# Simulation and Optimization in an AGC System after Deregulation

Vaibhav Donde, M. A. Pai, Fellow, IEEE, and Ian A. Hiskens, Senior Member, IEEE

*Abstract*—In this paper, the traditional AGC two-area system is modified to take into account the effect of bilateral contracts on the dynamics. The concept of DISCO participation matrix to simulate these bilateral contracts is introduced and reflected in the two-area block diagram. Trajectory sensitivities are used to obtain optimal parameters of the system using a gradient Newton algorithm.

*Index Terms*—Automatic generation control, bilateral contracts, deregulation, optimization, power system control, trajectory sensitivity.

#### I. INTRODUCTION

I N A restructured power system, the engineering aspects of planning and operation have to be reformulated although essential ideas remain the same. With the emergence of the distinct identities of GENCOS, TRANSCOS, DISCOS and the ISO, many of the ancillary services of a vertically integrated utility will have a different role to play and hence have to be modeled differently. Among these ancillary services is the automatic generation control (AGC). In the new scenario, a DISCO can contract individually with a GENCO for power and these transactions are done under the supervision of the ISO or the RTO.

In this paper, we formulate the two area dynamic model following the ideas presented by Kumar *et al.* [1], [2]. Specifically we focus on the dynamics, trajectory sensitivities and parameter optimization. The concept of a DISCO participation matrix (DPM) is proposed which helps the visualization and implementation of the contracts. The information flow of the contracts is superimposed on the traditional AGC system and the simulations reveal some interesting patterns. The trajectory sensitivities are helpful in studying the effects of parameters as well as in optimization of the ACE parameters *viz.* tie line bias *K* and frequency bias parameter *B*. The traditional AGC is well discussed in the papers of Elgerd and Fosha [3], [4]. Research work in deregulated AGC is contained in [1], [2], [5], [6].

The paper is organized as follows. In Section II, we explain how the bilateral transactions are incorporated in the traditional AGC system leading to a new block diagram. Simulation results are presented in Section III. In Section IV, we discuss trajectory sensitivities and the optimization of K and B parameters using these sensitivities. Section V presents numerical results on optimization and a comparison of the responses using optimal and

The authors are with the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801.

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nominal values of the parameters. This is followed by conclusions in Section VI.

#### **II. RESTRUCTURED SYSTEM**

#### A. Traditional vs. Restructured Scenario

The traditional power system industry has a "vertically integrated utility" (VIU) structure. In the restructured or deregulated environment, vertically integrated utilities no longer exist. The utilities no longer own generation, transmission, and distribution; instead, there are three different entities, *viz.*, GENCOs (generation companies), TRANSCOs (transmission companies) and DISCOs (distribution companies).

As there are several GENCOs and DISCOs in the deregulated structure, a DISCO has the freedom to have a contract with any GENCO for transaction of power. A DISCO may have a contract with a GENCO in another control area. Such transactions are called "bilateral transactions." All the transactions have to be cleared through an impartial entity called an independent system operator (ISO). The ISO has to control a number of so-called "ancillary services," one of which is AGC. For an in-depth discussion of implications of restructuring the power industry, refer to [7]–[9].

#### B. DISCO Participation Matrix

In the restructured environment, GENCOs sell power to various DISCOs at competitive prices. Thus, DISCOs have the liberty to choose the GENCOs for contracts. They may or may not have contracts with the GENCOs in their own area. This makes various combinations of GENCO-DISCO contracts possible in practice. We introduce the concept of a "DISCO participation matrix" (DPM) to make the visualization of contracts easier. DPM is a matrix with the number of rows equal to the number of GENCOs and the number of columns equal to the number of DISCOs in the system. Each entry in this matrix can be thought of as a fraction of a total load contracted by a DISCO (column) toward a GENCO (row). Thus, the ijth entry corresponds to the fraction of the total load power contracted by DISCO j from a GENCO *i*. The sum of all the entries in a column in this matrix is unity. DPM shows the participation of a DISCO in a contract with a GENCO; hence the name "DISCO participation matrix." The notation follows along the lines of [1], [2]. Consider a two-area system in which each area has two GENCOs and two DISCOs in it. Let GENCO<sub>1</sub>, GENCO<sub>2</sub>, DISCO<sub>1</sub>, and DISCO<sub>2</sub> be in area I and GENCO<sub>3</sub>, GENCO<sub>4</sub>, DISCO<sub>3</sub>, and DISCO<sub>4</sub> be in area II as shown in Fig. 1.

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*which* GENCO has to follow a load demanded by *which* DISCO. The scheduled steady state power flow on the tie line is given as

$$\Delta P_{tie1-2, scheduled}$$
= (demand of DISCOs in area II from  
GENCOs in area I)  
- (demand of DISCOs in area I from  
GENCOs in area II). (3)

At any given time, the tie line power error  $\Delta P_{tie1-2, error}$  is defined as

$$\Delta P_{tie1-2,\,error} = \Delta P_{tie1-2,\,actual} - \Delta P_{tie1-2,\,scheduled}.$$
 (4)

 $\Delta P_{tie1-2, error}$  vanishes in the steady state as the actual tie line power flow reaches the scheduled power flow. This error signal is used to generate the respective ACE signals as in the traditional scenario

$$ACE_1 = B_1 \Delta f_1 + \Delta P_{tie1-2,error}$$
$$ACE_2 = B_2 \Delta f_2 + \Delta P_{tie2-1,error}$$
(5)

where

$$\Delta P_{tie2\text{-}1,\,error} = -\frac{P_{r1}}{P_{r2}}\,\Delta P_{tie1\text{-}2,\,error}$$

and  $P_{r1}$ ,  $P_{r2}$  are the rated powers of areas I and II, respectively. Therefore

$$ACE_2 = B_2 \Delta f_2 + \alpha_{12} \Delta P_{tie1-2, error}$$

where

$$\alpha_{12} = -\frac{P_{r1}}{P_{r2}}.$$

The block diagram for AGC in a deregulated system is shown in Fig. 2. Structurally it is based upon the idea of [1], [2]. Dashed lines show the demand signals. The local loads in areas I and II are denoted by  $\Delta P_{L1, LOC}$  and  $\Delta P_{L2, LOC}$ , respectively.

# D. State Space Characterization of the Two-Area System in Deregulated Environment

The closed loop system in Fig. 2 is characterized in state space form as

$$\dot{x} = A^{cl}x + B^{cl}u \tag{6}$$

where x is the state vector and u is the vector of power demands of the DISCOs.  $A^{cl}$  and  $B^{cl}$  matrices are constructed from Fig. 2. The state matrices are given by (7) and (8).

# III. SIMULATION RESULTS OF A TWO-AREA SYSTEM IN THE DEREGULATED ENVIRONMENT

A two-area system is used to illustrate the behavior of the proposed AGC scheme. The same data as in [3], [4] is used for simulations. Both the areas are assumed to be identical. The governor-turbine units in each area are assumed to be identical.



The corresponding DPM will become

where cpf refers to "contract participation factor." Suppose that DISCO<sub>3</sub> demands 0.1 pu MW power, out of which 0.025 pu MW is demanded from GENCO<sub>1</sub>, 0.03 pu MW from GENCO<sub>2</sub>, 0.035 pu MW from GENCO<sub>3</sub> and 0.01 pu MW from GENCO<sub>4</sub>. Then column 3 entries in (1) are easily defined as

$$cpf_{13} = \frac{0.025}{0.1} = 0.25, \quad cpf_{23} = \frac{0.03}{0.1} = 0.3,$$
  
 $cpf_{33} = \frac{0.035}{0.1} = 0.35, \quad cpf_{43} = \frac{0.01}{0.1} = 0.1.$  (2)

Other cpfs are defined similarly to obtain the entire DPM. It is noted that  $\sum_i cpf_{ij} = 1$ . The block diagonals of DPM correspond to local demands. Off diagonal blocks correspond to the demands of the DISCOs in one area to the GENCOs in another area.

#### C. Block Diagram Formulation

In this section, we formulate the block diagram for a two-area AGC system in the deregulated scenario. Whenever a load demanded by a DISCO changes, it is reflected as a local load in the area to which this DISCO belongs. This corresponds to the local loads  $\Delta P_{L1}$  and  $\Delta P_{L2}$  and should be reflected in the deregulated AGC system block diagram at the point of input to the power system block. As there are many GENCOs in each area, ACE signal has to be distributed among them in proportion to their participation in the AGC. Coefficients that distribute ACE to several GENCOs are termed as "ACE participation factors" (apfs). Note that  $\sum_{j=1}^{m} apf_j = 1$  where m is the number of GENCOs. Unlike in the traditional AGC system, a DISCO asks/demands a particular GENCO or GENCOs for load power. These demands must be reflected in the dynamics of the system. Turbine and governor units must respond to this power demand. Thus, as a particular set of GENCOs are supposed to follow the load demanded by a DISCO, information signals must flow from a DISCO to a particular GENCO specifying corresponding demands. Here, we introduce the information signals which were absent in the traditional scenario. The demands are specified by  $cpf_s$  (elements of DPM) and the pu MW load of a DISCO. These signals carry information as to





Fig. 2. Two-area AGC system block diagram in restructured scenario.

#### A. Case 1: Base Case

Consider a case where the GENCOs in each area participate equally in AGC; i.e., ACE participation factors are  $apf_1 = 0.5$ ,  $apf_2 = 1 - apf_1 = 0.5$ ;  $apf_3 = 0.5$ ,  $apf_4 = 1 - apf_3 = 0.5$ . Assume that the load change occurs only in area I. Thus, the load is demanded only by DISCO<sub>1</sub> and DISCO<sub>2</sub>. Let the value of this load demand be 0.1 pu MW for each of them. Referring to (1), DPM becomes,

$$DPM = \begin{bmatrix} 0.5 & 0.5 & | & 0 & 0 \\ 0.5 & 0.5 & | & 0 & 0 \\ - & - & - & - & - \\ 0 & 0 & | & 0 & 0 \\ 0 & 0 & | & 0 & 0 \end{bmatrix}.$$

Note that DISCO<sub>3</sub> and DISCO<sub>4</sub> do not demand power from any GENCOs, and hence the corresponding participation factors (columns 3 and 4) are zero. DISCO<sub>1</sub> and DISCO<sub>2</sub> demand identically from their local GENCOs, *viz.*, GENCO<sub>1</sub> and GENCO<sub>2</sub>. Fig. 3 shows the results of this load change: area frequency deviations, actual power flow on the tie line (in a direction from area I to area II), and the generated powers of various GENCOs, following a step change in the load demands of DISCO<sub>1</sub> and DISCO<sub>2</sub>. The frequency deviation in each area goes to zero in the steady state [Fig. 3(a)]. As only the DISCOs in area I, *viz.* DISCO<sub>1</sub> and DISCO<sub>2</sub>, have nonzero load demands, the transient dip in frequency of area I is larger than that of area II. Since the off diagonal blocks of DPM are zero, i.e., there are no contracts of power between a GENCO in one area and a DISCO in another area, the scheduled steady state power flow over the tie line is zero. The actual power on the tie line goes to zero.

In the steady state, generation of a GENCO must match the demand of the DISCOs in contract with it. This desired generation of a GENCO in pu MW can be expressed in terms of cpfs and the total demand of DISCOs as

$$\Delta P_{Mi} = \sum_{j} cp f_{ij} \Delta P_{Lj} \tag{9}$$

where  $\Delta P_{Lj}$  is the total demand of DISCO j and cpfs are given by DPM. In the two-area case,

$$\Delta P_{Mi} = cpf_{i1}\Delta P_{L1} + cpf_{i2}\Delta P_{L2} + cpf_{i3}\Delta P_{L3} + cpf_{i4}\Delta P_{L4}.$$
(10)

For the case under consideration, we have,

$$\Delta P_{M1} = 0.5 \times \Delta P_{L1} + 0.5 \times \Delta P_{L2} = 0.1$$
 pu MW

and similarly,

$$\Delta P_{M2} = 0.1$$
 pu MW,  
 $\Delta P_{M3} = 0$  pu MW,  
 $\Delta P_{M4} = 0$  pu MW.

As Fig. 3(c) shows, the actual generated powers of the GENCOs reach the desired values in the steady state. GENCO<sub>3</sub> and GENCO<sub>4</sub> are not contracted by any DISCO for a transaction of power; hence, their change in generated power is zero in the steady state.

## B. Case 2

Consider a case where all the DISCOs contract with the GENCOs for power as per the following DPM:

$$DPM = \begin{bmatrix} 0.5 & 0.25 & | & 0 & 0.3 \\ 0.2 & 0.25 & | & 0 & 0 \\ - & - & - & - & - \\ 0 & 0.25 & | & 1 & 0.7 \\ 0.3 & 0.25 & | & 0 & 0 \end{bmatrix}.$$

It is assumed that each DISCO demands 0.1 pu MW power from GENCOs as defined by cpfs in DPM matrix and each GENCO participates in AGC as defined by following apfs:  $apf_1 = 0.75$ ,  $apf_2 = 1 - apf_1 = 0.25$ ;  $apf_3 = 0.5$ ,  $apf_4 = 1 - apf_3 = 0.5$ . ACE participation factors affect only the transient behavior of the system and not the steady state behavior when uncontracted loads are absent.

The system in Fig. 2 is simulated using this data and the results are depicted in Fig. 4. The off diagonal blocks of the DPM correspond to the contract of a DISCO in one area with a GENCO in another area. From (1) and (3), the scheduled power on the tie line in the direction from area I to area II is

$$\Delta P_{tie1-2, scheduled} = \sum_{i=1}^{2} \sum_{j=3}^{4} cp f_{ij} \Delta P_{Lj} - \sum_{i=3}^{4} \sum_{j=1}^{2} cp f_{ij} \Delta P_{Lj}.$$
(11)

Hence,  $\Delta P_{tie1-2, scheduled} = -0.05$  pu MW. Fig. 4(b) shows the actual power on the tie line. It is to be observed that it settles to -0.05 pu MW, which is the scheduled power on the tie line in the steady state.

As given by (10), in the steady state, the GENCOs must generate

$$\Delta P_{M1} = 0.5(0.1) + 0.25(0.1) + 0 + 0.3(0.1)$$
  
= 0.105 pu MW

and

$$\Delta P_{M2} = 0.045$$
 pu MW,  $\Delta P_{M3} = 0.195$  pu MW,  
 $\Delta P_{M4} = 0.055$  pu MW.

Fig. 4(c) shows actual generated powers of the GENCOs. The trajectories reach respective desired generations in the steady state.

### C. Case 3: Contract Violation

It may happen that a DISCO violates a contract by demanding more power than that specified in the contract. This excess power is not contracted out to any GENCO. This uncontracted power must be supplied by the GENCOs in the same area as the DISCO. It must be reflected as a local load of the area but not as the contract demand. Consider case 2 again with a modification that  $DISCO_1$  demands 0.1 pu MW of excess power.

The total local load in area I ( $\Delta P_{L1, LOC}$ ) = Load of DISCO<sub>1</sub> + load of DISCO<sub>2</sub> = (0.1 + 0.1) + 0.1 pu MW = 0.3 pu MW

Similarly, the total local load in area II ( $\Delta P_{L2,LOC}$ )

= Load of 
$$DISCO_3 + load$$
 of  $DISCO_4$ 

= 0.2 pu MW (no uncontracted load)

The frequency deviations vanish in the steady state [Fig. 5(a)]. As DPM is the same as in case 2 and the excess load is taken up by GENCOs in the same area, the tie line power is the same as in case 2 in steady state. The generation of GENCOs 3 and 4 is not affected by the excess load of DISCO<sub>1</sub> (refer case 2). The uncontracted load of DISCO<sub>1</sub> is reflected in the generations of GENCO<sub>1</sub> and GENCO<sub>2</sub>. ACE participation factors decide the distribution of uncontracted load in the steady state. Thus, this excess load is taken up by the GENCOs in the same area as that of the DISCO making the uncontracted demand.

Discussion: In the proposed AGC implementation, contracted load is fed forward through the DPM matrix to GENCO setpoints. This is shown in Fig. 2 by dotted lines. The actual loads affect system dynamics via the inputs  $\Delta P_{L, LOC}$  to the power system blocks. Any mismatch between actual and contracted demands will result in a frequency deviation that will drive AGC to redispatch GENCOs according to apfs.

Comments:

- 1) It is assumed that each control area contains at least one GENCO that participates in AGC, i.e., has nonzero *apf*.
- 2) The proposed AGC scheme does not require measurement of actual loads. The inputs  $\Delta P_{L, LOC}$  in the block diagram of Fig. 2 are part of the power system model, not part of AGC.

#### IV. TRAJECTORY SENSITIVITIES AND OPTIMIZATION

#### A. Trajectory Sensitivities

The two-area system in the deregulated case with identical areas can be optimized with respect to system parameters to obtain the best response. The parameters involved in the feedback are the integral feedback gains  $(K_1 = K_2 = K)$  and the frequency bias  $(B_1 = B_2 = B)$ . The optimal values of K and B



Fig. 3. (a) Frequency deviations (rad/s). (b) Tie line power (pu MW). (c) Generated power (pu MW).



Fig. 4. (a) Frequency deviations (rad/s). (b) Tie line power (pu MW). (c) Generated power (pu MW).

depend on the cost function used for optimization [10]. The integral of square error criterion is chosen for this case [3],

$$C = \int_0^\infty \left[ \alpha (\Delta P_{tie, \, error})^2 + \beta (\Delta f_1)^2 \right] dt.$$
 (12)

The "equi-B" cost curves can be plotted as shown in [3], [4] for  $\alpha = 1$  and  $\beta = 1$ . This was done for the DPM given for case



Fig. 5. (a) Frequency deviations (rad/s). (b) Tie line power (pu MW). (c) Generated power (pu MW).

1, with the contracted load in area I as 0.1 pu MW and an uncontracted load of 0.1 pu MW in the same area. The optimum values for K and B were found to be the same as for the traditional two-area case, namely, K = 0.6588 and B = 0.439.

TABLE I NUMERICAL RESULTS OF OPTIMIZATION

Iteration	$ \lambda_{k+1} - \lambda_k $	$\lambda = \begin{bmatrix} K & B \end{bmatrix}^T$	
		K	B/B <sub>nom</sub>
1	0.1211	1	1
2	0.0942	0.914	0.9088
3	0.0752	0.8453	0.7523
4	0.06429	0.6584	1.0135
5	0.034062	0.7261	0.9326
6	0.003982	0.6615	1.0298
7	0.002186	0.6586	1.0249
8	0.000358	0.6588	1.0335

A more systematic approach to the optimization can be achieved by using trajectory sensitivities in conjunction with agradient type Newton algorithm [11]. Equation (6) is the state space form of the system. Differentiating with respect to a parameter  $\lambda$ ,

$$\dot{x}_{\lambda} = \left(A^{cl}\right)_{\lambda} x + A^{cl} x_{\lambda} + \left(B^{cl}\right)_{\lambda} u \tag{13}$$

where the subscript " $\lambda$ " denotes a derivative. Equations (6) and (13) can be solved simultaneously to obtain trajectory sensitivities  $x_{\lambda}$ .

- B. Gradient Type Newton Algorithm
  - 1) Initialize  $\lambda$

$$\lambda = \lambda_0 = \begin{bmatrix} K_0 & B_0 \end{bmatrix}^T.$$
 (14)

2) Simulate the system given by (6) and (13) to obtain x and  $x_{\lambda}$  where

$$x_{\lambda} = \begin{bmatrix} x_K & x_B \end{bmatrix}^T.$$
(15)

3) Obtain the gradient of the cost function as

$$\nabla C(\lambda) = \begin{bmatrix} \frac{\partial C}{\partial K} & \frac{\partial C}{\partial B} \end{bmatrix}^T \tag{16}$$

where  $\partial C/\partial K$  and  $\partial C/\partial B$  are obtained by differentiating the cost function partially w.r.t. K and B, respectively as

$$\frac{\partial C}{\partial K} = 2 \int_0^\infty \left( \alpha \Delta P_{tie,1} \frac{\partial \Delta P_{tie,1}}{\partial K} + \beta \Delta f_1 \frac{\partial \Delta f_1}{\partial K} \right) dt$$
$$\frac{\partial C}{\partial B} = 2 \int_0^\infty \left( \alpha \Delta P_{tie,1} \frac{\partial \Delta P_{tie,1}}{\partial B} + \beta \Delta f_1 \frac{\partial \Delta f_1}{\partial B} \right) dt.$$
(17)

4) Compute the Hessian of the cost function as

$$H_C(\lambda) = \begin{bmatrix} \frac{\partial^2 C}{\partial K^2} & \frac{\partial}{\partial B} \left( \frac{\partial C}{\partial K} \right) \\ \frac{\partial}{\partial K} \left( \frac{\partial C}{\partial B} \right) & \frac{\partial^2 C}{\partial B^2} \end{bmatrix}.$$
 (18)

The Hessian is computed from the gradient by numerical differencing.

6) If  $\|\lambda_{k+1} - \lambda_k\| \le \varepsilon$ , go to step 7), else go to step 2).  $\varepsilon$  is a small positive number that defines the accuracy of the result  $\lambda$ . 7) END This "trajectory sensitivity approach" to optimization will be useful for any general control strategy, particularly when nonlinearities are involved. V. NUMERICAL RESULTS OF OPTIMIZATION Initial values are assumed to be  $K_0 = 1$  and  $B_0 = B_{nom} =$ D + (1/R) = 0.425 pu MW/Hz. The numerical results from the iterations are listed in Table I. The minimum obtained (Table I) is consistent with that obtained from equi-B curves. The system is simulated with the nominal as well as the optimum values of K and B and the trajectories are compared with each other in Fig. 6. It is evident that the optimal trajectories reach the steady state faster, thus, providing better performance.

## VI. CONCLUSION

The important role of AGC will continue in restructured electricity markets, but with modifications. Bilateral contracts can exist between DISCOs in one control area and GENCOs in other control areas. The use of a "DISCO Participation Matrix" facilitates the simulation of bilateral contracts. Using trajectory sensitivities, optimum values of K and B are obtained.

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Vaibhav Donde received the B.E. degree in electrical engineering from Veermata Jijabai Technological Institute, Mumbai (India) in 1998. He is currently working toward his Ph.D. degree at the University of Illinois at Urbana-Champaign.

Fig. 6. (a) Trajectories with nominal K and B. (b) Trajectories with optimal K and B.

Tie line power (pu MW)

(b)

30 Lim + (# )

5) Update the parameter vector  $\lambda$  using Newton iterations

$$\lambda_{k+1} = \lambda_k - H_C^{-1}(\lambda_k) \nabla C(\lambda_k).$$
<sup>(19)</sup>

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**M. A. Pai** (F'86) received the B.E. degree from the University of Madras, Madras, India, in 1953 and the M.S. and Ph.D. degrees from the University of California, Berkeley, in 1958 and 1961, respectively. He was on the faculty of the Indian Institute of Technology, Kanpur from 1963 to 1981. Since 1981, he has been on the faculty of the University of Illinois at Urbana-Champaign as a Professor of Electrical and Computer Engineering. **Ian A. Hiskens** (S'78–M'84–SM'96) received the B.Eng. and B.App.Sc. degrees from the Capricornia Institute of Advanced Education, Rockhampton, Australia, in 1980 and 1983, respectively, and the Ph.D. degree from the University of Newcastle, Newcastle, Australia, in 1990. From 1980 to 1992, he was with the Queensland Electricity Supply Industry. He was a Senior Lecturer in the Department of Electrical and Computer Engineering at the University of Newcastle from 1992 to 1999 and is currently a Visiting Associate Professor at the University of Illinois at Urbana-Champaign.