*(Mainly)*

# Linear Models

EECS 442 – Prof. David Fouhey

Winter 2019, University of Michigan

http://web.eecs.umich.edu/~fouhey/teaching/EECS442_W19/

# Administrivia

- ***60 people:*** please sign up for AWS educate. If you run into issues, please email them.
- HW2 Due Tonight; HW3 Out Tonight but not due for a fairly long time (March 14) due to Spring Break and Midterm.
- Remember: you have late days, and late days are for spending
- Start ***early***. Be inspired by RANSAC: Trying randomly will do the right thing if you try often.

# Midterm

- Covers up to February 19 (next class)
- We’ll have review sessions
- In class but randomized
- Please tell us about accomodations

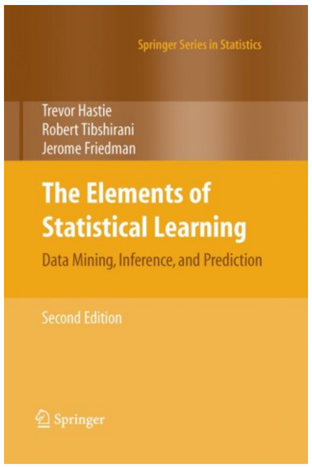
# Today and Next Tuesday

- Machine Learning (ML) Crash Course
- I can’t cover everything
- If you can, take a ML course or *learn online*
- ML really won’t solve all problems and is incredibly dangerous if misused
- But ML is a powerful tool and not going away

# Terminology

- ML is incredibly messy terminology-wise.
- Most things have at lots of names.
- I will try to write down multiple of them so if you see it later you’ll know what it is.

# Pointers




Useful book (Free too!):
The Elements of Statistical Learning
Hastie, Tibshirani, Friedman
https://web.stanford.edu/~hastie/ElemStatLearn/

Useful set of data:
UCI ML Repository
https://archive.ics.uci.edu/ml/datasets.html

A lot of important and hard lessons summarized:
https://homes.cs.washington.edu/~pedrod/papers/cacm12.pdf

# Machine Learning (ML)

- Goal: make “sense” of data
- Overly simplified version: transform vector **x** into vector **y**=T**(x)** that’s somehow better
- Potentially you fit T using pairs of datapoints and desired outputs ($\mathbf{x}_i$,$\mathbf{y}_i$), or just using a set of datapoints ($\mathbf{x}_i$)
- Always are trying to find some transformation that minimizes or maximizes some **objective function** or goal.

# Machine Learning

Input: **x**

**Feature vector/Data point:**
Vector representation of datapoint. Each dimension or "**feature**" represents some aspect of the data.

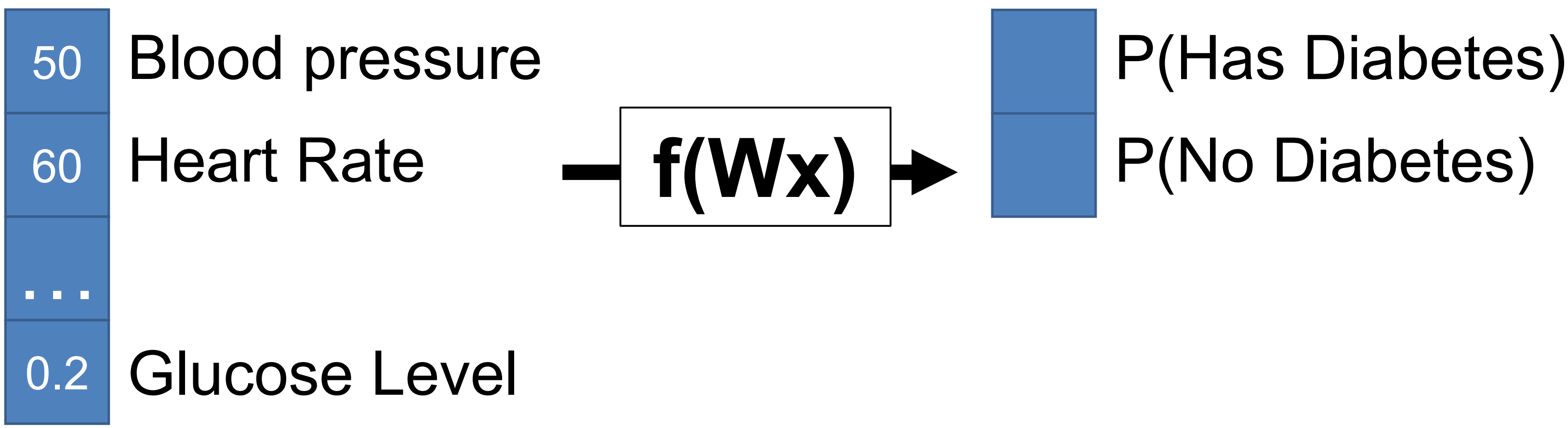
Output: **y**

**Label / target:**
Fixed length vector of desired output. Each dimension represents some aspect of the output data

**Supervised**: we are given y.
**Unsupervised**: we are not, and make our own ys.

# Example – Health

Input: **x** in $R^N$ Output: **y**

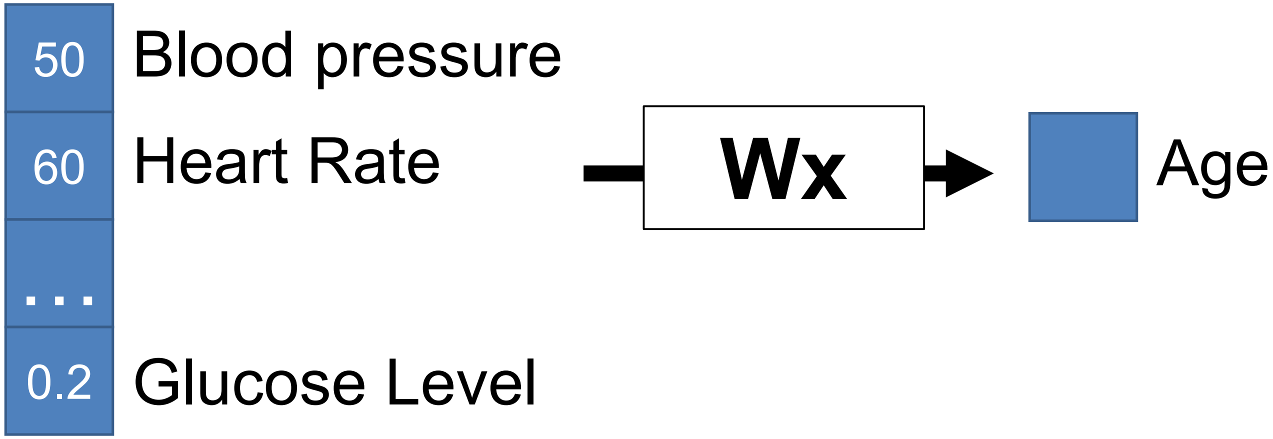



*Intuitive objective function*: Want correct category to be likely with our model.

# Example – Health

Input: **x** in $R^N$

Output: **y**

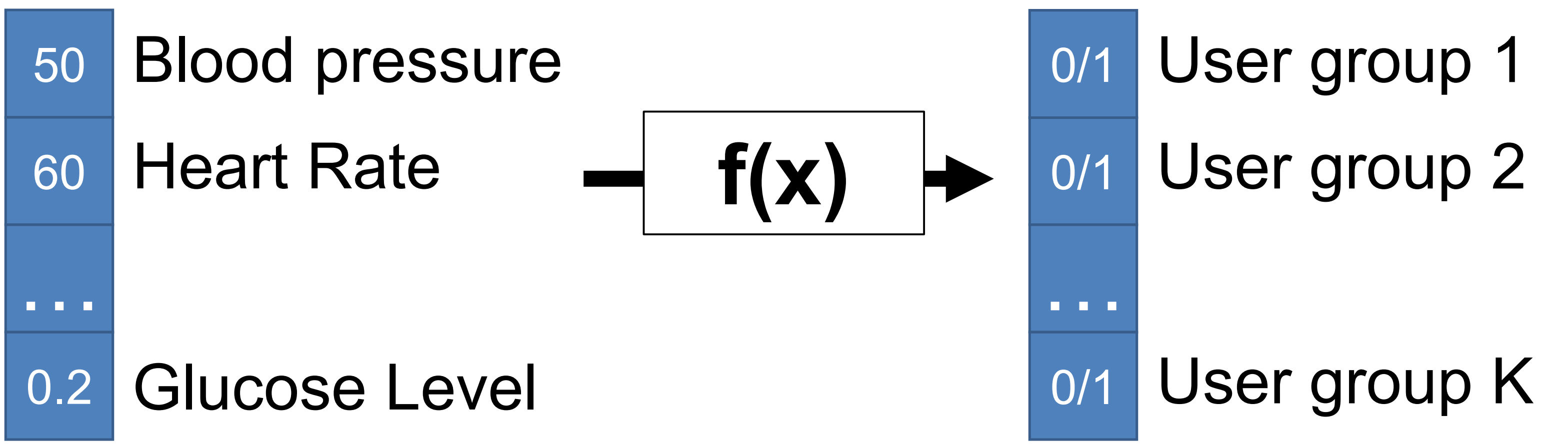



*Intuitive objective function*: Want our prediction of age to be "close" to true age.

# Example – Health

Input: **x** in $R^N$

Output: **discrete y (unsupervised)**

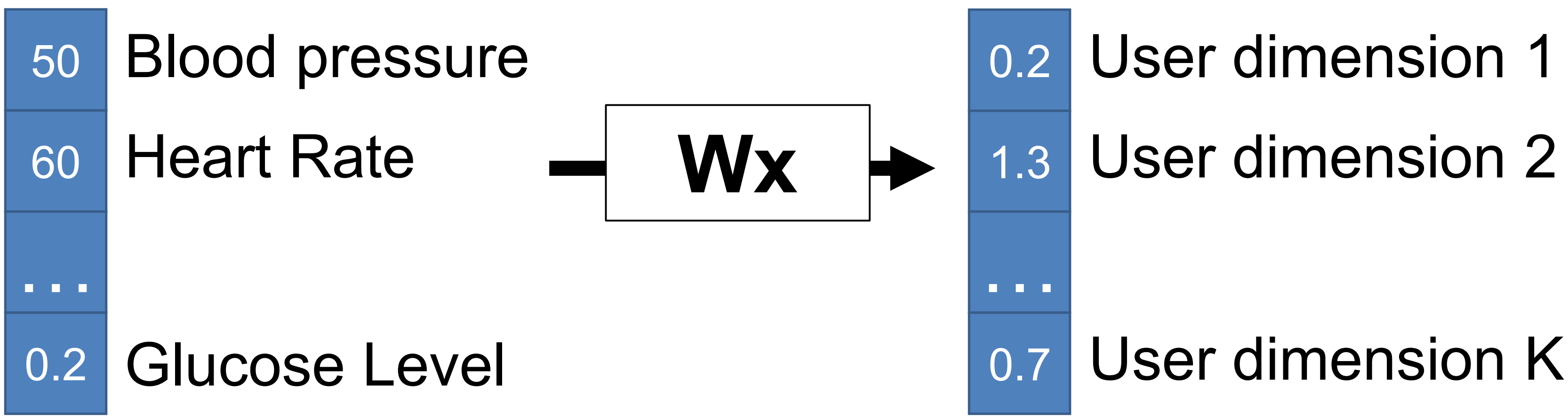



*Intuitive objective function*: Want to find K groups that explain the data we see.

# Example – Health

Input: **x** in $R^N$

Output: **continuous y (discovered)**

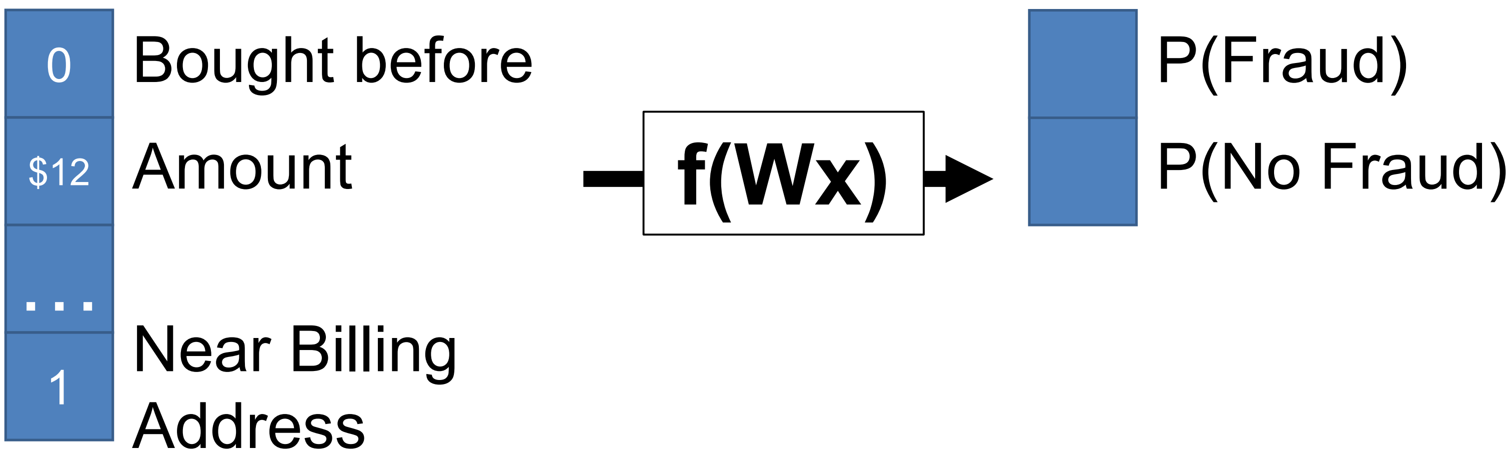



*Intuitive objective function*: Want to K dimensions (often two) that are easier to understand but capture the variance of the data.

# Example – Credit Card Fraud

Input: **x** in $R^N$

Output: **y**

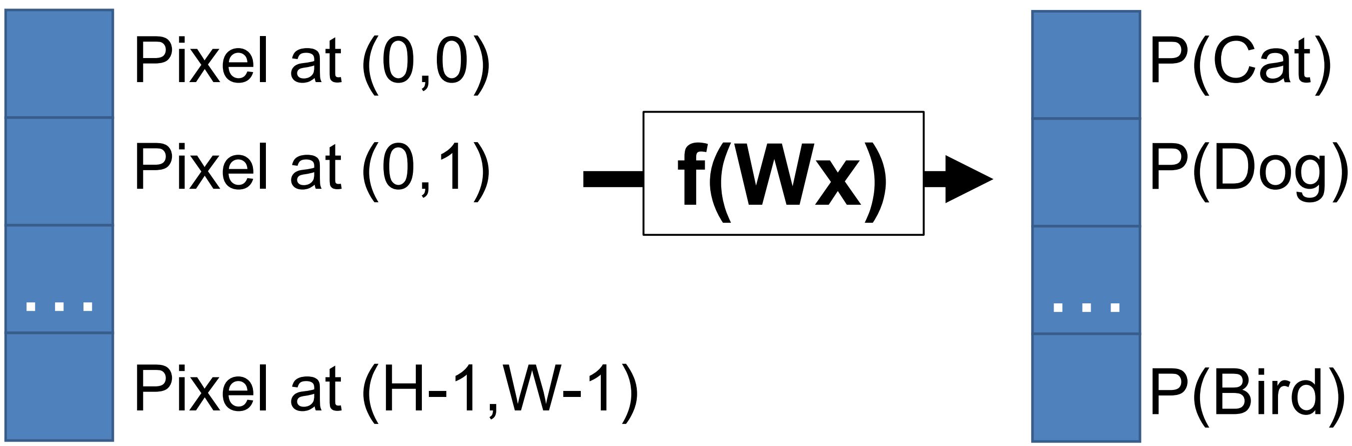



*Intuitive objective function*: Want correct category to be likely with our model.

# Example – Computer Vision

Input: **x** in $R^N$

Output: **y**

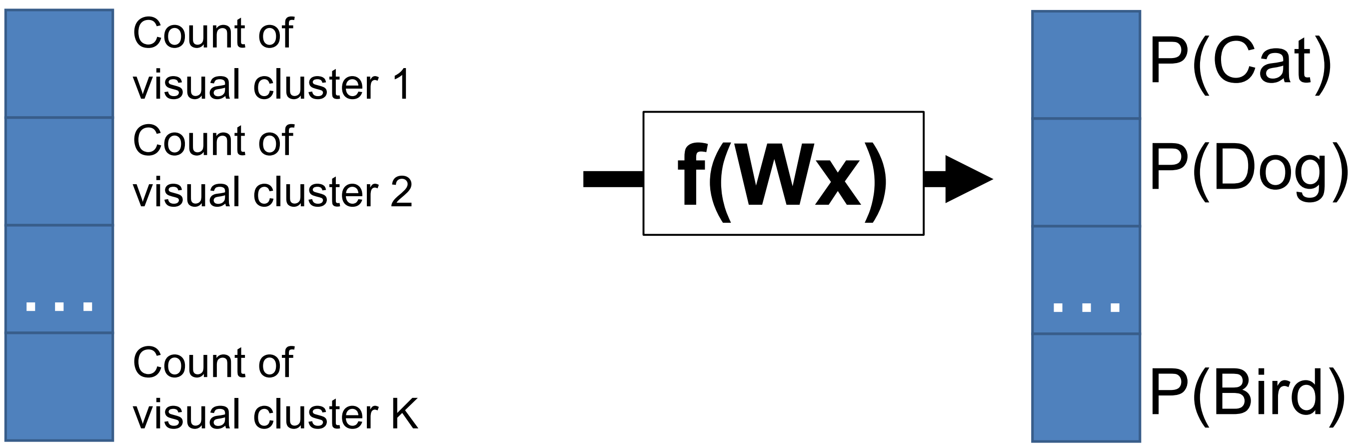



*Intuitive objective function*: Want correct category to be likely with our model.

# Example – Computer Vision

Input: **x** in $R^N$

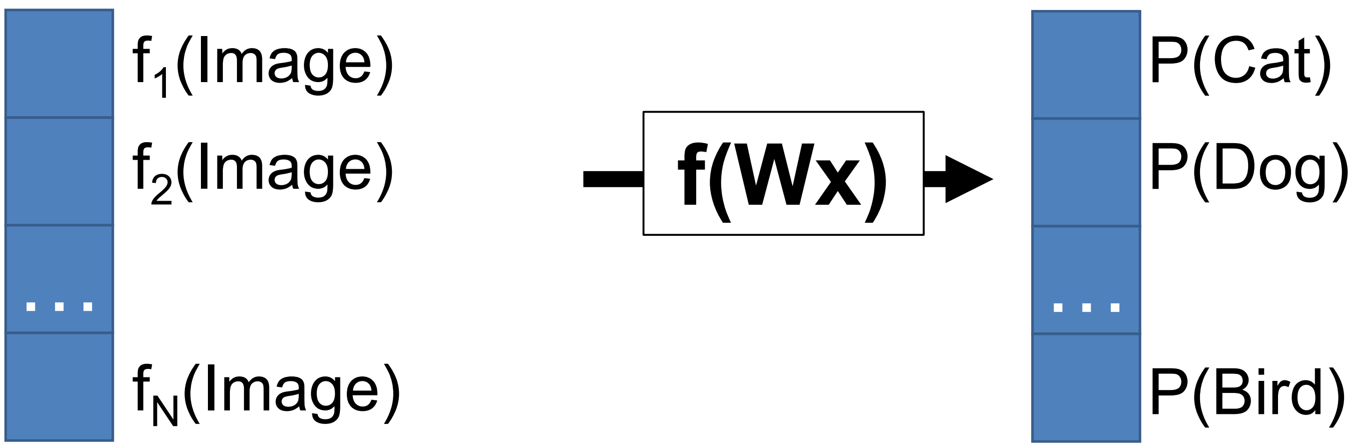
Output: **y**




*Intuitive objective function*: Want correct category to be likely with our model.

# Example – Computer Vision

Input: **x** in $R^N$ Output: **y**




*Intuitive objective function*: Want correct category to be likely with our model.

# Abstractions

- Throughout, assume we've converted data into a fixed-length feature vector. There are well-designed ways for doing this.
- But remember it could be big!
  - Image (e.g., 224x224x3): 151K dimensions
  - Patch (e.g., 32x32x3) in image: 3072 dimensions

# ML Problems in Vision


Image credit: Wikipedia

# ML Problem Examples in Vision

| | Supervised (Data+Labels) | Unsupervised (Just Data) |
|---|---|---|
| **Discrete Output** | Classification/ Categorization | |
| **Continuous Output** | | |

Slide adapted from J. Hays

# ML Problem Examples in Vision

## ***Cat*egorization/Classification**

Binning into K mutually-exclusive categories

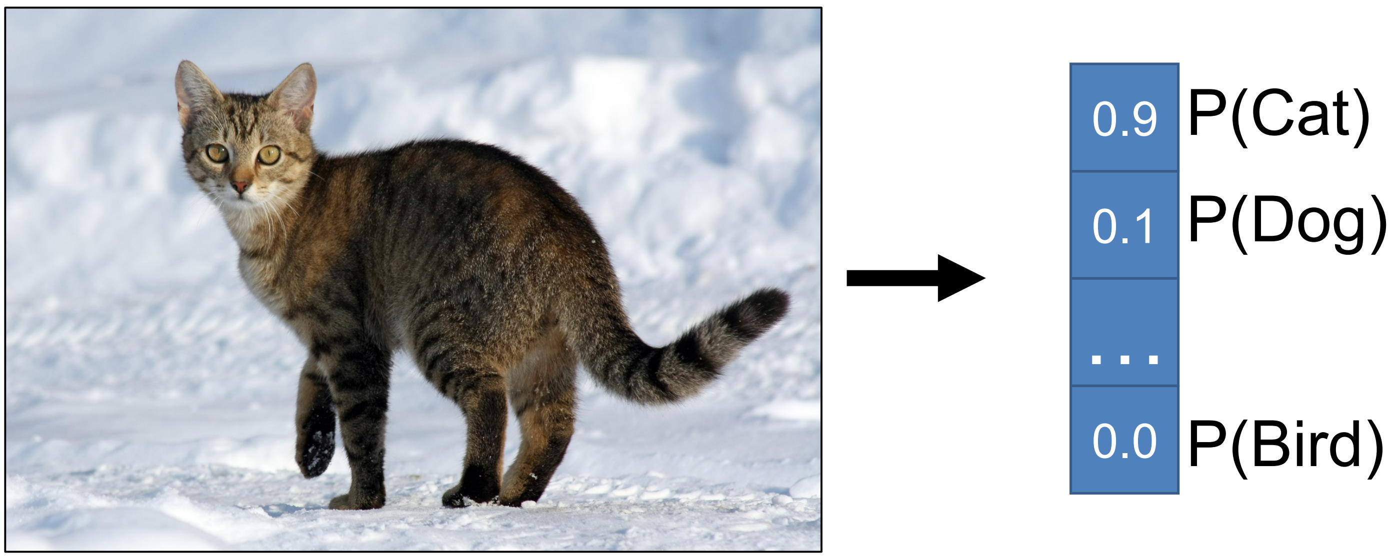



Image credit: Wikipedia

# ML Problem Examples in Vision

| | **Supervised (Data+Labels)** | **Unsupervised (Just Data)** |
|---|---|---|
| **Discrete Output** | Classification/ Categorization | |
| **Continuous Output** | **Regression** | |

Slide adapted from J. Hays

# ML Problem Examples in Vision

**Regression**

Estimating continuous variable(s)

→ 3.6 kg Cat weight

Image credit: Wikipedia

# ML Problem Examples in Vision

| | **Supervised (Data+Labels)** | **Unsupervised (Just Data)** |
|---|---|---|
| **Discrete Output** | Classification/ Categorization | **Clustering** |
| **Continuous Output** | Regression | |

Slide adapted from J. Hays

# ML Problem Examples in Vision

## **Clustering**

Given a set of cats, automatically discover clusters or *cat*egories.

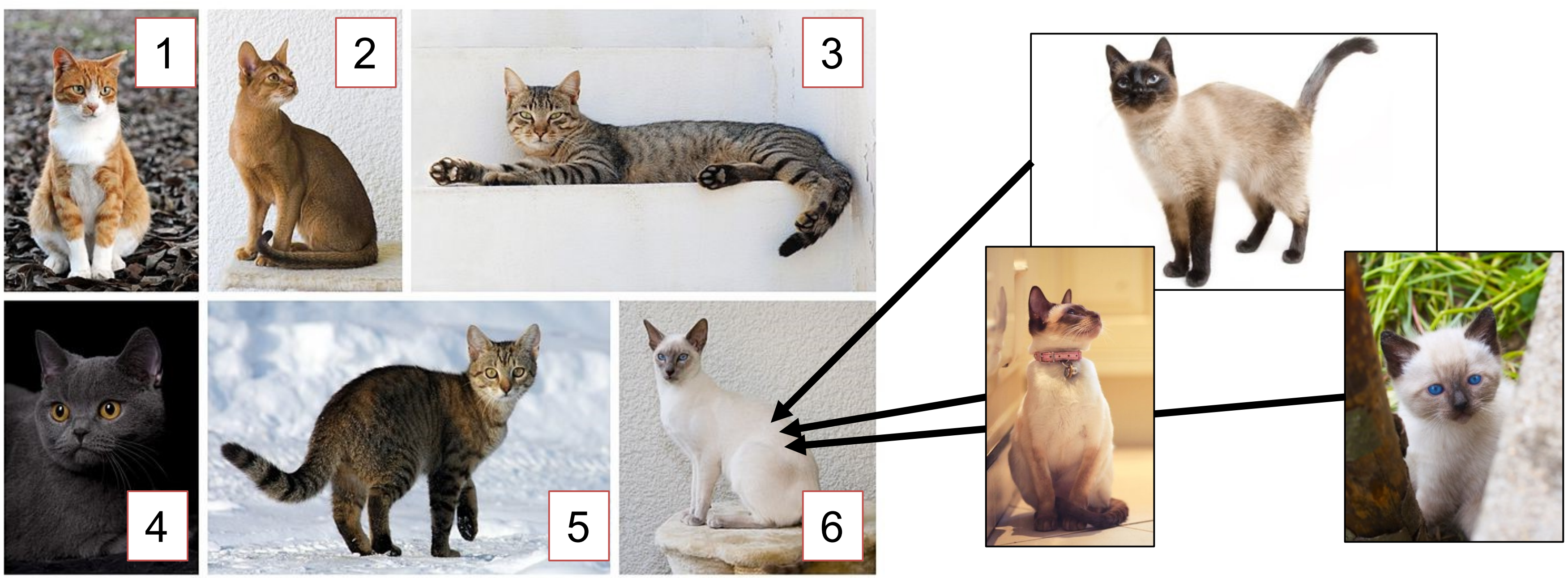



Image credit: Wikipedia, cattime.com

# ML Problem Examples in Vision

| | **Supervised (Data+Labels)** | **Unsupervised (Just Data)** |
|---|---|---|
| **Discrete Output** | Classification/ Categorization | Clustering |
| **Continuous Output** | Regression | **Dimensionality Reduction** |

Slide adapted from J. Hays

# ML Problem Examples in Vision

**Dimensionality Reduction**

Find dimensions that best explain the whole image/input

Cat size in image

Location of cat in image

For ordinary images, this is currently a totally hopeless task. For certain images (e.g., faces, this works reasonably well)

Image credit: Wikipedia

# Practical Example

- ML has a tendency to be mysterious
- Let’s start with:
  - A model you learned in middle/high school (a line)
  - A fitting method you find in a 200-level math course
- One thing to remember:
  - N eqns, <N vars = overdetermined (will have errors)
  - N eqns, N vars = exact solution
  - N eqns, >N vars = underdetermined (infinite solns)

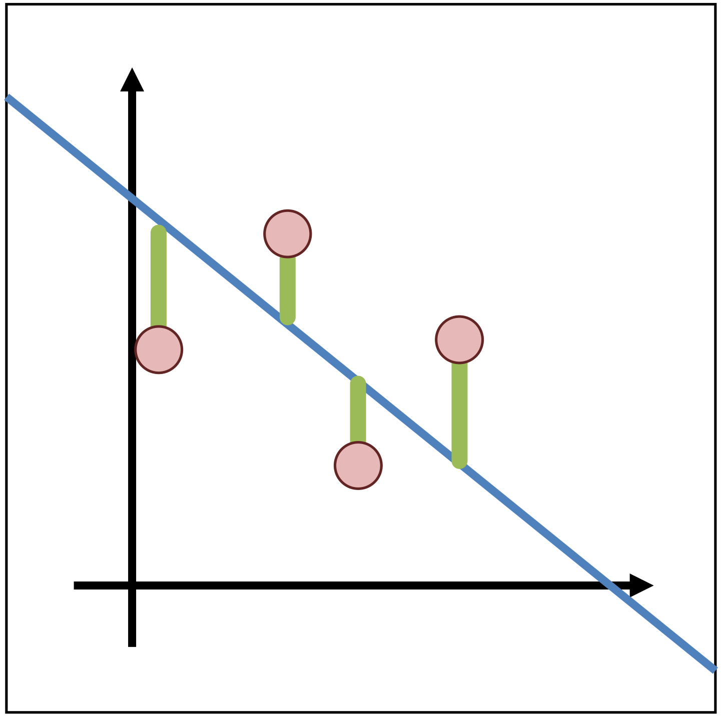
# Example – Least Squares

Let's make the world's **worst** weather model

Data: $(x_1,y_1)$, $(x_2,y_2)$, …, $(x_k,y_k)$

Model: $(m,b)$ $y_i=mx_i+b$
Or $(\mathbf{w})$ $y_i = \mathbf{w}^T\mathbf{x}_i$

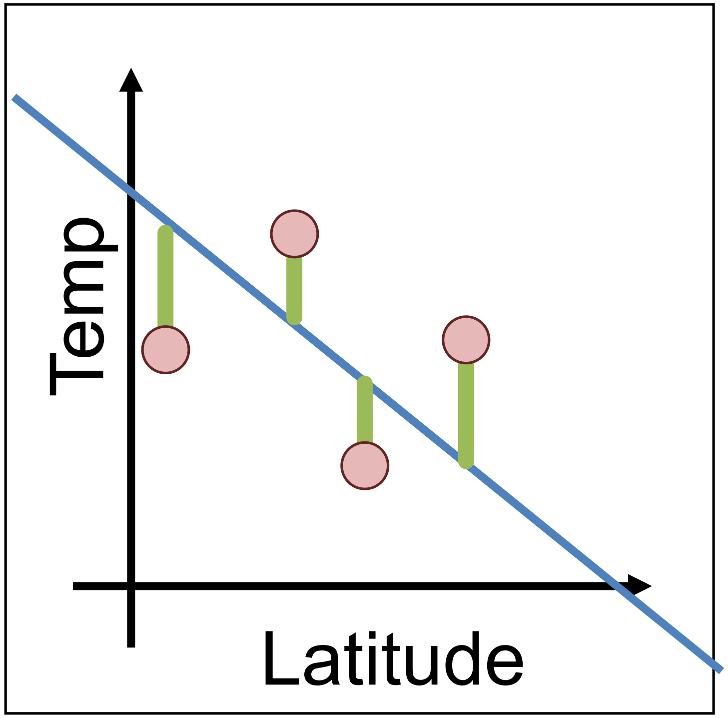
Objective function:
$(y_i - \mathbf{w}^T\mathbf{x}_i)^2$



# World's Worst Weather Model

Given latitude (distance above equator), predict temperature by fitting a line

| *City* | *Latitude (°)* | *Temp (F)* |
|---|---|---|
| Ann Arbor | 42 | 33 |
| Washington, DC | 39 | 38 |
| Austin, TX | 30 | 62 |
| Mexico City | 19 | 67 |
| Panama City | 9 | 83 |



# Example – Least Squares

$$\sum_{i=1}^{k} (y_i - \boldsymbol{w}^T \boldsymbol{x_i})^2 \longrightarrow \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_2^2$$

**Output**:
Temperature

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_k \end{bmatrix}$$

**Inputs**:
Latitude, 1

$$\boldsymbol{X} = \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_k & 1 \end{bmatrix}$$

**Model/Weights**:
Latitude, “Bias”

$$\boldsymbol{w} = \begin{bmatrix} m \\ b \end{bmatrix}$$

# Example – Least Squares

$$\sum_{i=1}^{k}(y_i - \boldsymbol{w}^T\boldsymbol{x_i})^2 \quad \longrightarrow \quad \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_2^2$$

**Output:**
Temperature

$$\boldsymbol{y} = \begin{bmatrix} 33 \\ \vdots \\ 83 \end{bmatrix}$$

**Inputs:**
Latitude, 1

$$\boldsymbol{X} = \begin{bmatrix} 42 & 1 \\ \vdots & \vdots \\ 9 & 1 \end{bmatrix}$$

**Model/Weights:**
Latitude, "Bias"

$$\boldsymbol{w} = \begin{bmatrix} m \\ b \end{bmatrix}$$

**Intuitively why do we add a one to the inputs?**

# Example – Least Squares

Training ($\mathbf{x}_i$,$y_i$):

$$\arg\min_{\boldsymbol{w}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_2^2 \quad \textbf{or}$$

$$\arg\min_{\boldsymbol{w}} \sum_{i=1}^{n} \|\boldsymbol{w}^T \boldsymbol{x}_{\boldsymbol{i}} - y_i\|^2$$

**Loss function/objective**: evaluates correctness.
Here: Squared L2 norm / Sum of Squared Errors

**Training/Learning/Fitting:** try to find model that *optimizes*/*minimizes* an objective / loss function

*Recall*: optimal **w*** is $\boldsymbol{w}^* = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y}$

# Example – Least Squares

Training ($\mathbf{x}_i$,$y_i$):

$$\arg\min_{\boldsymbol{w}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_2^2 \quad \textbf{or}$$

$$\arg\min_{\boldsymbol{w}} \sum_{i=1}^{n} \|\boldsymbol{w}^T \boldsymbol{x_i} - y_i\|^2$$

Inference (x):

$$\boldsymbol{w}^T \boldsymbol{x} = w_1 x_1 + \cdots + w_F x_F$$

**Testing/Inference:** Given a new output, what's the prediction?

# Least Squares: Learning

## Data

| *City* | *Latitude* | *Temp* |
|---|---|---|
| Ann Arbor | 42 | 33 |
| Washington, DC | 39 | 38 |
| Austin, TX | 30 | 62 |
| Mexico City | 19 | 67 |
| Panama City | 9 | 83 |

## Model

Temp =
-1.47*Lat + 97

$$\boldsymbol{X}_{5x2} = \begin{bmatrix} 42 & 1 \\ 39 & 1 \\ 30 & 1 \\ 19 & 1 \\ 9 & 1 \end{bmatrix} \quad \boldsymbol{y}_{5x1} = \begin{bmatrix} 33 \\ 38 \\ 62 \\ 67 \\ 83 \end{bmatrix}$$

$$(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y}$$

$$w_{2x1} = \begin{bmatrix} -1.47 \\ 97 \end{bmatrix}$$

# Let's Predict The Weather

The EECS 442 Weather Channel

| *City* | *Latitude* | *Temp* | *Temp* | *Error* |
|---|---|---|---|---|
| Ann Arbor | 42 | 33 | 35.3 | 2.3 |
| Washington, DC | 39 | 38 | 39.7 | 1.7 |
| Austin, TX | 30 | 62 | 52.9 | 10.9 |
| Mexico City | 19 | 67 | 69.1 | 2.1 |
| Panama City | 9 | 83 | 83.8 | 0.8 |

# Is This a Minimum Viable Product?

| The EECS 442 Weather Channel | The Weather Channel |
|---|---|
|  *Pittsburgh*: Temp = -1.47*40 + 97 = 38 | *Actual Pittsburgh:* *45* |
|  *Berkeley*: Temp = -1.47*38 + 97 = 41 | *Actual Berkeley:* *53* |
|  *Sydney*: Temp = -1.47*-33 + 97 = **146** | *Actual Sydney:* *74* |

Won't do so well in the Australian market…

# Where Can This Go Wrong?

# Where Can This Go Wrong?

## Data

| *City* | *Latitude* | *Temp* |
|---|---|---|
| Ann Arbor | 42 | 33 |
| Washington, DC | 39 | 38 |

## Model

Temp =
-1.66*Lat + 103

**How well can we predict Ann Arbor and DC and why?**

# Always Need Separated Testing

Model might be fit data too precisely "*overfitting*"
Remember: #datapoints = #params = perfect fit

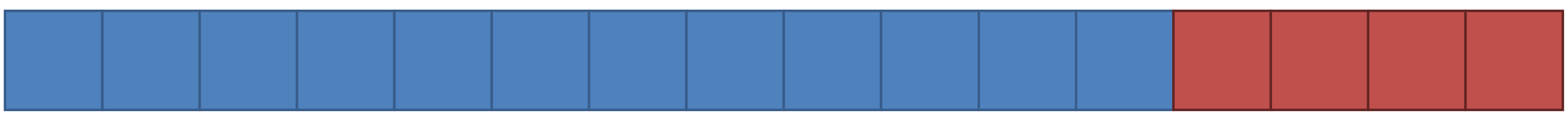
Model may only work under some conditions (e.g., trained on northern hemisphere).

*Sydney*:
Temp = -1.47*-33 + 97 = **146**

# Training and Testing

Fit model parameters on **training** set;
evaluate on *entirely unseen* **test** set.

Training Test



"It's tough to make predictions, especially about the future"
-Yogi Berra

Nearly any model can predict data it's seen. If your model can't accurately interpret "unseen" data, it's probably useless. We have no clue whether it has just memorized.

# Let's Improve Things

## If one feature does ok, what about more features!?

| *City Name* | *Latitude (deg)* | *Avg July High (F)* | *Avg Snowfall* | *Temp (F)* |
|---|---|---|---|---|
| Ann Arbor | 42 | 83 | 58 | 33 |
| Washington, DC | 39 | 88 | 15 | 38 |
| Austin, TX | 30 | 95 | 0.6 | 62 |
| Mexico City | 19 | 74 | 0 | 67 |
| Panama City | 9 | 93 | 0 | 83 |

$\boldsymbol{X}_{5x4}$ 4 features + a feature of 1s for intercept/bias

$\boldsymbol{y}_{5x1}$

# Let's Improve Things

All the math works out!

Data

$\boldsymbol{X}_{5x4}$ $\boldsymbol{y}_{5x1}$

$$\boxed{\boldsymbol{w}^* = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y}}$$

Model

$\boldsymbol{w}_{4x1}$

New EECS 442 Weather Rule:

$w_1$*latitude + $w_2$*(avg July high) + $w_3$*(avg snowfall) + $w_4$*1

*In general called linear regression*

# Let’s Improve Things More

## If one feature does ok, what about **LOTS** of features!?

| *City Name* | *Latitude (deg)* | *Avg July High (F)* | *Avg Snowfall* | *Day of Year* | *Elevation (ft)* | *% Letter M* | *Temp (F)* |
|---|---|---|---|---|---|---|---|
| Ann Arbor | 42 | 83 | 58 | 45 | 840 | 100 | 33 |
| Washington, DC | 39 | 88 | 15 | 45 | 409 | 3 | 38 |
| Austin, TX | 30 | 95 | 0.6 | 45 | 489 | 2 | 62 |
| Mexico City | 19 | 74 | 0 | 45 | 7200 | 4 | 67 |
| Panama City | 9 | 93 | 0 | 45 | 7 | 1 | 83 |

$\boldsymbol{X}_{5x7}$

6 features + a feature of 1s for intercept/bias

$\boldsymbol{y}_{5x1}$

# Let’s Improve Things More

Data

$\boldsymbol{X}_{5x7}$ $\boldsymbol{y}_{5x1}$

$$\boxed{\boldsymbol{w}^* = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y}}$$

Model

$\boldsymbol{w}_{7x1}$

$$\boldsymbol{w}^* = \underbrace{\left(\boldsymbol{X}^T\boldsymbol{X}\right)}^{-1}\boldsymbol{X}^T\boldsymbol{y}$$

$\mathbf{X^TX}$ is a 7x7 matrix but is **rank deficient** (rank 5) *and has no inverse. There are an infinite number of solutions.*

Have to express some preference for which of the infinite solutions we want.

Exercise for the mathematically-inclined folks: derive what the space of solutions looks like.

# The Fix – Regularized Least Squares

Add **regularization** to objective that prefers some solutions:

Before: $\arg\min_{\boldsymbol{w}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_2^2 \rightarrow$ Loss

After: $\arg\min_{\boldsymbol{w}} \underbrace{\|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_2^2}_{\text{Loss}} + \underbrace{\lambda}_{\text{Trade-off}} \underbrace{\|\boldsymbol{w}\|_2^2}_{\text{Regularization}}$

Want model "smaller": pay a penalty for **w** with big norm

Intuitive Objective: accurate model (low loss) but not too complex (low regularization). λ controls how much of each.

# The Fix – Regularized Least Squares

Objective: $\arg\min_{\boldsymbol{w}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_2^2 + \lambda\|\boldsymbol{w}\|_2^2$

Loss

Trade-off

Regularization

Take $\frac{\partial}{\partial \boldsymbol{w}}$, set to **0**, solve

$$\boldsymbol{w}^* = \underbrace{\left(\boldsymbol{X}^T\boldsymbol{X} + \boldsymbol{\lambda}\boldsymbol{I}\right)}^{-1}\boldsymbol{X}^T\boldsymbol{y}$$

**$\mathbf{X}^T\mathbf{X}+\lambda\mathbf{I}$** is full-rank (and thus invertible) for $\lambda>0$

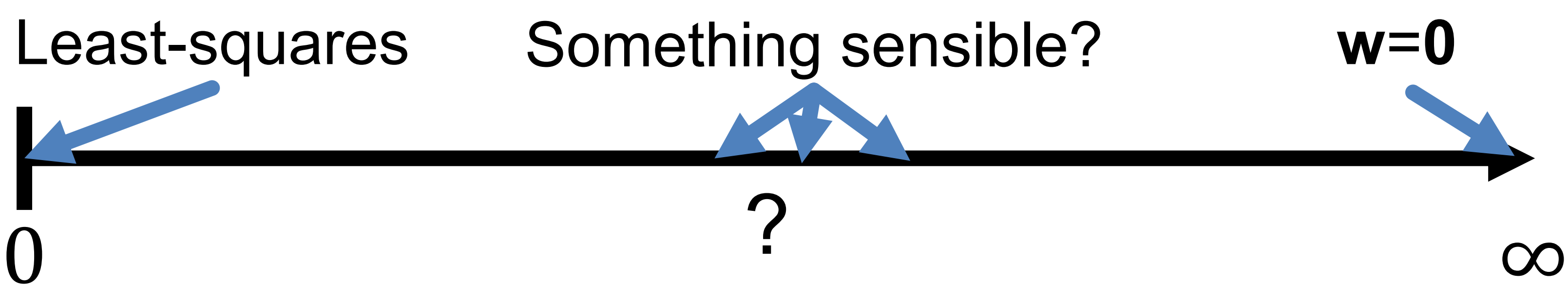
Called *lots of things:* regularized least-squares, Tikhonov regularization (after Andrey Tikhonov), ridge regression, Bayesian linear regression with a multivariate normal prior.

# The Fix – Regularized Least Squares

Objective: $\underbrace{\arg\min_{\boldsymbol{w}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_2^2}_{\text{Loss}} + \underbrace{\lambda}_{\text{Trade-off}} \underbrace{\|\boldsymbol{w}\|_2^2}_{\text{Regularization}}$

## What happens (and why) if:

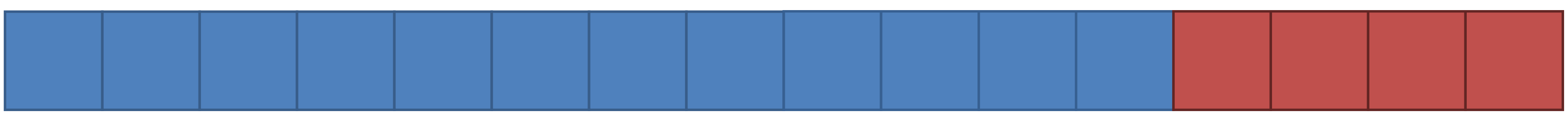
- λ=0
- λ=∞



# Training and Testing

Fit model parameters on training set;
evaluate on *entirely unseen* test set.

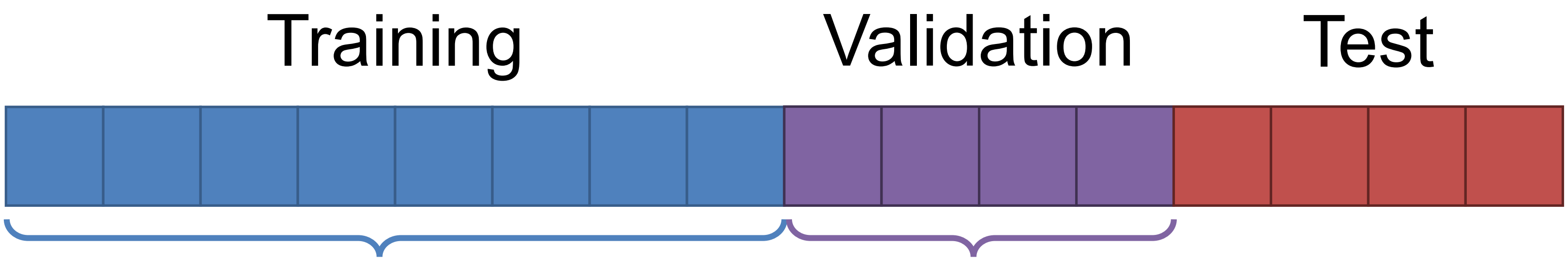
Training Test



**How do we pick λ?**

# Training and Testing

Fit model parameters on training set;
find *hyperparameters* by testing on validation set;
evaluate on *entirely unseen* test set.



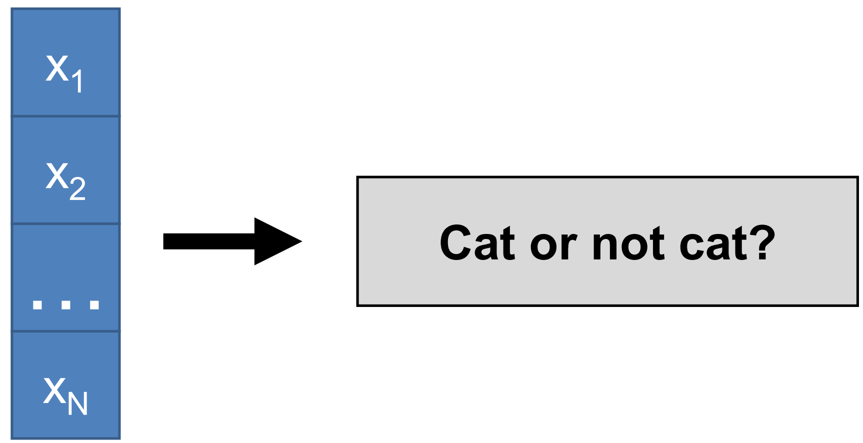

Use these data points to fit $\mathbf{w}^*=(\mathbf{X}^T\mathbf{X}+\lambda\mathbf{I})^{-1}\mathbf{X}^T\mathbf{y}$

Evaluate on these points for different $\lambda$, pick the best

# Classification

Start with simplest example: binary classification

Actually: a feature vector representing the image



# Classification by Least-Squares

Treat as regression: $x_i$ is image feature; $y_i$ is 1 if it's a cat, 0 if it's not a cat. Minimize least-squares loss.

Training ($\mathbf{x}_i$,$y_i$):

$$\arg\min_{\boldsymbol{w}} \sum_{i=1}^{n} \|\boldsymbol{w}^T \boldsymbol{x_i} - y_i\|^2$$

Inference (x):

$$\boldsymbol{w}^T \boldsymbol{x} > t$$

Unprincipled in theory, but often effective in practice
The reverse (regression via discrete bins) is also common


Rifkin, Yeo, Poggio. *Regularized Least Squares Classification* (http://cbcl.mit.edu/publications/ps/rlsc.pdf). 2003
Redmon, Divvala, Girshick, Farhadi. *You Only Look Once: Unified, Real-Time Object Detection.* CVPR 2016.

# Easiest Form of Classification

Just **memorize** (as in a Python dictionary)
Consider cat/dog/hippo classification.

If this:
cat.

If this:
dog.

If this:
hippo.

# Easiest Form of Classification

## Where does this go wrong?

Rule: if this, then cat

Hmmm. Not quite the same.

# Easiest Form of Classification

Known Images
Labels


$\boldsymbol{x}_1 \leftarrow D(\boldsymbol{x}_1, \boldsymbol{x}_T) \rightarrow \boldsymbol{x}_T$

Test
Image


…

$D(\boldsymbol{x}_N, \boldsymbol{x}_T)$


$\boldsymbol{x}_N$

(1) Compute distance between feature vectors (2) find nearest (3) use label.

# Nearest Neighbor

## "Algorithm"

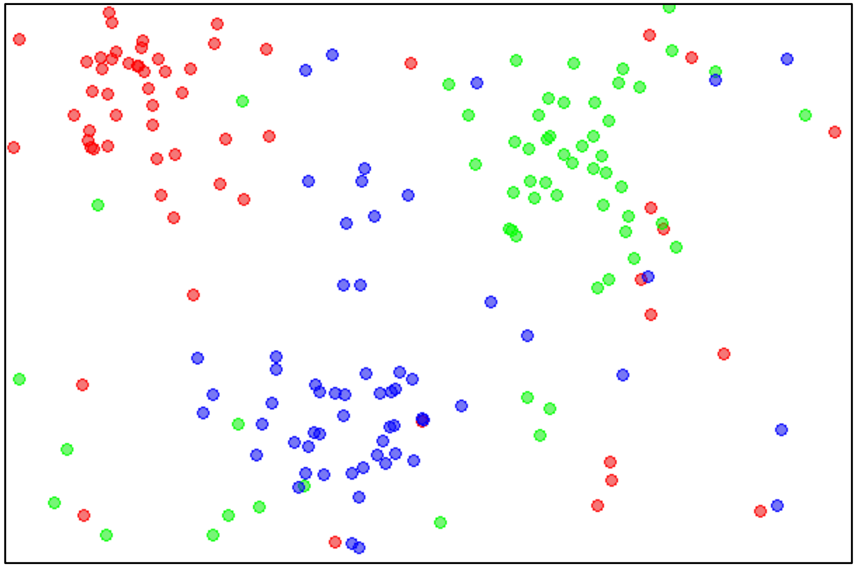
Training ($\mathbf{x}_i$,$y_i$): Memorize training set

Inference (x):

bestDist, prediction = Inf, None
for i in range(N):
    if dist($x_i$,x) < bestDist:
        bestDist = dist($x_i$,x)
        prediction = $y_i$

# Nearest Neighbor

2D Datapoints
(colors = labels)



2D Predictions
(colors = labels)

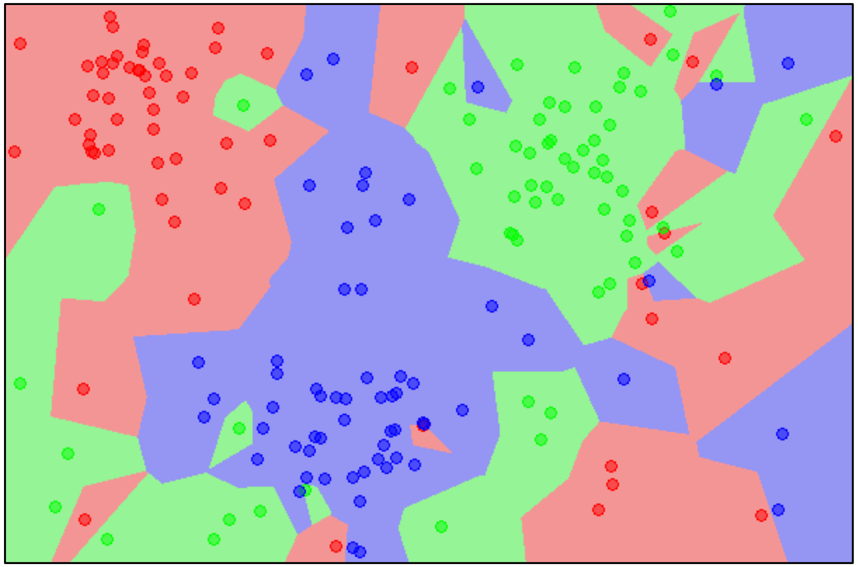
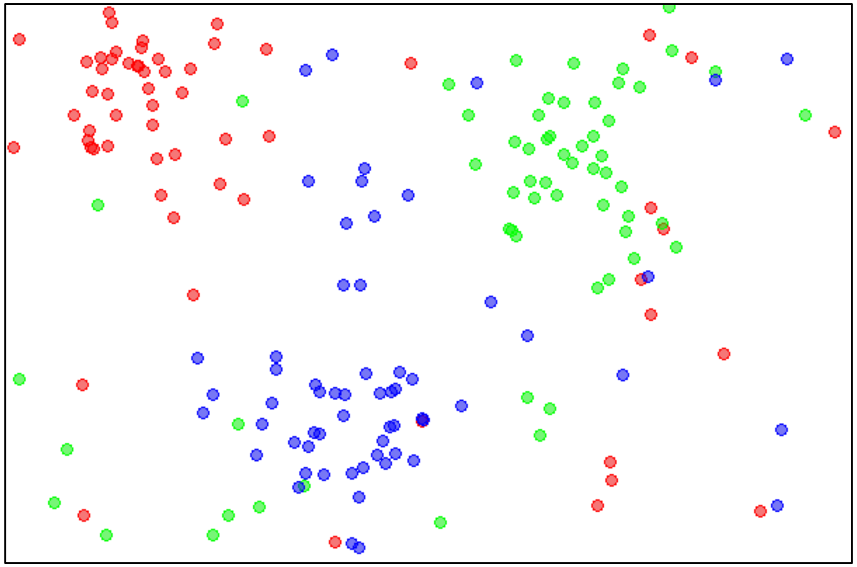


Diagram Credit: Wikipedia

# K-Nearest Neighbors

Take top K-closest points, vote

2D Datapoints
(colors = labels)



2D Predictions
(colors = labels)

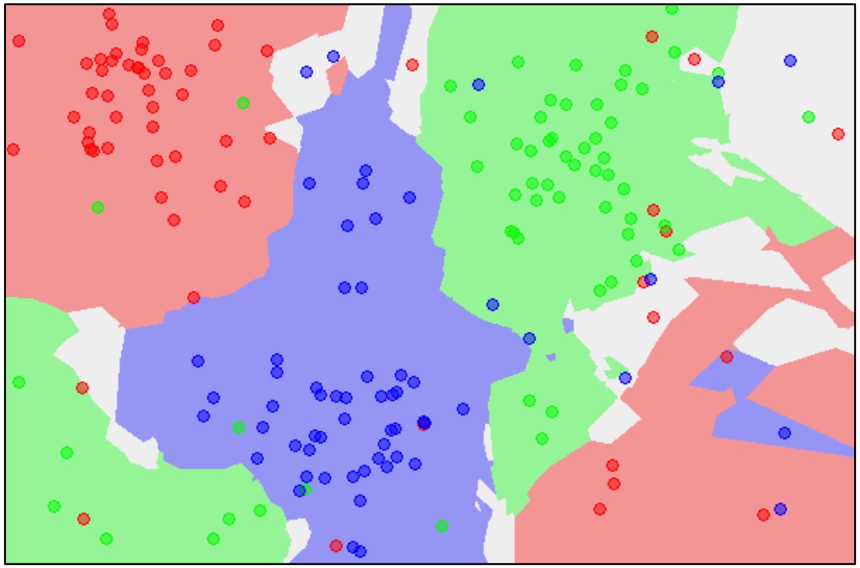
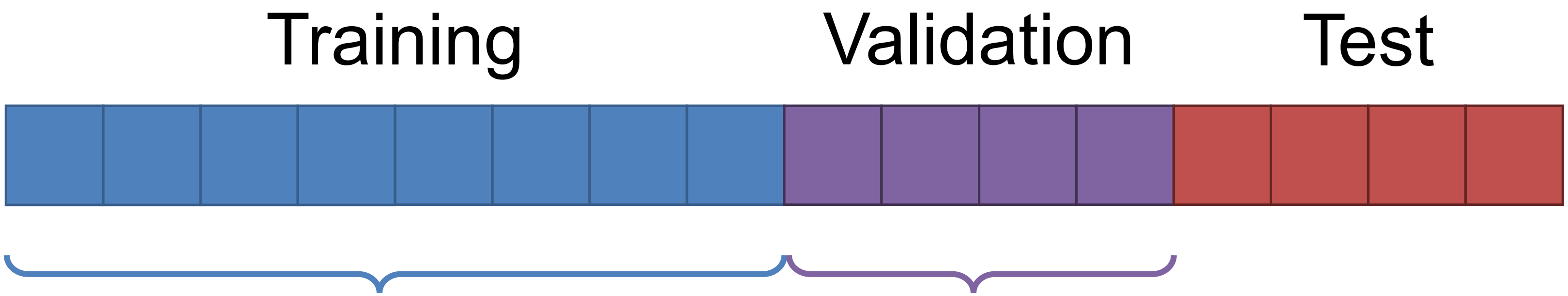


Diagram Credit: Wikipedia

# K-Nearest Neighbors

What distance? What value for K?




Use these data points for lookup

Evaluate on these points for different k, distances

# K-Nearest Neighbors

- No learning going on but usually effective
- Same algorithm for every task
- As number of datapoints $\rightarrow \infty$, error rate is guaranteed to be at most 2x worse than optimal you could do on data

# Linear Models

## Example Setup: 3 classes

Model – one weight per class: $\boldsymbol{w}_0, \boldsymbol{w}_1, \boldsymbol{w}_2$

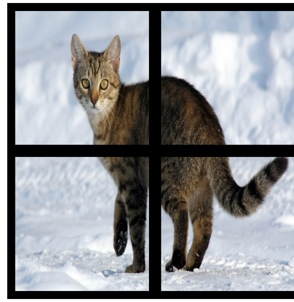
$\boldsymbol{w}_0^T\boldsymbol{x}$ big if cat

$\boldsymbol{w}_1^T\boldsymbol{x}$ big if dog

$\boldsymbol{w}_2^T\boldsymbol{x}$ big if hippo

Stack together: $\boldsymbol{W}_{\boldsymbol{3xF}}$ where **x** is in R$^F$

# Linear Models



| | | | | | | | $x_i$ | | $Wx_i$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| Cat weight vector | 0.2 | -0.5 | 0.1 | 2.0 | 1.1 | | 56 | | -96.8 | Cat score |
| Dog weight vector | 1.5 | 1.3 | 2.1 | 0.0 | 3.2 | | 231 | → | 437.9 | Dog score |
| Hippo weight vector | 0.0 | 0.3 | 0.2 | -0.3 | -1.2 | | 24 | | 61.95 | Hippo score |
| | | | | | | | 2 | | | |
| | | | | | | | 1 | | | |

$\boldsymbol{W}$

$\boldsymbol{Wx_i}$

$\boldsymbol{x_i}$

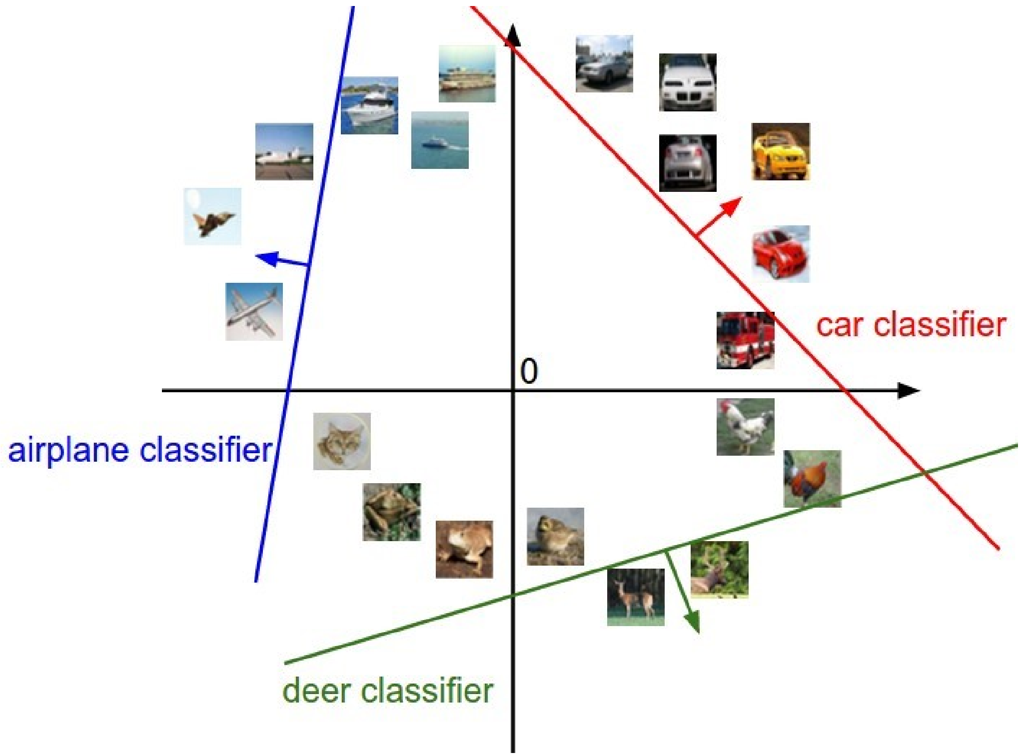
Weight matrix a collection of scoring functions, one per class

Prediction is vector where jth component is “score” for jth class.

Diagram by: Karpathy, Fei-Fei

# Geometric Intuition*

## What does a linear classifier look like* in 2D?

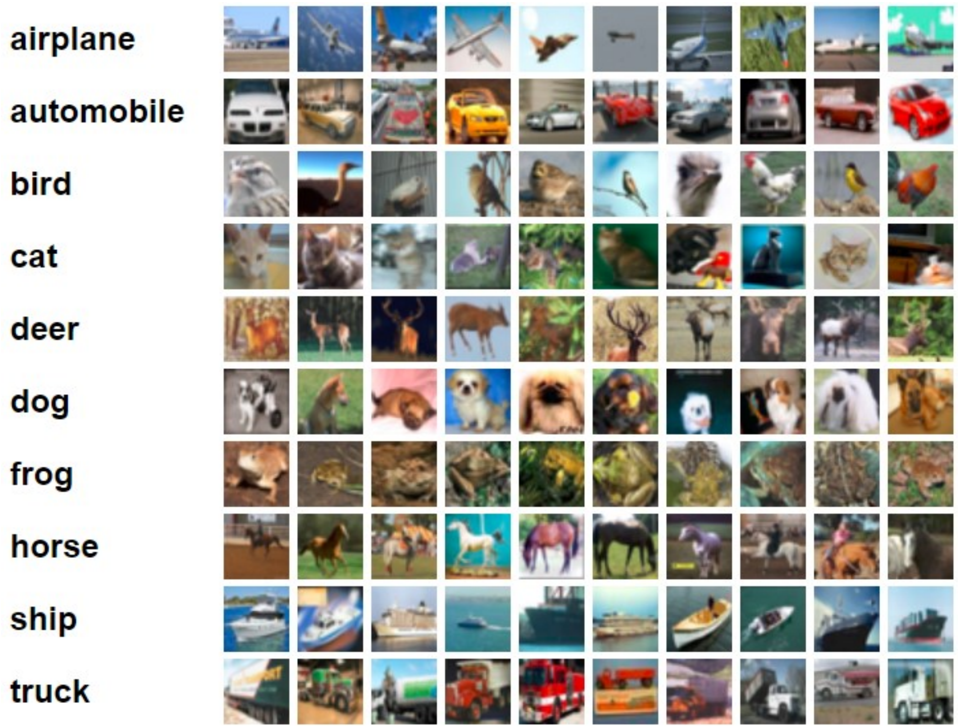



*2D is good for vague intuitions, but ML typically deals with at least dozens if not *thousands* of dimensions. Your intuitions about space and geometry from living in 3D are **completely wrong** in high dimensions. Never trust people who show you 2D diagrams and write "Intuition" in the slide title. See: *On the Surprising Behavior of Distance Metrics in High Dimensional Space.* Charu, Hinneburg, Keim. ICDT 2001

Diagram credit: Karpathy & Fei-Fei. 12-point font mini-rant: me

# Visual Intuition

CIFAR 10:
32x32x3 Images, 10 Classes




- Turn each image into feature by unrolling all pixels
- Fit 10 linear models

Slide credit: Karpathy & Fei-Fei

# Guess The Classifier

Decision rule is $\mathbf{w}^T\mathbf{x}$. If $\mathbf{w}_i$ is big, then big values of $x_i$ are indicative of the class.

# Deer or Plane?

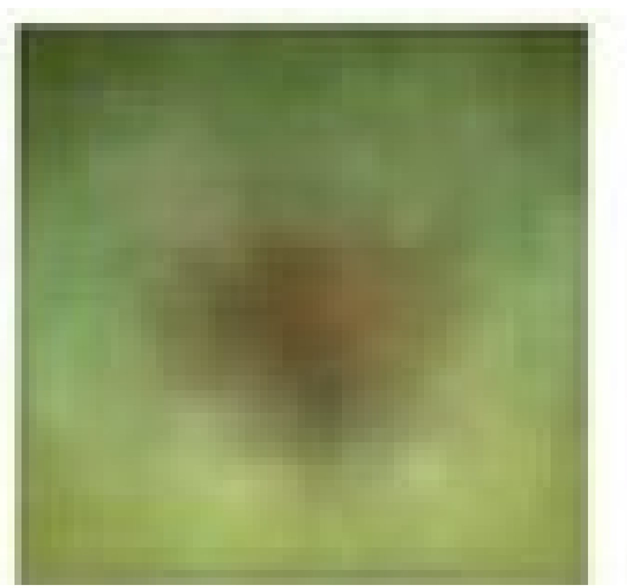


Diagram credit: Karpathy & Fei-Fei

# Guess The Classifier

Decision rule is $\mathbf{w}^T\mathbf{x}$. If $\mathbf{w}_i$ is big, then big values of $x_i$ are indicative of the class.

## Ship or Dog?

Diagram credit: Karpathy & Fei-Fei

# Interpreting a Linear Classifier

Decision rule is $\mathbf{w}^T\mathbf{x}$. If $\mathbf{w}_i$ is big, then big values of $x_i$ are indicative of the class.


Diagram credit: Karpathy & Fei-Fei

# Objective 1: Multiclass SVM

Inference (x): $\arg\max_{k} \; (\boldsymbol{W}\boldsymbol{x})_k$

(Take the class whose weight vector gives the highest score)

# Objective 1: Multiclass SVM

Inference (x,y): $\arg\max_{k} \ (\boldsymbol{W}\boldsymbol{x})_k$

(Take the class whose weight vector gives the highest score)

Training ($\mathbf{x}_i$,$y_i$):

$$\arg\min_{\boldsymbol{W}} \lambda \|\boldsymbol{W}\|_2^2 + \sum_{i}^{n} \sum_{j \neq y_i} \underbrace{\max(0, (\boldsymbol{W}\boldsymbol{x}_i)_j - (\boldsymbol{W}\boldsymbol{x}_i)_{y_i} + m)}$$

Regularization

Over all data points

For every class j that's NOT the correct one ($y_i$)

Pay no penalty if prediction for class yi is bigger than j by m ("margin"). Otherwise, pay proportional to the score of the wrong class.

# Objective: Multiclass SVM

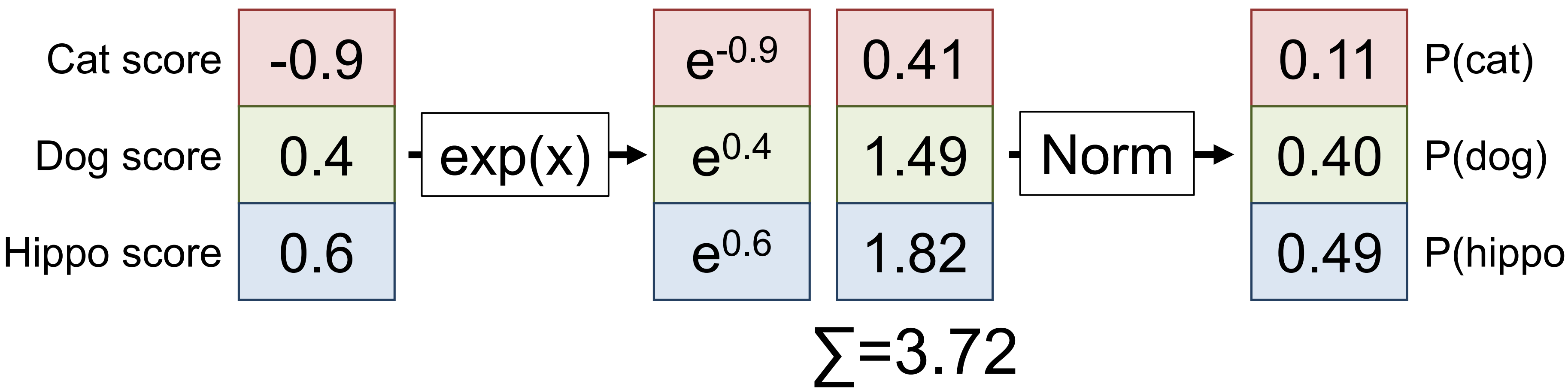
How on earth do we optimize:

$$\arg\min_{\boldsymbol{W}} \boldsymbol{\lambda}||\boldsymbol{W}||_{\mathbf{2}}^{\mathbf{2}} + \sum_{i}^{n} \sum_{j \neq y_i} \max(0, (\boldsymbol{W}\boldsymbol{x}_i)_j - (\boldsymbol{W}\boldsymbol{x}_{\boldsymbol{i}})_{y_i} + m)$$

Hold that thought!

# Preliminaries

## Converting Scores to "Probability Distribution"




Generally P(class j): $$\frac{\exp((Wx)_j)}{\sum_k \exp((Wx)_k)}$$

# Objective 2: Softmax

Inference (x): $\arg\max_{k} \ (\boldsymbol{Wx})_k$

(Take the class whose weight vector gives the highest score)

P(class j): $\dfrac{\exp((Wx)_j)}{\sum_k \exp((Wx)_k)}$

**Why can we skip the exp/sum exp thing to make a decision?**

# Objective 2: Softmax

Inference (x): $\arg\max_{k} \ (\boldsymbol{Wx})_k$

(Take the class whose weight vector gives the highest score)

Training ($\mathbf{x}_i$, $y_i$):

$$\arg\min_{\boldsymbol{W}} \underbrace{\lambda\|\boldsymbol{W}\|_2^2}_{\text{Regularization}} + \sum_{i}^{n} \underbrace{-\log\left(\frac{\exp((Wx)_{y_i})}{\sum_k \exp((Wx)_k)}\right)}_{\text{Pay penalty for negative log-likelihood of correct class}}$$

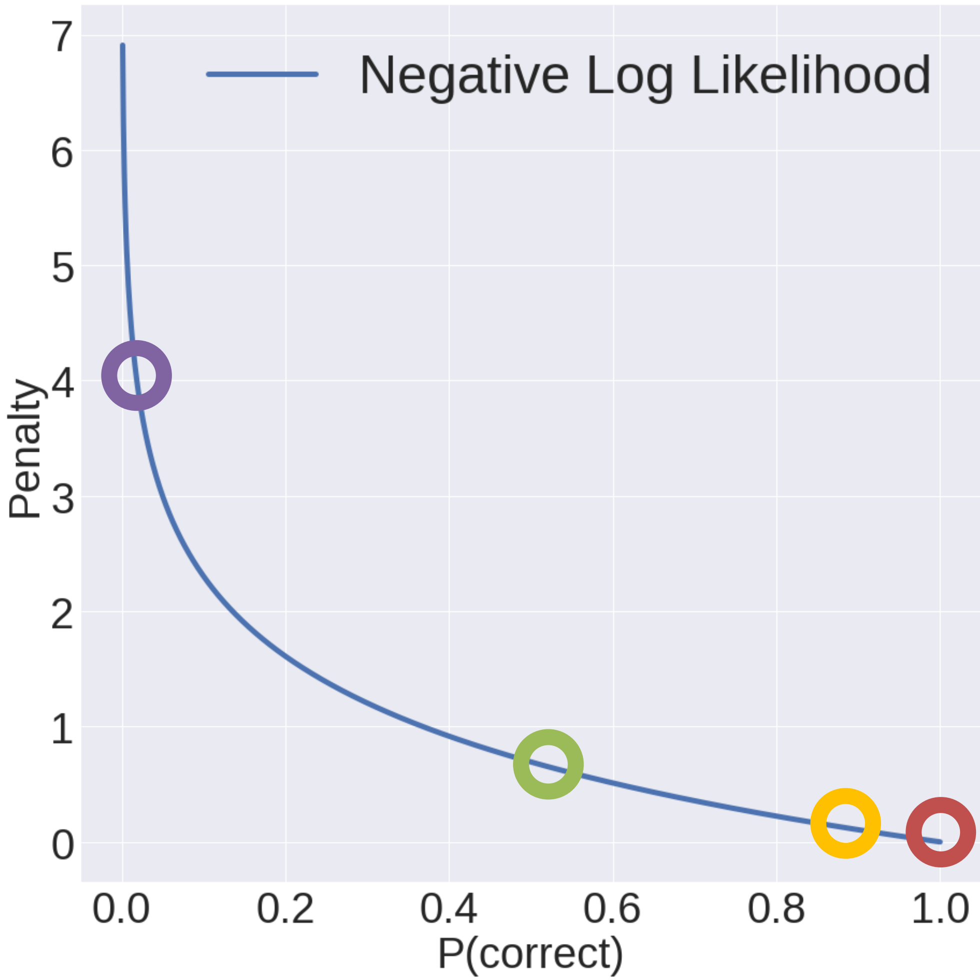
P(correct class)

Regularization

Over all data points

Pay penalty for negative log-likelihood of correct class

# Objective 2: Softmax




**P(correct) = 0.05:
3.0 penalty**

**P(correct) = 0.5:
0.11 penalty**

**P(correct) = 0.9:
0.11 penalty**

**P(correct) = 1:
No penalty!**

# Next Class

- How do we optimize more complex stuff?
- A bit more ML

# ML Problem Examples In Vision

## Clustering

Given a set of vectors $\mathbf{x}_i$, find clusters $\mathbf{c}_j$ and assignments $d_{ij}$ that minimizes

$$c^*, d^* = \arg\min_{\mathrm{c,d}} \frac{1}{N} \sum_{j}^{N} \sum_{i}^{K} d_{ij} \left( \boldsymbol{c}_j - \boldsymbol{x}_i \right)^2$$

If point i is assigned to cluster j, minimize squared distance between $\mathbf{c}_j$ and $\mathbf{x}_i$