SENSITIVITY ANALYSIS OF POWER SYSTEM TRAJECTORIES: RECENT RESULTS

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ABSTRACT

The development of trajectory sensitivity analysis for power systems is presented in the paper. A hybrid system model which has a differential-algebraic-discrete structure is proposed. Crucial to the analysis is the development of jump conditions describing the behaviour of sensitivities at discrete events, such as switching and state resetting. A power system example which involves a mix of continuous and discrete behaviour is presented to illustrate various aspects of the theory. It is shown that trajectory sensitivities provide insights into system behaviour which cannot be obtained from traditional simulations.

1. INTRODUCTION

Power systems exhibit dynamic behaviour which is governed by a mix of constrained continuous-time dynamics, discrete-time and discrete-event dynamics, switching action and jump phenomena. Analysis of this dynamic behaviour is vital in system design and operation. However the nonlinear nonsmooth nature of dynamic behaviour generally precludes the use of simple systematic analysis techniques. Power system analysts therefore rely largely on simulation.

The advantage of simulation is that it is applicable for arbitrarily complicated models. A disadvantage is that it provides information about a single scenario. Generally it is not possible to confidently extrapolate results, even for small changes in system conditions. Each change to the system requires another simulation. For large systems, such as power systems, this often involves a large computational cost.

Trajectory sensitivity analysis offers some relief from the rigors of repetitive simulation. The approach is based upon linearizing the system around a nominal *trajectory*, rather than around an equilibrium point [1, 2, 3]. It is therefore possible to determine directly the change in a trajectory due to a (small) change in initial conditions and/or parameters. The analysis is straightforward for smooth systems, but can also be applied to systems which contain discontinuities [1, 3, 4].

Trajectory sensitivities were originally associated with a number of areas in control and parameter estimation [1]. More recent applications have included stability assessment of power systems [5, 6].

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2. MODEL

Power systems can be generically described by a parameter dependent differential-algebraic-discrete (DAD) model of the form,

$$\dot{x} = f(x, y, z; \lambda) \tag{1}$$

$$0 = g^{(0)}(x, y, z; \lambda) \tag{2}$$

$$0 = \begin{cases} g^{(i-)}(x, y, z; \lambda) & y_{d,i} < 0 \\ g^{(i+)}(x, y, z; \lambda) & y_{d,i} > 0 \end{cases} \qquad i = 1, ..., d \tag{3}$$

$$z^+ = h_j(x^-, y^-, z^-; \lambda) \qquad y_{e,j} = 0 \qquad j \in \{1, ..., e\} \tag{4}$$

$$\dot{z} = 0 \qquad y_{e,j} \neq 0 \qquad \forall j \in \{1, ..., e\} \tag{5}$$

where

$$x \in X \subseteq \mathbb{R}^n, y \in Y \subseteq \mathbb{R}^m, z \in Z \subseteq \mathbb{R}^l, \lambda \in L \subseteq \mathbb{R}^p$$
$$y_d = Dy$$
$$y_e = Ey$$
$$f : \mathbb{R}^{n+m+l+p} \to \mathbb{R}^n$$
$$g = \begin{bmatrix} g^{(0)} \\ g^{(1)} \\ \vdots \\ g^{(d)} \end{bmatrix} : \mathbb{R}^{n+m+l+p} \to \mathbb{R}^m$$
$$h_j : \mathbb{R}^{n+m+l+p} \to \mathbb{R}^l \qquad j = 1, ..., e$$

and $D \in \mathbb{R}^{d\times m}$, $E \in \mathbb{R}^{e\times m}$ are matrices of zeros, except that each row of each matrix has a single 1 in an appropriate location. There is no restriction on y_d and y_e sharing some common elements. In (4), x^-, y^-, z^- refer to the values of x, y, and z just prior to the reset condition, whilst z^+ denotes the value of z just after the reset event.

In this model, which is similar to a model proposed in [7], x are continuous dynamic state variables, y are algebraic state variables, z are discrete state variables, and λ are parameters. In the power system context x would include machine dynamic states such as angles, velocities and fluxes, y would include network variables such as load bus voltage magnitudes and angles, z could represent transformer tap positions and/or relay internal states, and λ may be parameters such as loads and/or fault clearing time.

Note that the model does not allow discontinuities in the dynamic states. This is not a restriction forced by the analysis; in fact later analysis is directly applicable to cases where x undergoes jumps. The model adopts the philosophy that the dynamic states of *real* systems cannot undergo step changes.

However the proposed model (1)-(5) captures all the important aspects of power system behaviour, namely the interaction between continuous and discrete states as they evolve over time. The continuous states are driven by the differential equations (1), with the discrete states evolving according to the reset condition (4). The algebraic equations (2),(3) establish the interconnections between continuous and discrete components of the system, and ensure the system satisfies physical constraints. They also allow the event variables y_d and y_e to describe arbitrarily complicated sets of conditions.

At an event given by $y_{d,i} = 0$, the algebraic states y often undergo a discontinuity. However the discrete states z remain constant and the dynamic states x are continuous through the event. At an event described by $y_{e,j} = 0$, at least one of the discrete states is reset, i.e., undergoes a step change. This may result in a discontinuity in y, but the x are again continuous through the event. Let the times at which events occur be given by $\{\tau_k : t_0 < \tau_1 < \tau_2 < ...\}$.

Initial conditions for the model (1)-(5) are given by

$$x(t_0) = x_0, \quad y(t_0) = y_0, \quad z(t_0) = z_0 \tag{6}$$

where y_0 is a solution of

$$g(x_0, y_0, z_0; \lambda) = 0. \tag{7}$$

Note that in solving for y_0, the constraint switching described by (3) must be taken into account.

Trajectories of the DAD system (1)-(5) describe the behaviour of the dynamic states x, the algebraic states y, and the discrete states z over time. To formalize these concepts we define the flows of x, y and z respectively as

$$\begin{aligned} x(t) &= \phi_x(x_0, z_0, t, \lambda) \\ y(t) &= \phi_y(x_0, z_0, t, \lambda) \\ z(t) &= \phi_z(x_0, z_0, t, \lambda) \end{aligned}$$

where

$$\begin{aligned} &\frac{d}{dt}\left(\phi_x(x_0, z_0, t, \lambda)\right) \\ &\quad = f\left(\phi_x(x_0, z_0, t, \lambda), \phi_y(x_0, z_0, t, \lambda), \phi_z(x_0, z_0, t, \lambda); \lambda\right) \\ 0 &= g\left(\phi_x(x_0, z_0, t, \lambda), \phi_y(x_0, z_0, t, \lambda), \phi_z(x_0, z_0, t, \lambda); \lambda\right) \end{aligned}$$

and $\phi_z(x_0, z_0, t, \lambda)$ is piece-wise constant, with step transitions between the constant sections described by the reset equations (4). From the definitions of the flows, it is clear that $\phi_x(x_0, z_0, t_0, \lambda) = x_0$, $\phi_y(x_0, z_0, t_0, \lambda) = y_0$ and $\phi_z(x_0, z_0, t_0, \lambda) = z_0$.

Notice that ϕ_y has been defined in terms of x_0 and z_0 rather than y_0. This reflects the dependence of y_0 on x_0, z_0 and λ, as described by (7). Therefore the definitions of ϕ_x, ϕ_y and ϕ_z establish the dependence of the flows on x_0, z_0 and λ.

It is clear that the notation can quickly become unwieldy. Therefore in the sequel we will generally write the model more compactly as

$$\dot{\underline{x}} = \underline{f}(\underline{x}, y) \tag{8}$$

$$0 = g^{(0)}(\underline{x}, y) \tag{9}$$

$$0 = \begin{cases} g^{(i-)}(\underline{x}, y) & y_{d,i} < 0 \\ g^{(i+)}(\underline{x}, y) & y_{d,i} > 0 \end{cases} \qquad i = 1, ..., d \tag{10}$$

$$\underline{x}^+ = \underline{h}_j(\underline{x}^-, y^-) \qquad y_{e,j} = 0 \qquad j \in \{1, ..., e\} \tag{11}$$

where

$$\underline{x} = \begin{bmatrix} x \\ z \\ \lambda \end{bmatrix} \in \underline{X} = X \times Z \times L \subseteq \mathbb{R}^{\underline{n}=n+l+p}$$

$$\underline{f} = \begin{bmatrix} f \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{h}_j = \begin{bmatrix} x \\ h_j \\ \lambda \end{bmatrix}.$$

The system flow is defined accordingly as

$$\phi(\underline{x}_0, t) = \begin{bmatrix} \phi_{\underline{x}}(\underline{x}_0, t) \\ \phi_y(\underline{x}_0, t) \end{bmatrix} = \begin{bmatrix} \underline{x}(t) \\ y(t) \end{bmatrix}. \tag{12}$$

Notice that the definition of $\underline{f}$ ensures that z and λ remain constant away from reset events (11). Further, $\underline{h}_j$ ensures that x and λ remain unchanged at a reset event. Over each of the open time intervals (τ_k, τ_{k+1}) the system is described by a smooth DA model

$$\begin{aligned} \dot{\underline{x}} &= \underline{f}(\underline{x}, y) && (13) \\ 0 &= g(\underline{x}, y) && (14) \end{aligned}$$

where g is composed of (9) together with functions from (10) chosen depending on the signs of the elements of y_d. (Recall that the definition of the τ_k ensures that no elements of y_d can change sign during the period (τ_k, τ_{k+1}).)

3. TRAJECTORY SENSITIVITY ANALYSIS

3.1. General concepts

The flow ϕ of a system will generally vary with changes in parameters and/or initial conditions. Trajectory sensitivity analysis provides a way of quantifying the changes in the flow that result from (small) changes in parameters and initial conditions. The development of these sensitivity concepts will be based upon the compact form of the DAD model (8)-(11). Recall that in this model, $\underline{x}_0$ incorporates the initial conditions x_0 and z_0, as well as the parameters λ. Therefore the sensitivity of the flow to $\underline{x}_0$ fully describes its sensitivity to x_0, z_0 and λ.

In Section 2 we defined the system flow ϕ in terms of $\underline{x}_0$. The dependence of ϕ on y_0 is not explicit, but follows from (7). Therefore, in determining trajectory sensitivities, we will not directly establish the sensitivity of flows to changes in y_0. Rather, such sensitivity is given implicitly by sensitivity to $\underline{x}_0$.

Trajectory sensitivities follow from a Taylor series expansion of the flows $\phi_{\underline{x}}$ and ϕ_y. Referring to (12), the expansion for $\phi_{\underline{x}}$ can be expressed as

$$\begin{aligned} \Delta\underline{x}(t) &= \Delta\phi_{\underline{x}}(\underline{x}_0, t) \\ &= \frac{\partial\phi_{\underline{x}}(\underline{x}_0, t)}{\partial\underline{x}_0}\Delta\underline{x}_0 + \text{ higher order terms.} \end{aligned}$$

Neglecting the higher order terms and using (12), we obtain

$$\begin{aligned} \Delta\underline{x}(t) &\approx \frac{\partial\underline{x}(t)}{\partial\underline{x}_0}\Delta\underline{x}_0 \\ &\equiv \underline{x}_{\underline{x}_0}(t)\Delta\underline{x}_0 && (15) \end{aligned}$$

where $\underline{x}_{\underline{x}_0} \in \mathbb{R}^{\underline{n}\times\underline{n}}$. From (15), the sensitivity of the flow $\phi_{\underline{x}}$ to (small) changes $\Delta\underline{x}_0$ is given by the trajectory sensitivities $\underline{x}_{\underline{x}_0}(t)$.

A similar Taylor series expansion of ϕ_y yields

$$\begin{aligned}\Delta y(t) &= \Delta\phi_y(\underline{x}_0, t) \\ &= \frac{\partial\phi_y(\underline{x}_0, t)}{\partial\underline{x}_0}\Delta\underline{x}_0 + \text{ higher order terms.}\end{aligned}$$

Again neglecting the higher order terms and using (12) results in

$$\begin{aligned}\Delta y(t) &\approx \frac{\partial y(t)}{\partial\underline{x}_0}\Delta\underline{x}_0 \\ &\equiv y_{\underline{x}_0}(t)\Delta\underline{x}_0\end{aligned} \tag{16}$$

where $y_{\underline{x}_0} \in \mathbb{R}^{m\times\underline{n}}$. In this case the sensitivity of the flow ϕ_y to (small) changes $\Delta\underline{x}_0$ is given by the trajectory sensitivities $y_{\underline{x}_0}(t)$.

Once the trajectory sensitivities $\underline{x}_{\underline{x}_0}(t)$ and $y_{\underline{x}_0}(t)$ are known, the sensitivity of the system flow ϕ to small changes in initial conditions and parameters, which are described by $\Delta\underline{x}_0$, can be determined from

$$\Delta\phi(\underline{x}_0, t) = \begin{bmatrix} \Delta\underline{x}(t) \\ \Delta y(t) \end{bmatrix} = \begin{bmatrix} \underline{x}_{\underline{x}_0}(t) \\ y_{\underline{x}_0}(t) \end{bmatrix}\Delta\underline{x}_0. \tag{17}$$

We have yet to consider the calculation of the trajectory sensitivities. Details are provided in the following sections.

3.2. Sensitivity Evolution Away from Events

In this section we discuss the calculation of the trajectory sensitivities $\underline{x}_{\underline{x}_0}(t)$ and $y_{\underline{x}_0}(t)$ over the open time intervals (τ_k, τ_{k+1}), i.e., away from events. The behaviour of sensitivities at switching and reset events is presented in Section 3.3.

Away from events, the system model is given by (13),(14). Differentiating this DA system with respect to the initial conditions $\underline{x}_0$ results in

$$\dot{\underline{x}}_{\underline{x}_0} = \underline{f}_{\underline{x}}(t)\underline{x}_{\underline{x}_0} + \underline{f}_y(t)y_{\underline{x}_0} \tag{18}$$

$$0 = g_{\underline{x}}(t)\underline{x}_{\underline{x}_0} + g_y(t)y_{\underline{x}_0}. \tag{19}$$

Note that $\underline{f}_{\underline{x}}$, $\underline{f}_y$, $g_{\underline{x}}$, g_y are evaluated along the flow $\phi(\underline{x}_0, t)$, and hence are time varying matrices.

Initial conditions for $\underline{x}_{\underline{x}_0}$ on the first time interval $[t_0, \tau_1)$ are obtained by differentiating the x and z conditions of (6) with respect to $\underline{x}_0$,

$$\underline{x}_{\underline{x}_0}(t_0) = I \tag{20}$$

where I is the identity matrix. Initial conditions for $y_{\underline{x}_0}$ follow from (19),

$$0 = g_{\underline{x}}(t_0)\underline{x}_{\underline{x}_0}(t_0) + g_y(t_0)y_{\underline{x}_0}(t_0).$$

On other time intervals, say (τ_k, τ_{k+1}), the initial sensitivities $\underline{x}_{\underline{x}_0}(\tau_k^+)$, $y_{\underline{x}_0}(\tau_k^+)$ are given by the jump conditions described in Section 3.3.

3.3. Sensitivity Behaviour at Events

In Section 3.2 we established equations (18),(19) describing the evolution of the sensitivities $\underline{x}_{\underline{x}_0}$ and $y_{\underline{x}_0}$ over the intervals between switching and reset events. To fully describe the sensitivities though, we must quantify their behaviour at these discrete events that are characteristic of hybrid systems. To determine this behaviour, we will consider the system at a single event. Accordingly, attention is focused on the model

$$\dot{\underline{x}} = \underline{f}(\underline{x}, y) \tag{21}$$

$$0 = \begin{cases} g^-(\underline{x}, y) & s(\underline{x}, y) < 0 \\ g^+(\underline{x}, y) & s(\underline{x}, y) > 0 \end{cases} \tag{22}$$

$$\underline{x}^+ = \underline{h}(\underline{x}^-, y^-) \qquad s(\underline{x}, y) = 0 \tag{23}$$

which is directly related to the compact DAD model (8)-(11). A number of comments should be made about this model:

- In this model, the switching and reset events are triggered by the condition $s(\underline{x}, y) = 0$ rather than by an element of y passing through zero. This modification helps to identify the role of the triggering condition. We will later revert to the situation where the event is described by a condition $y_k = 0$.
- Notice that the model describes a coincident switching and reset event when $s(\underline{x}, y) = 0$. This is the most general case. Sensitivity behaviour at independent switching and reset events follows from this general case.
- We are investigating a single event. However the extension to the usual case where there are multiple events, each separated by a finite time interval, is straightforward.

Define the triggering hypersurface as

$$\mathcal{S} = \{(\underline{x}, y) \in \mathcal{C}\backslash\mathcal{I} : s(\underline{x}, y) = 0\}.$$

We are interested in the sensitivity of trajectories which pass through $\mathcal{S}$. It is convenient to assume that the trajectory starts from a point where $s(\underline{x}, y) < 0$, passes through $\mathcal{S}$, and proceeds to a point where $s(\underline{x}, y) > 0$. There is no loss of generality in this assumption. Let $\phi(\underline{x}_0, t) = [\underline{x}(t)^t \;\; y(t)^t]^t$ be such a trajectory, which starts from $\phi(\underline{x}_0, t_0) = [\underline{x}_0^t \;\; y_0^t]^t$, intersects $\mathcal{S}$ at the point $\phi(\underline{x}_0, \tau) = [\underline{x}(\tau)^t \; y(\tau)^t]^t$, and proceeds to the point $\phi(\underline{x}_0, t_1) = [\underline{x}_1^t \;\; y_1^t]^t$. The intersection point $(\underline{x}(\tau), y(\tau))$ is called the *junction point*, and τ is called the *junction time*.

The concept of trajectories passing through $\mathcal{S}$ is important. Sensitivities cannot be defined for trajectories which are tangential to $\mathcal{S}$. Consider such a trajectory. Then there exists an incremental change in the initial conditions $\underline{x}_0$, such that the intersection point disappears. But for a different small change in $\underline{x}_0$, the intersection point persists. Therefore at the tangent point, the trajectory is infinitely sensitive to initial conditions. To overcome this difficulty we make the following assumption.

Assumption 1 *Trajectories are transversal to the triggering hypersurface $\mathcal{S}$.*

It is also necessary to make the assumption,

Assumption 2 *The triggering function $s(\underline{x}, y)$ has a unique normal $\nabla s(\underline{x}, y)$ at points in $\mathcal{S}$.*

The transversality condition of Assumption 1 ensures that the junction point depends continuously on initial conditions $\underline{x}_0$ [2].

We also need to ensure that the switching and reset events are consistent with $s(\underline{x}, y)$ changing sign as $\mathcal{S}$ is crossed. The following assumption is therefore made.

Assumption 3 *At a junction point* $(\underline{x}(\tau), y(\tau)) \in \mathcal{S}$,

$$s(\underline{x}(\tau^-), y(\tau^-)) \times s(\underline{x}(\tau^+), y(\tau^+)) < 0.$$

This assumption is generically satisfied for realistic systems. If it was not satisfied, then the trajectory could reach an 'impasse' at the triggering hypersurface. Upon encountering $\mathcal{S}$, the algebraic equations would switch from g^- to g^+. But then if $s(\underline{x}(\tau^+), y(\tau^+)) \not> 0$, the model would be forced to switch back to g^-, which may result in switching to g^+ again, and so on.

Based on the model (21)-(23), it is shown in [8] that the jump conditions are given by

$$\underline{x}_{\underline{x}_0}(\tau^+) = \underline{h}^*_{\underline{x}}\, \underline{x}_{\underline{x}_0}(\tau^-) - \left(\underline{f}^+ - \underline{h}^*_{\underline{x}}\, \underline{f}^-\right) \tau_{\underline{x}_0} \qquad (24)$$

where

$$\underline{h}^*_{\underline{x}} = \left(\underline{h}_{\underline{x}} - \underline{h}_y (g_y^-)^{-1} g_{\underline{x}}^-\right)\Big|_{\tau^-} \qquad (25)$$

$$\tau_{\underline{x}_0} = -\frac{\left(s_{\underline{x}} - s_y (g_y^-)^{-1} g_{\underline{x}}^-\right)\big|_{\tau^-} \underline{x}_{\underline{x}_0}(\tau^-)}{\left(s_{\underline{x}} - s_y (g_y^-)^{-1} g_{\underline{x}}^-\right)\big|_{\tau^-} \underline{f}^-} \qquad (26)$$

and $\underline{f}^+$, $\underline{f}^-$ are given by

$$\begin{aligned} \underline{f}^- &= \underline{f}(\underline{x}(\tau^-), y^-(\tau^-)) \\ \underline{f}^+ &= \underline{f}(\underline{x}(\tau^+), y^+(\tau^+)). \end{aligned} \qquad (27)$$

In developing the jump conditions, we chose to use an arbitrary trigger function $s(\underline{x}, y)$. It can be seen that this function influences the jump conditions through $\tau_{\underline{x}_0}$. Reverting back to the original system description (8)-(11), the trigger function becomes

$$s(\underline{x}, y) = y_k$$

for some k. Therefore $s_{\underline{x}} = 0$ and $s_y = [0 \ldots \overset{k}{1} \ldots 0]^t = \underline{1}_k$. Substituting into (26) gives

$$\tau_{\underline{x}_0} = -\frac{\underline{1}_k \left((g_y^-)^{-1} g_{\underline{x}}^-\right)\big|_{\tau^-} \underline{x}_{\underline{x}_0}(\tau^-)}{\underline{1}_k \left((g_y^-)^{-1} g_{\underline{x}}^-\right)\big|_{\tau^-} \underline{f}^-}.$$

4. EXAMPLE

The small power system of Figure 1 provides a practical example of a system where continuous and discrete dynamics interact. The real power load has recovery dynamics [9], and is modelled by

$$\dot{x}_p = \frac{1}{T_p}(P_s^0 - P_d) \qquad (28)$$

$$P_d = x_p + P_t V_3^2 \qquad (29)$$

where x_p is the load state driving the actual load demand P_d. The rate of recovery is dictated by the load time constant T_p.

An important aim of this example is to illustrate the ability of the DAD structure (1)-(5) to model logic-based systems. Therefore a relatively detailed representation of the automatic voltage regulator (AVR) of the tap changing transformer has been adopted. The

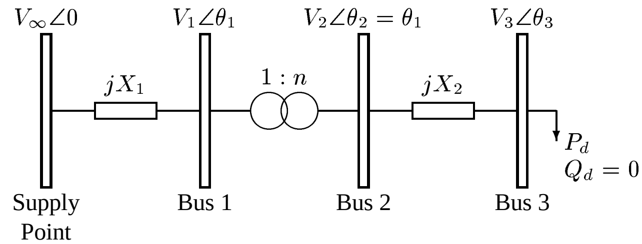

Figure 1: Power system example.

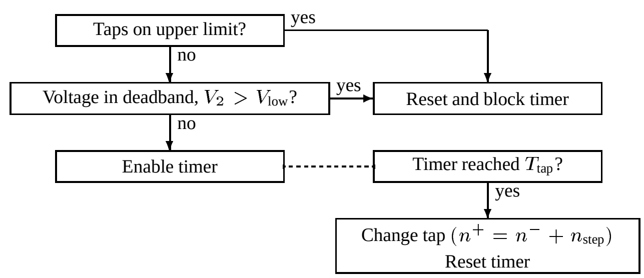

Figure 2: Tap changing transformer AVR logic.

logic flow of the AVR for low voltages, i.e., for increasing tap ratio, is outlined in Figure 2. It is shown in [8] that this AVR logic can be modelled by (1)-(5).

The system was disturbed at $t = 10$ seconds by increasing the impedance X_1. This simulated the loss of a feeder from the supply point to the transformer. The behaviour of the voltage at bus 3 is shown in Figure 3, along with the load demand P_d. The system was clearly stable, though the voltage underwent a large excursion. The voltage stabilized to a value that was below the pre-disturbance level because the transformer encountered its maximum tap.

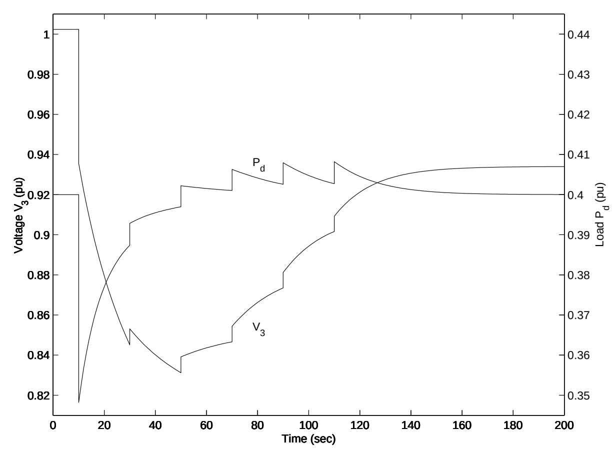

Figure 3: Voltage and load behaviour.

Figure 4 shows the sensitivity of the voltage V_3 trajectory to the parameters T_p and T_{tap}. These sensitivities are used in Figure 5 to approximate voltage behaviour when both T_p and T_{tap} are perturbed. Figure 5 shows the trajectory $V_3(5, 20)$ corresponding to the nominal parameter values $T_p = 5$, $T_{\text{tap}} = 20$. It also shows the trajectory $V_3(5.5, 22)$ which corresponds to perturbed parameters $T_p = 5.5$ and $T_{\text{tap}} = 22$. The third curve,

$$V_3^{\text{approx}}(5, 20) = V_3(5, 20) + 0.5 \frac{\partial V_3}{\partial T_p}(5, 20) + 2 \frac{\partial V_3}{\partial T_{\text{tap}}}(5, 20)$$

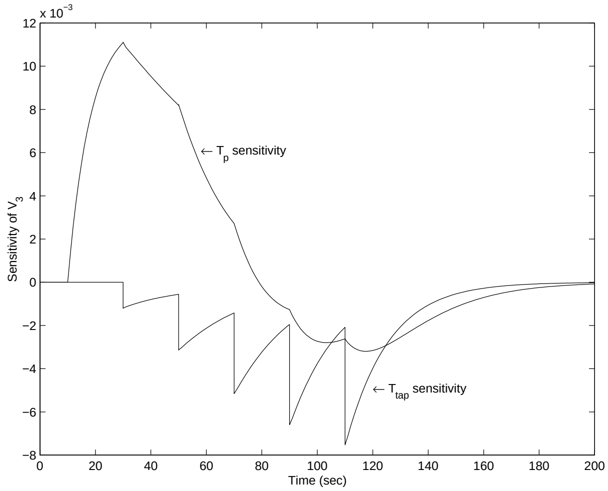

Figure 4: Trajectory sensitivities.

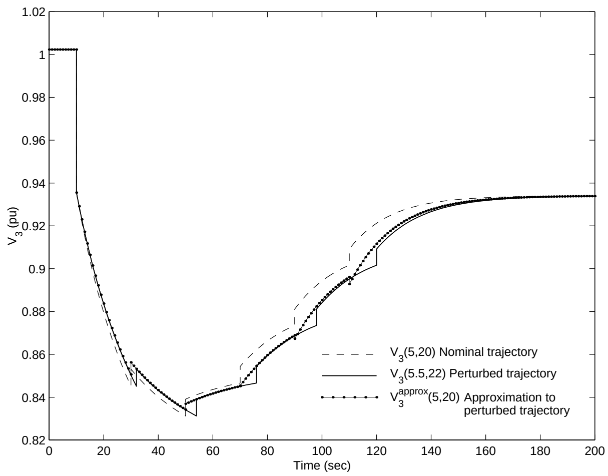

Figure 5: Trajectory approximation.

uses the sensitivities $\frac{\partial V_3}{\partial T_p}$ and $\frac{\partial V_3}{\partial T_{\text{tap}}}$, evaluated for the nominal case, to approximate $V_3(5.5, 22)$. The approximation is very close, except around the times where tapping occurs. As discussed in detail in [8], if parameter variations cause a change $\Delta\tau$ in the timing of an event, then the sensitivities cannot capture behaviour during that $\Delta\tau$ period. In this case, the increase in T_{tap} results directly in delaying the tap changing events.

Trajectory sensitivities provide helpful insights in the analysis of system behaviour. Consider first the sensitivity with respect to T_p. It can be seen from Figure 4 that an increase in T_p will lead to an increase in the voltage over the first 80 seconds of the trajectory, but after that it will result in a decrease in voltage. This is consistent with physical intuition. An increase in T_p corresponds to slower load recovery. During the initial voltage drop, the load is less than its steady state value P_s^0; see Figure 3. Therefore slower load recovery means the load is smaller for longer, so the voltage is higher. However over the latter section of the transient, whenever the voltage steps up due to a tap change, the load overshoots P_s^0. So the slower recovery corresponds to the load staying higher for longer, and hence to reduced voltage.

Now consider T_{tap}. From Figure 4 it can be seen that an increase in T_{tap} leads to a decrease in voltage. Again this is consistent with intuition. It is clear that the voltage recovery is due to the increase in the tap ratio. Increasing T_{tap} delays the tap changes, so the voltage stays lower for longer. The tap delay due to an increased T_{tap} accumulates with each tap change. Therefore the effect on the voltage becomes more pronounced with each subsequent tap change. This is evident in Figure 4.

For this simple example, the sensitivities do not provide qualitative information beyond that which is intuitively obvious. (Though they do provide quantitative information which is not otherwise available. For example, it can be seen from Figure 4 that a 1 second change in T_p would have a larger effect on the voltage trajectory than a 1 second change in T_{tap}.) However for more complicated systems, where the interpretation of parameter influences is not so straightforward, sensitivities can be extremely useful. Such a situation is explored in [6].

5. CONCLUSIONS

In this paper the concept of trajectory sensitivities has been extended to differential-algebraic-discrete systems. Application to a small power system containing a tap changing transformer and a dynamic load has been shown. Future work involves the extension to larger systems and resolution of the associated computational issues.

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