Bounding Uncertainty in Power System Dynamic Simulations

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Abstract— **Parameters of power system models can never be known exactly. Yet dynamic security assessment relies upon the simulations derived from those uncertain models. This paper proposes an approach to quantifying the uncertainty in simulations of power system dynamic behaviour. It is shown that trajectory sensitivities can be used to generate an accurate first order approximation of the trajectory corresponding to a perturbed parameter set. The computational cost of obtaining the sensitivities and perturbed trajectory is minimal. Therefore it is feasible to quickly generate many approximate trajectories from a single nominal case. To quantify the effect of parameter uncertainty on the nominal case, parameter sets are randomly generated according to their underlying statistical distribution. An approximate trajectory is obtained for each set. The collection of trajectories provides a bound within which the actual system dynamic behaviour should lie.**

Keywords: Uncertain systems; power system simulation; power system security; sensitivity.

I. Introduction

Analysis of power system dynamic behaviour is largely model based. Actual system behaviour is inferred from the simulated response of system models formed from many component models. Planning and operating decisions are influenced by simulated behaviour, therefore model accuracy is very important. Model validation clearly plays a vital role [1], [2]. However many parameters can never be known with absolute certainty; load modelling provides a classic example [3].

Ideally, multiple studies should be undertaken over a range of parameter values. However simulation of large power systems is computationally intensive. Such an investigation would be extremely time-consuming. The more practical approach is to assume that a *nominal* set of parameters provides a good representation of behaviour over the full range of values. This may not always be a good assumption though.

An approach to quantifying the uncertainty in power system simulation is presented in this paper. The ideas build on trajectory sensitivity analysis. The paper is structured as follows. Modelling issues are discussed in Section II, with a general model structure given in Appendix A. Trajectory sensitivities are reviewed in Section III. The algorithm for quantifying trajectory uncertainty is presented in Section IV. Conclusions are provided in Section V.

II. Modelling

Power systems are composed of many diverse components. Interactions between the components can result in complicated forms of dynamic behaviour. The influences on system response include:

- continuous nonlinear dynamics, examples include electrical machines and their associated controllers and mechanical plant;
- algebraic constraints, primarily current balance at network nodes;
- discontinuities, for example control and physical limits, and feeder switching;
- discrete-event dynamics, such as protection relaying and tap-changer operation.

Nonlinear differential-algebraic models provide a familiar framework for representing the continuous dynamics of power systems. However less attention has been focussed on rigorous modelling of discontinuities and discrete-event dynamics. For example, Figure 1 is representative of the rules governing tap-changer operation. The mapping of these rules into an analytical model is not straightforward.

The development of a model that can capture the full range of continuous/discrete (hybrid) dynamic behaviour is outside the scope of this paper. Details can be found in [4]. For completeness however, a model is provided in Appendix A. This model forms the basis for the trajectory sensitivity analysis presented in Section III.

Trajectories of the model (7)-(10) describe the behaviour of the dynamic states $\underline{x}$ and algebraic states y over time. To faciliate the discussion of trajectory sensitivities and trajectory uncertainty, this dynamic behaviour shall be defined in terms of the flow

$$\phi(\underline{x}_0, t) = \begin{bmatrix} \phi_{\underline{x}}(\underline{x}_0, t) \\ \phi_y(\underline{x}_0, t) \end{bmatrix} = \begin{bmatrix} \underline{x}(t) \\ y(t) \end{bmatrix} \qquad (1)$$

where $\underline{x}_0$ are initial conditions, i.e.,

$$\phi(\underline{x}_0, t_0) = \begin{bmatrix} \underline{x}(t_0) \\ y(t_0) \end{bmatrix} = \begin{bmatrix} \underline{x}_0 \\ y_0 \end{bmatrix}. \qquad (2)$$

III. Trajectory Sensitivities

A. Background

The flow ϕ of a system will generally vary with changes in parameters and/or initial conditions. Trajectory sensitivity analysis provides a way of quantifying the changes in the flow that result from (small) changes in parameters

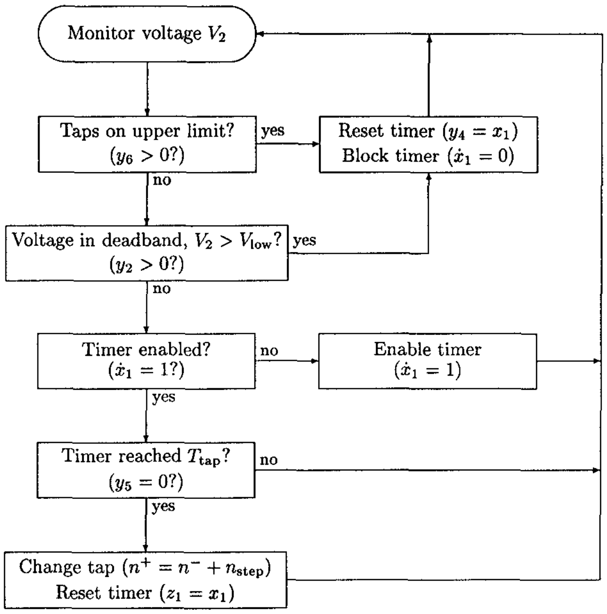

Fig. 1. Transformer AVR logic for increasing tap.

and initial conditions. The development of these sensitivity concepts will be based upon the DAD model (7)-(10). In this model, $\underline{x}_0$ incorporates the initial conditions x_0 and z_0, as well as the parameters λ. Therefore the sensitivity of the flow to $\underline{x}_0$ fully describes its sensitivity to x_0, z_0 and λ.

Trajectory sensitivities follow from a Taylor series expansion of the flows $\phi_{\underline{x}}$ and ϕ_y. Referring to (1), the expansion for $\phi_{\underline{x}}$ can be expressed as

$$\begin{aligned}\Delta\underline{x}(t) &= \Delta\phi_{\underline{x}}(\underline{x}_0, t)\\ &= \frac{\partial\phi_{\underline{x}}(\underline{x}_0, t)}{\partial\underline{x}_0}\Delta\underline{x}_0 + \text{ higher order terms.}\end{aligned}$$

Neglecting the higher order terms and using (1), we obtain

$$\begin{aligned}\Delta\underline{x}(t) &\approx \frac{\partial\underline{x}(t)}{\partial\underline{x}_0}\Delta\underline{x}_0\\ &\equiv \underline{x}_{\underline{x}_0}(t)\Delta\underline{x}_0\end{aligned} \tag{3}$$

i.e., the sensitivity of the flow $\phi_{\underline{x}}$ to (small) changes $\Delta\underline{x}_0$ is given by the trajectory sensitivities $\underline{x}_{\underline{x}_0}(t)$. A similar Taylor series expansion of ϕ_y yields

$$\begin{aligned}\Delta y(t) &\approx \frac{\partial y(t)}{\partial\underline{x}_0}\Delta\underline{x}_0\\ &\equiv y_{\underline{x}_0}(t)\Delta\underline{x}_0.\end{aligned} \tag{4}$$

In this case the sensitivity of the flow ϕ_y to (small) changes $\Delta\underline{x}_0$ is given by the trajectory sensitivities $y_{\underline{x}_0}(t)$.

Once the trajectory sensitivities $\underline{x}_{\underline{x}_0}(t)$ and $y_{\underline{x}_0}(t)$ are known, the sensitivity of the system flow ϕ to small changes in initial conditions and parameters, which are described by $\Delta\underline{x}_0$, can be determined from

$$\Delta\phi(\underline{x}_0, t) = \begin{bmatrix}\Delta\underline{x}(t)\\ \Delta y(t)\end{bmatrix} = \begin{bmatrix}\underline{x}_{\underline{x}_0}(t)\\ y_{\underline{x}_0}(t)\end{bmatrix}\Delta\underline{x}_0. \tag{5}$$

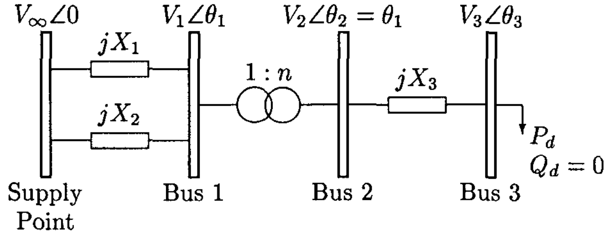

Fig. 2. Transformer-load example.

The computation of trajectory sensitivities is outlined in Appendix B. Complete details can be found in [4].

B. *First order approximation*

A change in parameters and/or initial conditions will usually generate a new trajectory. Based on the Taylor series expansion, a first order approximation of the new trajectory can be obtained from (5) as

$$\phi(\underline{x}_{0,2}, t) = \phi(\underline{x}_{0,1}, t) + \begin{bmatrix}\underline{x}_{\underline{x}_0}(t)\\ y_{\underline{x}_0}(t)\end{bmatrix}(\underline{x}_{0,2} - \underline{x}_{0,1}) \tag{6}$$

where $\underline{x}_{\underline{x}_0}(t)$, $y_{\underline{x}_0}(t)$ are computed along the original trajectory $\phi(\underline{x}_{0,1}, t)$. In other words, if the trajectory sensitivities $\underline{x}_{\underline{x}_0}(t)$, $y_{\underline{x}_0}(t)$ are available for a nominal trajectory, then (6) can be used to provide a good estimate of the trajectory $\phi(\underline{x}_{0,2}, t)$ corresponding to another (nearby) parameter set. This is illustrated in the following example.

Example 1

The small system of Figure 2 provides a case where continuous and discrete-event dynamics interact. The transformer tap-changer logic is described by Figure 1. The load exhibits dynamic recovery. Figure 3 shows the response of the voltage at bus 3 following the tripping of one of the feeders between the supply point and bus 1. The nominal trajectory, corresponding to a load time constant $T_p = 5$s and a tapping delay of $T_{\mathrm{tap}} = 20$s, is shown as a dashed line. This trajectory $V_3(t; 5, 20)$, along with the corresponding trajectory sensitivities $\frac{\partial V_3}{\partial T_p}$ and $\frac{\partial V_3}{\partial T_{\mathrm{tap}}}$, was used to generate a first order approximation of the voltage response when $T_p = 5.5$s and $T_{\mathrm{tap}} = 22$s, according to

$$\begin{aligned}V_3^{\mathrm{approx}}(t; 5.5, 22) =& V_3(t; 5, 20)\\ &+ 0.5\frac{\partial V_3}{\partial T_p}(t; 5, 20) + 2\frac{\partial V_3}{\partial T_{\mathrm{tap}}}(t; 5, 20).\end{aligned}$$

The approximate trajectory is shown in Figure 3, along with the actual perturbed trajectory $V_3(t; 5.5, 22)$. It can be seen that the first order approximation is very close.

□

The computational burden involved in generating the approximate trajectories is negligible. Therefore given the nominal trajectory and associated trajectory sensitivities, new (approximate) trajectories can be obtained for many parameter sets. This forms the basis of the procedure given in Section IV for quantifying trajectory uncertainty.

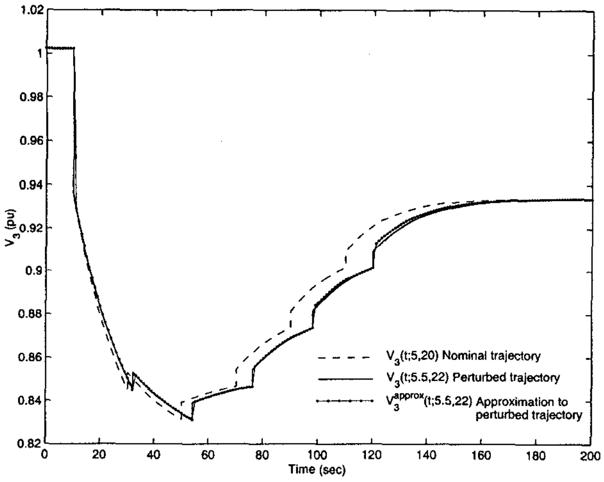

Fig. 3. Trajectory approximation for Example 1.

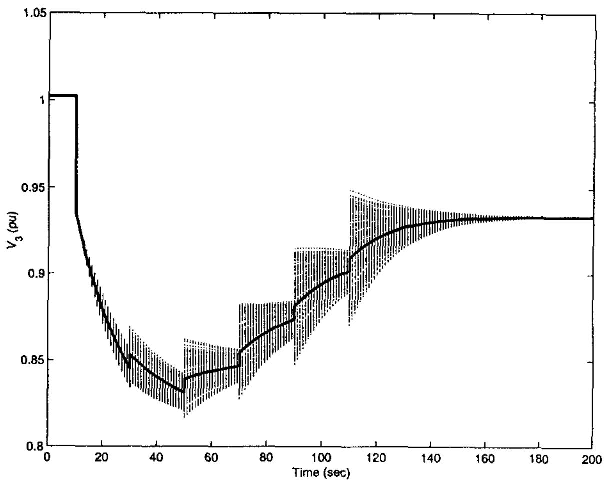

Fig. 4. Trajectory bounds for Example 1.

C. *Efficient calculation of sensitivities*

The feasibility of generating approximate trajectories using (6) rests on the efficient computation of the trajectory sensitivities. It appear from (13),(14) that the solution of high order differential equations is required. Fortunately that is not the case.

If an implicit numerical integration technique such as trapezoidal integration [5], [6] is used to obtain the nominal trajectory, then at each time step a Jacobian matrix must be built and factorized. It is shown in [4] that exactly the same Jacobian is required for numerical integration of the trajectory sensitivity equations (13),(14). Because this matrix is already factorized, little extra computation is involved in numerically integrating (13),(14).

IV. Bounding Uncertainty

Exact knowledge of parameter values is unlikely. It is more common for the range of a parameter value to be known. Also, an estimate of the statistical distribution may be available. For example, it might be assumed that parameters are uniformly distributed between some lower and upper bounds. Alternatively, there may be sufficient data to assume a normal distribution with some mean and variance. In any case, such distributions allow parameter sets to be randomly chosen.

As indicated earlier, simulation of a large number of parameter sets is not feasible. However generation of the first order approximation of a perturbed trajectory is feasible for a large number of cases. Therefore a Monte-Carlo technique can be employed to quantify the uncertainty in a trajectory:

- parameter sets are randomly generated,
- first order approximations are obtained using (6).

The following examples illustrate this process.

Example 1 (cont)

The parameters of interest in this case were the load time constant T_p and the transformer tapping delay T_{tap}. The nominal case, with $T_p = 5$s and $T_{\text{tap}} = 20$s is shown in Figure 4 as a dark line. It was assumed that these parameters were uniformly distributed over the ranges $T_p \in [4\ 6]$ and $T_{\text{tap}} \in [15\ 25]$. Two hundred sets of parameters $\{T_p, T_{\text{tap}}\}$ were randomly chosen, and an approximate trajectory generated for each case. These trajectories are shown in Figure 4 by dotted lines. The bound on the nominal trajectory is clearly evident.

Notice that the bound becomes wider with each tap change, before shrinking as steady-state is approached. This occurs as a result of the errors due to the uncertainty in T_p and T_{tap} accumulating over time.

□

Example 2

The IEEE 39-bus 10-machine case illustrates trajectory bounds for a larger system. Data for this example can be found in [7]. A fault was applied at bus 17. The response of the voltage at bus 8 is plotted in Figure 5. The nominal trajectory, which is shown as a solid line, corresponds to constant admittance loads, i.e., the active and reactive power voltage indices were $\zeta = 2.0$ and $\eta = 2.0$ respectively at all buses. To introduce parameter uncertainty, the load indices of five major load buses were assumed to be uniformly distributed over the ranges $\zeta_i \in [1.5\ 2.5]$ and $\eta_i \in [1.5\ 2.5]$. One hundred sets of parameters were randomly chosen, and an approximate trajectory generated for each case. These trajectories are shown as dotted lines in Figure 5.

The approximate trajectories show a bound around the nominal trajectory. The bound is more pronounced at the voltage extremes, where the uncertain load indices are more influential.

This example illustrates the importance of error bounds in assessing risks such as voltage sag. The nominal trajectory may indicate an acceptable voltage dip. However the error bound may fall below the viability threshold, indicating vulnerability of the actual system.

□

In these examples, the whole trajectory was generated for each parameter set. However if only particular sections of

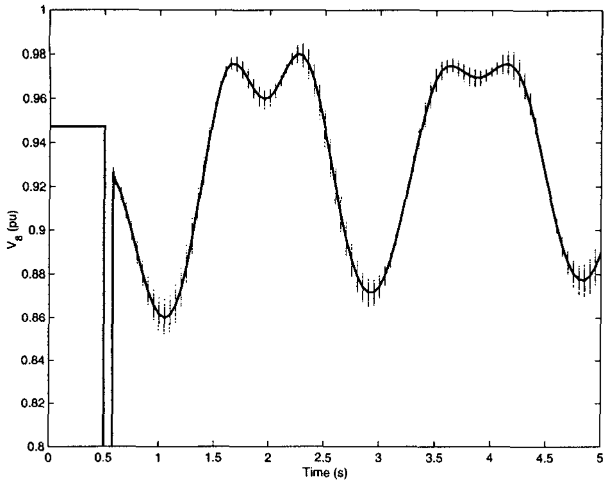

Fig. 5. Trajectory bounds for Example 2.

a trajectory were of interest, (6) could be used to generate just those pieces. Storage of the trajectory sensitivities would only be required for those periods.

V. Conclusions

Parameters of power system models can never be known exactly. Yet the results of power system simulation studies are routinely used in planning and operating decisions. Therefore it is important to quantify the errors induced by parameter uncertainty. This paper provides a technique for bounding the errors.

Trajectory sensitivities can be efficiently computed for a nominal trajectory. They can then be used to generate first order approximations of perturbed trajectories. Error bounds around a nominal trajectory can be obtained by a Monte-Carlo process consisting of,
1. generating random sets of parameters, and
2. computing a first order approximation of the trajectory corresponding to each parameter set.

The error bounds allow better assessment of risks such as voltage sag and unplanned protection operation.

Appendix

A. Model Structure

Systems which exhibit complex interactions between continuous and discrete-event dynamics can be modelled by a parameter dependent differential-algebraic-discrete (DAD) model of the form,

$$\dot{\underline{x}} = \underline{f}(\underline{x}, y) \tag{7}$$

$$0 = g^{(0)}(\underline{x}, y) \tag{8}$$

$$0 = \begin{cases} g^{(i-)}(\underline{x}, y) & y_{d,i} < 0 \\ g^{(i+)}(\underline{x}, y) & y_{d,i} > 0 \end{cases} \qquad i = 1, \ldots, d \tag{9}$$

$$\underline{x}^+ = \underline{h}_j(\underline{x}^-, y^-) \qquad y_{e,j} = 0 \qquad j \in \{1, \ldots, e\} \tag{10}$$

where

$$\underline{x} = \begin{bmatrix} x \\ z \\ \lambda \end{bmatrix}, \qquad \underline{f} = \begin{bmatrix} f \\ 0 \\ 0 \end{bmatrix}, \qquad \underline{h}_j = \begin{bmatrix} x \\ h_j \\ \lambda \end{bmatrix}.$$

In this model, x are continuous dynamic state variables, y are algebraic state variables, z are discrete state variables, and λ are parameters. As an example, in the power system context x would include machine dynamic states such as angles, velocities and fluxes, y would include network variables such as load bus voltage magnitudes and angles, z could represent transformer tap positions and relay internal states, and λ could be chosen from a diverse range of parameters, from loads through to fault clearing time. In (10), $\underline{x}^-$ and y^- refer to the values of $\underline{x}$ and y just prior to the reset condition, whilst $\underline{x}^+$ denotes the value of $\underline{x}$ just after the reset event.

Notice that the definition of $\underline{f}$ ensures that z and λ remain constant away from reset events (10). Further, $\underline{h}_j$ ensures that x and λ remain unchanged at a reset event.

Away from switching and reset events, the system is described by a smooth differential-algebraic model

$$\dot{\underline{x}} = \underline{f}(\underline{x}, y) \tag{11}$$

$$0 = g(\underline{x}, y) \tag{12}$$

where g is composed of (8) together with functions from (9) chosen depending on the signs of the elements of y_d.

B. Trajectory Sensitivity Computation

Away from events, where system dynamics evolve smoothly, the sensitivities $\underline{x}_{\underline{x}_0}$ and $y_{\underline{x}_0}$ are obtained by differentiating (11),(12) with respect to $\underline{x}_0$. This gives

$$\dot{\underline{x}}_{\underline{x}_0} = \underline{f}_{\underline{x}}(t)\underline{x}_{\underline{x}_0} + \underline{f}_y(t) y_{\underline{x}_0} \tag{13}$$

$$0 = g_{\underline{x}}(t)\underline{x}_{\underline{x}_0} + g_y(t) y_{\underline{x}_0} \tag{14}$$

where $\underline{f}_{\underline{x}} \equiv \frac{\partial \underline{f}}{\partial \underline{x}}$, and likewise for the other Jacobian matrices. Note that $\underline{f}_{\underline{x}}$, $\underline{f}_y$, $g_{\underline{x}}$, g_y are evaluated along the trajectory, and hence are time varying matrices. It is shown in [4] that the solution of this (potentially high order) DA system can be obtained as a by-product of solving the original DA system (11),(12).

Initial conditions for $\underline{x}_{\underline{x}_0}$ are obtained from (2) as

$$\underline{x}_{\underline{x}_0}(t_0) = I$$

where I is the identity matrix, and for $y_{\underline{x}_0}$ from (14),

$$0 = g_{\underline{x}}(t_0) + g_y(t_0) y_{\underline{x}_0}(t_0).$$

Equations (13),(14) describe the evolution of the sensitivities $x_{\underline{x}_0}$ and $y_{\underline{x}_0}$ between events. However at an event, the sensitivities are generally not continuous. It is necessary to calculate *jump conditions* describing the step change in $x_{\underline{x}_0}$ and $y_{\underline{x}_0}$. For clarity, consider a single switching/reset event, so the model (7)-(10) reduces to the form

$$\dot{\underline{x}} = \underline{f}(\underline{x}, y) \tag{15}$$

$$0 = \begin{cases} g^-(\underline{x}, y) & s(\underline{x}, y) < 0 \\ g^+(\underline{x}, y) & s(\underline{x}, y) > 0 \end{cases} \tag{16}$$

$$\underline{x}^+ = \underline{h}(\underline{x}^-, y^-) \qquad s(\underline{x}, y) = 0. \tag{17}$$

Let $(\underline{x}(\tau), y(\tau))$ be the point where the trajectory encounters the hypersurface $s(\underline{x}, y) = 0$, i.e., the point where an event is triggered. This point is called the *junction point* and τ is the *junction time.*

Just prior to event triggering, at time τ^-, we have

$$\begin{aligned} \underline{x}^- &= \underline{x}(\tau^-) &&= \phi_{\underline{x}}(\underline{x}_0, \tau^-) \\ y^- &= y^-(\tau^-) &&= \phi_y(\underline{x}_0, \tau^-) \end{aligned}$$

where

$$0 = g^-(\underline{x}^-, y^-).$$

Similarly, $\underline{x}^+, y^+$ are defined for time τ^+, just after the event has occurred. It is shown in [4] that the jump conditions for the sensitivities $\underline{x}_{\underline{x}_0}$ are given by

$$\underline{x}_{\underline{x}_0}(\tau^+) = \underline{h}^*_{\underline{x}}\, \underline{x}_{\underline{x}_0}(\tau^-) - \left(\underline{f}^+ - \underline{h}^*_{\underline{x}}\, \underline{f}^-\right) \tau_{\underline{x}_0} \tag{18}$$

where

$$\begin{aligned} \underline{h}^*_{\underline{x}} &= \left(\underline{h}_{\underline{x}} - \underline{h}_y (g_y^-)^{-1} g_{\underline{x}}^-\right)\Big|_{\tau^-} \\ \tau_{\underline{x}_0} &= -\frac{\left(s_{\underline{x}} - s_y (g_y^-)^{-1} g_{\underline{x}}^-\right)\Big|_{\tau^-} \underline{x}_{\underline{x}_0}(\tau^-)}{\left(s_{\underline{x}} - s_y (g_y^-)^{-1} g_{\underline{x}}^-\right)\Big|_{\tau^-} \underline{f}^-} \\ \underline{f}^- &= \underline{f}(\underline{x}(\tau^-), y^-(\tau^-)) \\ \underline{f}^+ &= \underline{f}(\underline{x}(\tau^+), y^+(\tau^+)). \end{aligned}$$

The sensitivities $y_{\underline{x}_0}$ immediately after the event are given by

$$y_{\underline{x}_0}(\tau^+) = -\left(g_y^+(\tau^+)\right)^{-1} g_{\underline{x}}^+(\tau^+) \underline{x}_{\underline{x}_0}(\tau^+).$$

Following the event, i.e., for $t > \tau^+$, calculation of the sensitivities proceeds according to (13),(14), until the next event. The jump conditions provide the initial conditions for the post-event calculations.

Actual power systems involve many discrete events. The more general case follows naturally though, and is presented in [4].

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Biographies

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