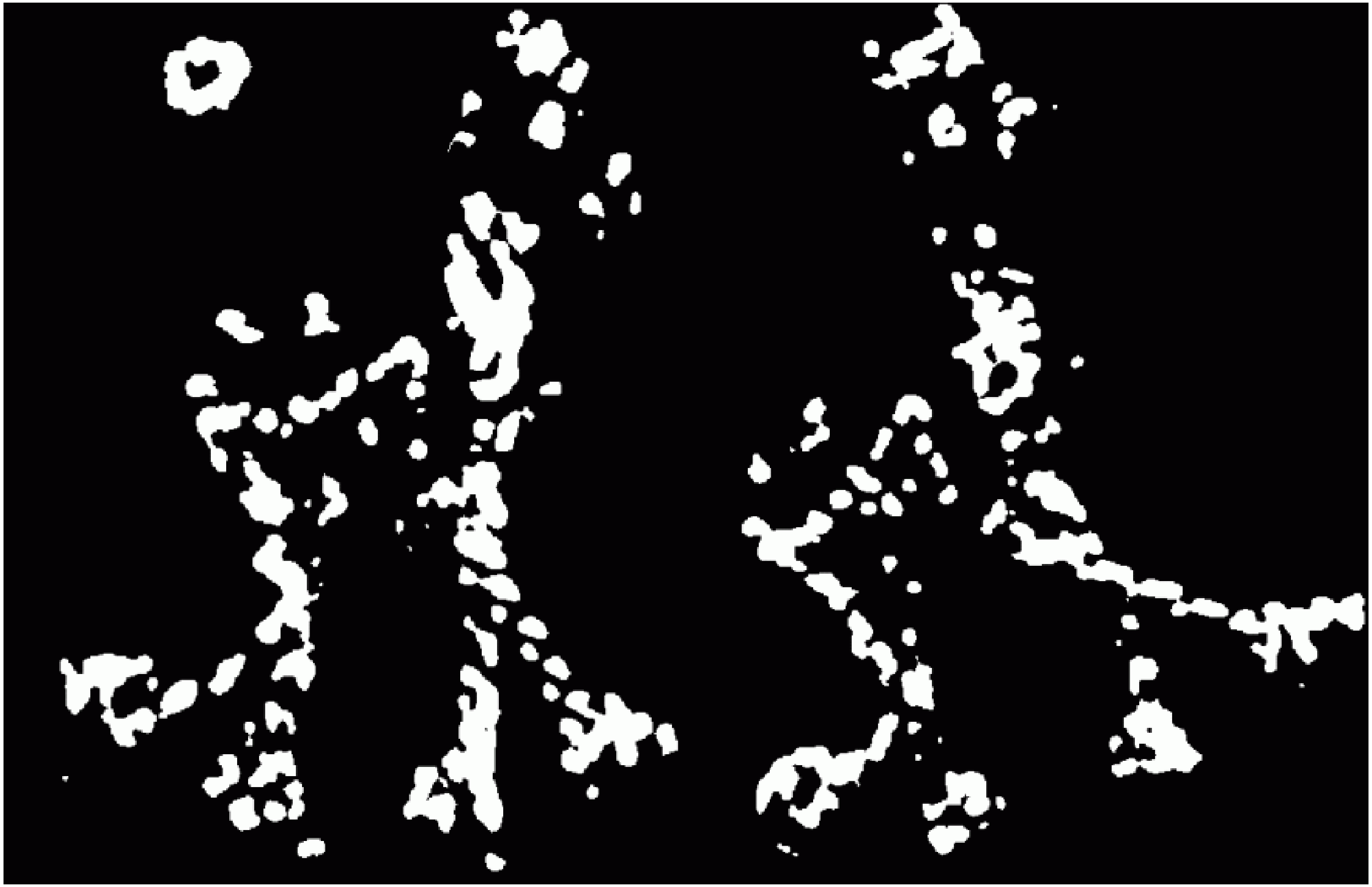
In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w
3. Compute response function R
4. Threshold R

C.Harris and M.Stephens. “A Combined Corner and Edge Detector.” *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Slide credit: S. Lazebnik

Thresholded R



Slide credit: S. Lazebnik

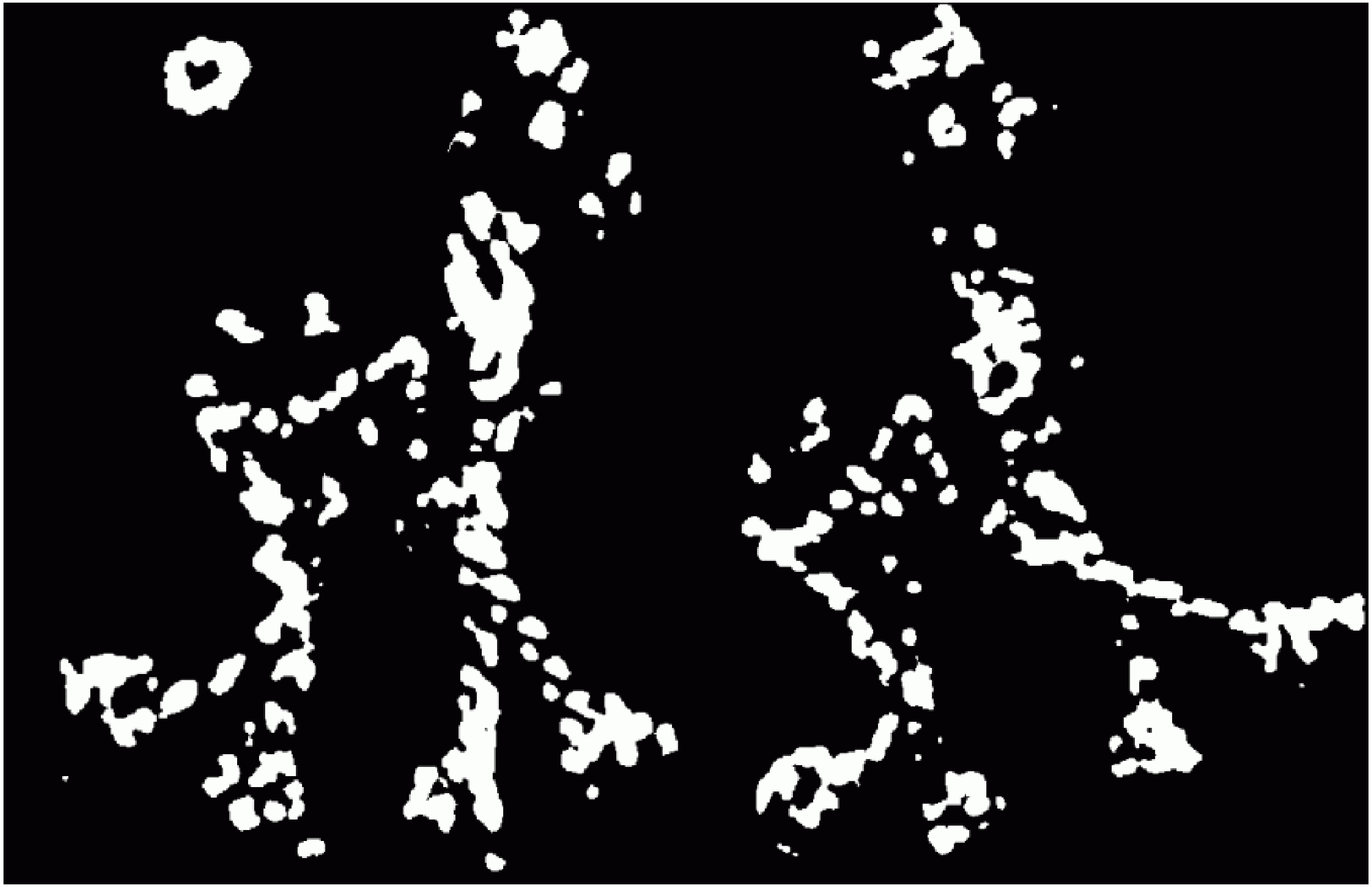
In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w
3. Compute response function R
4. Threshold R
5. Take only local maxima (called non-maxima suppression)

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Slide credit: S. Lazebnik

Thresholded, NMS R



Slide credit: S. Lazebnik

Final Results

Slide credit: S. Lazebnik

Desirable Properties

If our detectors are repeatable, they should be:

- **Invariant** to some things: image is transformed and corners remain the same
- **Covariant/equivariant** with some things: image is transformed and corners transform with it.

Slide credit: S. Lazebnik

Recall Motivating Problem

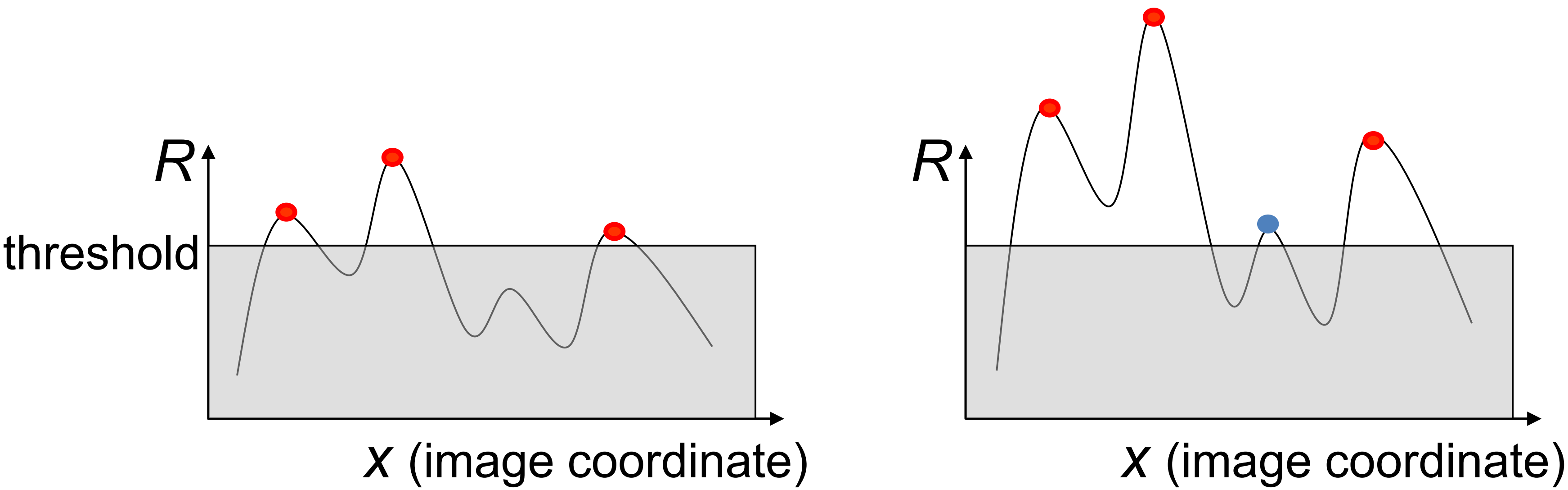
Images may be different in lighting and geometry

Affine Intensity Change

$$I_{new} = aI_{old} + b$$

M only depends on derivatives, so b is irrelevant

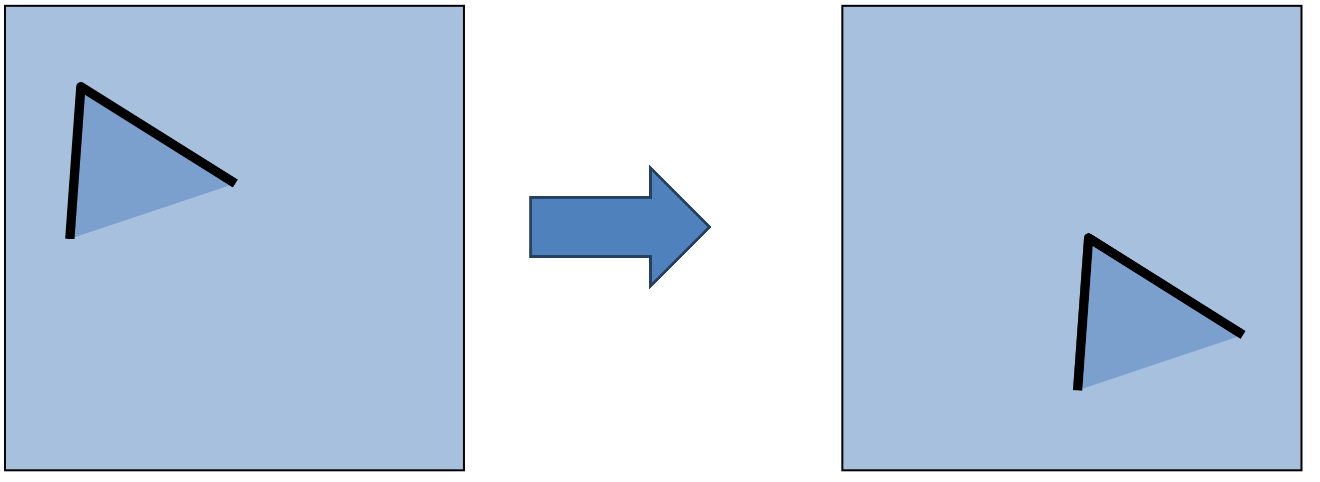
But a scales derivatives and there's a threshold



Partially invariant to affine intensity changes

Slide credit: S. Lazebnik

Image Translation

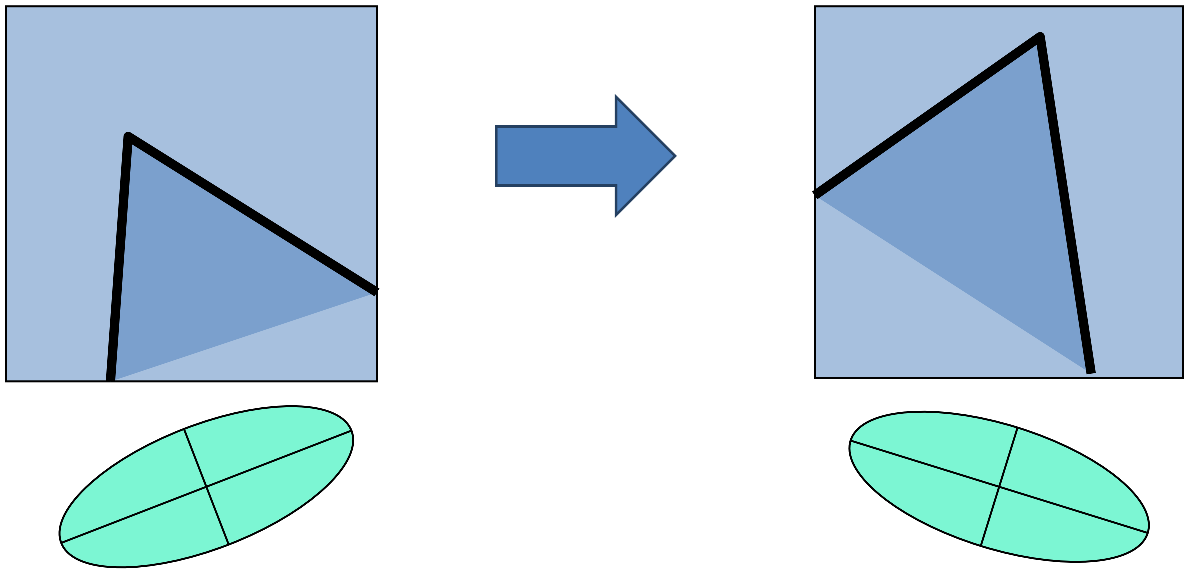


All done with convolution. Convolution is translation invariant.

Equivariant with translation

Slide credit: S. Lazebnik

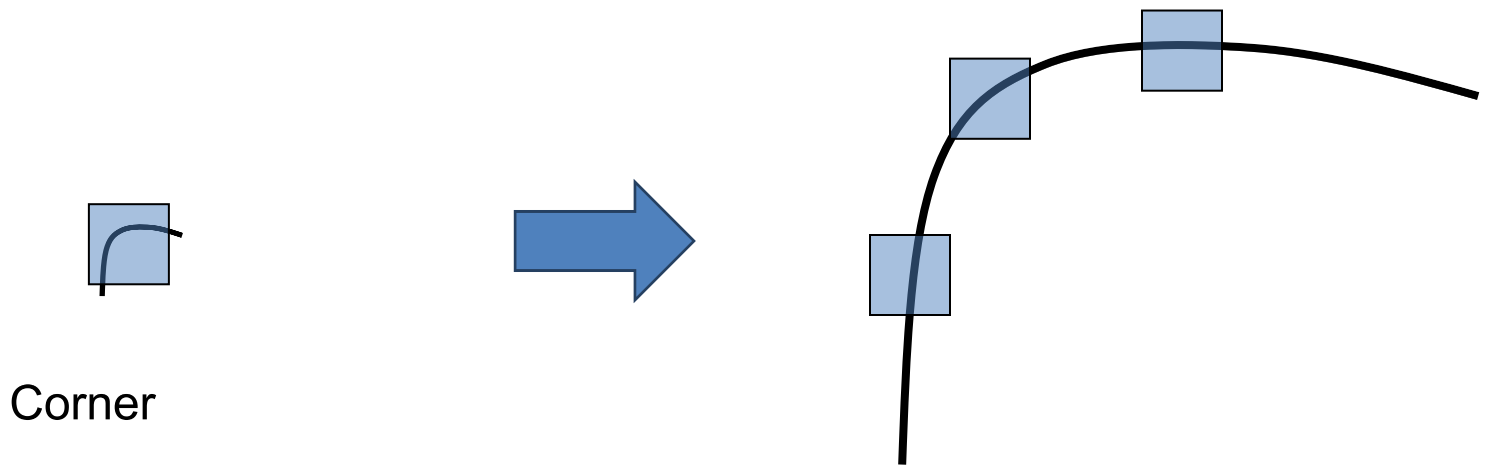
Image Rotation



Rotations just cause the corner rotation to change.
Eigenvalues remain the same.

Equivariant with rotation

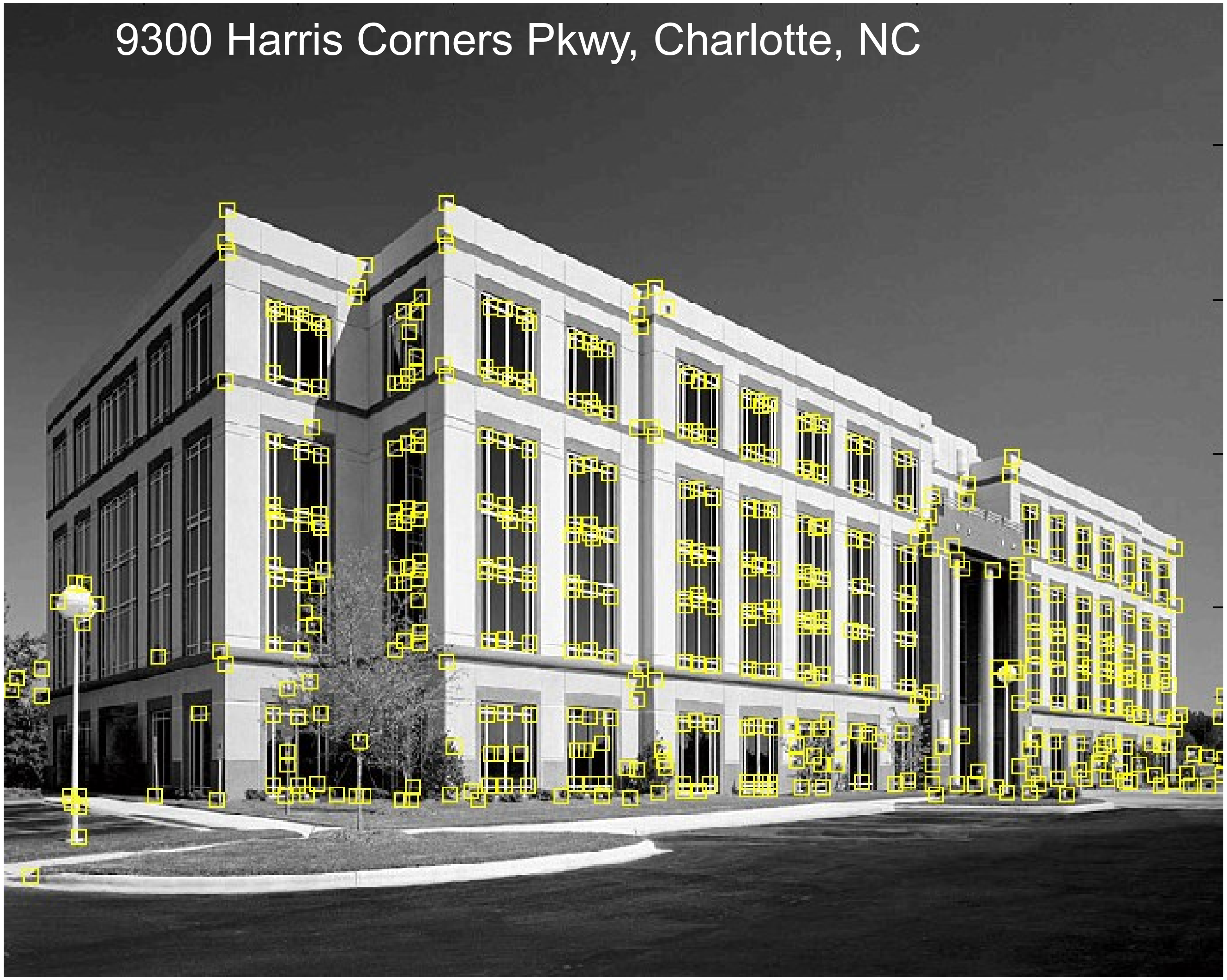
Image Scaling



One pixel can become many pixels and vice-versa.

Not equivariant with scaling

Corners



Slide Credit: S. Lazebnik

Derivatives Review

Given quadratic function f(x)

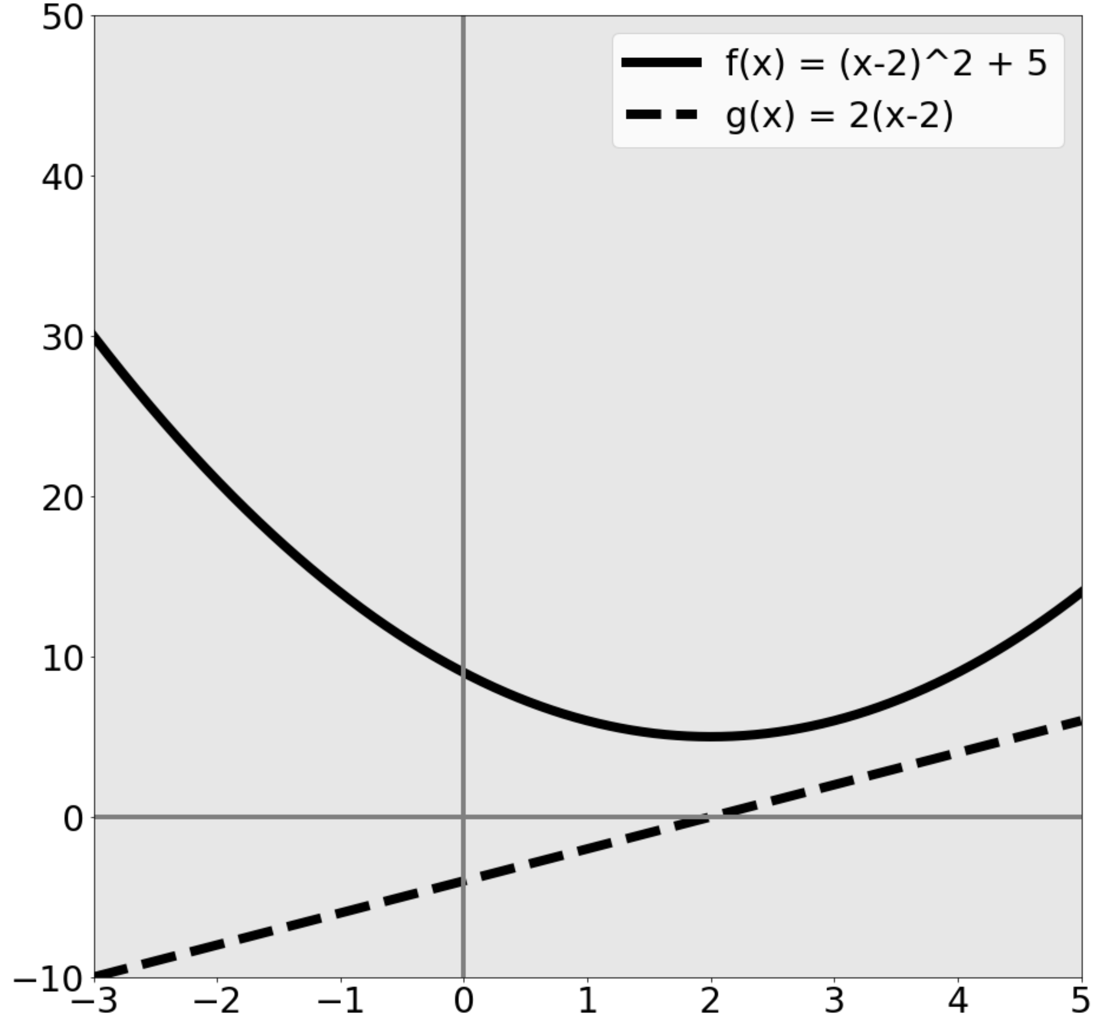
$$f(x) = (x-2)^2 + 5$$

$f(x)$ is function

$$g(x) = f'(x)$$

aka

$$g(x) = \frac{d}{dx} f(x)$$



Given quadratic function f(x)

$$f(x) = (x-2)^2 + 5$$

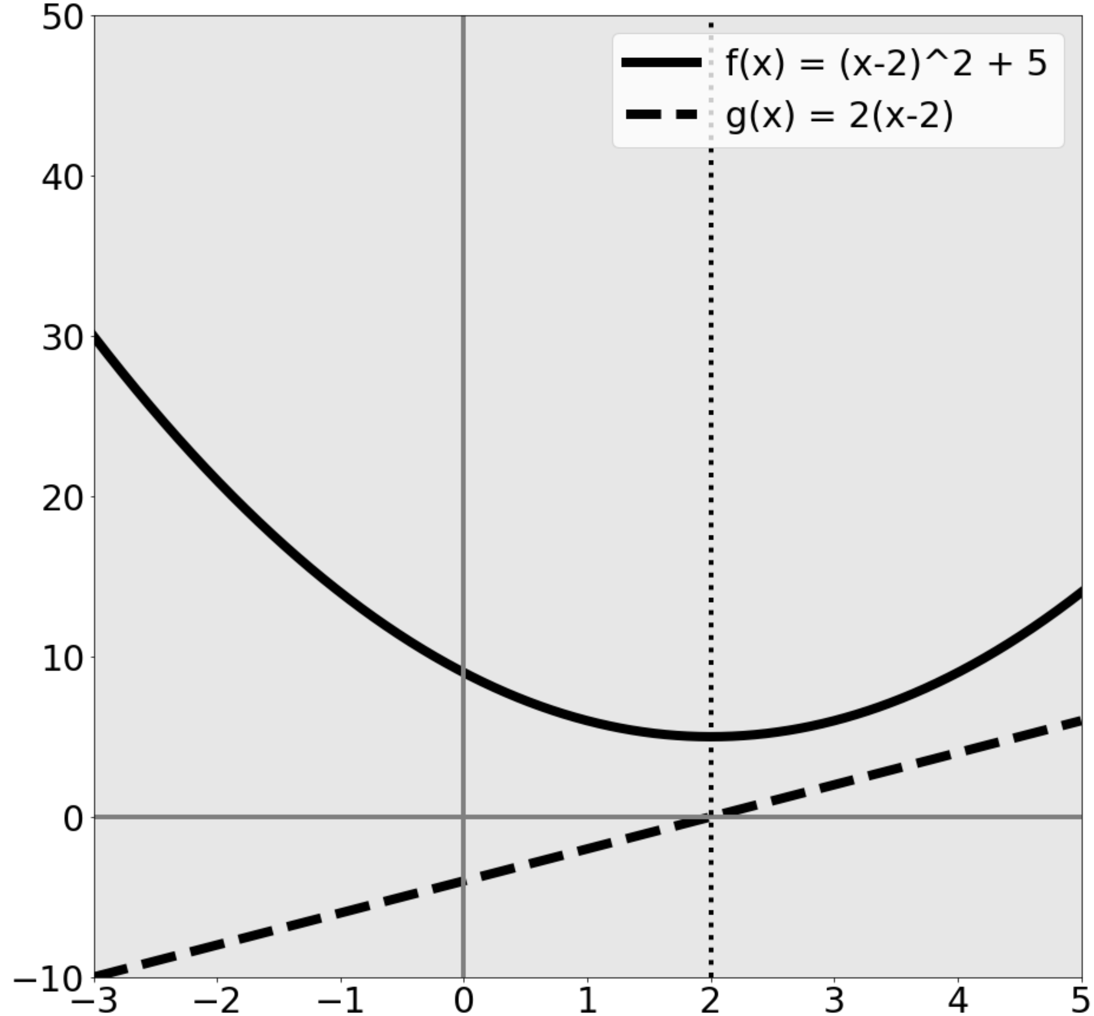
What's special about x=2?

$f(x)$ minim. at 2

$g(x) = 0$ at 2

a = minimum of f → $g(a) = 0$

Reverse is ***not true***



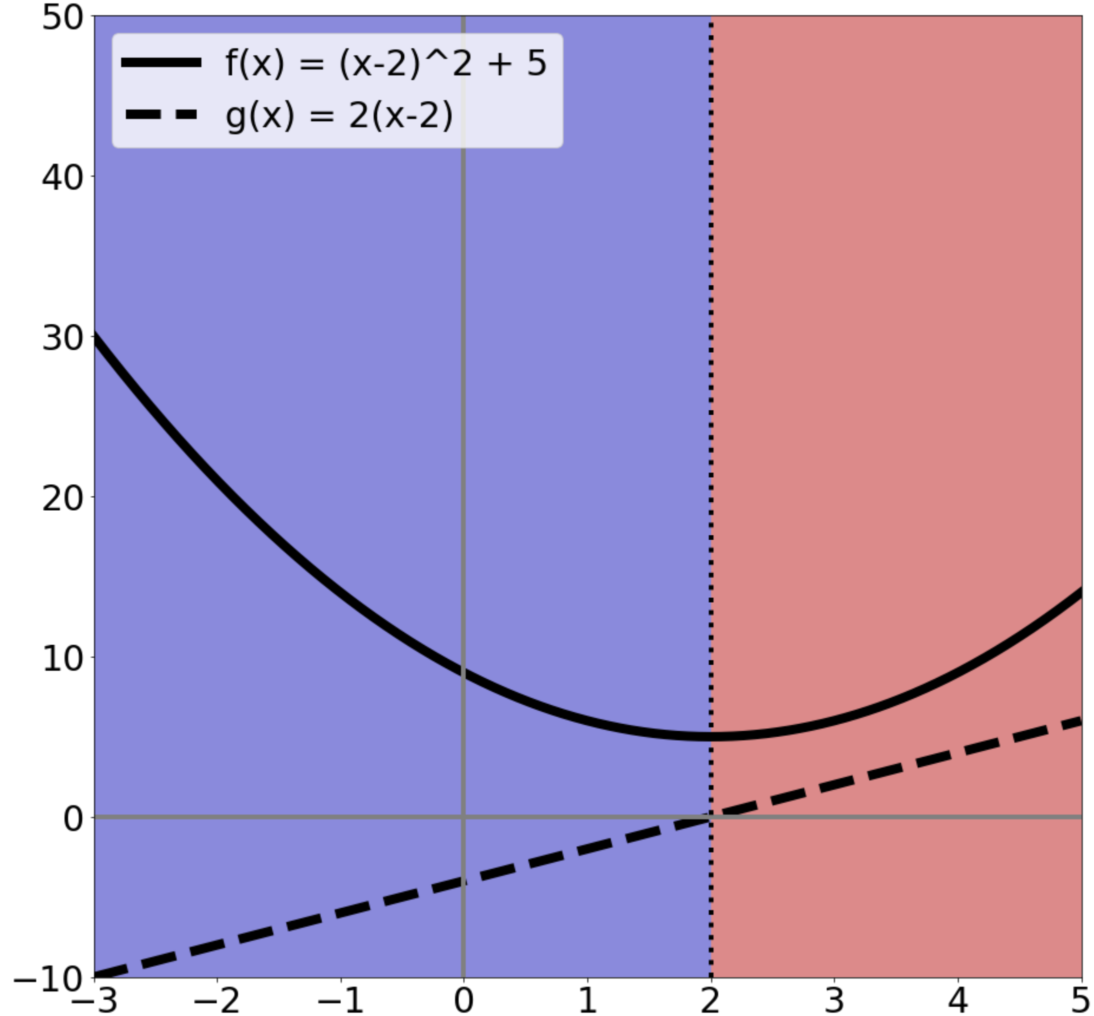
Rates of change

$$f(x) = (x-2)^2 + 5$$

Suppose I want to increase f(x) by changing x:

Blue area: move left
Red area: move right

Derivative tells you direction of ascent and rate



What Calculus Should I Know

- Really need intuition
- Need chain rule
- Rest you should look up / use a computer algebra system / use a cookbook
- Partial derivatives (and that's it from multivariable calculus)

Partial Derivatives

- Pretend other variables are constant, take a derivative. That's it.
- Make our function a function of two variables

$$f(x) = (x-2)^2 + 5$$

$$\frac{\partial}{\partial x} f(x) = 2(x-2) * 1 = 2(x-2)$$

$$f_2(x,y) = (x-2)^2 + 5 + \boxed{(y+1)^2}$$

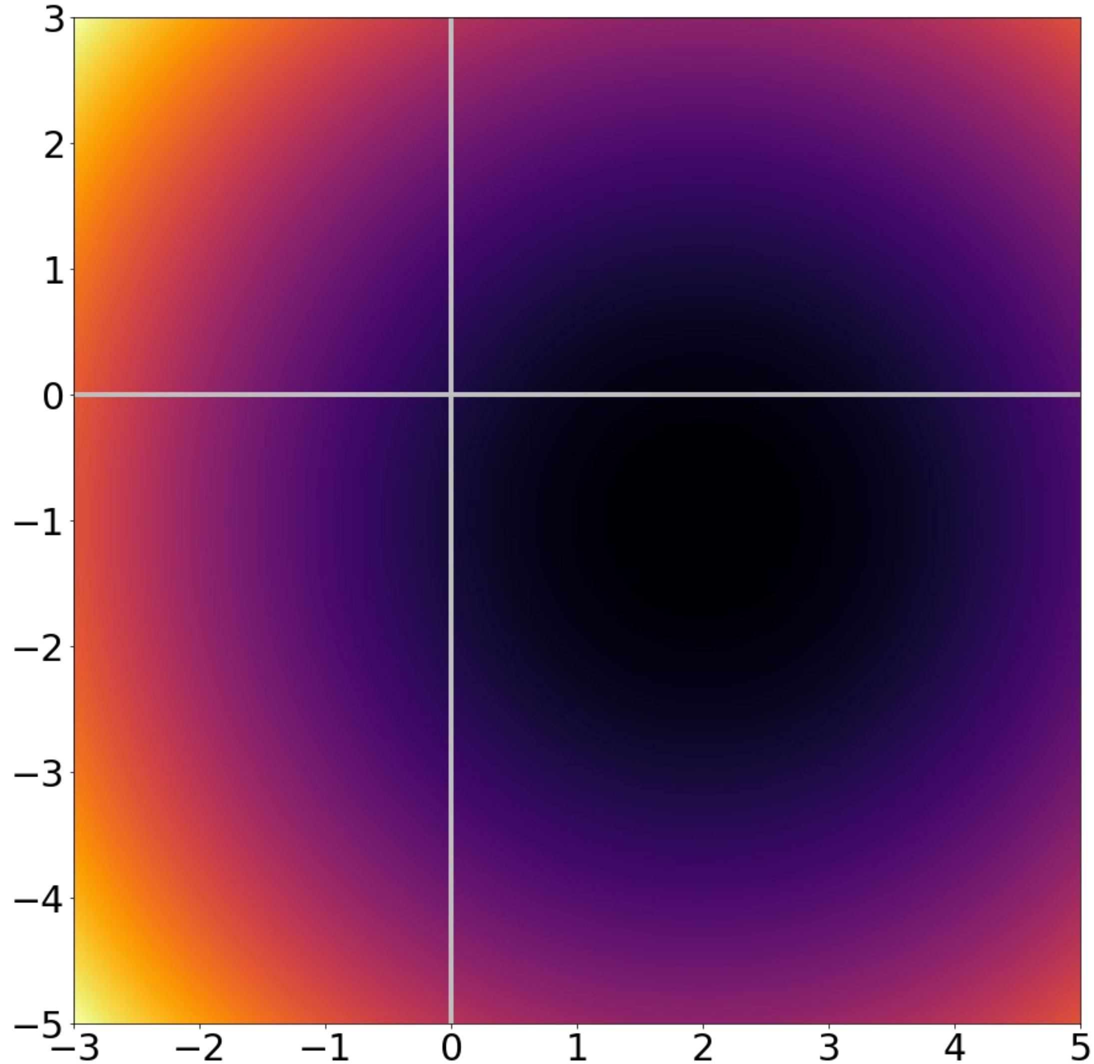
Pretend it's constant → derivative = 0

$$\frac{\partial}{\partial x} f_2(x) = 2(x-2)$$

Zooming Out

$$f_2(x, y) = (x - 2)^2 + 5 + (y + 1)^2$$

Dark = f(x,y) low
Bright = f(x,y) high



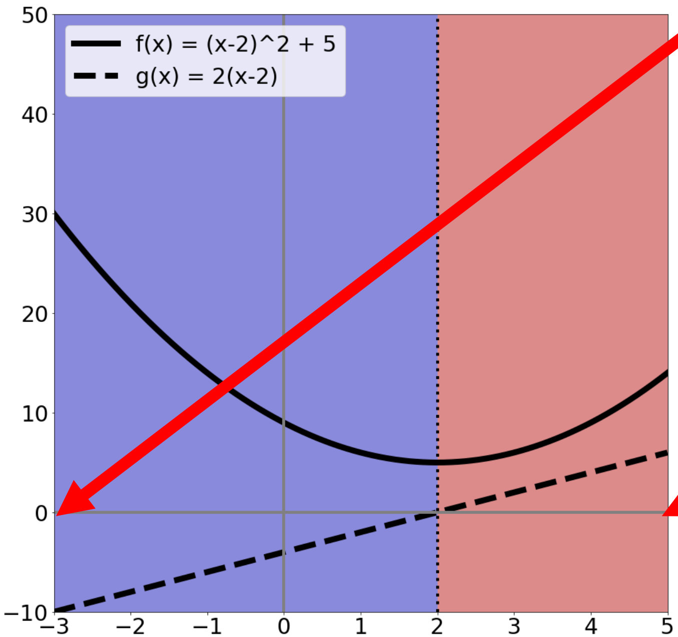
Taking a slice of

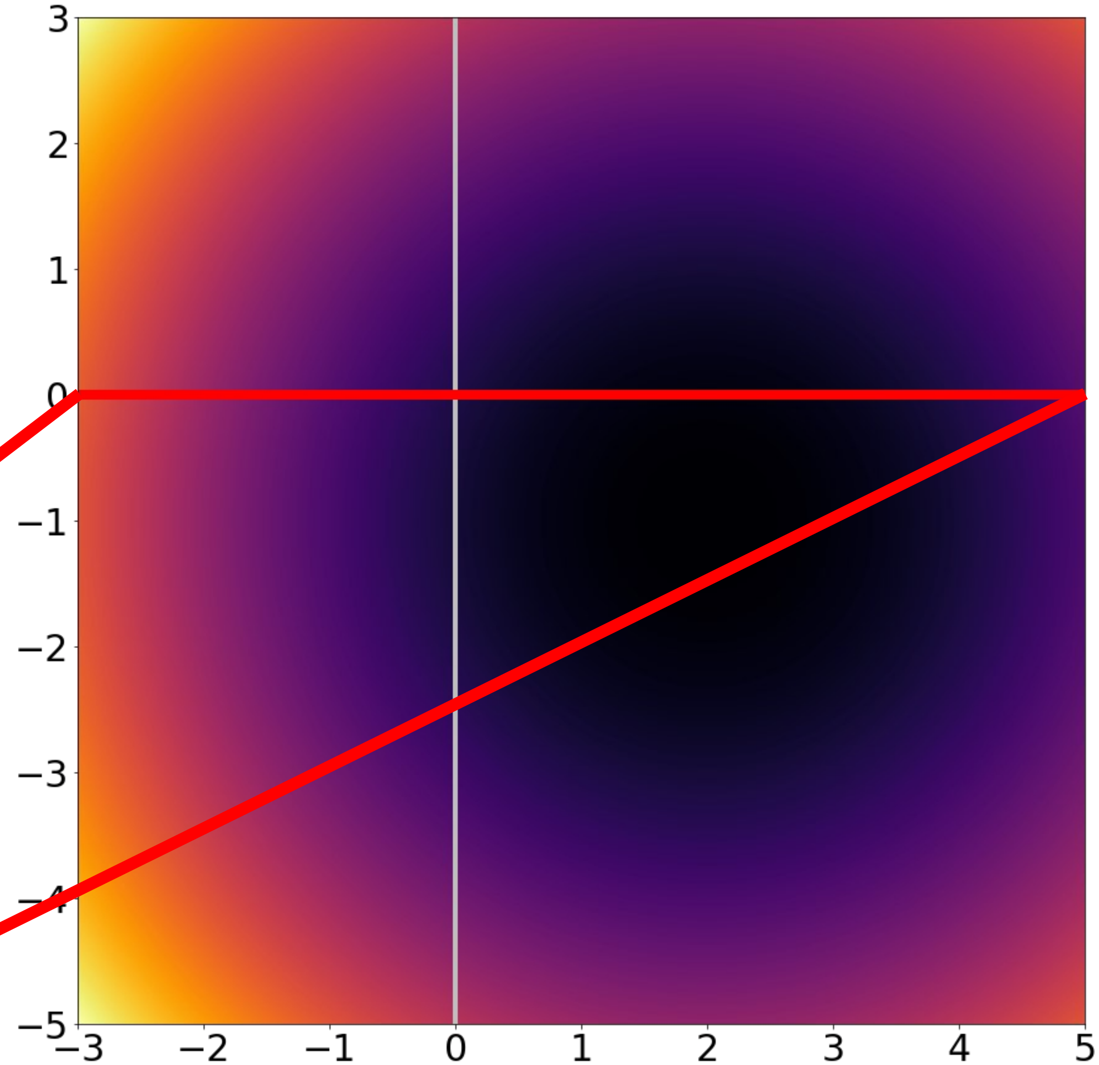
$$f_2(x, y) = (x-2)^2 + 5 + (y+1)^2$$

Slice of y=0 is the function from before:

$$f(x) = (x-2)^2 + 5$$

$$f'(x) = 2(x-2)$$

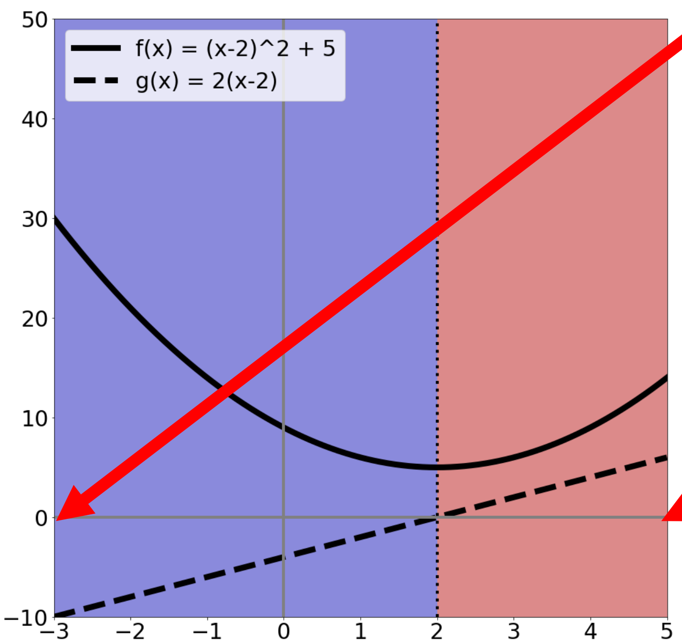


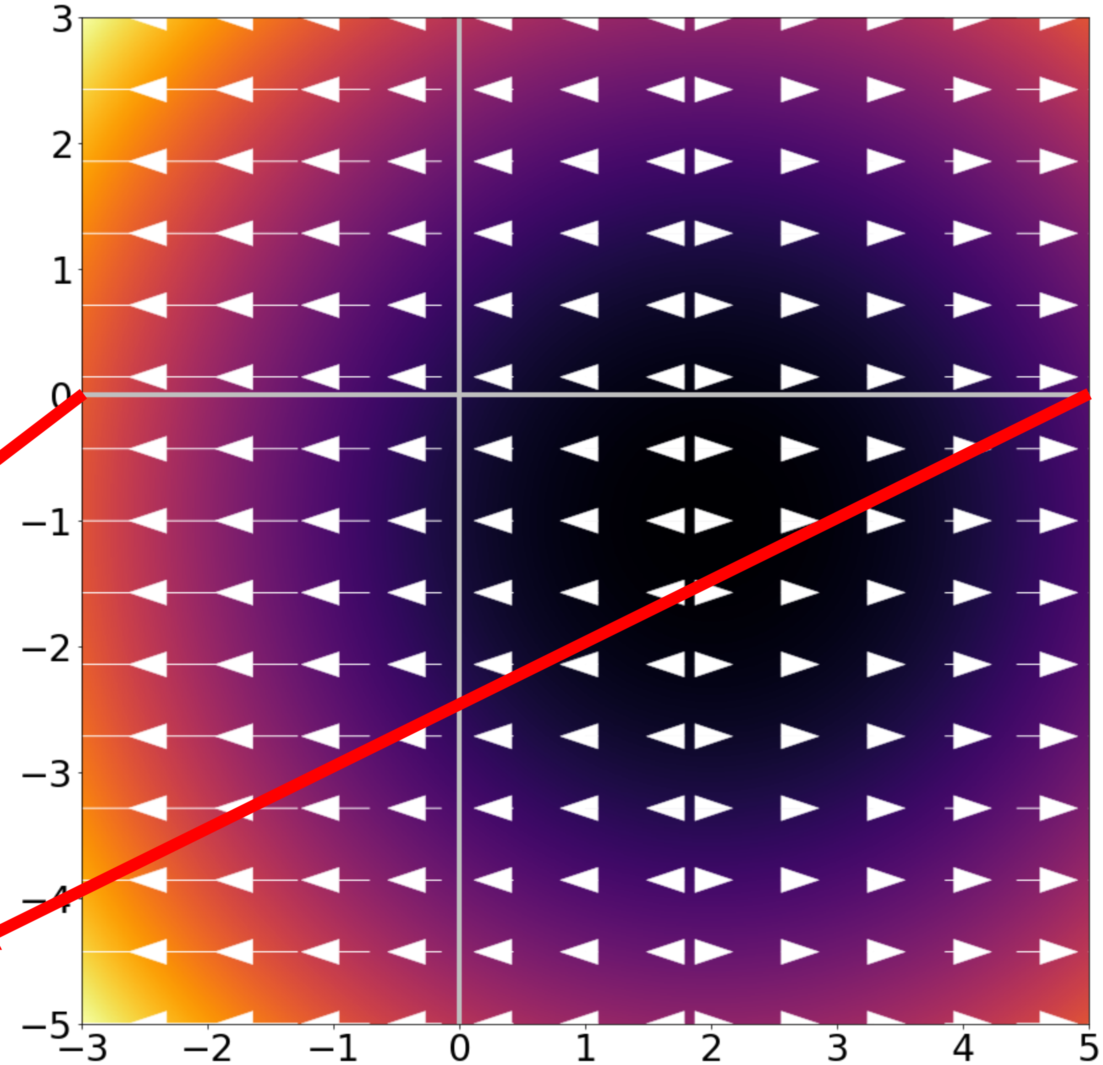


Taking a slice of

$$f_2(x, y) = (x-2)^2 + 5 + (y+1)^2$$

$\frac{\partial}{\partial x} f_2(x, y)$ is rate of change & direction in x dimension





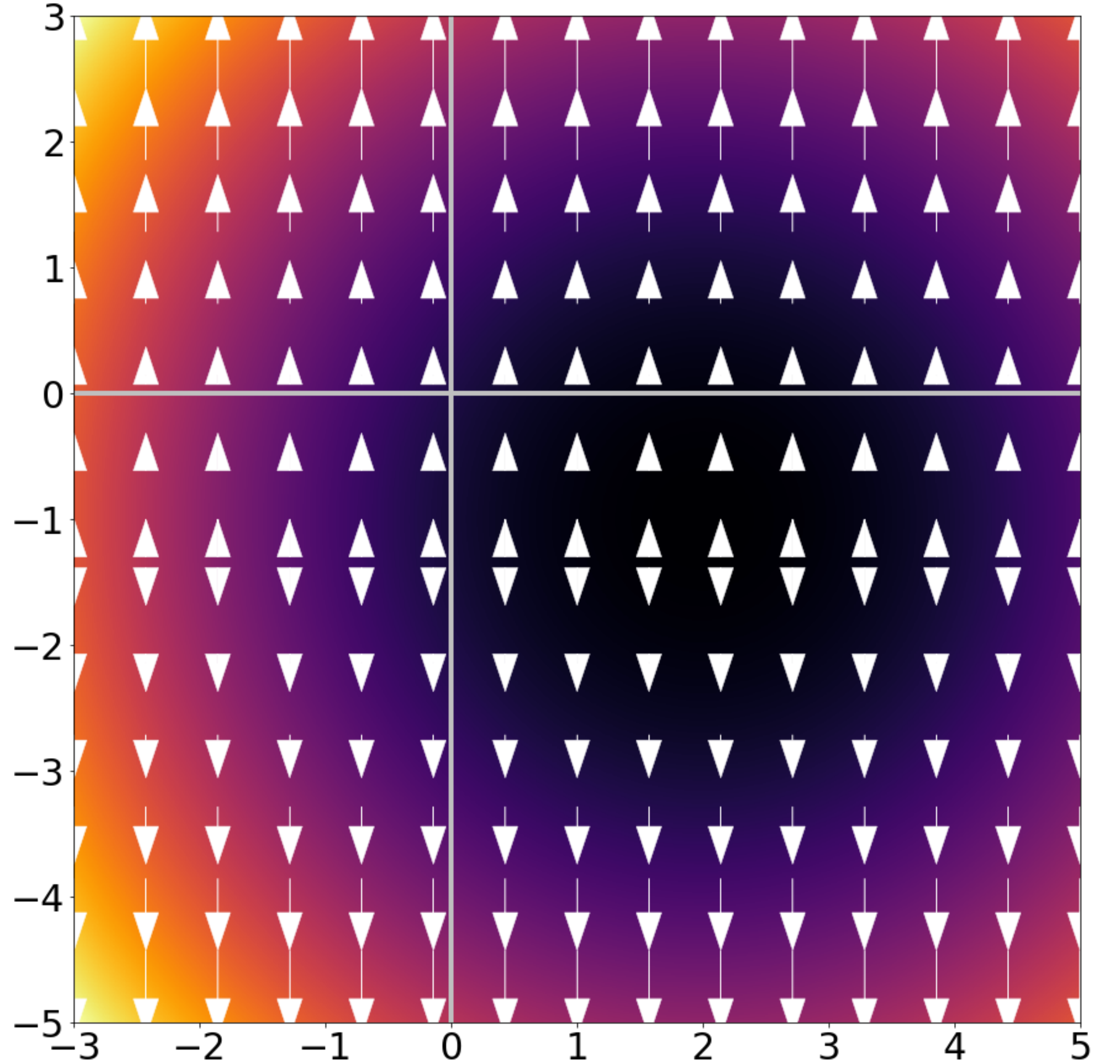
Zooming Out

$$f_2(x, y) = (x-2)^2 + 5 + (y+1)^2$$

$\frac{\partial}{\partial y} f_2(x, y)$ is

$2(y+1)$

and is the rate of change & direction in y dimension



Zooming Out

$$f_2(x, y) = (x-2)^2 + 5 + (y+1)^2$$

Gradient/Jacobian:
Making a vector of

$$\nabla_f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

gives rate and direction of change.

Arrows point OUT of minimum / basin.