A NEW METHOD FOR ADAPTIVE WIDEBAND BEAMFORMING

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ABSTRACT

Classical AOA estimation methods assume incoming signals are either purely narrowband or purely wideband in nature. There are many physical scenarios where this may not be the case. We call such scenarios mixed, in that the incoming signals contain a combination of both wideband and narrowband components.

The paper introduces a new method for performing AOA estimation using a wideband adaptive beamsummer in the presence of mixed (narrowband and wideband) signals. The method uses an adaptation criterion which is a function of both the mean and variance of the beamsummer output. The method is an extension of a similar narrowband beamsummer method introduced in [1]. Simulation results are presented which compare the performance of the new criterion to the standard narrowband and wideband adaptation criteria. These results show that the new method can perform as well or better than the classical methods in terms of target resolution and angle of arrival estimator variance.

1. INTRODUCTION

The use of beamsumming arrays for determining the angle of arrival (AOA) of an impinging signal has been studied extensively in recent years. Excellent reviews of the general concept of AOA estimation (both narrowband and wideband) using beamsumming arrays are given in [2,3]. Several new methods have also been proposed [3-8]. In the majority of treatments, signals are assumed to be purely wideband or narrowband. However, there are many scenarios when impinging signals may have both wideband and narrowband characteristics. For example, narrowband signals in the presence of fast fading exibit a strong component at the narrowband carrier frequency as well as a wideband modulation from the fading channel. As another example, in a multiple access communications channel, an array may simultaneously receive coherent narrowband and coherent wideband type signals. In cases such as these, classical methods based on purely wideband or purely narrowband signal models will be suboptimal. Indeed, for the case of a narrowband signal in the presence of fading, wideband methods might work better during times of deep fading, while narrowband methods may work better during shallow fades. In this paper we review the differences between the classical narrowband and wideband AOA estimation methods, and propose a new generalized wideband beamsumming method which handles narrowband, wideband, and mixed signals.

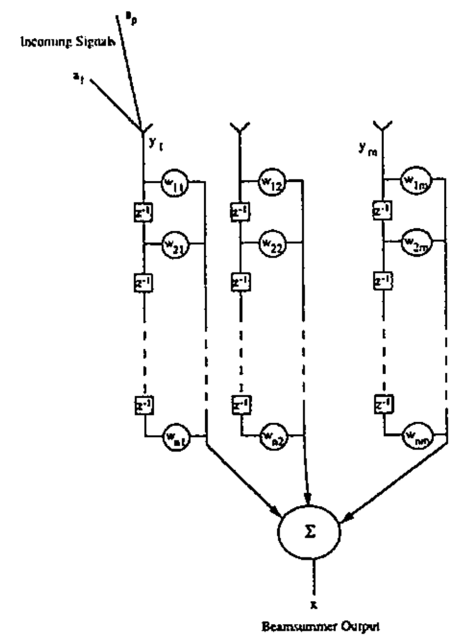

Figure 1: Beamsummer Configuration

2. PROBLEM STATEMENT

We begin by looking at the vector output of the m-element linear wideband antenna array of Figure 1. In this figure, a_1 through a_p represent the amplitudes of p incoming signals arriving at angles θ_1 through θ_p. These signals may be narrowband, wideband, or mixed. At the kth time instant, we take a snapshot $\underline{Y}^k = [y_1^k, ..., y_m^k]^T$ of the outputs of the antenna elements. We make the following definitions and assumptions. The snapshots, $\{\underline{Y}^k\}$ consist of independent identically distributed complex baseband data. The signal amplitudes may or may not have significant variation from element to element. We characterize this variation by letting the signal amplitudes at each element be random variables with unknown mean μ, variance σ^2, and correlation coefficient ρ. There are three general signal categories associated with the random signal amplitudes: $\rho = 1$, $\sigma^2 = 0$, $|\mu| > 0$ for perfectly coherent narrowband signals; $\rho = 0$, $\sigma^2 > 0$, and $\mu = 0$ for incoherent wideband signals, and $0 < \rho < 1$, $\sigma^2 > 0$, and $|\mu| > 0$ for mixed signals. The output of the antenna array at time k is given by

$$\underline{Y}^k = \begin{bmatrix} y_1^k \\ y_2^k \\ \cdot \\ \cdot \\ y_m^k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^p a_{i1}^k + n_1^k \\ \sum_{i=1}^p a_{i2}^k e^{-ju_i} + n_2^k \\ \cdot \\ \cdot \\ \sum_{i=1}^p a_{im}^k e^{-j(m-1)u_i} + n_m^k \end{bmatrix} \quad (1)$$

where a_{iq}^k represents the input at antenna element q due to the incoming signal i at time instant k, and $\underline{N}^k = [n_1^k, ..., n_m^k]^T$ is a spatially incoherent and temporally uncorrelated noise vector.

The beamsummer output, is given by

$$x^k = \sum_{i=1}^{m} \sum_{j=1}^{n} w_{ij}^* y_i^{k-i+1} \quad (2)$$

where w_{ij} is a complex weight associated with the i^{th} tap of the tapped delay line applied to sensor j.

By defining the 1xp dimensional steering vectors $\underline{S}_q = [e^{-j(q-1)u_1}, ..., e^{-j(q-1)u_p}], q \in 1, ..., m$, the mxpm dimensioned
block diagonal steering matrix S with diagonal elements $\underline{S}_q$, and $\underline{A}^k = [a_{11}^k, ..., a_{p1}^k, a_{12}^k, ..., a_{p2}^k, ..., a_{1m}^k, ..., a_{pm}^k]^T$, we can rewrite (1) as

$$\underline{Y}^k = \mathrm{S}\underline{A}^k + \underline{N}^k \quad (3)$$

By further defining

$$\underline{\mathrm{W}} = [w_{11}, ..., w_{1m}, w_{21}, ..., w_{2m}, ..., w_{nm}]^H, \quad (4)$$

$$\underline{\mathcal{Y}}^k = [\underline{Y}^k, \underline{Y}^{k-1}, ..., \underline{Y}^{k-n}]^T, \quad (5)$$

$$\underline{\mathcal{A}}^k = [\underline{A}^k, ..., \underline{A}^{k-n}]^T, \quad (6)$$

$$\underline{\mathcal{N}} = [\underline{N}^k, ..., \underline{N}^{k-n}]^T, \quad (7)$$

and the block diagonal matrix $\mathcal{S}$ with n diagonal elements, each of which is an S matrix as in (3), we have the vector forms of (2) :

$$\begin{aligned} x^k &= \underline{\mathrm{W}}^H \underline{\mathcal{Y}}^k \\ &= \underline{\mathrm{W}}^H \mathcal{S}\mathcal{A} + \mathcal{N} \end{aligned} \quad (8)$$

The beamsummer output x^k has mean and variance

$$\begin{aligned} E[x^k] &= \underline{\mathrm{W}}^H E[\underline{\mathcal{Y}}] = \underline{\mathrm{W}}^H \underline{\mu}_{\mathcal{Y}} & (9) \\ &= \underline{\mathrm{W}}^H \mathcal{S} \underline{\mu}_{\mathcal{A}} & (10) \\ cov[x^k] &= E[(x^k - \mu_x)(x^k - \mu_x)^*)] = \underline{\mathrm{W}}^H \Lambda_{\mathcal{Y}} \underline{\mathrm{W}} & (11) \\ &= \underline{\mathrm{W}}^H \mathcal{S} \Lambda_{\mathcal{A}} \mathcal{S}^H \underline{\mathrm{W}} & (12) \end{aligned}$$

where $\Lambda_{\mathcal{Y}}$ is the covariance of the $\underline{\mathcal{Y}}$ vector.

While most wideband treatments assume $\underline{\mu}_{\mathcal{Y}}$ is zero, we assume that $\underline{\mu}_{\mathcal{Y}}$ may be non-zero due to the mixed nature of the signals. This implies that there may be information about the signal AOAs in *both* the mean and the variance of the beamsummer output (from (10) and (12)).

3. CLASSICAL BEAMSUMMING METHODS

To perform AOA estimation using a beamsumming array, one forms the beamsums, $x^k = \underline{\mathrm{W}}^H \underline{\mathcal{Y}}^k, k = 1, \ldots, l$, where $\underline{\mathrm{W}}$ is normalized so that $\underline{\mathrm{W}}^H \underline{\mathrm{W}} = 1$. The weight vector $\underline{\mathrm{W}}$ is then adjusted to maximize a signal quality index (SQI), e.g. the power of x^k, which is computed using the samples $[x^k]_{k=1}^l$.

The classical narrowband beamsummer [9], assumes the signals have variance $\sigma^2 = 0$, and maximizes the squared mean beamsummer output, which we denote as λ^2:

$$\lambda^2 = |E[x^k]|^2 = \underline{\mathrm{W}}^H E[\underline{\mathcal{Y}}] E[\underline{\mathcal{Y}}]^H \underline{\mathrm{W}} \quad (13)$$

The classical wideband beamsummer [10] assumes the signals have mean $\mu = 0$, and maximizes the variance of the beamsummer output, which we denote as α^2:

$$\alpha^2 = E[xk^2] = \underline{\mathrm{W}}^H \Lambda_{\mathcal{Y}} \underline{\mathrm{W}} \quad (14)$$

Note that for this case $\lambda^2 = 0$, and the wideband beamsummer maximizes the mean-squared beamsummer output, $\lambda^2 + \alpha^2$.

4. A NEW METHOD

Looking at the classical wideband and narrowband beamsumming methods, we see that each uses one of two available statistical properties of the incoming signals, the mean or the variance. Looking at equations (10) and (12), it becomes apparent that if the signals are modeled as having a non-zero mean *and* a nonzero variance, a new beamsumming criterion might be appropriate. One such criterion which we have investigated extensively is a convex combination of the narrowband and wideband criteria, given by:

$$\gamma = (1 - \epsilon)\lambda^2 + \epsilon(\alpha^2 + \lambda^2) \quad (15)$$

where $\epsilon = \frac{\alpha^2}{1+\alpha^2}$. Looking at (15), we see that for signals with a very small variance (e.g. narrowband signals), $\alpha^2 \approx 0$ and γ reduces to the classical narrowband beamformer criterion. On the other hand, for signals with very small mean (e.g. wideband signals), $\lambda \approx 0$ and γ is equivalent to the classical wideband criterion. For cases that fall between these extremes, we have shown [1] that the new criterion is inversely proportional to the Cramer-Rao bound (CRB) on AOA estimator mean squared error (MSE) for the special case of no delay taps and $\rho = 1$. Other cases are simulated in the following section.

5. SIMULATION RESULTS

The new criterion is easily implemented by extension of an algorithm introduced in [1]. The algorithm, which is designed for a wideband beamsummer, operates as follows:

We wish to maximize our new SQI over the weight vector $\underline{W}$:

$$\begin{aligned} \gamma &= (1-\epsilon)\lambda^2 + \epsilon(\alpha^2+\lambda^2) \\ &= \lambda^2 + \frac{\alpha^2}{1+\alpha^2}\alpha^2 \end{aligned} \tag{16}$$

Using Lagrange multipliers, it can be shown that (16) is maximized by iteratively maximizing the following two functions over $\underline{W}$:

$$(\mathcal{S}\underline{\mu}_{\mathcal{A}}\underline{\mu}_{\mathcal{A}}^H\mathcal{S}^H + \tilde{K}\Lambda_{\mathcal{Y}})\underline{W} = \lambda\underline{W}, \tag{17}$$

where

$$\begin{aligned} \tilde{K} &= F(\underline{W}, \Lambda_{\mathcal{Y}}) \\ &\stackrel{\text{def}}{=} \frac{2\underline{W}^H\Lambda_{\mathcal{Y}}\underline{W} + (\underline{W}^H\Lambda_{\mathcal{Y}}\underline{W})^2}{(1+\underline{W}^H\Lambda_{\mathcal{Y}}\underline{W})^2} \end{aligned} \tag{18}$$

The alorithm is initialized by choosing an initial guess for the weight vector $\underline{W}$. Methods for finding an initial guess are discussed in [1]. This $\underline{W}$ is used to calculate $\tilde{K}$. $\tilde{K}$ is then substituted into (17), and a new $\underline{W}$ is found by letting $\underline{W}$ be the maximum eigenvector of the matrix $(\mathcal{S}\underline{\mu}_{\mathcal{A}}\underline{\mu}_{\mathcal{A}}^H\mathcal{S}^H + \tilde{K}\Lambda_{\mathcal{Y}})$. This $\underline{W}$ value is then used to update $\tilde{K}$, and the procedure is repeated until the algorithm converges, typically after two or three iterations.

We note here that the values $\Lambda_{\mathcal{Y}}$ and $\underline{\mu}_{\mathcal{A}}$ are generally unknown and must be estimated from the data. This is the algorithm that is used to calculate optimal weights in the following subsections. We used the sample mean and variance as estimates of the true mean and variance of $\mathcal{Y}$. Monte-Carlo simulations were performed to assess the performance of the new criterion as compared to the classical criterion using the wideband beamsummer array of Figure 1. Based on these simulations we conclude that for mixed wideband and narrowband signals:

1. The new criterion appears to be more likely to resolve closely spaced targets.
2. The new criterion appears to yield lower variance in the antenna beam pattern and in the AOA estimates.

On the other hand, when the signals are purely narrowband or purely wideband, the new criterion appears to perform reasonably close to the classical narrowband or wideband criterion, respectively.

5.1 Multiple Signal Resolution

An example of the simulations we performed is as follows. We simulated two equal power incoming signals at angles of +/− .25 radians with 5 antenna elements, and two delay taps per antenna element. The incoming signals had a squared-mean SNR, $|\mu_{a_i}|^2/\sigma^2$, of 1, and a mean-squared SNR, $(|\mu_{a_i}|^2 + \sigma_{a_i}^2)/\sigma^2$ of 2, $i = 1, 2$. The spatial correlation coefficient, ρ, was .5. As snapshots at times $k = 1, 2, \ldots l$ were generated we computed a running sample mean and running sample variance of the array outputs which were used to find the optimal weight vector sequence $\{\underline{W}^k\}_{k=1}^l$ under both the classical(wideband) and the new criteria. For each $k = 1, 2, \ldots l$ we computed and plotted the associated antenna gain patterns as a function of angle. As estimates of signal angle-of-arrivals we chose the angles of maximum array gain.

In Figures 2.1 a-e we plot the average antenna gain pattern over the course of 50 snapshots. Each of the plots in Figure 2.1 are based on 200 trials. The dashed lines correspond to the beam pattern generated by the weights obtained from the new SQI criterion, while the solid lines correspond to that generated by the classical weights. Figure 2.1a shows the average beam patterns generated using 10 snapshots, Figure 2.1b 20 snapshots, and so on. While it appears from this figure that both methods are resolving the two signals on average, the new method appears to have sharper peaks and deeper nulls. This is because the new method consistently resolves both of the signals, while the classical method frequently resolves only one of the two signals, or fails to resolve either one. This can be seen in Figure 2.2 a-e, which represents a single realization.

5.2 Variance in AOA Estimates

For this simulation we repeated the experiment of the previous section and for 30 snapshots and varied ρ from zero to 1. When ρ is close to zero, and when ρ is close to one, the signals are almost perfectly correlated. For each of these extremes, $\rho = 0$ and $\rho = 1$, the information about the signals coherence is contained in the mean, i.e. the narrowband regime, although the effective coherent SNR is less for $\rho = 0$ than for $\rho = 1$. for $0 < \rho < 1$ this information is contained in both the mean *and* the variance. For each value of ρ we ran 200 trials and computed the variance in the AOA estimates using the classical wideband, narrowband, and the new methods. The results are shown in Figure 3. We see that the new criterion (dashed line) outperforms the classical wideband criterion (solid line) in most cases, except when ρ becomes very small and the new criterion outperforms the classical narrowband criterion (dotted line) except as ρ approaches 1.

CONCLUSIONS

We have presented a new criterion for generating wideband adaptive antenna array weight vectors. This method is relatively easy to implement and appears to give better AOA estimation performance than the classical method, particularly when incoming signals are mixed. We intend to continue this research by comparing the new criterion to the more complicated (and theoretically more accurate) MLE methods.

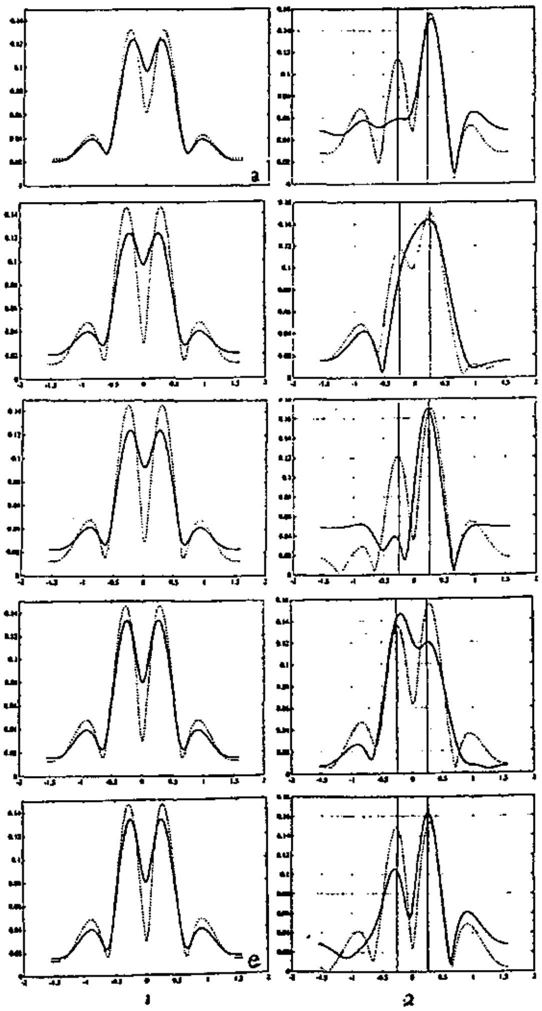

Figure 2: Antenna Patterns - Classical Vs New

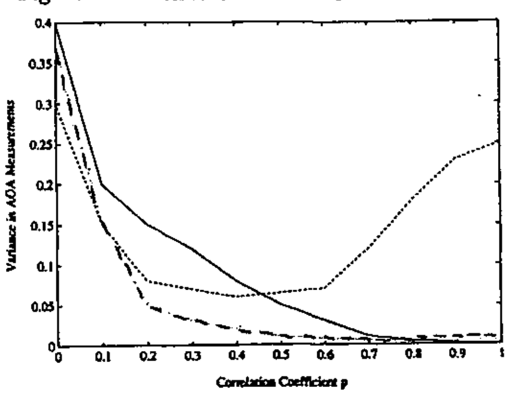

Figure 3: Variance in AOA estimates Vs ρ

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