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Uncertainty Evaluation and Mitigation in Electrical Power Systems

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SUMMARY

Planning and operation of electrical power networks involves extensive simulation-based evaluation of system behavior, in order to determine operating limits that are robust to credible contingencies. The resulting decisions have tremendous economic and reliability implications. It is well known, however, that the parameters of many dynamic models are quite uncertain. Load model parameters, in particular, are often a source of appreciable error.

Load parameter uncertainty can be reduced by structuring models so that they adequately capture the load's physical characteristics. A further reduction in uncertainty can often be achieved by estimating parameter values from disturbance measurements. It is common for parameter estimation algorithms to be based on a nonlinear least-squares formulation that seeks to minimize discrepancies between measurements and simulation. Reliable estimates can only be attained for identifiable parameters. Identifiability may be assessed using a subset selection algorithm that considers the conditioning of a matrix built from trajectory sensitivities. There are two causes for parameter ill-conditioning (non-identifiability), 1) the parameter has negligible influence on system behavior, and 2) the influences exerted by various parameters are closely coupled. This second situation, where parameters are influential yet they cannot be estimated, is particularly troubling.

A number of approaches are available for assessing and reducing the impact of uncertainty. Trajectory sensitivities can be used to form approximate trajectories for parameter sets that are perturbations from the nominal parameter values. The influence of parameter uncertainty can then be expressed as a bound that is mapped along with the nominal trajectory. The Probabilistic Collocation Method (PCM) uses Gaussian quadrature concepts to select appropriate points from the set of uncertain parameters, in order to approximate the mapping between parameters and an output of the simulation. Such outputs can take many forms, including the maximum voltage dip or proximity to protection operation. Grazing concepts can be used to determine the smallest changes in parameters that would cause an event such as protection operation. Such information provides another mechanism for assessing whether system behavior is robust to uncertainty in parameters.

KEYWORDS

Parameter uncertainty; load modeling; parameter estimation; dynamic performance assessment.

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1 Introduction

Analysis of power system dynamic behavior requires models that capture the phenomena of interest, together with parameter values that ensure those models adequately replicate reality. It is important to distinguish between model fidelity and parameter accuracy. Models are always an approximation. In many cases, the level of approximation is determined by the nature of the study. For example, phasor-based models that are used for dynamic security assessment ignore electromagnetic transient phenomena. In other cases, however, model approximation is a matter of convenience, with the outcome not necessarily providing a good reflection of reality. Load modeling provides an example. It is common for the aggregate behavior of loads to be represented by a voltage dependent model, such as the ZIP model. This is a gross approximation, given the complex composition of loads on most distribution feeders. This deficiency is particularly evident in distribution systems that supply a significant motor load, as the ZIP model cannot capture the delayed voltage recovery associated with induction motors re-accelerating or stalling.

The choice of models is a decision that should be made based on knowledge of the actual system composition and the phenomena that are being studied. Determining parameters for those models, on the other hand, usually relies on comparison of model response with actual measured behavior. Parameter estimation processes seek to minimize the difference between measured and simulated behavior. Different choices for model structure will usually result in different parameter values. This is a consequence of the estimation process trying to compensate for unmodeled, or poorly modeled, effects. In all cases, the models and associated parameter sets are approximations, though the goal should always be to obtain the best possible approximations.

Load models are further complicated by the fact that load composition is continually changing. Even if it were possible to obtain a load model that was perfectly accurate at a particular time, it would be inaccurate a short while later. Developing load models is not a futile exercise though, as overall load composition tends to behave fairly predictably. For example, the composition of a residential feeder will (approximately) follow a 24 hour cycle. But, while composition from one day to the next may be roughly equivalent, morning load conditions may well differ greatly from those in the evening. Seasonal variations may be even more pronounced.

As mentioned previously, all models are approximate to some extent. Model structures for large dominant components, such as synchronous generators, are well established, as are procedures for determining the associated parameter values. Furthermore, parameter values for such devices remain fairly constant over their lifetime. Models that represent an aggregation of many distributed components are much more contentious though, given the inherent uncertainty in the overall composition of the model. This paper focuses on uncertainty associated with load modeling. Similar issues arise in the modeling of other power system components though, with wind generation being a particularly topical example.

2 Sources of Uncertainty

Loads form the major source of uncertainty in power system modeling. Loads are highly distributed, and quite variable, so detailed modeling is impossible. Aggregation provides the only practical approach to incorporating loads into power system studies. For static (power flow) analysis, the approximations inherent in aggregate load models are largely unimportant, as the composition of the load has little impact on results. On the other hand, load composition is very important in the analysis of system dynamic behavior. Different types of loads exhibit quite diverse responses to disturbances. For example, lighting loads vary statically (almost) with voltage, whereas motor loads exhibit dynamic behavior, perhaps even stalling. In fact, each different load type displays unique characteristics. Aggregate load models attempt to blend all those differing responses.

In many cases aggregate load models are required to represent loads that are widely distributed, physically and electrically. Because of this electrical separation, the voltages seen by loads may differ greatly. Such voltage differences may critically affect the response of loads to large disturbances, resulting in diverse load behavior. It is difficult for aggregate load models to capture such diversity. At best, those topological influences can only be crudely approximated.

Accounting for switching-type behavior in aggregate load models is also challenging. When residential air-conditioning compressor motors experience a voltage dip to around 0.6 pu, they almost instantaneously stall. This can be modeled as a mode switch, from running to stalled. As mentioned previously, voltage is usually not uniform across a distribution system. Therefore voltage dips may result in some compressor motors stalling and others not. As motors stall, the resulting high currents will further depress voltages, possibly inducing further stalling. The proportion of stalled motors will depend nonlinearly and temporally on many factors, including the severity of the initiating voltage dip, and the strength and topology of the distribution system. These attributes are difficult to capture, with any degree of certainty, in aggregate load models.

Other devices may also switch under disturbed voltage conditions. Contactors provide an example. They use an electromechanical solenoid to hold a switch in the closed position. When a disturbance depresses the voltage, the solenoid may not be able to hold the switch closed, resulting in unintended tripping of the associated load. The voltage threshold at which such action occurs varies widely. Precise modeling is not possible.

Looking to the future, a number of trends are likely to increase the level of uncertainty associated with aggregate load models. Distribution systems will see a greater penetration of distributed generation as fuel cells and solar cells, for example, become commercially viable. Plug-hybrid electric vehicles (PHEVs) will certainly gain in popularity, and may well become a significant feature of distribution systems. Not only do PHEVs present a load that moves from one location to another, but their vehicle-to-grid capability offers the possibility of highly dispersed generation. All these trends suggest that methods for assessing the impact of uncertainty are set to become increasingly important.

3 Reducing Uncertainty

3.1 Model structure

Load parameter uncertainty can be reduced by structuring models so that they adequately capture the physical characteristics of the actual loads. A ZIP model, for example, provides a poor representation of loads that include a significant proportion of air-conditioner motors. Attempting to replicate motor-induced delayed voltage recovery using such a model is futile. Tuning the ZIP parameters to best match one disturbance would provide no guarantee that the parameters were appropriate for another event. The WECC model of Figure 1, on the

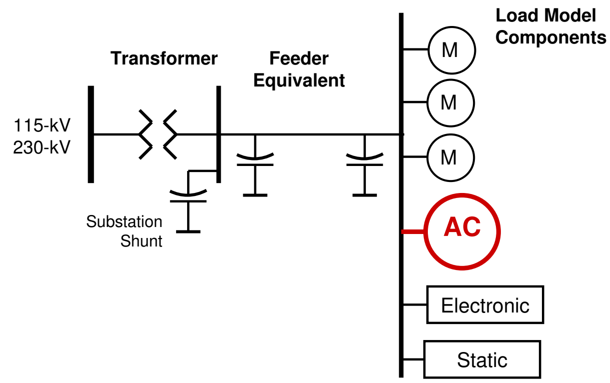

Figure 1: WECC load model structure.

other hand, provides a versatile structure that is capable of representing various different load types. The issue with this latter model is one of identifying the multitude of parameters associated with the more complete model structure.

3.2 Parameter estimation

It is often possible to estimate parameter values from disturbance measurements. For example, simply measuring the active and reactive power consumed by a load during a disturbance may yield sufficient information to accurately estimate several model parameters. The aim of parameter estimation is to determine parameter values that achieve the closest match between the measured samples and the model trajectory.

Disturbance measurements are obtained from data acquisition systems that record sampled system quantities. Let a measurement of interest be given by the sequence of samples $m = [m_0\ m_1\ \dots\ m_N]$, with the corresponding simulated trajectory being given by $\mathbf{x} = [x(t_0)\ x(t_1)\ \dots\ x(t_N)]$. The mismatch between the measurement and its corresponding (discretized) model trajectory can be written in vector form as $e(\theta) = \mathbf{x}(\theta) - m$, where a slight abuse of notation has been used to show the dependence of the trajectory on the parameters θ.

The best match between model and measurement is obtained by varying the parameters so as to minimize the error vector $e(\theta)$. It is common for the size of the error vector to be expressed in terms of the 2-norm cost,

$$\mathcal{C}(\theta) = \|e(\theta)\|_2^2 = \sum_{k=0}^{N} e_k(\theta)^2. \tag{1}$$

The desired parameter estimate is then given by minimizing $\mathcal{C}(\theta)$. This nonlinear least squares problem can be solved using a Gauss-Newton iterative procedure [1]. At each iteration j of this procedure, the parameter values are updated according to

$$\Phi(\theta^j)^T \Phi(\theta^j) \Delta\theta^{j+1} = -\Phi(\theta^j)^T e(\theta^j)^T \tag{2}$$

$$\theta^{j+1} = \theta^j + \alpha^{j+1} \Delta\theta^{j+1} \tag{3}$$

where Φ is constructed from trajectory sensitivities [2], and α^{j+1} is a suitable scalar step [3].

An estimate of θ which (locally) minimizes the cost function $\mathcal{C}(\theta)$ is obtained when $\Delta\theta^{j+1}$ is close to zero. Note that this procedure will only locate local minima though, as it is based on a first-order approximation of $e(\theta)$. However if the initial guess for θ is good, which is generally possible using engineering judgement, then a local minimum is usually sufficient.

3.3 Parameter conditioning

The information content of a measured trajectory determines which parameters may be estimated. Parameters that have a significant effect on the trajectory are generally identifiable. Conversely, parameters that have little effect on trajectory shape are usually not identifiable. This information is captured in the trajectory sensitivities Φ.

When developing a parameter estimation algorithm, it is necessary to separate identifiable parameters from those that are not, in order to avoid spurious results. This can be achieved using a *subset selection* algorithm [4, 5]. This algorithm considers the conditioning of the matrix $\Phi^T\Phi$ that appears in (2). If it is well conditioned, then its inverse will be well defined, allowing (2) to be reliably solved for $\Delta\theta^{j+1}$. On the other hand, ill-conditioning of $\Phi^T\Phi$ introduces numerical difficulties in solving for $\Delta\theta^{j+1}$, with the Gauss-Newton process becoming unreliable.

The subset selection algorithm considers the eigenvalues of $\Phi^T\Phi$ (which are the square of the singular values of Φ.) Small eigenvalues are indicative of ill-conditioning. The algorithm separates parameters into those associated with large eigenvalues (identifiable parameters) and the rest which cannot be identified. The latter parameters are subsequently fixed at their original values.

In summary, two situations lead to parameter ill-conditioning (non-identifiability). The first is where the trajectory sensitivities, corresponding to available disturbance measurements, are small relative to other sensitivities. This group of parameters cannot be estimated from available measurements. That may not be particularly troublesome though, if this is the only disturbance of interest, as their influence on behavior is negligible anyway. However, they may be influential for other disturbances. This should be assessed by considering a variety of viable disturbance scenarios. The second case arises when the trajectory sensitivities are highly correlated. Consequently, the influence of numerous parameters cannot be separated. This would be the situation, for example, when varying two parameters in unison gave no overall change in behavior. Both parameters are influential, but neither can be estimated without fixing the other. This dilemma may be resolvable by considering various disturbances, in the hope of finding cases where the parameters exert differing influences.

4 Approaches to assessing and reducing the impact of uncertainty

4.1 Trajectory approximation using sensitivities

The dependence of a trajectory on parameter values can be expressed mathematically as the *flow*, $x(t) = \phi(t, \theta)$. By expanding the flow as a Taylor series, and neglecting the higher order terms, trajectories arising from perturbing parameters by $\Delta\theta$ can be approximated as

$$\phi(t, \theta + \Delta\theta) \approx \phi(t, \theta) + \Phi(t, \theta)\Delta\theta \tag{4}$$

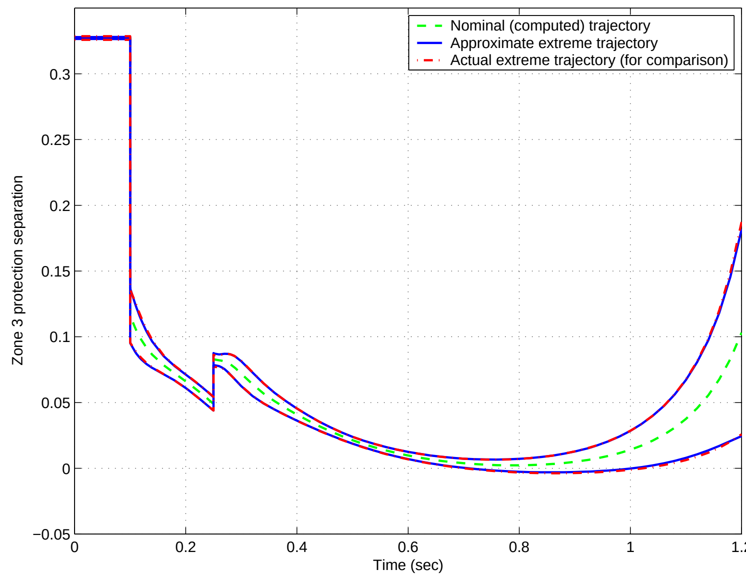

Figure 2: Zone 3 protection operation assessment, worst-case bounds.

where $\phi(t, \theta)$ is the trajectory obtained using the nominal set of parameters θ, and the corresponding trajectory sensitivities are given by $\Phi(t, \theta)$. If the perturbations $\Delta\theta$ are relatively small, then the approximation (4) is quite accurate. This accuracy is difficult to quantify though. It is shown in [6] that the higher order terms neglected in (4) become increasingly significant as the system becomes less stable. Nevertheless, the approximations generated by (4) are generally quite accurate.

The affine nature of (4) can be exploited to establish two straightforward approaches to mapping parameter uncertainty through to bounds around the nominal trajectory [6]. The first approach assumes that each uncertain parameter is uniformly distributed over a specified range. Multiple uncertain parameters are therefore uniformly distributed over a multidimensional hyperbox. As time progresses, the affine transformation (4) distorts that hyperbox into a multidimensional parallelotope. A simple algorithm is proposed in [6] for determining the vertices of the time-varying parallelotope that correspond to worst-case behavior.

An example, based on the IEEE 39 bus system, can be used to illustrate this process. Parameters describing the load composition were assumed to satisfy a uniform distribution over a range of ± 0.2 around their nominal values. Zone 3 protection on one of the major lines was considered, with Figure 2 showing the separation[1] between the zone 3 mho characteristic [7] and the apparent impedance seen by the relay. The dashed line in Figure 2 was obtained using the nominal set of load parameters. It suggests the zone 3 characteristic is not entered. The sensitivity-derived worst-case bounds on behavior are shown as solid lines, and the true (simulated) bounds are shown as dash-dot lines. The sensitivity-based predictions are very accurate over this crucial time period. Every selection of uncertain load parameters results in a trajectory that lies within the bounds shown in Figure 2. Notice that the lower bound passes below zero, indicating the possibility of a zone 3 trip.

Often parameter values are not uniformly distributed over the range of uncertainty, but are better described by a normal distribution. Under those conditions, worst-case analysis gives

[1] This distance goes negative when the apparent impedance enters the mho characteristic.

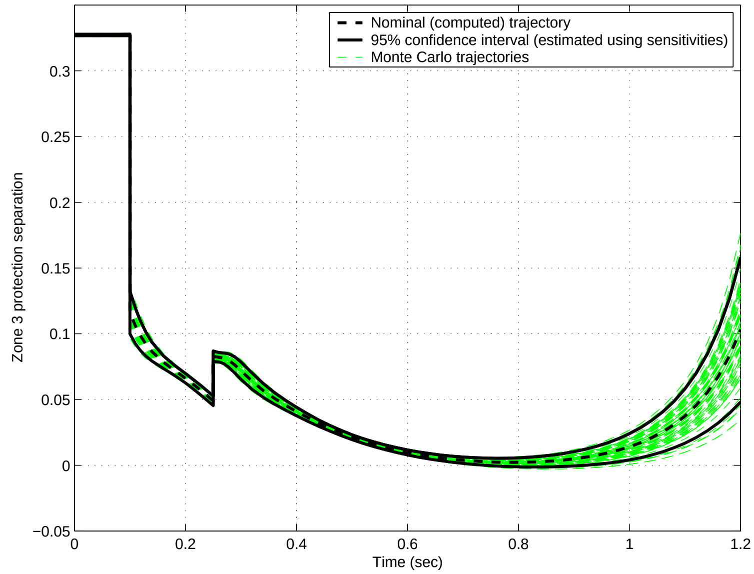

Figure 3: Zone 3 protection operation assessment, 95% confidence interval bounds.

a conservative view of parametric influences. Less conservatism is achieved with probabilistic assessment.

A probabilistic approach to assessing the influence of uncertainty assumes θ is a random vector with mean μ and covariance matrix Σ. It follows that deviations $\Delta\theta = \theta - \mu$ have zero mean and covariance Σ. The nominal flow and corresponding trajectory sensitivities are generated with parameters set to μ. From (4), perturbations in the trajectory at time t are given (approximately) by $\Delta x(t) = \Phi(t, \theta)\Delta\theta$. It follows from basic statistical properties [8] that perturbations in state i will have mean and variance

$$E[\Delta x_i(t)] = \Phi_i(t, \theta)\, E[\Delta\theta] = 0 \tag{5}$$

$$\mathrm{Var}[\Delta x_i(t)] = \Phi_i(t, \theta)\, \Sigma\, \Phi_i(t, \theta)^T. \tag{6}$$

Furthermore, if the elements of random vector $\Delta\theta$ are statistically independent, then Σ will be diagonal with elements $\sigma_1^2 \ldots \sigma_n^2$. In this case, (6) reduces to

$$\mathrm{Var}[\Delta x_i(t)] = \sum_{j=1}^{n} \Phi_{ij}(t, \theta)^2\, \sigma_j^2. \tag{7}$$

Referring back to the earlier example, the load composition parameters were assumed normally distributed, with mean $\mu = 0.5$ and variance $\sigma^2 = 0.01$. Equation (7) was used to determine the variance of the zone 3 protection signal at each time step along the trajectory. The bounds shown by solid lines in Figure 3 were constructed from points that are ± 1.96 times the standard deviation away from the nominal trajectory. The choice of 1.96 corresponds to the 95% confidence interval.

4.2 Probabilistic collocation method

The probabilistic collocation method (PCM) provides a computationally efficient approach to building an approximate relationship between random variables and outputs that depend upon those variables. In assessing the impact of parameter uncertainty, it is assumed that the parameters of interest satisfy given probability density functions $f(\lambda)$. The desired outputs are obtained by running a simulation for each randomly chosen set of parameters. Any feature of the trajectory could be chosen as an output, for example the values of states at certain times, and/or the maximum voltage dip.

This section provides an overview of PCM. More complete details are presented in [9]. In order to simplify notation, the discussion will assume a single uncertain parameter. The ideas extend to larger numbers of parameters, though with increased computations.

For a given probability density function $f(\lambda)$, a set of orthonormal polynomials $h_i(\lambda)$ can be determined. The subscript i refers to the order of the polynomial, and orthogonality is defined in terms of the inner product

$$< h_i(\lambda), h_j(\lambda) > \; = \int f(\lambda) h_i(\lambda) h_j(\lambda) d\lambda.$$

Underlying PCM is the assumption that the uncertain parameter λ and the output of interest are related by a polynomial $g(\lambda)$ of order $2n-1$. This is generally not strictly true, though such polynomial approximation is not unusual. Given this "true" relationship $g(\lambda)$ between parameter and output, PCM determines a lower order polynomial $\hat{g}(\lambda)$ such that the mean value for $\hat{g}(\lambda)$ coincides with that of $g(\lambda)$,

$$E[\hat{g}(\lambda)] = E[g(\lambda)].$$

If $g(\lambda)$ is of order $2n-1$, then $\hat{g}(\lambda)$ has order $n-1$, and can be written in terms of the orthonormal polynomials $h_i(\lambda)$ as

$$\hat{g}(\lambda) = g_0 h_0(\lambda) + g_1 h_1(\lambda) + ... + g_{n-1} h_{n-1}(\lambda). \tag{8}$$

The coefficients $g_0, ..., g_{n-1}$ are obtained by solving

$$\begin{bmatrix} g(\lambda_1) \\ \vdots \\ g(\lambda_n) \end{bmatrix} = \begin{bmatrix} h_{n-1}(\lambda_1) & \cdots & h_0(\lambda_1) \\ \vdots & \ddots & \vdots \\ h_{n-1}(\lambda_n) & \cdots & h_0(\lambda_n) \end{bmatrix} \begin{bmatrix} g_{n-1} \\ \vdots \\ g_0 \end{bmatrix} \tag{9}$$

where the λ_i are the roots of $h_n(\lambda)$.

In summary, for a given probability density function $f(\lambda)$ for the uncertain parameter, PCM requires the following computations. The set of orthonormal polynomials $h_0, ..., h_n$, corresponding to the given $f(\lambda)$, can be obtained using a straightforward recursive algorithm [10]. The roots of $h_n(\lambda)$ provide the values $\lambda_1, ..., \lambda_n$ which are used in simulations to obtain the output values $g(\lambda_1), ..., g(\lambda_n)$. Also, $h_0, ..., h_{n-1}$ are evaluated at $\lambda_1, ..., \lambda_n$ to establish the matrix in (9), which is subsequently inverted to obtain the coefficients $g_0, ..., g_{n-1}$. These coefficients are used in (8) to give the desired lower-order approximation $\hat{g}(\lambda)$.

4.3 Grazing analysis

Many power system disturbances escalate through events such as operation of protection devices. In order to assess vulnerability to events, triggering conditions such as protection operating characteristics can be conceptualized as hypersurfaces in state space. A trajectory that passes close by a hypersurface, but does not encounter it, will not initiate an event.

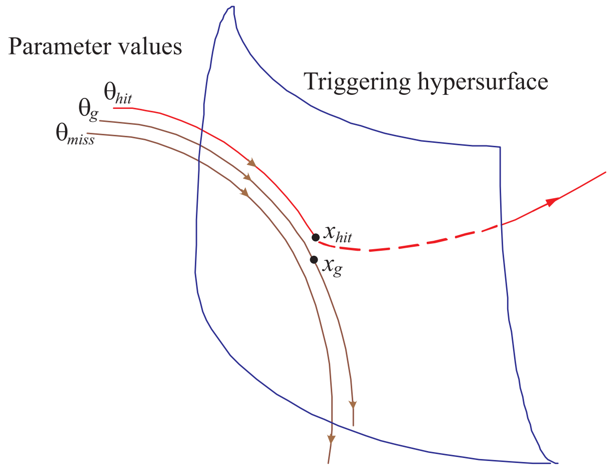

Figure 4: Illustration of grazing.

On the other hand, if the trajectory does encounter the hypersurface, an event will occur, possibly with detrimental consequences.

Trajectories $\phi(t, \theta)$ are parameter dependent. For a certain set of parameters, the trajectory may miss the event triggering hypersurface. The hypersurface may be encountered for a different set of parameters though. These two situations are separated by trajectories that only just touch the hypersurface. This is illustrated in Figure 4. The critical condition, which separates two different forms of behavior, is referred to as *grazing* [11].

It is shown in [11] that grazing conditions establish a set of algebraic equations that can be solved using a Newton process. Each iteration of the Newton algorithm requires simulation to obtain the trajectory and associated sensitivities. Such solution processes are known as shooting methods [12]. Full details for grazing applications can be found in [11, 13].

Referring to the example illustrated in Figures 2 and 3, grazing analysis can be used to determine the smallest changes in load-composition parameters that cause the apparent impedance trajectory to just touch the mho characteristic. Such information provides another mechanism for assessing whether system behavior is robust to uncertainty in parameters.

Figure 5 provides a parameter-space view of grazing conditions. The line is composed of three sets of parameters, each of which results in grazing. Further grazing points could be found, using a continuation process, to establish a smoother line in parameter space. Proximity to that line would suggest vulnerability to grazing, and hence to event triggering. This is illustrated in Figure 5. The point corresponding to the nominal parameter values is shown, together with a dashed line that indicates uncertainty of ± 0.15. The region describing parameter uncertainty overlaps the line of grazing points. This suggests a finite probability that the mho characteristic will be encountered, and hence that protection will operate.

This grazing-based approach to assessing robustness to uncertainty can be extended to an arbitrary number of parameters. The information derived from such analysis is useful for exploring the relative impact of uncertainty in the different parameters. For example, it may show that a small variation in one of the parameters may induce grazing, whereas a much larger variation could be tolerated in a different parameter. These concepts are explored in [13] in the context of power electronic circuits. Adaptation to power system applications is conceptually straightforward, though has not yet been undertaken.

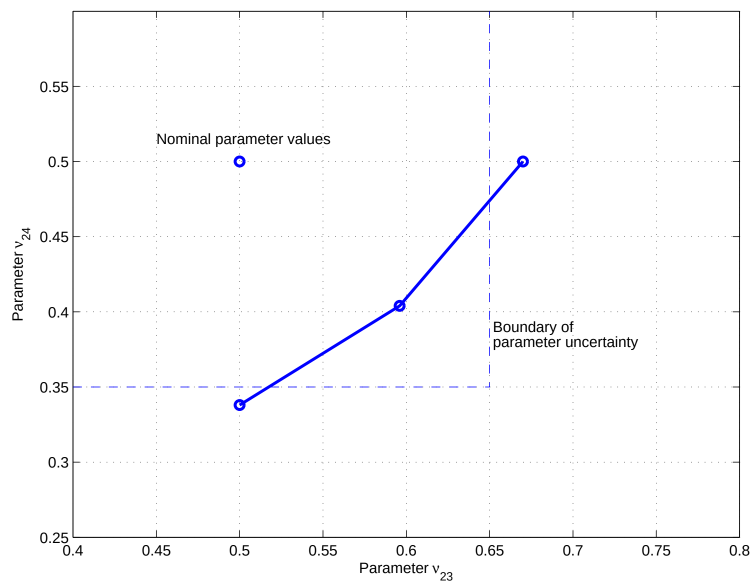

Figure 5: Parameter-space view of vulnerability to grazing conditions.

5 Conclusions

All models are an approximation, to some extent. Uncertainty in model-based analysis is therefore unavoidable. Model design should take into account the nature of the phenomena under investigation, with well designed models minimizing the impact of unmodeled effects and of uncertainty. In power systems, the major source of uncertainty arises from the modeling of loads. Accurate modeling is particularly challenging due to the continual variation in load composition.

Trajectory sensitivities provide a numerically tractable approach to assessing the impact of uncertainty in parameters. Such sensitivities describe the variation in the trajectory resulting from perturbations in parameters. Small sensitivities indicate that uncertainty in the respective parameters has negligible impact on behavior. Large sensitivities, on the other hand, suggest that the respective parameters exert a measurable influence on behavior. It is important to minimize the uncertainty in the latter group of parameters. This can be achieved by estimating parameter values from measurements of system disturbances. The parameter estimation process seeks to minimize the difference between measured behavior and simulated response. This difference can be formulated as a nonlinear least squares problem, with the solution obtained via a Gauss-Newton process. Trajectory sensitivities provide the gradient information that underlies that process.

The impact of uncertain parameters is generally not significant for systems that are unstressed. As the stability margin reduces, however, system behavior becomes much more sensitive to parameter perturbations. It is particularly important to consider cases that are on the verge of protection operation. In such cases, uncertainty may make the difference between protection operating or remaining inactive, with the consequences being vastly different.

Various numerical techniques are available for assessing the impact of parameter uncertainty. Trajectory sensitivities can be used to generate approximate trajectories, which in turn allow parameter uncertainty to be mapped to a bound around the nominal trajectory. The probabilistic collocation method can be used to determine (approximately) the statistical distribution associated with important features of a trajectory. This method can also be used to establish an uncertainty bound around the nominal trajectory. The likelihood that uncertain parameters may induce undesirable events, such as reactionary protection operation, can be assessed using techniques that build on grazing concepts.

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