BLIND TRACKING USING SPARSITY PENALIZED MULTIDIMENSIONAL SCALING

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ABSTRACT

In this paper, we consider the problem of target tracking using sensor network measurements. We assume no prior knowledge of the sensor locations and so we refer to this tracking as 'blind'. Since any sensor localization algorithm can only find the sensor location estimates up to a rotation and translation, we propose a novel sparsity penalized multidimensional scaling (MDS) algorithm to align the current time sensor location estimates to those of the previous time-frames. In the presence of a target, only location estimates of those sensors in the vicinity of a target vary from their initially estimated values. Based on the differences in the sensor location estimates between two time-frames, we design a perturbation based algorithm naturally rising from the sparsity penalized MDS for tracking multiple targets relative to the initial sensor location estimates. Through a detailed numerical analysis, we show that the tracking algorithm based on sparsity penalized MDS outperforms the conventional likelihood ratio test (LRT) based tracking.

Index Terms— Target tracking, sensor localization, sensor networks, distributed detection

1. INTRODUCTION

Target tracking has been of significant interest in many military and civilian applications such as surveillance, vehicle tracking, robotics, biological research, and automotive collision warning systems. Depending on the models for the target trajectory and sensor measurements, tracking algorithms based on the Kalman Filter [1], extended Kalman filter [2], and Gaussian sum approximations [3] have been proposed. Particle filtering methods were then proposed for tracking, where the probability density of the state of the target (e.g., physical coordinates, velocity) is approximated on a set of discrete points [4]. Most prior work on tracking consider a model-based approach, which requires a detailed probabilistic model of the unknown target dynamics, more sensed information, and is computationally intensive.

A link level tracking algorithm localizes the target to within a small set of sensor links. Link level tracking has many attractive features, the most important of which is that it does not require a physical model for the target. This approach for a simple binary sensing measurement model is shown to require minimal power and is also analytically tractable [5]. Moreover, the goal of certain sensor networks is to obtain an estimate of the location of the targets, or detect changes in the network. For example, in military applications, the sensors can locate a target relative to the network and the network can activate the appropriate sensors to identify the target. For animal tracking in biological research, it is sufficient to have a low resolution tracking algorithm to monitor animal behavior and interactions with their own clan and with other species.

Most tracking algorithms assume knowledge of the sensor locations or estimate the sensor locations separately before employing the tracking algorithm. The process of estimating the sensor locations using a set of inter-sensor measurements is called sensor localization. Prior work on sensor localization assume the presence of anchor nodes, i.e., certain sensors which have knowledge of their positions in the network. In the absence of anchor nodes, the sensor location estimates are only accurate up to a rotation and translation. The intuition behind this result is as follows: consider the problem of estimating n sensor locations given the $n(n-1)/2$ inter-sensor distance measurements. The distance information depends only on the differences in the sensor locations so that the positions of the n sensors in the network can be rotated and translated without changing these distances.

In this paper, we propose the sparsity penalized distributed weighted multidimensional scaling (dwMDS) algorithm which simultaneously localizes the sensor nodes in the absence of anchors and tracks multiple targets. The principle behind our proposed algorithm is the following: in the 'acquisition phase' or initialization, an initial estimate of sensor locations is acquired. Once the sensors have been initially localized, it is only the network topology that is critical to the problem of tracking. Hence, during the tracking phase, we introduce a sparsity constraint to the cost function of a localization algorithm, which attempts to align the current time sensor location estimates to that of the previous time-frame. By doing so, we keep monitoring the network with respect to a fixed geometry obtained by the localization algorithm at the first time instance. The sparsity constraint only reassigns a small fraction of the sensor locations, while the rest of the sensor location estimates remain unchanged from their previously estimated values. When the sensor network is then used for tracking, only the sensors affected by the presence of a target are perturbed. Based on the differences in the sensor location estimates between two time-frames, we propose a novel perturbation based link level tracking algorithm, which accurately localizes a target to within a small set of sensor links. Since this tracking method arises naturally from the sparsity penalized MDS algorithm, it is able to perform spatial and temporal smoothing unlike the more conventional LRT based tracking. We present a detailed numerical analysis to illustrate the advantages of the perturbation based tracking method when compared to LRT based tracking. In the absence of a target trajectory model, we also suggest methods for translating this link level estimate to actual target coordinates.

The paper is organized as follows: in Section 2, we formulate the problem of sensor and target localization. In Section 3, we present the sparsity penalized MDS algorithm for sensor location alignment. We describe the LRT and the sparse MDS based tracking algorithms in Section 4 and present a numerical study of their performance in Section 5. We conclude this paper in Section 6.

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2. PROBLEM FORMULATION

The goal of this paper is to simultaneously localize the sensors and targets. Consider a network of $N = n + m$ sensors tracking the targets. Let $\mathbf{x}_1, \ldots, \mathbf{x}_N$ denote the true sensor locations. The m sensor nodes $\{\mathbf{x}_i\}_{i=n+1}^{n+m}$ are the anchor nodes, i.e., sensors which know their actual locations. Later, we set $m = 0$ for anchor free localization. Denote $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N]$ as the matrix of actual sensor locations. Let $\mathbf{D} = (d_{i,j})_{i,j=1}^{N}$ be the matrix of the true inter-sensor distances, where $d_{i,j}$ denotes the Euclidean distance between sensor i and sensor j. In some cases, there is imperfect a priori knowledge of certain sensor locations. This information is given by $\{\bar{\mathbf{x}}_i\}_{i=1}^{n}$ and the corresponding set of confidence weights is $\{r_i\}_{i=1}^{n}$. When $\bar{\mathbf{x}}_i$ is unavailable, we set $r_i = 0$. We obtain M inter-sensor received signal strength (RSS) measurements $\{P_{i,j}^{(t),k}\}_{k=1}^{M}$ for pairs of sensors i, j at time t. The indices (i, j) run over a subset of $\{1, 2, \ldots, N\} \times \{1, 2, \ldots, N\}$. Sensor localization is the process of estimating the location of the n sensor nodes $\{\mathbf{x}_i\}_{i=1}^{n}$ given $\{\mathbf{x}_i\}_{i=n+1}^{n+m}$, $\{r_i\}$, $\{\bar{\mathbf{x}}_i\}$, and $\{P_{i,j}^{(t),k}\}$. Furthermore, given the RSS measurements, the objective of the tracking algorithm is to identify the set of links i, j which indicate a presence of a target. These binary outputs are then used to obtain an estimate of the physical location of the target.

3. SPARSITY PENALIZED MDS

Sensor localization algorithms can be broadly classified into two categories: centralized strategies and decentralized strategies. In a centralized algorithm such as MDS, a fusion center estimates the sensor locations using the measurement data received from the sensors. In a decentralized algorithm, the localization of the sensor nodes is performed locally, i.e., each sensor estimates its location based on the information communicated from its neighbors. This distributive strategy limits power consumption and conserves bandwidth for large scale sensor networks. An example of decentralized localization is the dwMDS algorithm proposed in [6]. However, consistent reconstruction of the sensor locations is attainable only in the presence of anchor nodes. If the current localization algorithms are implemented for anchor free localization, the geometry of the sensor network assumes different alignments as localization is performed over various time instants. This makes it impossible to track changes in the network. To overcome this problem, we present a sparsity penalized dwMDS algorithm that aligns the current sensor location estimates to those of previous time-frames.

Consider using the MDS algorithm independently to obtain the sensor location estimates at time t and at time $t - 1$. Alignment between these two sets of points can be performed in various ways. For example, in Procrustes analysis [7] alignment is performed by finding the optimal affine transformation of one set of nodes that yields the set closest to the second set of points in the least squares sense. However, this procedure cannot guarantee that many sensor locations estimates will remain unchanged from their previously estimated values. The errors in the sensor location estimates between two time steps may accumulate over time resulting in alignment errors. In contrast, we introduce a sparseness penalty on the distances between the sensor location estimates at time t ($\mathbf{x}_i$) and at time $t-1$ ($\mathbf{x}_i^{(t-1)}$) directly to the sensor localization algorithm. Construct a vector of Euclidean distances between the location estimates at time t and at time $t - 1$

$$\mathbf{g}^{(t)} = \left[\|\mathbf{x}_1 - \mathbf{x}_1^{(t-1)}\|, \ldots, \|\mathbf{x}_n - \mathbf{x}_n^{(t-1)}\|\right]^T, \tag{1}$$

where $\|\cdot\|$ denotes the l_2-norm, i.e., $\|\mathbf{x}\| = \sqrt{\mathbf{x}^T\mathbf{x}}$. Define the l_0-measure of a vector $\mathbf{v} = [v_1, v_2, \ldots, v_n]$ as the number of nonzero elements given by

$$\|\mathbf{v}\|_0 \triangleq \sum_{i=1}^{n} I(v_i \neq 0), \tag{2}$$

where $I(\cdot)$ is the indicator function. Using an l_0-constraint on the distance vector $\mathbf{g}^{(t)}$ of the form $\|\mathbf{g}^{(t)}\|_0 \leq q$, we guarantee that no more than q of the location estimates will vary from their previous time-frame values. Minimizing a cost function under the l_0-constraint requires a combinatorial search which is computationally infeasible. Define the l_p-measure of a vector $\mathbf{v}$ as

$$\|\mathbf{v}\|_p \triangleq \Big(\sum_{i=1}^{n} |v_i|^p\Big)^{1/p}. \tag{3}$$

For a quadratic cost function, an l_p-constraint ($0 < p \leq 1$) induces a sparse solution. Among all l_p sparsifying constraints, only $p = 1$ offers a convex relaxation to the l_0-constraint [8]. To promote sparsity, we next advocate the use of the l_p-constraint as a penalty term via the Lagrange multiplier in the dwMDS algorithm to solve for the sensor location estimates. Hence the term *sparsity penalized MDS*.

The cost function of the dwMDS algorithm [6] is motivated by the variational formulation of the classical MDS, which attempts to find sensor location estimates that minimize the inter-sensor distance errors. Keeping in mind that it is the geometry of the sensor network which is crucial for tracking, we present a novel extension of the dwMDS algorithm through the addition of the sparseness inducing l_p-constraint. At any time t, we seek to minimize the overall cost function $C^{(t)}$ given by

$$C^{(t)} = \sum_{1\leq i\leq n} \sum_{i\leq j\leq n+m} \sum_{1\leq l\leq M} w_{i,j}^{(t),l} \left(\delta_{i,j}^{(t),l} - d_{i,j}(\mathbf{X})\right)^2 + \sum_{i=1}^{n} r_i\|\bar{\mathbf{x}}_i - \mathbf{x}_i\|^2 + \lambda\|\mathbf{g}^{(t)}\|_p^p. \tag{4}$$

For each time t, $\delta_{i,j}^{(t),l}$ is an estimate of the distance between sensor i and sensor j obtained from RSS measurement $P_{i,j}^{(t),l}$. The weights $\{w_{i,j}^{(t),l}\}$ are chosen to quantify the accuracy of the predicted distances. When no measurement is made between sensor i and sensor j, $w_{i,j}^{(t),l} = 0$. Furthermore, the weights are symmetric and $w_{i,i}^{(t),l} = 0$. If available, the a priori information of sensor locations is encoded through the penalty terms $\{r_i\|\bar{\mathbf{x}}_i - \mathbf{x}_i\|^2\}$. Finally, we introduce an l_p-constraint ($0 \leq p \leq 1$) on the distances between the sensor locations at time t and the estimated sensor locations at time $t - 1$. The Lagrange multiplier of the sparseness penalty is denoted as λ. We can tune the value of λ to yield the desired sparsity level in $\mathbf{g}^{(t)}$. Later, when we apply the algorithm for tracking, the sparseness will be advantageous as only those sensors which are highly affected by the target will vary from their initial positions, thereby allowing for a detection of the target through the process of relative sensor localization. The sensor location estimates are found by minimizing $C^{(t)}$ in (4) using optimization transfer. Closed-form iterations for a distributive implementation of the sparsity penalized MDS algorithm is derived in [9].

To find the maximum likelihood (ML) estimate of the distance from the RSS measurements, we assume the RSS to be log-normal in its distribution [10], i.e., if $P_{i,j}$ is the measured power by sensor

i transmitted by sensor j in milliWatts, then $10\log_{10}(P_{i,j})$ is Gaussian. Thus $P_{i,j}$ in dBm is typically modeled as

$$\begin{aligned} P_{i,j} &\sim \mathcal{N}(\bar{P}_{i,j}, \sigma_0^2) \\ \bar{P}_{i,j} &= P_0 - 10n_p \log\left(\frac{d_{i,j}}{d_0}\right), \end{aligned} \tag{5}$$

where $\bar{P}_{i,j}$ is the mean received power at distance $d_{i,j}$, σ_0 is the standard deviation of the received power in dBm, and P_0 is received power in dBm at a reference distance d_0. n_p is referred to as the path-loss exponent that depends on the multipath in the environment. Given the received power, we use maximum likelihood estimation to compute the range, i.e., distance between the sensor nodes i and j. The ML estimator of $d_{i,j}$ is given by

$$\delta_{i,j} = d_0 10^{\left((P_0 - P_{i,j})/10n_p\right)}. \tag{6}$$

4. TRACKING USING SPARSE MDS

Given the alignment of sensor location estimates between two time-frames, we now present an algorithm for performing link level tracking using the sparsity constrained MDS algorithm. Link level tracking does not require a physical model for a target. However, it is important to know the effect of the target on the inter-sensor measurements. Researchers have proposed various models for the RSS measurements ranging from the traditional linear Gaussian model to the binary sensing models. These are approximate statistical models and the distribution of the measurements in the presence of a target remains an open question.

To model the statistics under the setting of vehicle tracking, we conducted experiments using RF sensors hardware in the presence of a target [9]. We constructed a fine grid of locations, where the target was placed and RSS measurements were recorded between two static sensors for positions on the grid. Upon gathering the data, we fit the following statistical model in the presence of target. The RSS measurements under this H_1 hypothesis at sensor link i, j are distributed as

$$\begin{aligned} P_{i,j}^k | \hat{P}_{i,j} &\sim \mathcal{N}(\hat{P}_{i,j}, \sigma_0^2), \text{ i.i.d, } k = 1, 2, \ldots, M \\ \hat{P}_{i,j} &\sim \mathcal{N}(\bar{P}_{i,j}, \sigma_1^2), \end{aligned} \tag{7}$$

where $P_{i,j}^k$ is the k^{th} inter-sensor measurement when the target is in the neighborhood of the sensors. The M sensor link measurements are correlated through the random variable $\hat{P}_{i,j}$. The values obtained from our actual experiments were $\sigma_0 \approx 0.1463$dBm and $\sigma_1 \approx 1.5$dBm. The noise variance in the measurements σ_1 was roughly an order of 10 times larger than σ_0. In other words, RSS measurements tend to have a larger variance due to scattering and attenuation of the signals in the presence of a target. A confidence measure for such a log-normal distribution of the RSS data is obtained using the Kolmogorov-Smirnov (KS) test in [11] and the model is shown to work well for sensor localization. We assume this statistical model for the RSS measurements, when the target is within a specified distance R of the sensor link i, j. The distance R depends on the reflectivity of the object. If the object is highly reflective, then the variation in the RSS measurements is detected by more links.

Based on the H_0 and H_1 hypothesis given in (5) and (7) respectively, we formulate the optimal decision statistic to detect a presence of a target in a particular sensor link using the LRT. The LRT for each link i, j is given by

$$\left| \frac{1}{M} \sum_{l=1}^{M} P_{i,j}^{(t),l} - P_{i,j}^{'} \right| \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \gamma, \tag{8}$$

where γ is chosen to satisfy a false alarm level and $P_{i,j}^{'}$ is the mean received power in the sensor link estimated using an initial set of range measurements. We assume that the sensor network is in its steady state operation mode. We do not consider the transient effects in the measured data when it is obtained in the absence of any target. A derivation of the decision rule and its performance can be found in Appendix 7. We show that the performance of the optimal detector is dependent on the number of samples M available for the inter-sensor measurements. As M becomes very large, the probability of correct detection β tends to 1. However, if only few samples are available, β may not approach 1 and misdetect type errors may become non negligible. In such a case, instead of using the LRT, we can use a test on the variation of the sensor location estimates at time t from their estimates at a previous time τ ($\tau < t$). In other words, we can perform a simple hypothesis test for each link of the form

$$\left\| d_{i,j}^{(t)} - d_{i,j}^{(\tau)} \right\| \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \gamma_{i,j}, \tag{9}$$

where $d_{i,j}^{(t)} = \|\mathbf{x}_i^{(t)} - \mathbf{x}_j^{(t)}\|$ and $\{\mathbf{x}_i^{(t)}\}$ are the sensor location estimates obtained from the sparsity penalized MDS algorithm.

5. NUMERICAL STUDY

We analyze the performance of the localization algorithms using ROC curves. We consider the following setup: we deploy a 10×10 uniform grid of sensors in a network (see Fig. 2). We consider anchor free localization, i.e., $m = 0$ and make a single inter-sensor measurement ($M = 1$) at each time frame. We assume no a priori knowledge of the sensor coordinates, i.e., $r_i = 0$. Each sensor communicates only to its 8 nearest neighbors and the weights for those links were chosen by the LOESS strategy [6]. The rest of the weights were set to zero. Furthermore, we set noise variances σ_0 and σ_1 defined in (5) and (7), respectively as $\sigma_0 = 1$ and $\sigma_1 = 5\sigma_0 = 5$. Sensor links within a radius $R = 1.5$ indicate the presence of a target, i.e., follow the H_1 hypothesis. We set the reference distance d_0 defined below (5) to be $d_0 = 1$ and the path loss exponent $\eta = 2$. We set the sparseness parameters $\lambda = 2.5$ and $p = 1$ to produce a change in the location estimates for only a small portion ($< 10\%$) of the sensors.

We begin by considering the case of random appearance of targets in the sensor network, i.e., targets appear at different locations every time instant. For the distance based target localization algorithm (DBT), we set $\tau = 0$ in (9), i.e., we compare our distance estimates to a fixed initial frame. For every time instant, the DBT and the LRT are performed on each active sensor link and the process is repeated for 5000 target locations. The resulting ROC curve is presented in Fig. 1. The ROC for the LRT using simulations is indicated using circles and the corresponding theoretical curve obtained from (14) is shown as a solid line. We observe that the simulation and the theoretical curves match for the LRT. The ROC for the DBT is shown using a dashed line. The DBT algorithm yields higher probability of correct detection than the LRT for most false alarm levels. For example, at false alarm level $\alpha = 0.3$, β for the DBT is approximately 0.89 which is 5% more than that of the LRT,

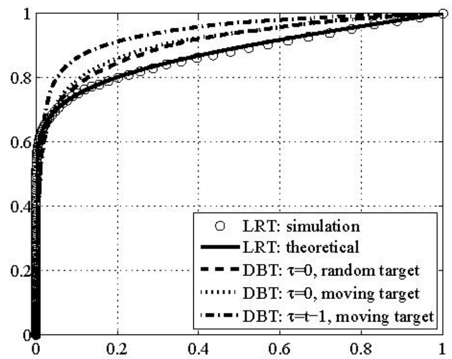

Fig. 1. ROC curve for the LRT and the DBT link level tracking algorithm. LRT (solid line), DBT for a random target with $\tau = 0$ (dashed), DBT for a moving target with $\tau = 0$ (dotted), and DBT for a moving target with $\tau = t - 1$ (dashed dotted).

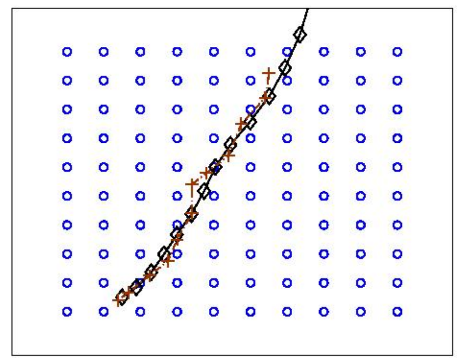

Fig. 2. A simple tracking algorithm based on link level tracking. True sensor locations (circle), true trajectory of the target (diamond), estimated trajectory (plus).

which yields $\beta \approx 0.84$. The intuition for this result is as follows: in the presence of a target, the RSS measurements of the sensor links are spatially-correlated. The presence of a target in a given link implies that with high probability the target is present in neighboring sensor links. However, the RSS model in (7) specifies only the distribution of the measurements independently on each link. The LRT makes complete use of the RSS measurements but is limited in its performance as the optimal decision statistic for each sensor link i, j is independent of other sensor link measurements. On the other hand, the DBT finds the active sensor links only based on the estimated distances through sparsity penalized MDS. However, since the inter-sensor distances are computed at each sensor using information from its nearest neighbors, this method makes an implicit use of the spatial correlation of the measurements in its decision statistic, which results in an improvement in performance.

Next, we consider the case of a moving target, where we assumed a standard state-space target motion model (for the purpose of a visually pleasing trajectory). We repeated the same algorithms for 5000 such trajectories. The LRT based algorithm yields the same performance curve as the test is independent of whether the target is moving or not. The resulting ROC curve for the DBT is presented as a dotted line in Fig. 1. Since we continue to base our decision rule on the fixed initial frame ($\tau = 0$), we observe that the performance of the DBT is also similar to the case of random target appearances.

In the case of a moving target, the RSS measurements are also temporally-correlated. Given a set of sensors indicating a presence of a target at a particular time, there is a high probability that the target is in the vicinity of these sensors at the next time frame. To make use of the temporal correlation, we can compare the current estimated distances to the estimated distances from the previous time-frame rather than the initial frame, i.e., set $\tau = t - 1$ instead of $\tau = 0$. The temporal correlation of the RSS measurements is captured in the DBT through the sparsity constraint used for aligning the sensors locations estimates. In other words, with high probability the sensor location estimates that are perturbed in the previous time-frame will also be perturbed in the current time-frame, thereby increasing the probability of detection. The results for $\tau = t - 1$ are presented in Fig. 1 using a dashed dotted line. We observe that the performance gains are higher than the DBT performed only with spatial smoothing ($\tau = 0$) as such a decision rule incorporates both spatial and temporal correlations of the target dynamics. For example, for $\alpha = 0.1$, β for the LRT is 0.75. The result of spatial smoothing alone yields $\beta \approx 0.79$. By performing both spatial and temporal smoothing, we can obtain $\beta \approx 0.86$ through our algorithm, which corresponds to a 15% increase in performance.

We make the following observations for the two proposed tests:

- The DBT for link level tracking outperforms the LRT as it can account for the spatial and the temporal correlations in the target motion.
- The LRT outperforms DBT for low false alarm levels ($\alpha < 0.01$) for the following reasons: first, the DBT we considered is suboptimal as we did not optimize the performance over the choice of sparsity (p, λ). Furthermore, the LRT uses an optimal decision statistic and the exact measurements to perform the test.
- Any scenario that exhibits high spatial correlations (e.g., highly reflective targets) can yield further improvement in performance of the DBT. If the sampling time for the sensors and the computation time of the DBT algorithm is much faster than the target motion, the DBT can yield better performance by taking advantage of more temporal correlations.
- The disadvantage of LRT in this setting is that the test is performed independently on each sensor link. Further improvements in the probability of detection can be achieved when the LRT is derived for the full spatio-temporal model.
- In the performance analysis, we assumed steady state operation, i.e., perfect knowledge of the inter-sensor distances are obtained a priori in the absence of target. If such knowledge is unavailable and distances need to be estimated, the LRT tracker must be modified to a generalized likelihood ratio test (GLRT). The DBT can estimate the initial set of distances more accurately from the RSS measurements by taking advantage of spatial correlations and hence can yield a higher probability of detection than the GLRT.

Given sensors localizing the target, there is a number of ways in which the sensor coordinates can be translated to target coordinates. For example, take the midpoint of the convex hull generated by the positions of those sensors that yield a high in the hypothesis test. Another estimate can be found by the intersection of convex regions corresponding to the sensor links that show the presence of the target through the optimal decision rule. An example of the midpoint tracking algorithm is shown in Fig. 2.

6. CONCLUSIONS

In this paper, we proposed a novel sparsity penalized MDS algorithm for simultaneous target and sensor localization. The subset selection capability of the sparsity constraint allowed us to find the set of sensors that have been perturbed in the presence of the target. Based on experimental results, we formulated statistical models for the RSS measurements in the presence and absence of targets. Using this model, we showed that for a large range of false alarm levels, the DBT outperforms the LRT as it is able to perform spatial and temporal smoothing without the need for target motion models. The nonparametric nature of our algorithm makes it attractive when RSS models are unavailable or inaccurate.

7. APPENDIX: OPTIMAL LIKELIHOOD RATIO TEST

To test the presence of a target on a sensor link i, j, we pose the following hypotheses testing problem

$$\begin{aligned} \mathrm{H}_0 &: P_1, \ldots, P_M \sim \mathcal{N}(\bar{P}, \sigma_0^2) \\ \mathrm{H}_1 &: P_1, \ldots, P_M | \hat{P} \sim \mathcal{N}(\hat{P}, \sigma_0^2), \text{ i.i.d}, \quad \hat{P} \sim \mathcal{N}(\bar{P}, \sigma_1^2), \end{aligned}$$

where $P_1, \ldots, P_M$ are the measurements made by a particular link i, j. We leave out the indices i, j in the measurements for brevity. $\bar{P}$ is the mean received power in the sensor link i, j. We assume it can be obtained during the system setup in the absence of targets. Denote the measurements by the M-element vector $\mathbf{p} = [P_1, P_2, \ldots, P_M]^T$. Then the hypotheses can be written as

$$\begin{aligned} \mathrm{H}_0 &: \mathbf{p} \sim \mathcal{N}(\bar{P}\mathbf{1}, \sigma_0^2 \mathbf{I}) \\ \mathrm{H}_1 &: \mathbf{p} \sim \mathcal{N}(\bar{P}\mathbf{1}, \sigma_1^2 \mathbf{1}\mathbf{1}^T + \sigma_0^2 \mathbf{I}). \end{aligned}$$

To construct the LRT, we first compute the log likelihood ratio as

$$\begin{aligned} \Lambda &= \log\left(\frac{f(\mathbf{p}|\mathrm{H}_1)}{f(\mathbf{p}|\mathrm{H}_0)}\right) \\ &= \frac{1}{2}(\mathbf{p} - \bar{P}\mathbf{1})^T(\mathbf{C}_0^{-1} - \mathbf{C}_1^{-1})(\mathbf{p} - \bar{P}\mathbf{1}) + \frac{1}{2}\log\left(\frac{|\mathbf{C}_0|}{|\mathbf{C}_1|}\right), \end{aligned} \tag{10}$$

where $\mathbf{C}_0 = \sigma_0^2 \mathbf{I}$, $\mathbf{C}_1 = \sigma_1^2 \mathbf{1}\mathbf{1}^T + \sigma_0^2 \mathbf{I}$ and $|\mathbf{C}|$ denotes the determinant of a matrix $\mathbf{C}$. The eigendecompositions of the covariance matrices $\mathbf{C}_0$ and $\mathbf{C}_1$ can be written as

$$\mathbf{C}_0 = \mathbf{V}_0 \mathbf{D}_0 \mathbf{V}_0^T, \quad \mathbf{C}_1 = \mathbf{V}_1 \mathbf{D}_1 \mathbf{V}_1^T,$$

where $\mathbf{D}_i$ is a diagonal matrix composed of the eigenvalues $\{\lambda_j^i\}_{j=1}^M$ and $\mathbf{V}_i$ is the matrix of corresponding eigenvectors. The eigenvalues of the covariance matrix $\mathbf{C}_1$ are given by $\lambda_1^1 = \sigma_1^2 M + \sigma_0^2$ and $\lambda_i^1 = \sigma_0^2$, $i = 2, \ldots, M$. The corresponding eigenvectors are $\mathbf{v}_1 = \mathbf{1}/\sqrt{M}, \mathbf{v}_2, \ldots, \mathbf{v}_M$, where $\{\mathbf{v}_i\}_{i=1}^M$ are a set of orthogonal unit norm vectors. The eigenvalues of $\mathbf{C}_0$ are all σ_0^2 and it is easy to verify that $\mathbf{v}_1, \ldots, \mathbf{v}_M$ are eigenvectors to $\mathbf{C}_0$, i.e., $\mathbf{V}_0 = \mathbf{V}_1$. Thus

$$\mathbf{C}_0^{-1} - \mathbf{C}_1^{-1} = \mathbf{V}_0 \mathrm{diag}\,(\psi, 0, \ldots, 0)\, \mathbf{V}_0^T = \psi \frac{\mathbf{1}\mathbf{1}^T}{M}, \tag{11}$$

where $\psi = \frac{\sigma_1^2 M}{\sigma_1^2 M + \sigma_0^2}$. Substituting (11) in (10) and collecting constant terms at the right hand side yields the optimal LRT as

$$|\bar{p} - \bar{P}| \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \gamma, \tag{12}$$

where $\bar{p} = \sum_{i=1}^M P_i/M$ is the minimal sufficient statistics of this test. Under H_0, $\bar{p}$ is distributed as $\mathcal{N}(\bar{P}, \sigma_0^2/M)$ and under H_1, $\bar{p}$ is $\mathcal{N}(\bar{P}, \sigma_0^2/M + \sigma_1^2)$. We find γ to satisfy a false alarm of level α, i.e.,

$$P\left(|\bar{p} - \bar{P}| > \gamma | \mathrm{H}_0\right) = 2Q\left(\frac{\sqrt{M}\gamma}{\sigma_0}\right) = \alpha, \tag{13}$$

which implies $\gamma = (\sigma_0/\sqrt{M})Q^{-1}(\alpha/2)$. The probability of correct decision, β is then given by

$$\begin{aligned} \beta &= P\left(|\bar{p} - \bar{P}| > \gamma | \mathrm{H}_1\right) \\ &= 2Q\left(\frac{\gamma}{\sqrt{\sigma_0^2/M + \sigma_1^2}}\right) \\ &= 2Q\left(Q^{-1}(\alpha/2)\sqrt{\frac{\sigma_0^2}{\sigma_0^2 + M\sigma_1^2}}\right). \end{aligned} \tag{14}$$

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