Terahertz Power-Combining with Coupled Oscillator Arrays

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Abstract — A quasi-optical method for solid-state power-combining is discussed, with application to high-power millimeter-wave generation. The approach uses two-dimensional planar arrays of coupled oscillators. These arrays are distinguished by the strength of the coupling, which is limited to weak interactions. This simplifies both the analysis and design of the arrays. A theoretical description of the coupled-oscillator arrays is briefly discussed, along with experimental results for two prototype arrays using Gunn and MESFET devices at X-band. Experiments indicate that mutual synchronization of the elements is facilitated by using a quasi-optical reflector to control the inter-element coupling.

Introduction

Devices such as resonant tunneling diodes and Josephson junctions have demonstrated modest power levels in oscillator circuits in the millimeter/sub-millimeter range. However, applications such as space communications or remote sensing and imaging will require much higher power levels than could be obtained from a single millimeter-wave device. A solid-state source for these applications must combine the output powers of thousands of devices. Clearly the traditional power-combining methods used for lower frequencies [1] are not feasible. For high efficiency and high power levels, a quasi-optical approach has been proposed in which the power-combining takes place in free-space [2].

This paper discusses one type of quasi-optical array, which incorporates many single-device oscillator elements in a classical antenna array. Weak mutual coupling synchronizes the array elements, enabling coherent summation of individual power outputs. This approach is not to be confused with another reported technique [10], using a distributed oscillator approach where the devices are mounted in a periodic grid structure and placed in an open quasi-optical cavity. The primary difference between the two methods is the amount of coupling between the elements. It is too early yet to articulate the relative merits of the two systems.

In the present approach, the strength of the coupling between elements is limited to ensure that each element operates close to its free-running state. In this configuration, the operating frequency is set by the design of the individual oscillator elements and biasing considerations. This technique is modular, as more elements can be added to increase the power without altering the operating frequency. Thus the important components in our power-combining arrays are the individual oscillator elements themselves.

The active radiating elements consist of a suitable device integrated into a printed radiating structure. The choice of device and radiating structure depends on many factors, such as operating frequency, power dissipation requirements, and substrate considerations. Several novel architectures have appeared in the literature [3-5] which creatively incorporate an active device in a planar microstrip antenna. The experimental arrays described later were constructed using two different active microstrip patch antenna designs, one with a Gunn diode [3] and the other with a MESFET [5]. The device is located at the point where it's impedance is matched to that of the antenna. Small signal device models, and simple analytical models for the antenna structures are usually sufficient for design purposes.

Coupled Oscillator Theory

The description of coupled oscillators is greatly simplified by the assumption of weak coupling between the elements. In this case, the individual oscillators are only slightly perturbed from their free-running state by the presence of the neighboring oscillators. It can then be argued that the steady-state behaviour is governed by the phase dynamics alone [7]. Similar arguments were put forth by Adler in connection with his famous injection-locking equation [6], which describes the phase dynamics of an oscillator under the influence of a weak injection signal. By suitably modifying Adler's equation, we can arrive at a dynamical equation for the coupled oscillator system.

Adler's equation for injection locking is

$$\frac{d\phi_0}{dt} = -\frac{A_{\text{inj}}}{A_0}\frac{\omega_0}{2Q}\sin(\phi_0 - \psi_{\text{inj}}) + (\omega_0 - \omega_{\text{inj}}) \qquad (1)$$

where ϕ_0 = phase of oscillator, ψ_{inj} = phase of injected signal, ω_0 = free-running frequency of oscillator, ω_{inj} = frequency of injected signal from an external or neighboring oscillator, A_0 = free-running amplitude of oscillator (voltage), A_{inj} = amplitude of injected signal (voltage), and Q = the external Q of the oscillator circuit. The oscillator phase is defined relative to the injected signal, so that $\omega = \omega_{\text{inj}} + d\phi_0/dt$ is the instantaneous frequency of the oscillator. Generalizing (1) to the case of several simultaneously injected signals, and describing the mutual coupling by a complex coupling coefficient, $\lambda_{ij}\ \exp(\jmath\Phi_{ij})$, gives the set of equations

$$\omega = \omega_i \left[1 - \sum_{j\neq i}^{N} \frac{\lambda_{ij}}{2Q_i}\frac{A_j}{A_i}\sin(\phi_i - \phi_j + \Phi_{ij})\right] \qquad i = 1, 2, \ldots, N \tag{2}$$

where N is the number of oscillators in the system. Note that the instantaneous frequency ω is the same for all oscillators in the system *at synchronization.* Equation (2) describes a set of equations which can be used to determine the steady-state of the system.

Some simple analytical results can be derived from (2) by considering a linear chain of oscillators with only nearest neighbor interactions. Assuming that the coupling is the same between adjacent elements in the array, $\lambda_{ij} \equiv \lambda$ and $\Phi_{ij} \equiv \Phi$. Furthermore, let $Q \equiv Q_i$, $\lambda' \equiv \lambda/2Q$, $\rho_i \equiv A_{i-1}/A_i$, and $\Delta\phi_i \equiv \phi_i - \phi_{i-1}$. The set of governing equations becomes

$$\omega = \omega_i \left[1 - \lambda'\rho_i \sin(\Phi + \Delta\phi_i) - \lambda' \sin(\Phi - \Delta\phi_{i+1})/\rho_{i+1}\right] \qquad n = 1, 2, \ldots, N \tag{3}$$

Note that $\rho_1 = 1/\rho_{N+1} = 0$. Despite the simplification, this is still a set of coupled nonlinear equations. In general there are many possible phase distributions which satisfy (3), but not all are stable solutions. Stability of these modes can be investigated using a perturbation analysis. Recalling that $\omega = \omega_{\text{inj}} + d\phi_i/dt$, gives

$$\begin{aligned}\frac{d}{dt}\Delta\phi_i = {} & \omega_i \left[1 - \lambda'\rho_i \sin(\Phi + \Delta\phi_i) - \lambda' \sin(\Phi - \Delta\phi_{i+1})/\rho_{i+1}\right] \\ & - \omega_{i-1}\left[1 - \lambda'\rho_{i-1}\sin(\Phi + \Delta\phi_{i-1}) - \lambda'\sin(\Phi - \Delta\phi_i)/\rho_i\right]\end{aligned} \tag{4}$$

The phase is then perturbed by a small amount by letting $\Delta\phi_i \to \Delta\phi_i + \delta_i$. After some algebra this leads to

$$\begin{aligned}\frac{d}{dt}\delta_i &= a_i\delta_{i-1} + b_i\delta_i + c_i\delta_{i+1} \qquad i = 2, 3, \ldots, N \\ \text{where} \quad a_i &= \lambda'\rho_{i-1}\omega_{i-1}\cos(\Phi + \Delta\phi_{i-1}) \\ b_i &= -\lambda'\omega_i\rho_i\cos(\Phi + \Delta\phi_i) - \lambda'\omega_{i-1}\cos(\Phi - \Delta\phi_i)/\rho_i \\ c_i &= \lambda'\omega_i\cos(\Phi - \Delta\phi_{i+1})/\rho_{i+1}\end{aligned} \tag{5}$$

This is of the form of a matrix equation, $d\underline{\delta}/dt = \mathbf{A}\underline{\delta}$, where $\mathbf{A}$ is a tridiagonal matrix, and $\underline{\delta}$ is a column vector with elements δ_i. A stable solution for the phase distribution requires $\underline{\delta}$ to decay with time. This is satisfied when all the eigenvalues of $\mathbf{A}$ have negative real parts.

For power combining applications, we desire a mode of operation where all elements are in phase ($\Delta\phi = 0$). Substituting this into (3) for a chain of identical oscillators, we find that the allowed values of coupling phase are determined by the boundary elements, and are given by $\Phi = n\pi$, where $n = 0, 1, 2 \ldots$. Note that for an infinite chain, any value of coupling phase would satisfy (3). From the stability analysis (5), we find that the eigenvalues of the matrix can have negative real parts only if $\cos\Phi > 0$, so a stable, in-phase mode is only possible if $\Phi = 0, 2\pi, \ldots$.

There are several ways of accomplishing the desired inter-element coupling. Simple proximty-coupling, through free-space and surface waves, could be used, but this means that the elements must be spaced at multiples of one wavelength. Such spacing is generally unacceptable because of grating lobes in the antenna patterns. Another possibility is to use a quasi-optical reflector element (such as a dielectric slab) over the array. This is an attractive alternative since the coupling can be varied by moving this reflector element above the array, thus introducing additional flexibility into the design. Consistent with the weak-coupling assumption, the reflectivity of this element must be small. Another alternative is to use a microstrip coupling circuit to provide a weak coupling signal at the required phase angle.

Since we have chosen to characterize our arrays by this complex coupling parameter, some mention should be made regarding its measurement. Recently a technique has been described [8] for the measurement of mutual coupling between two oscillators. Using this technique, it is possible to determine the complex coupling coefficient directly.

Experimental Array Results

The first experimental array of weakly coupled oscillators was a 16-element array using packaged Gunn diodes, shown in figure 1 [9]. This array design uses individual bias to each device, which was required due to device non-uniformities. Elements of the array are spaced a half free space wavelength apart, a distance which was initially selected based on curves

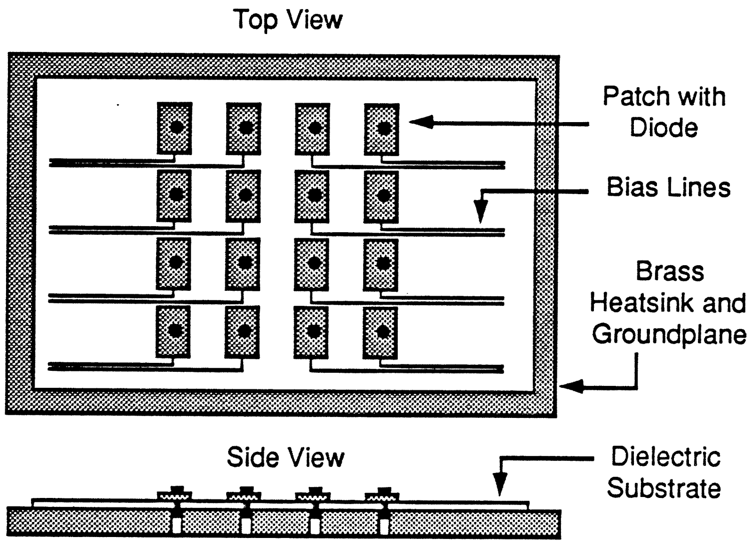

Figure 1 — Diagram of Gunn diodes mounted into a 4×4 array of microstrip patches. The brass block serves as a groundplane, heatsink, and DC bias return. Individual bias to each element is applied at an RF null.

in [2]. Thus a reflector proved necessary for proper operation. The array was designed on a 60 mil substrate with $\varepsilon_r = 4.1$.

Each diode was first biased, one at a time, to establish a common operating frequency. These individual biases were then applied simultaneously. Single frequency operation was verified with a spectrum analyzer, as shown in figure 2b. Spectra resembling figure 2a result when the elements are not all in synchronization. Radiation pattern measurements were made at the final oscillation frequency of 9.6 GHz. The total radiated power was estimated from these patterns to be 415 mW, giving an effective radiated power (ERP) of 22 Watts. The overall DC to RF conversion efficiency was low, typical of Gunn diodes, around 1%.

A second array was constructed with MESFET devices. Much higher efficiency, larger tuning range, and better noise properties can be obtained using FET devices. The experimental MESFET array is depicted in figure 3. The individual element design [5] uses a general purpose Fujitsu device (FSX02). Half as many bias lines were used in this array design, with bias isolation between the elements provided using a 6-turn coil. The gate resistor was used to supress bias circuit oscillations. Again, the spacing was chosen to be less than one wavelength to avoid grating lobes, which meant that a reflector element was required for in-phase operation. The array was fabricated on a 93 mil Duroid 5870 substrate ($\varepsilon_r = 2.33$).

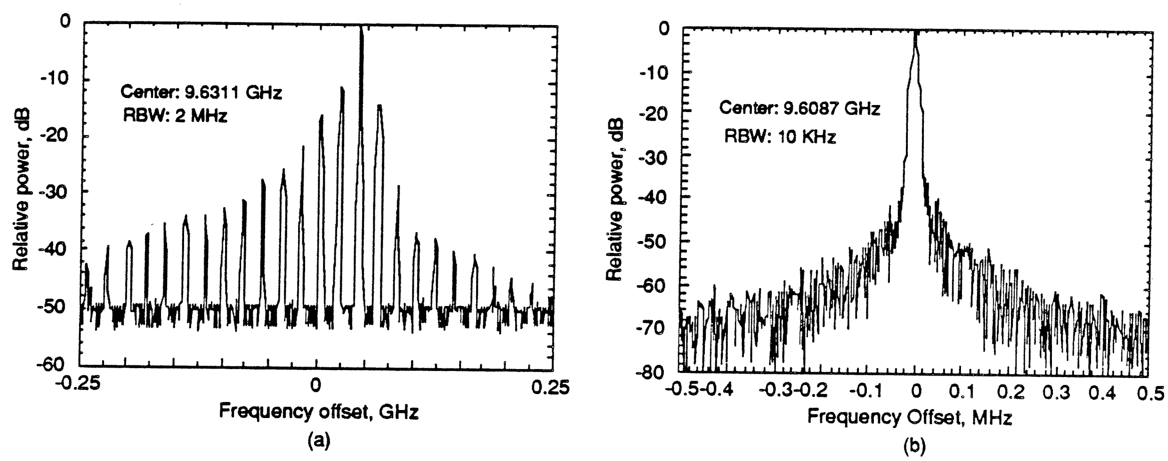

Figure 2 — (a) Spectrum of the Gunn array on the threshold of synchronization, and (b) at full synchronization. These measurements were made with the dielectric reflector in place.

As with the Gunn array, the power-on sequence was to tune each group of elements individually to set a common operating frequency, and then apply DC power to all elements at once. Varying the reflector element spacing is then usually sufficient to enforce mutual synchronization. With single frequency operation verified at 8.27 GHz , the patterns of figure 4 were measured. These patterns closely correspond to the expected pattern when the elements are all in phase. A total radiated power of 184 mW was estimated from the pattern measurements, giving a 10 Watt ERP and 11.5 mW per device at 26% efficiency. The results of the MESFET array are seen to be similar to that of the Gunn array, but with a much higher DC-to-RF efficiency, which makes it a more attractive design.

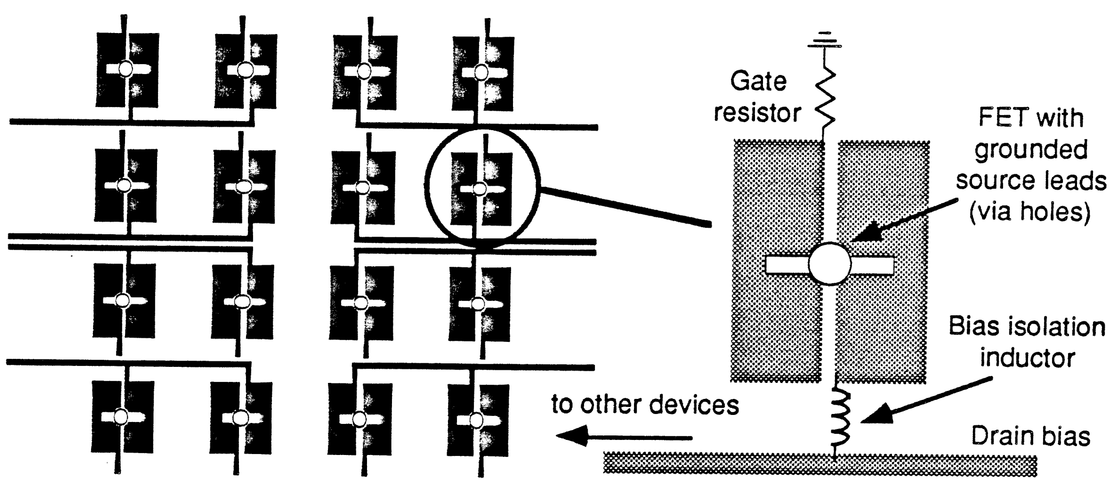

Figure 3 — Sketch of the array which uses Fujitsu fsx02 MESFETs, showing bias arrangement and individual element design. Elements measure 11 mm by 15 mm and the spacing of the elements is 0.67 λ_0 between centers. The bias inductor reduces element interactions along the bias line.

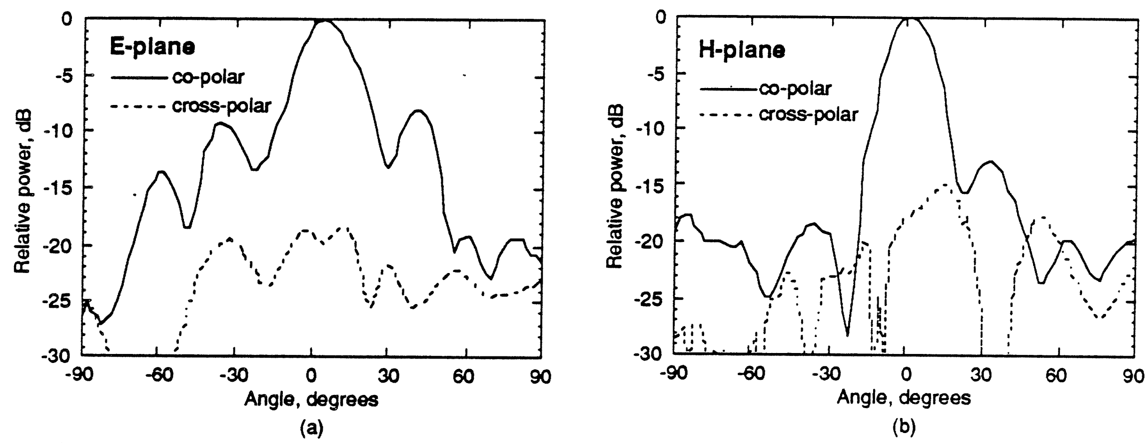

Figure 4 — (a) E-plane and (b) H-plane patterns for the 4×4 MESFET array. The measurements were made at 8.27 GHz, using a flat, 2.5 cm thick dielectric reflector with a dielectric constant of 4. The good patterns indicate in-phase operation.

Conclusion

An architecture for quasi-optical power combining has been discussed, which involves mutually synchronized arrays of single-device oscillators. The oscillators are allowed to interact weakly in order to synchronize the frequency and phase relationships. A quasi-optical reflector element was found to aid in establishing a desired phase relationship. A simple theory based on Adler's equation has also been presented, which establishes certain design guidelines for these types of arrays.

The proposed concepts have been verified using two 16-element X-band arrays. Both of the arrays were constructed using hybrid assembly techniques. For large arrays, biasing will be an important issue. Individual bias to each element also allows the system to degrade gracefully—for multiple devices on a single bias, failure of one device often leads to failure of all of the devices. However, individual bias to all the elements of a large array containing several hundred devices is impractical. Proper operation of the array requires that the elements have nearly identical characteristics, which would allow them to be biased from a common source. Monolithically fabricated arrays could yield the required uniformity between elements to make this technique possible, and are expected to be the best test of theory and design concepts presented here.

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