CSE 455/555 Spring 2013 Quiz 3 of 14

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Name:

ID#:

Section: 455 or 555

2	8	10

Directions – *The quiz is closed book/notes. You have 10 minutes to complete it; use this paper only.*

Problem 1: Recall (2pts) (Answer in one sentence only.)

What is zero-one loss?

A symmetrical loss function that assigns 1 as the loss for prediction error and 0 otherwise.

Problem 2: Work (8 pts) (Show all derivations/work and explain.)

Show that the following statement is true:

If the decision regions of a two-class classification are chosen to minimize the probability of misclassification, this probability will satisfy

$$p(mistake) \leq \int \{p(x,\omega_1)p(x,\omega_2)\}^{1/2}dx$$

(Hint: for two nonnegative numbers a and b, if $a \leq b$ then $a \leq (ab)^{1/2}$.)

Proof. Let R_1 be the distribution area of class ω_1, and R_2 be the area of class ω_2

$$p(mistake) = \int_{R_1} p(x,\omega_2)dx + \int_{R_2} p(x,\omega_1)dx$$

In the error made in R_1 we always have $p(\omega_1|x) \geq p(\omega_2|x)$. So we have the following for R_1,

$$p(\omega_2|x) \leq \{p(\omega_1|x)p(\omega_2|x)\}^{1/2}$$

$$\begin{aligned}
\int_{R_1} p(x,\omega_2)dx &= \int_{R_1} p(\omega_2|x)p(x)dx \\
&\leq \int_{R_1} \{p(\omega_1|x)p(\omega_2|x)\}^{1/2}p(x)dx \\
&= \int_{R_1} \{p(x,\omega_1)p(x,\omega_2)\}^{1/2}dx
\end{aligned}$$

and similar situations apply for errors in R_2, *i.e.* $p(\omega_2|x) \geq p(\omega_1|x)$ when in R_2

$$p(\omega_1|x) \leq \{p(\omega_1|x)p(\omega_2|x)\}^{1/2} \Rightarrow$$

$$\int_{R_2} p(x,\omega_1)dx \leq \int_{R_2} \{p(\omega_{|}x)p(\omega_2,x)\}^{1/2}dx$$

substitute $\int_{R_1} p(x,\omega_2)dx$ and $\int_{R_2} p(x,\omega_1)dx$ back to the original equation, we get

$$\begin{aligned}
p(mistake) &= \int_{R1} p(x,\omega_2)dx + \int_{R2} p(x,\omega_1)dx \\
&\leq \int_{R_1} \{p(x,\omega_1)p(x,\omega_2)\}^{1/2}dx + \int_{R_2} \{p(x,\omega_1)p(x,\omega_2)\}^{1/2}dx \\
&= \int \{p(x,\omega_1)p(x,\omega_2)\}^{1/2}dx
\end{aligned}$$

□