Filtering

EECS 442 – Prof. David Fouhey

Winter 2019, University of Michigan

http://web.eecs.umich.edu/~fouhey/teaching/EECS442_W19/

Note: I'll ask the front row on the right to participate in a demo. All you have to do is say a number that I'll give to you. If you don't want to, it's fine, but don't sit in the front.

Let’s Take An Image

Let’s Fix Things

- We have noise in our image
- Let’s replace each pixel with a *weighted* average of its neighborhood
- Weights are *filter kernel*

	Out	

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

Slide Credit: D. Lowe

1D Case

Signal/ Front Row	10	12	9	11	10	11	12
Filter/ David	1/3	1/3	1/3				
Output	10.33	10.66	10	10.66	11		

Applying a Linear Filter

Input

I11	I12	I13	I14	I15	I16
I21	I22	I23	I24	I25	I26
I31	I32	I33	I34	I35	I36
I41	I42	I43	I44	I45	I46
I51	I52	I53	I54	I55	I56

Filter

F11	F12	F13
F21	F22	F23
F31	F32	F33

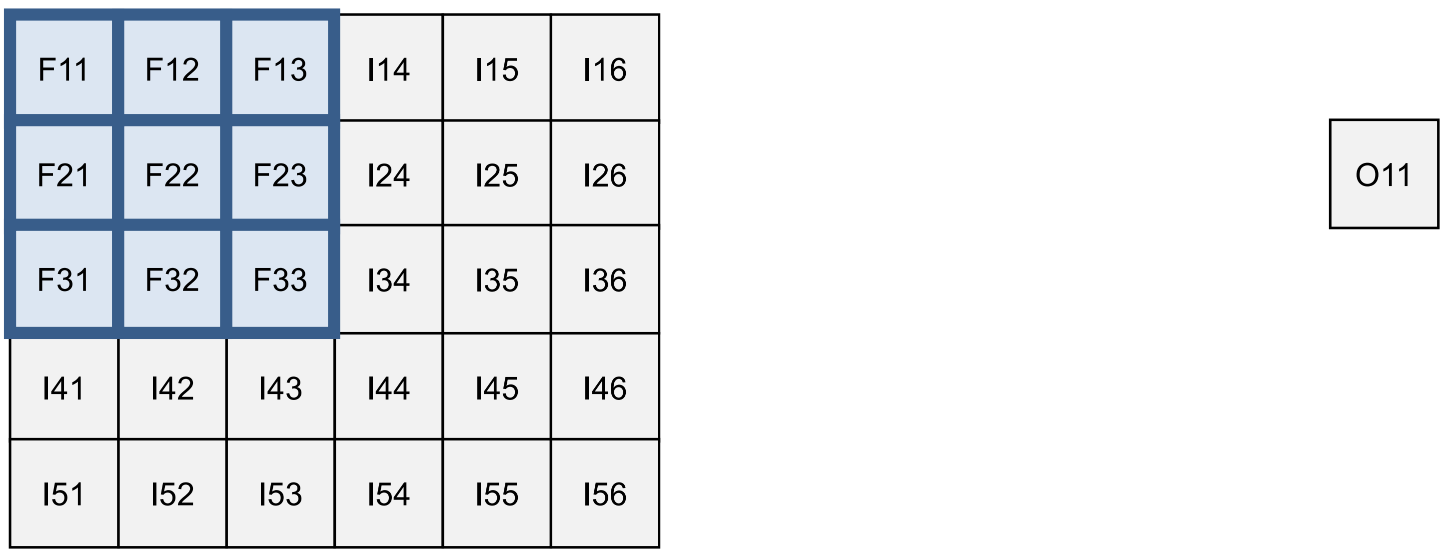
Output

O11	O12	O13	O14
O21	O22	O23	O24
O31	O32	O33	O34

Applying a Linear Filter

Input & Filter

Output

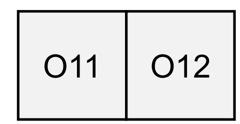


O11 = I11*F11 + I12*F12 + … + I33*F33

Applying a Linear Filter

Input & Filter

Output



$$O12 = I12*F11 + I13*F12 + \ldots + I34*F33$$

Applying a Linear Filter

Input

I11	I12	I13	I14	I15	I16
I21	I22	I23	I24	I25	I26
I31	I32	I33	I34	I35	I36
I41	I42	I43	I44	I45	I46
I51	I52	I53	I54	I55	I56

Filter

F11	F12	F13
F21	F22	F23
F31	F32	F33

Output

How many times can we apply a 3x3 filter to a 5x6 image?

Applying a Linear Filter

Input

I11	I12	I13	I14	I15	I16
I21	I22	I23	I24	I25	I26
I31	I32	I33	I34	I35	I36
I41	I42	I43	I44	I45	I46
I51	I52	I53	I54	I55	I56

Filter

F11	F12	F13
F21	F22	F23
F31	F32	F33

Output

O11	O12	O13	O14
O21	O22	O23	O24
O31	O32	O33	O34

$$Oij = Iij*F11 + Ii(j+1)*F12 + \ldots + I(i+2)(j+2)*F33$$

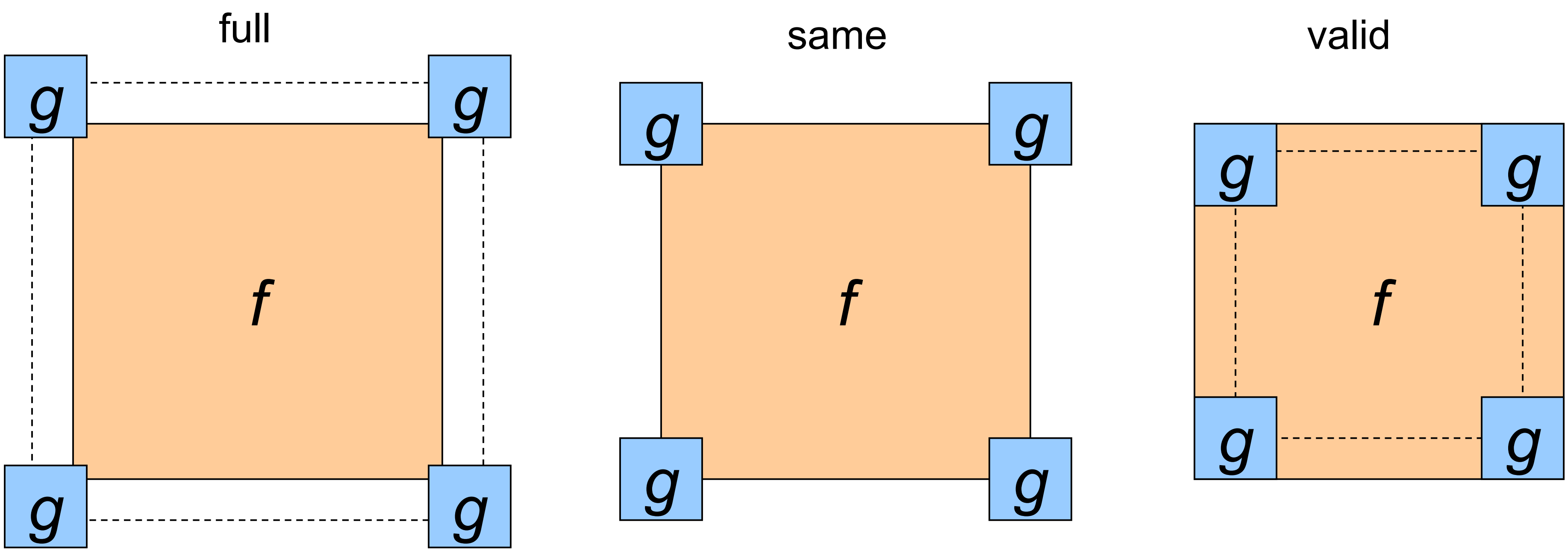
Painful Details – Edge Cases

Convolution doesn't keep the whole image.

Suppose **f** is the image and **g** the filter.

Full – any part of g touches f. **Same** – same size as f;

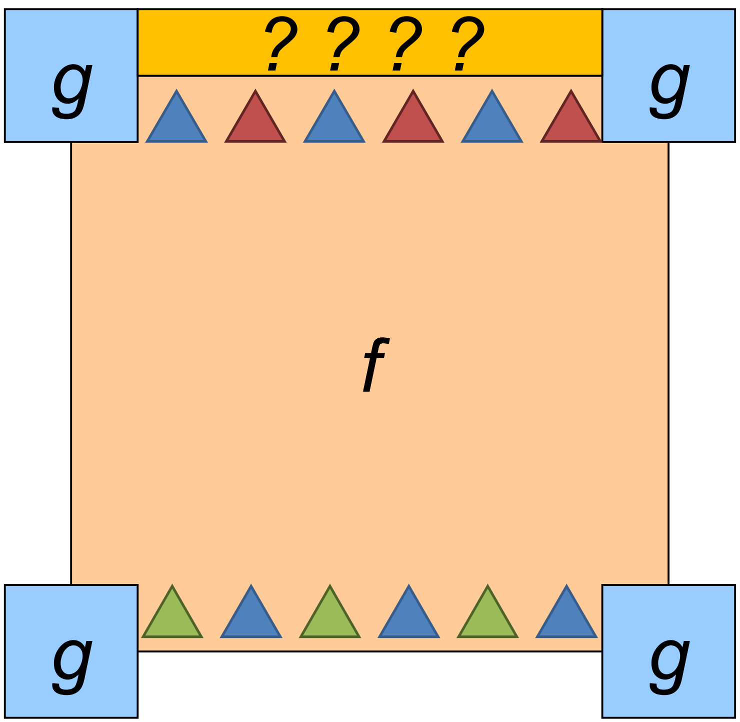
Valid – only when filter doesn't fall off edge.



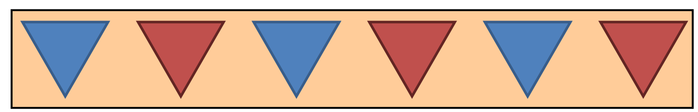
f/g Diagram Credit: D. Lowe

Painful Details – Edge Cases

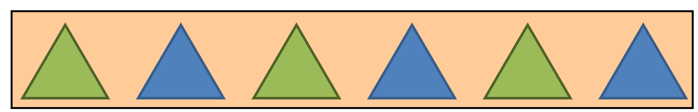
What to about the “?” region?



Symm: fold sides over



Circular/Wrap: wrap around



pad/fill: add value, often 0

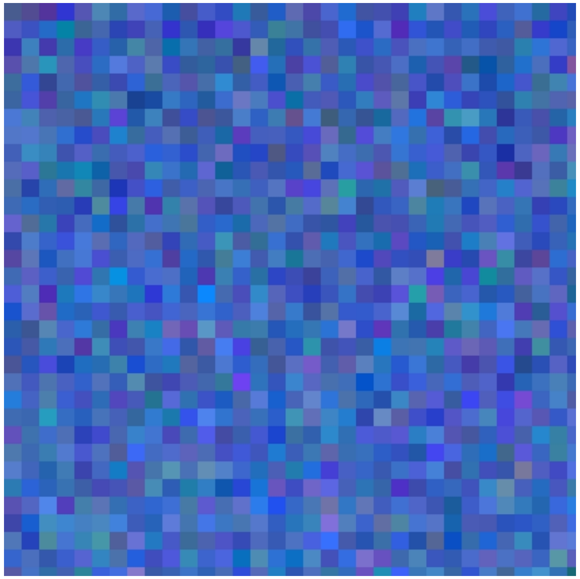
f/g Diagram Credit: D. Lowe

Painful Details – Does it Matter?

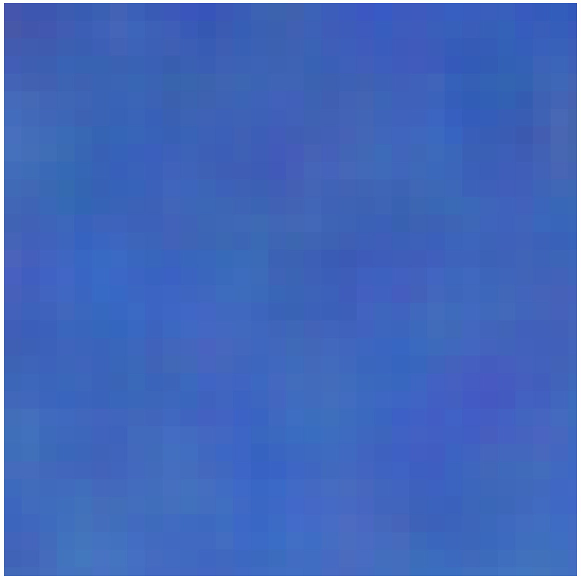
(I've applied the filter per-color channel)

Which padding did I use and *why*?

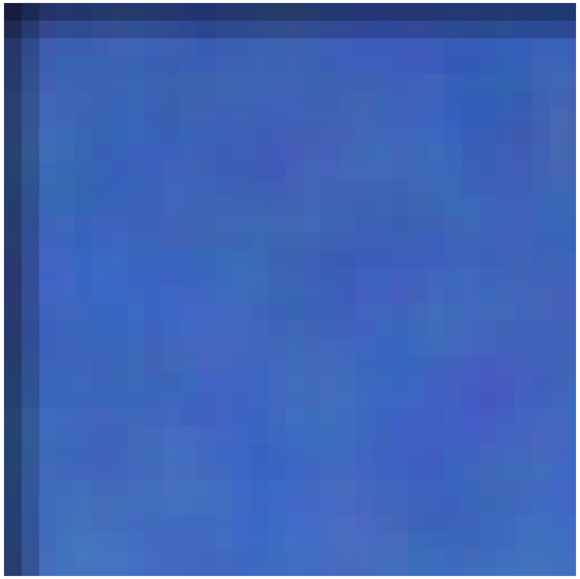
Input Image



Box Filtered ???



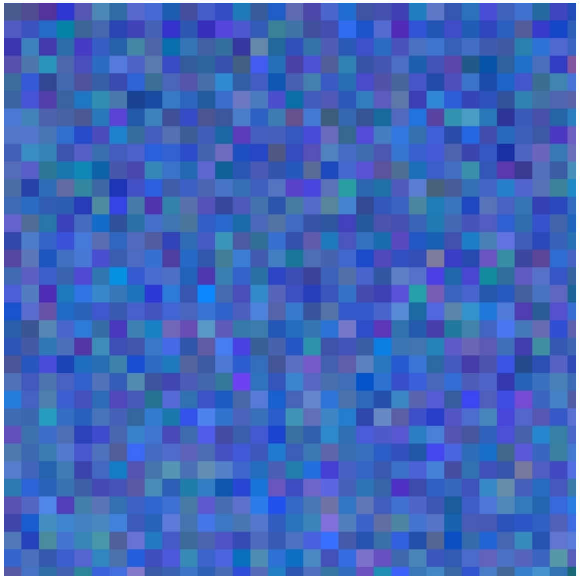
Box Filtered ???



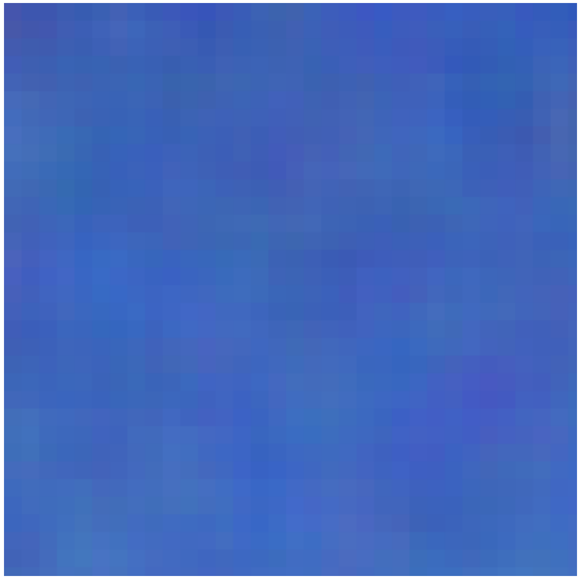
Painful Details – Does it Matter?

(I've applied the filter per-color channel)

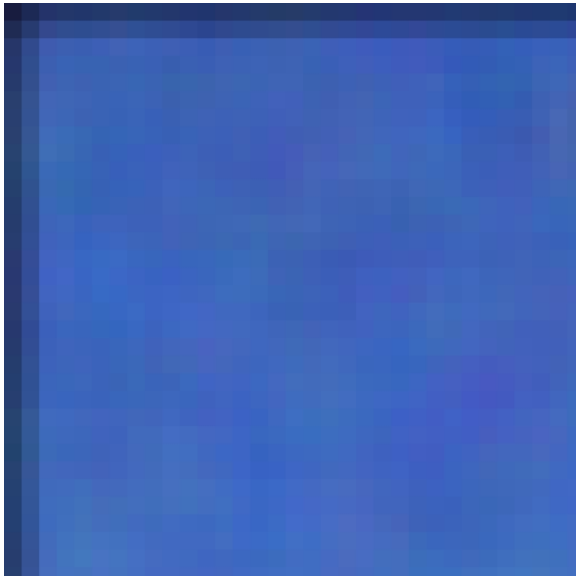
Input Image



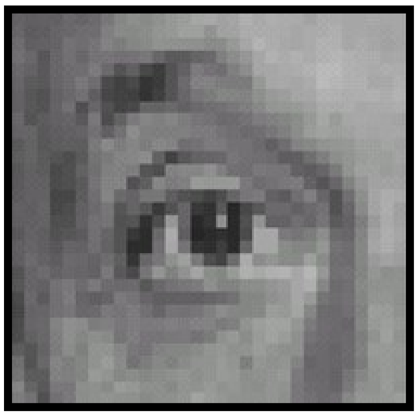
Box Filtered Symm Pad



Box Filtered Zero Pad



Practice with Linear Filters



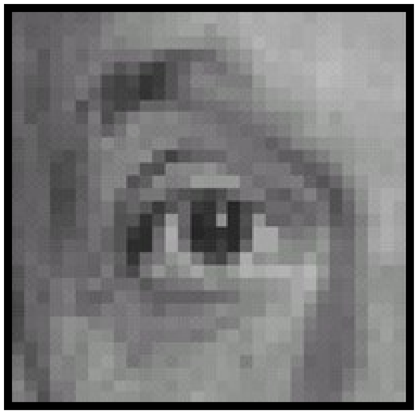
Original

0	0	0
0	1	0
0	0	0

?

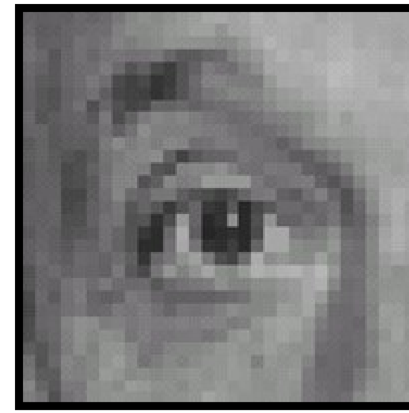
Slide Credit: D. Lowe

Practice with Linear Filters



Original

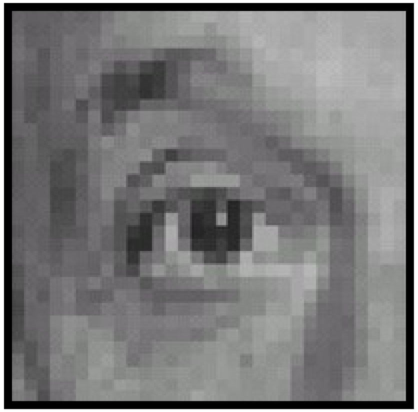
0	0	0
0	1	0
0	0	0



The Same!

Slide Credit: D. Lowe

Practice with Linear Filters



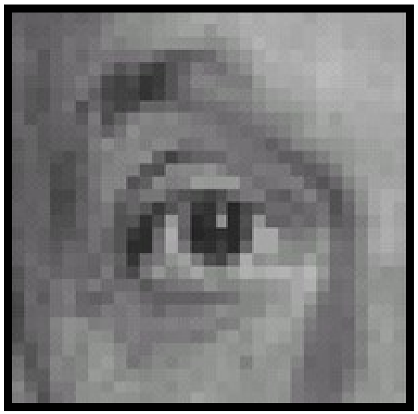
Original

0	0	0
0	0	1
0	0	0

?

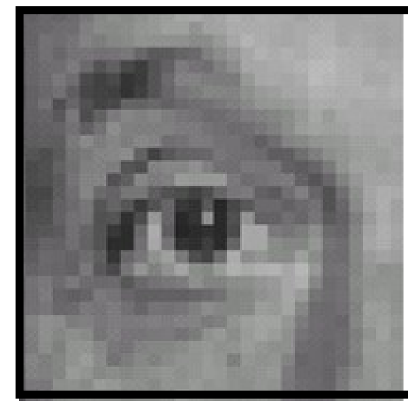
Slide Credit: D. Lowe

Practice with Linear Filters



Original

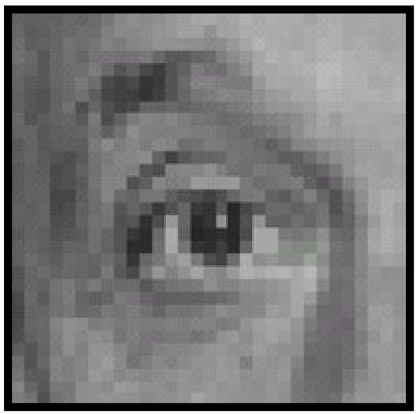
0	0	0
0	0	1
0	0	0



Shifted
LEFT
1 pixel

Slide Credit: D. Lowe

Practice with Linear Filters



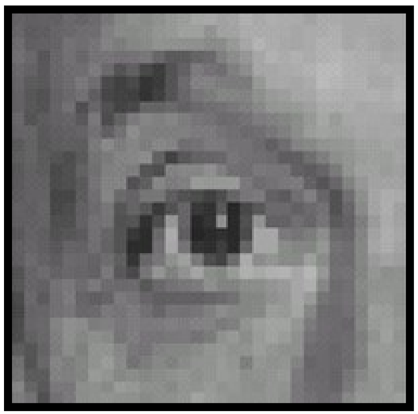
Original

0	1	0
0	0	0
0	0	0

?

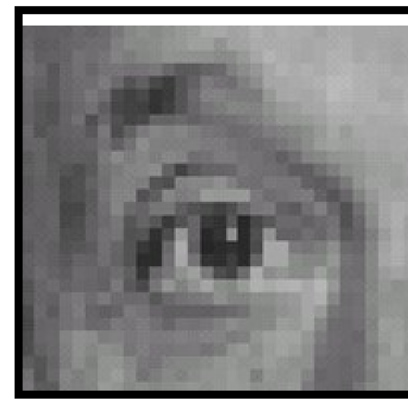
Slide Credit: D. Lowe

Practice with Linear Filters



Original

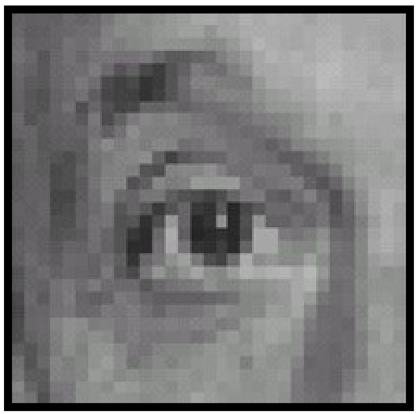
0	1	0
0	0	0
0	0	0



Shifted
DOWN
1 pixel

Slide Credit: D. Lowe

Practice with Linear Filters



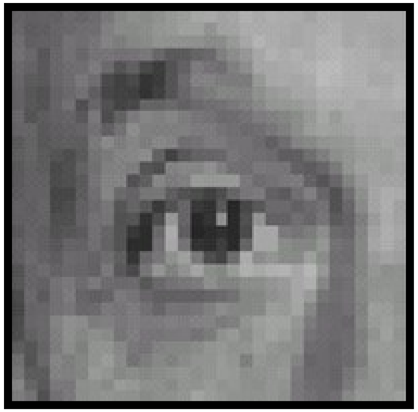
Original

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

?

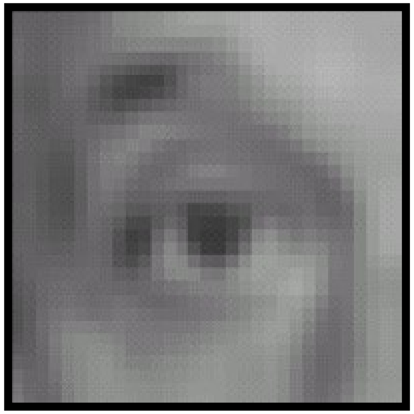
Slide Credit: D. Lowe

Practice with Linear Filters



Original

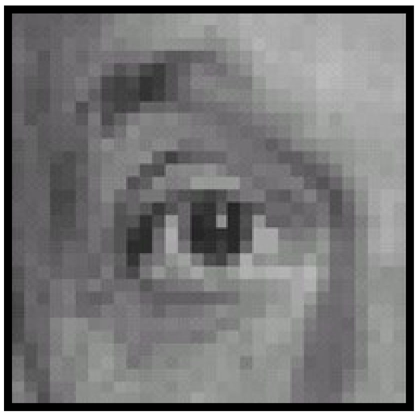
1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9



Blur
(Box Filter)

Slide Credit: D. Lowe

Practice with Linear Filters



Original

0	0	0
0	2	0
0	0	0

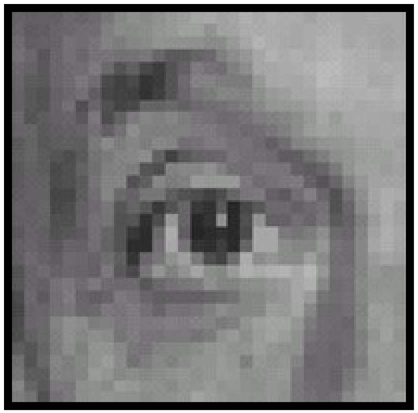
-

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

?

Slide Credit: D. Lowe

Practice with Linear Filters



Original

0	0	0
0	2	0
0	0	0

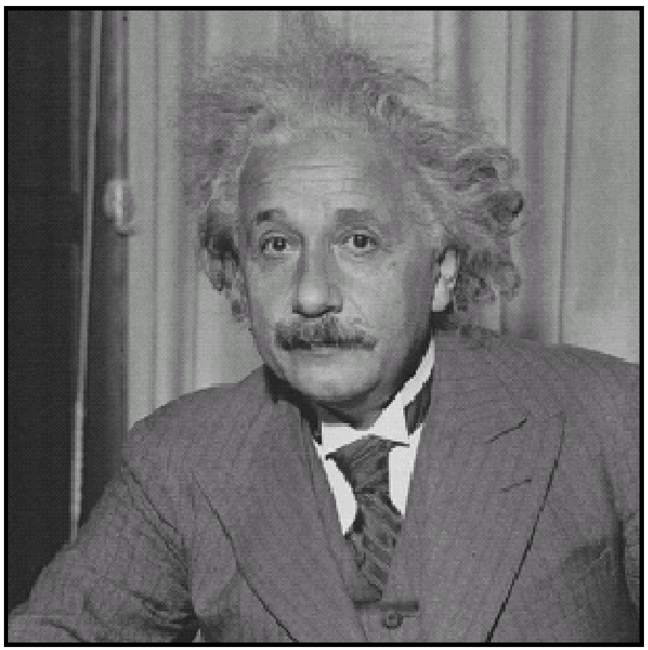
-

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

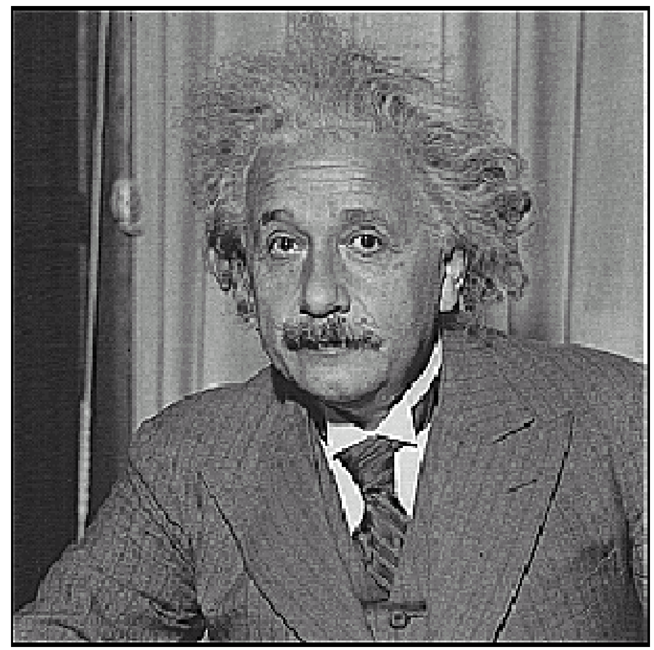
Sharpened
(Acccentuates difference from local average)

Slide Credit: D. Lowe

Sharpening



before



after

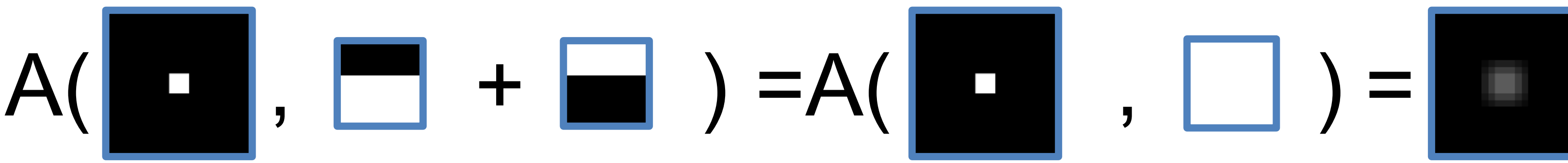
Slide Credit: D. Lowe

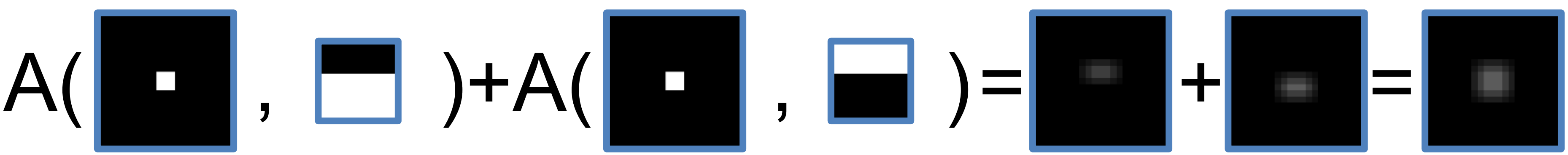
Properties – Linear

Assume: I image f1, f2 filters

Linear: apply(I,f1+f2) = apply(I,f1) + apply(I,f2)

I is a box on black, and and f1, f2 are boxes





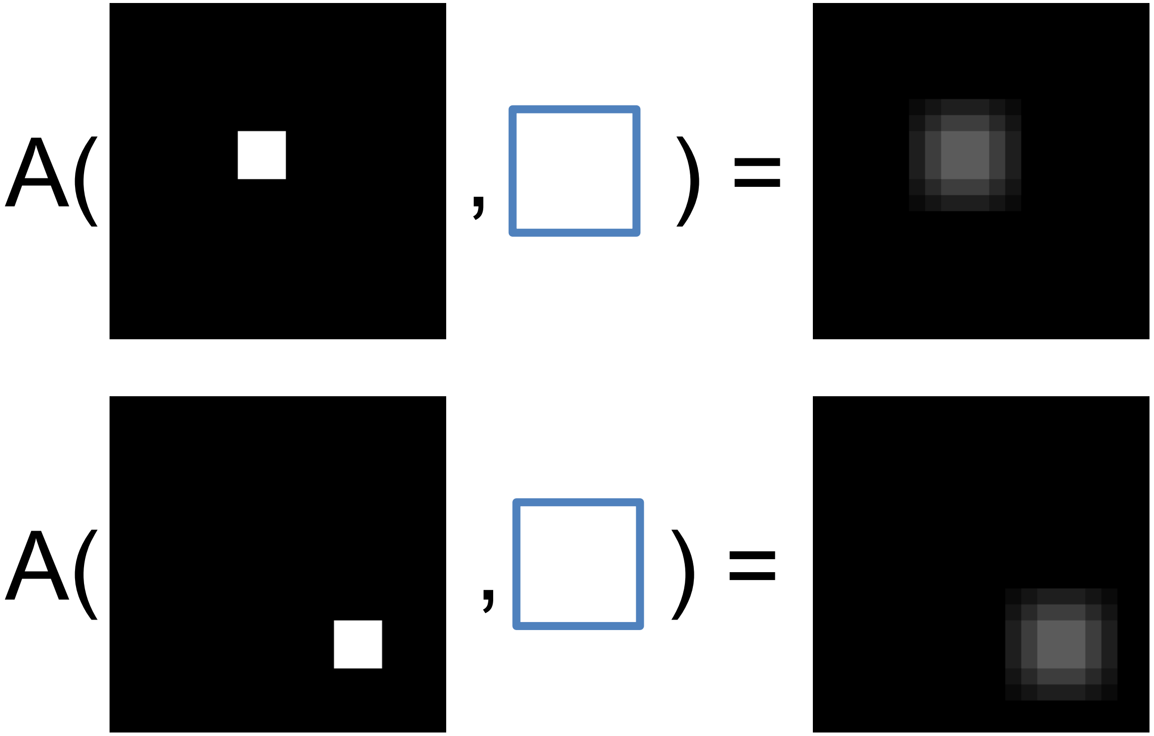
Note: I am showing filters un-normalized and blown up. They're a smaller box filter (i.e., each entry is 1/(size^2))

Properties – Shift-Invariant

Assume: I image, f filter

Shift-invariant: shift(apply(I,f)) = apply(shift(I,f))

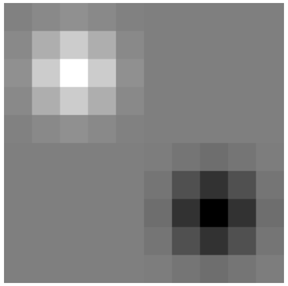
Intuitively: only depends on filter neighborhood



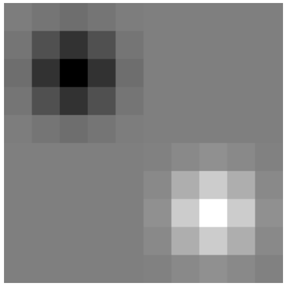
Painful Details – Signal Processing

Often called "convolution". *Actually* cross-correlation.

Cross-Correlation
(Original Orientation)

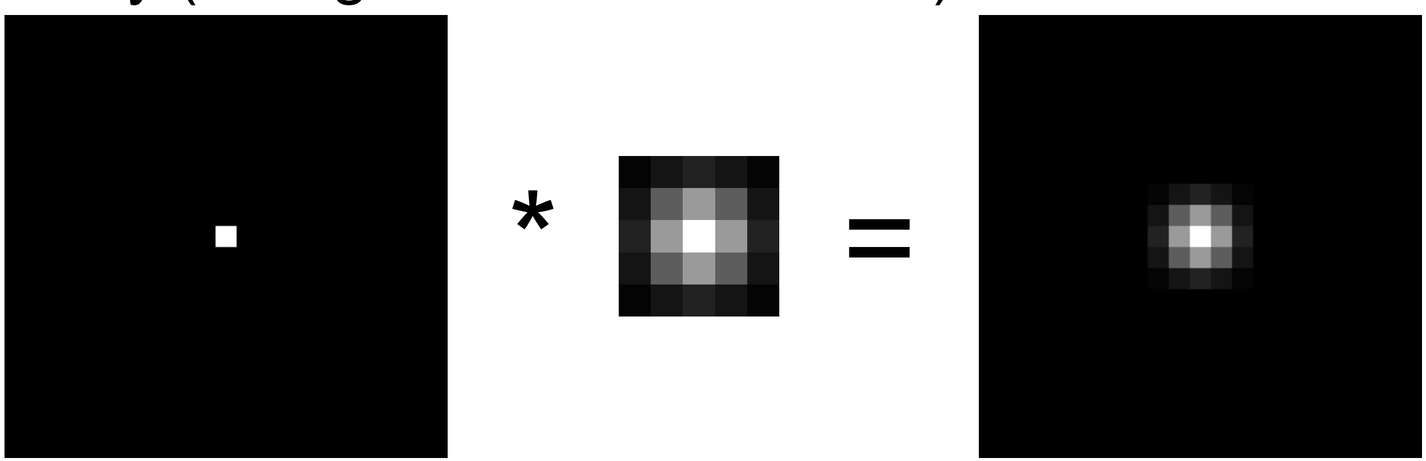


Convolution
(Flipped in x and y)



Properties of Convolution

- Any shift-invariant, linear operation is a convolution
- Commutative: $f * g = g * f$
- Associative: $(f * g) * h = f * (g * h)$
- Distributes over +: $f * (g + h) = f * g + f * h$
- Scalars factor out: $kf * g = f * kg = k (f * g)$
- Identity (a single one with all zeros):



Property List: K. Grauman

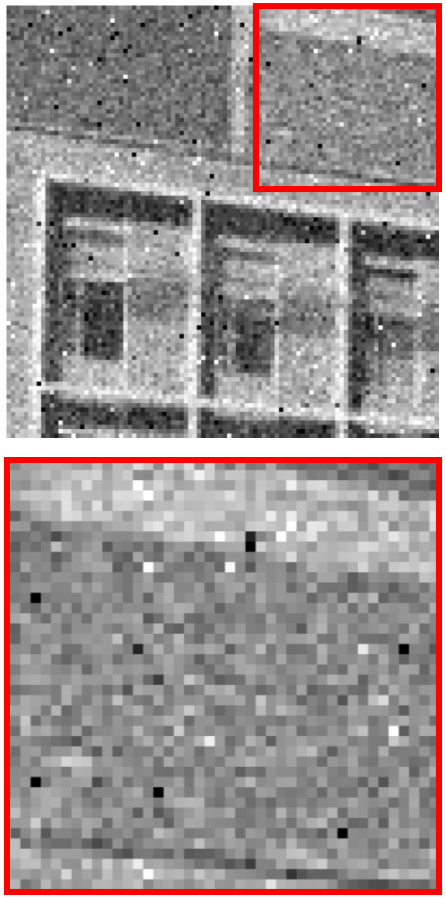
Questions?

- Nearly everything onwards is a convolution.
- This is important to get right.

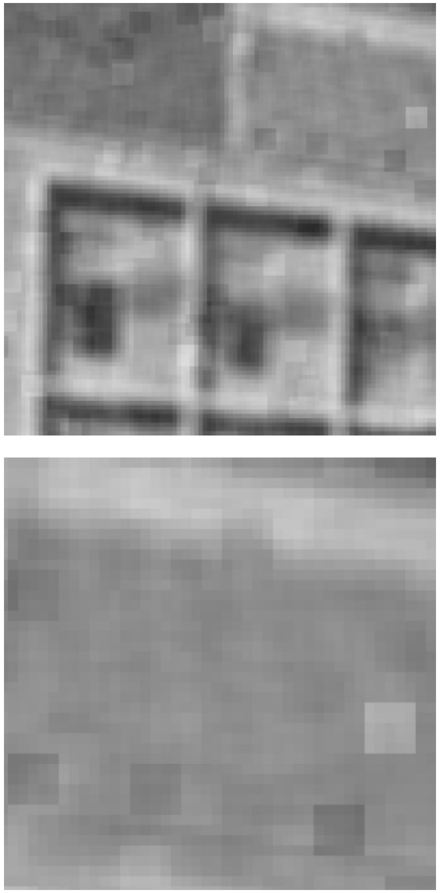
Smoothing With A Box

Intuition: if filter touches it, it gets a contribution.

Input



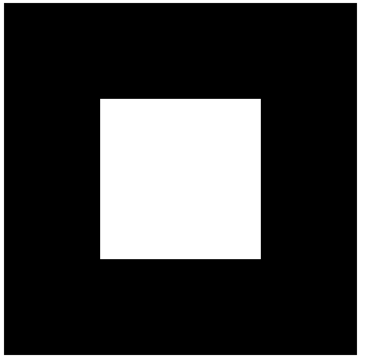
Box Filter



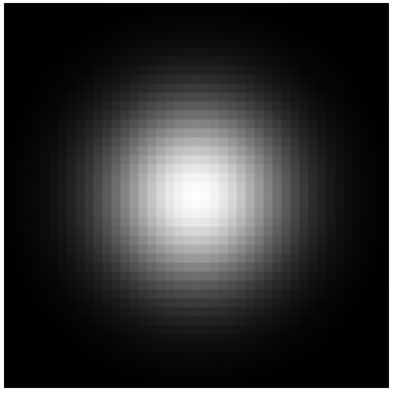
Solution – Weighted Combination

Intuition: weight contributions according to closeness to center.

$$Filter_{ij} \propto 1$$



$$Filter_{ij} \propto \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

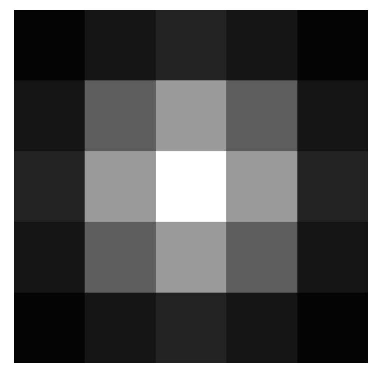


Recognize the Filter?

It's a Gaussian!

$$Filter_{ij} \propto \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

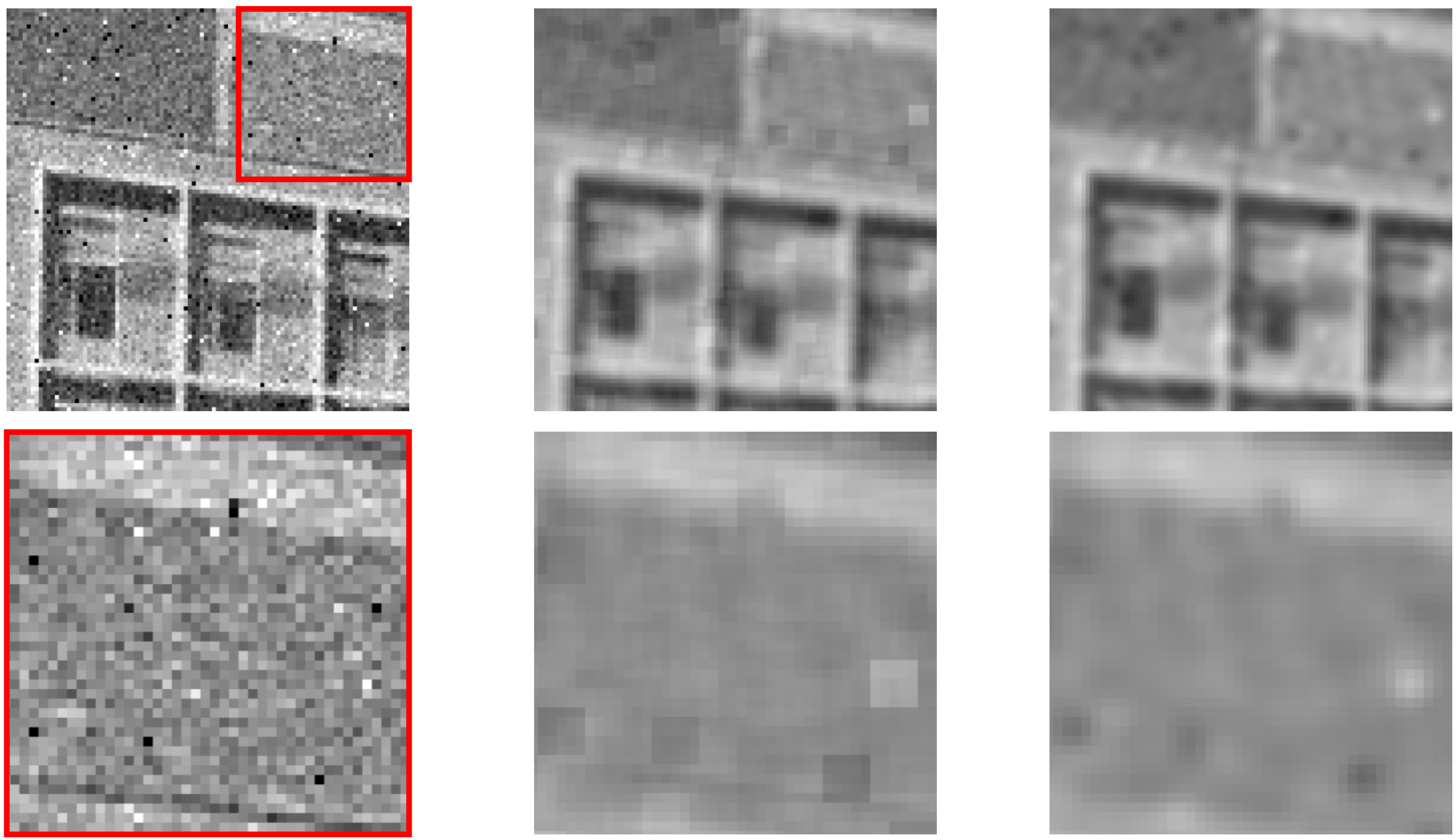
$$\begin{bmatrix} 0.003 & 0.013 & 0.022 & 0.013 & 0.003 \\ 0.013 & 0.060 & 0.098 & 0.060 & 0.013 \\ 0.022 & 0.098 & 0.162 & 0.098 & 0.022 \\ 0.013 & 0.060 & 0.098 & 0.060 & 0.013 \\ 0.003 & 0.013 & 0.022 & 0.013 & 0.003 \end{bmatrix}$$



Smoothing With A Box & Gauss

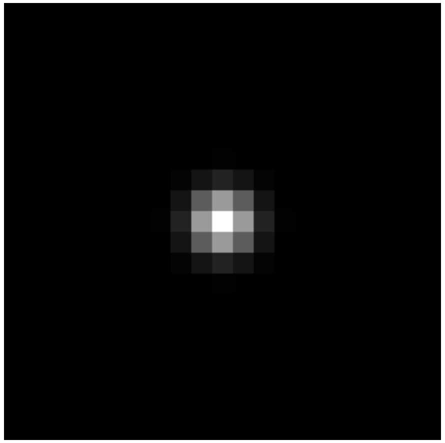
Still have some speckles, but it’s not a big box

Input | Box Filter | Gauss. Filter

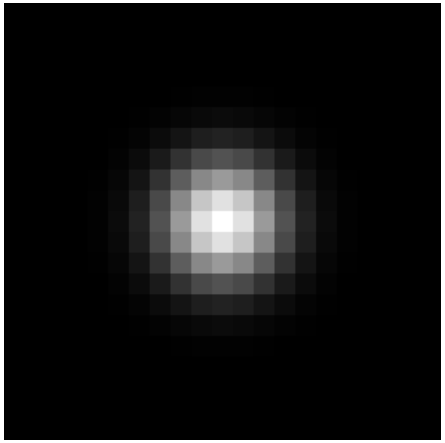


Gaussian Filters

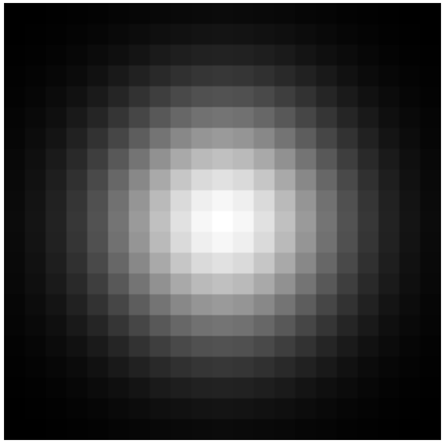
σ = 1
filter = 21x21



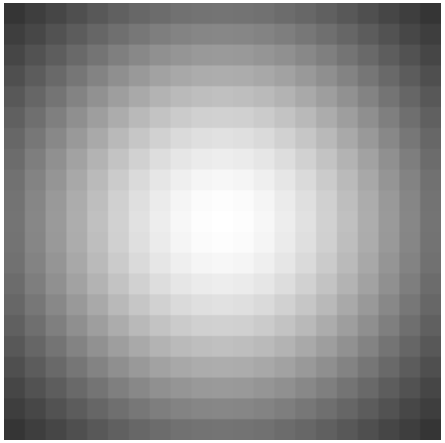
σ = 2
filter = 21x21



σ = 4
filter = 21x21

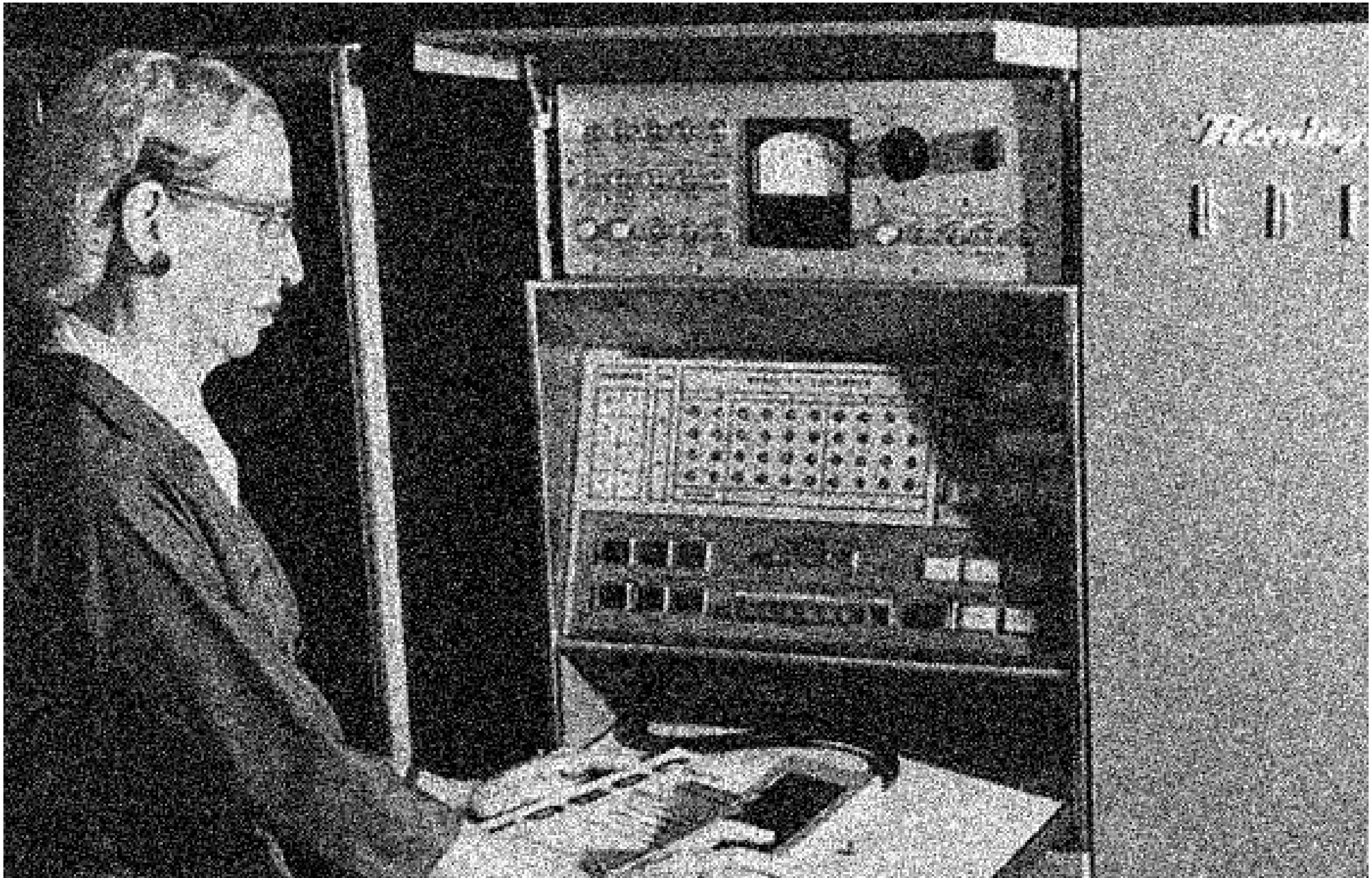


σ = 8
filter = 21x21



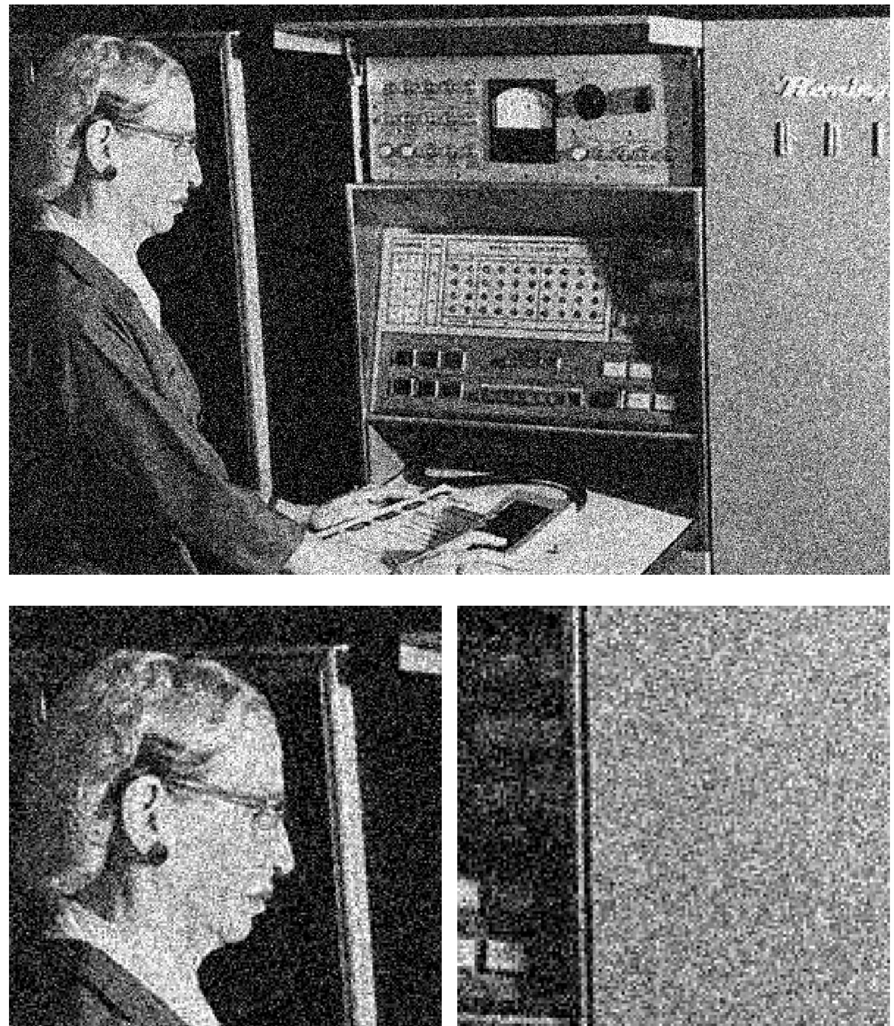
Note: filter visualizations are independently normalized throughout the slides so you can see them better

Applying Gaussian Filters



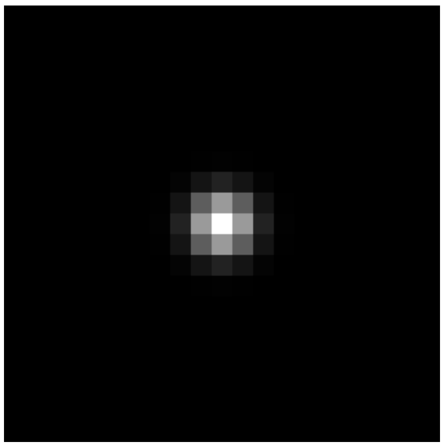
Applying Gaussian Filters

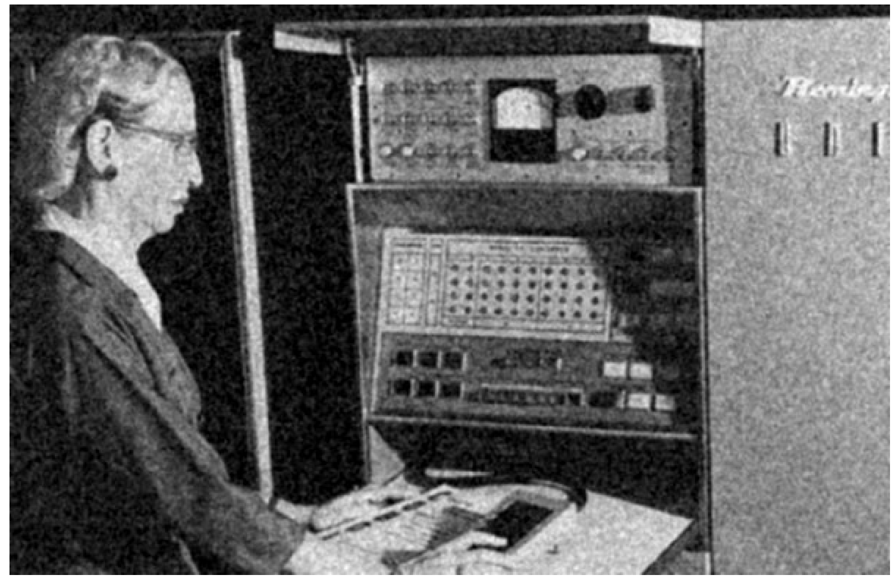
Input Image
(no filter)

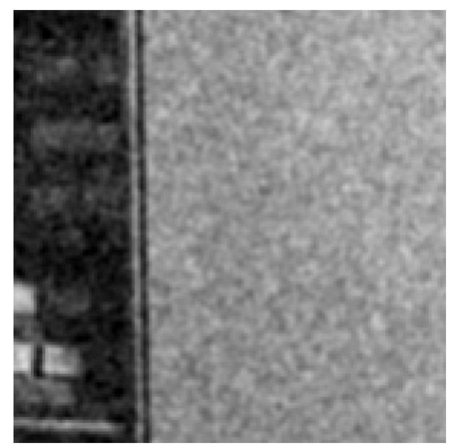


Applying Gaussian Filters

$\sigma = 1$

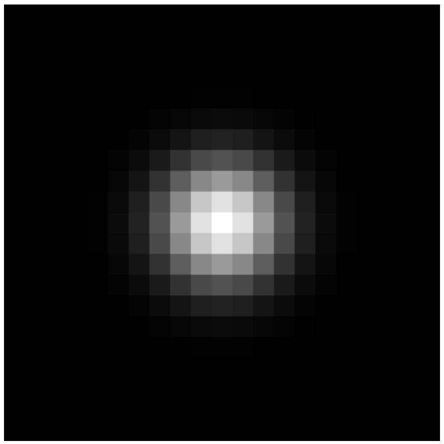


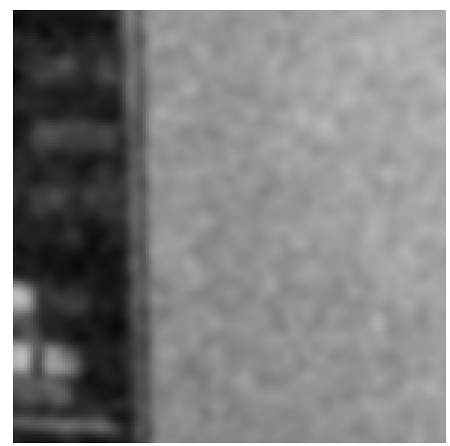




Applying Gaussian Filters

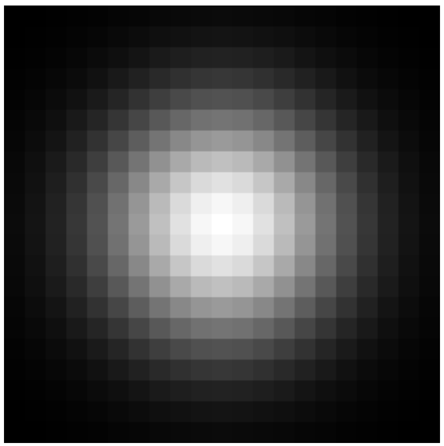
σ = 2

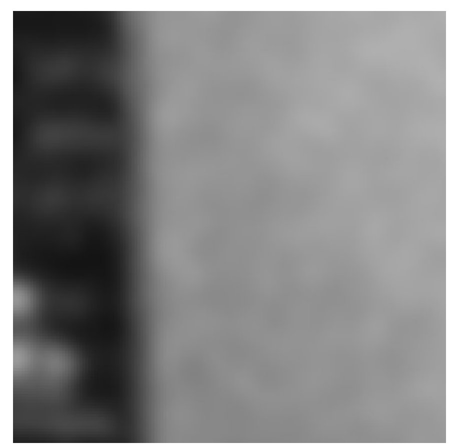




Applying Gaussian Filters

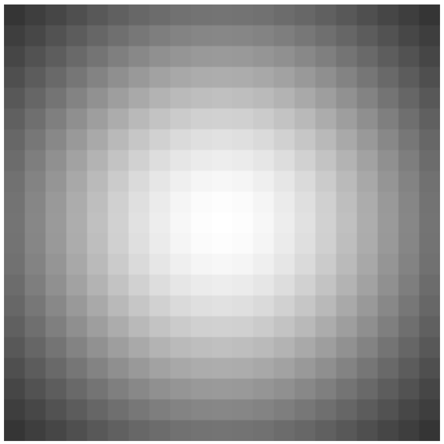
$\sigma = 4$





Applying Gaussian Filters

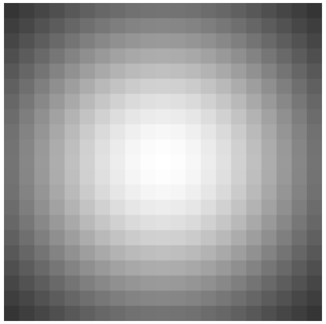
$\sigma = 8$



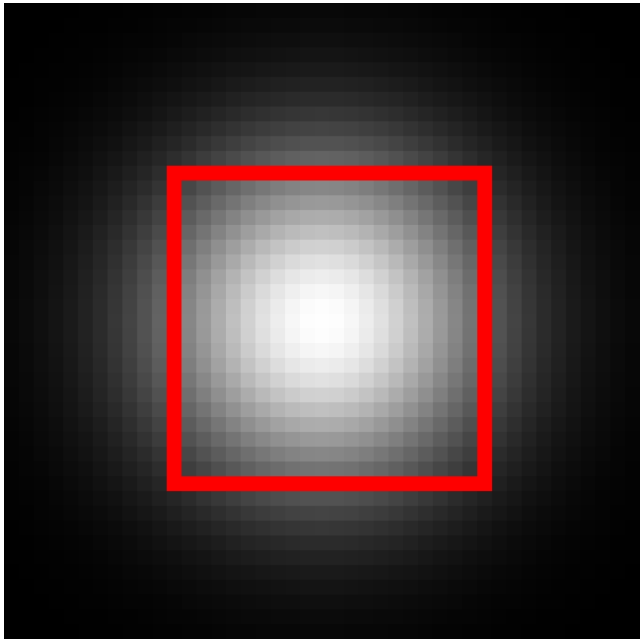
Picking a Filter Size

Too small filter → bad approximation
Want size ≈ 6σ (99.7% of energy)
Left far too small; right slightly too small!

σ = 8, size = 21



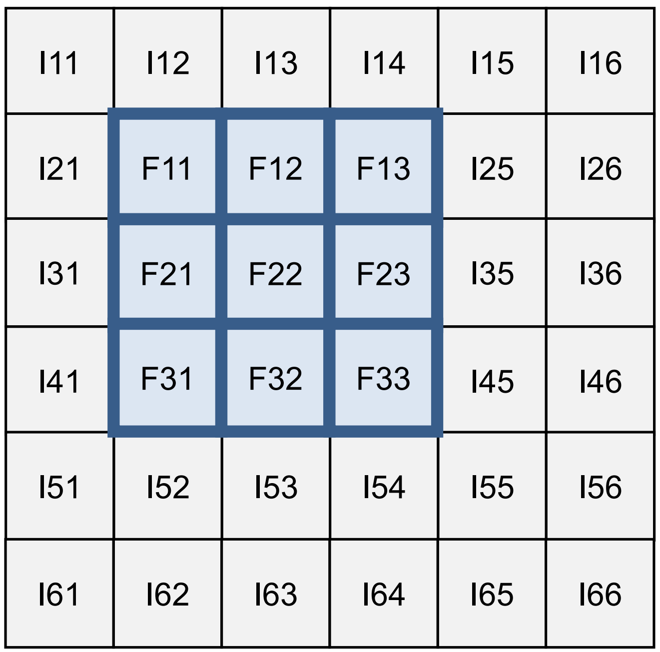
σ = 8, size = 43



Runtime Complexity

Image size = NxN = 6x6

Filter size = MxM = 3x3

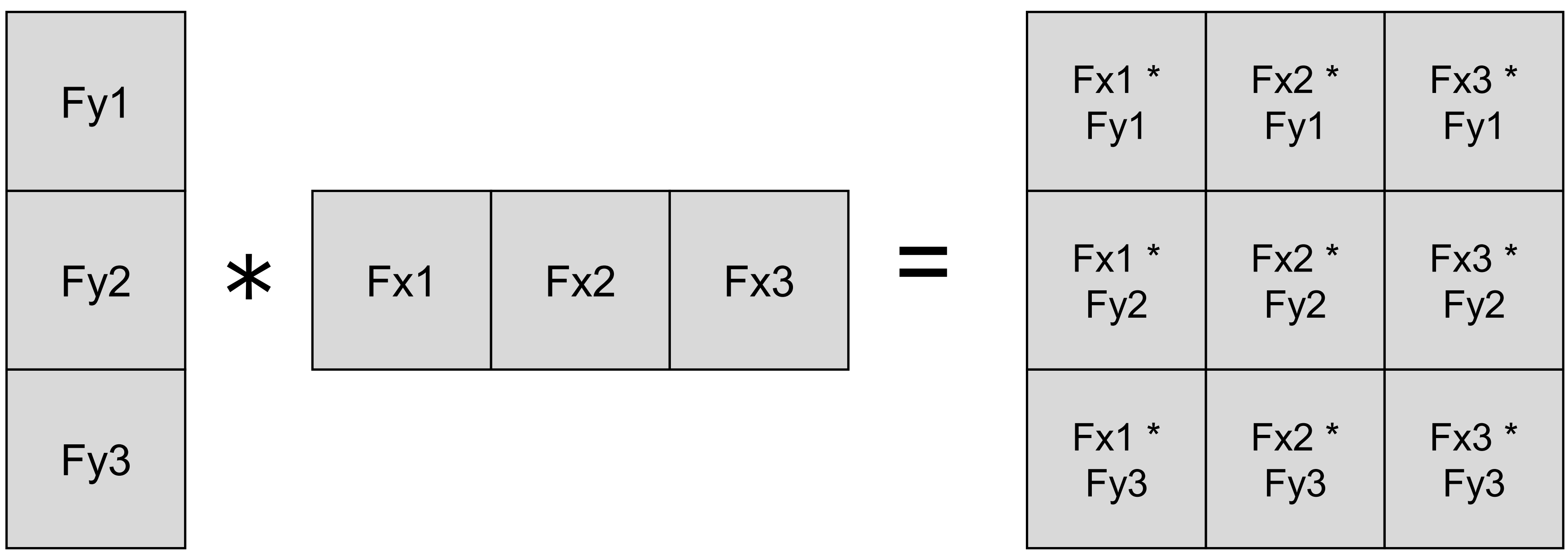



```
for ImageY in range(N):
    for ImageX in range(N):
        for FilterY in range(M):
            for FilterX in range(M):
                ...
```

Time: $O(N^2M^2)$

Separability

Conv(vector, transposed vector) → outer product



Separability

$$Filter_{ij} \propto \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

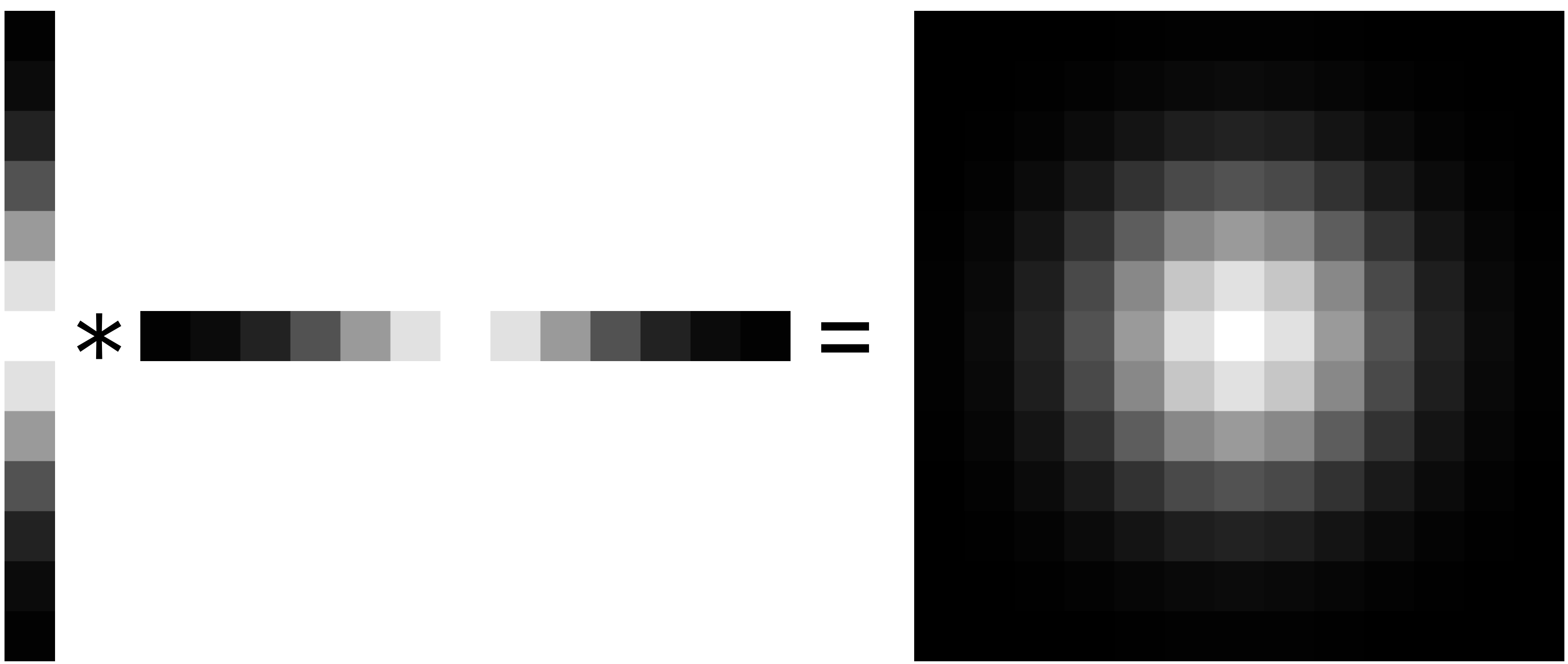
$$\rightarrow$$

$$Filter_{ij} \propto \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right)\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{y^2}{2\sigma^2}\right)$$

Separability

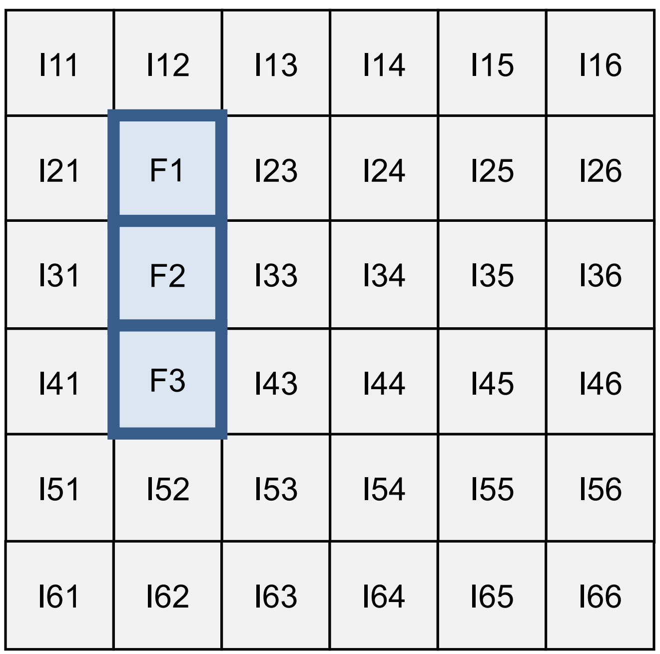
1D Gaussian $*$ 1D Gaussian = 2D Gaussian

Image $*$ 2D Gauss = Image $*$ (1D Gauss $*$ 1D Gauss)
= (Image $*$ 1D Gauss) $*$ 1D Gauss



Runtime Complexity

Image size = NxN = 6x6
Filter size = Mx1 = 3x1



What are my compute savings for a 13x13 filter?

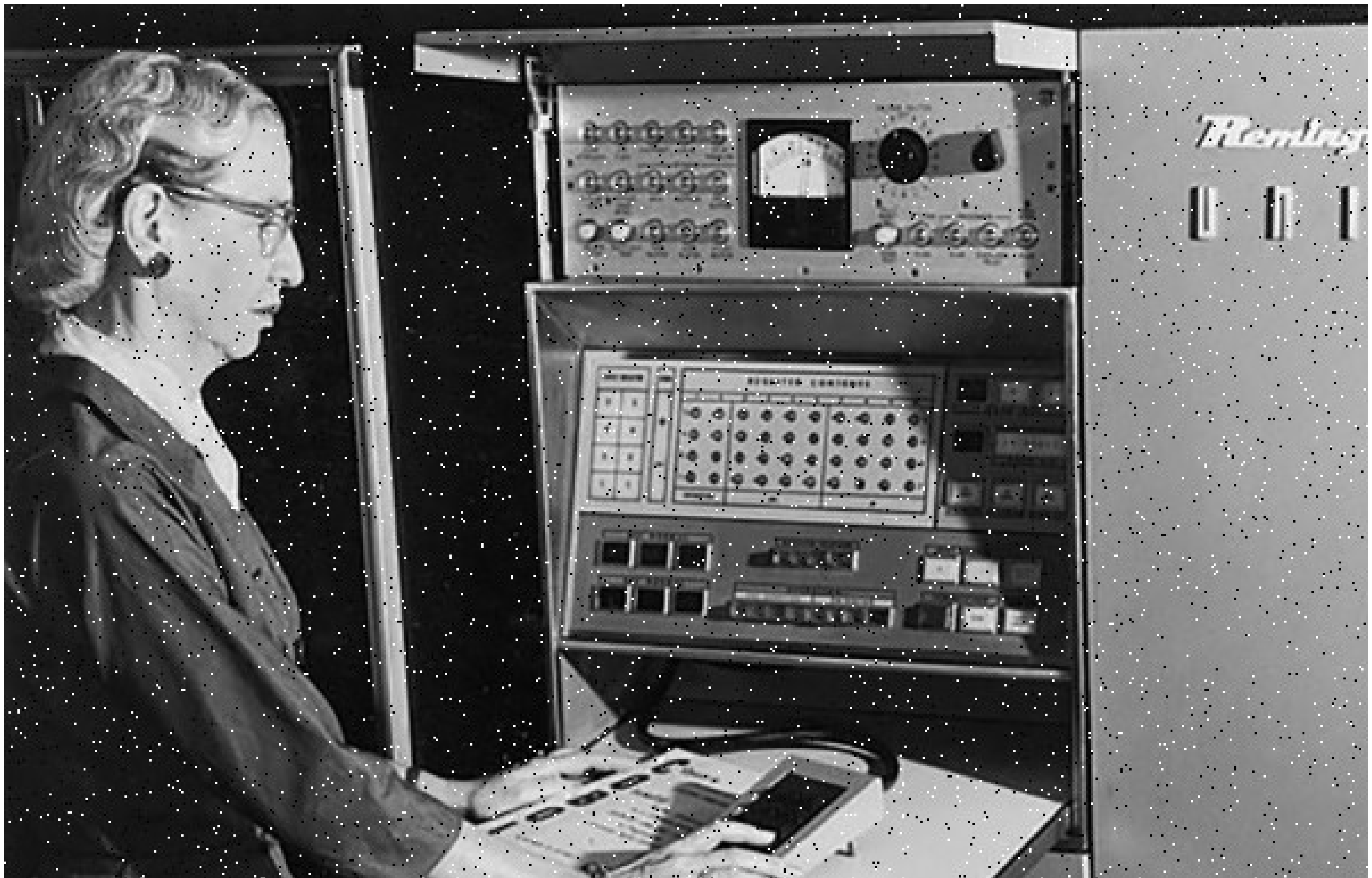
```
for ImageY in range(N):
    for ImageX in range(N):
        for FilterY in range(M):
            ...
for ImageY in range(N):
    for ImageX in range(N):
        for FilterX in range(M):
            ...
```

Time: $O(N^2M)$

Why Gaussian?

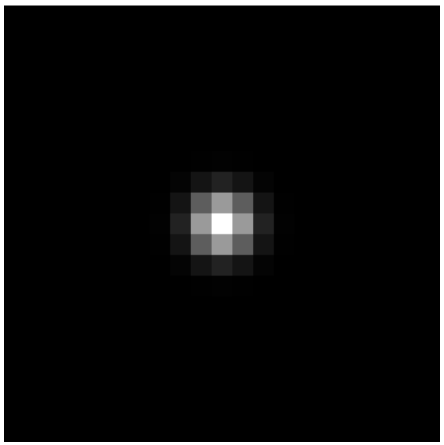
Gaussian filtering removes parts of the signal above a certain frequency. Often noise is high frequency and signal is low frequency.

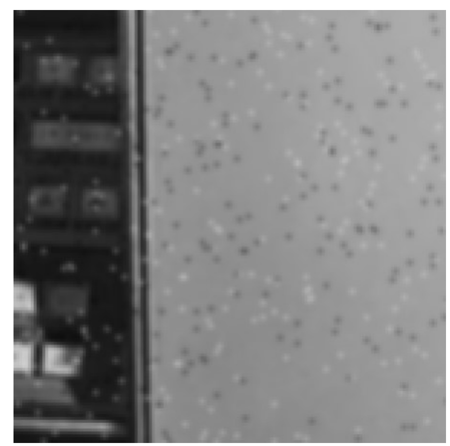
Where Gaussian Fails



Applying Gaussian Filters

σ = 1





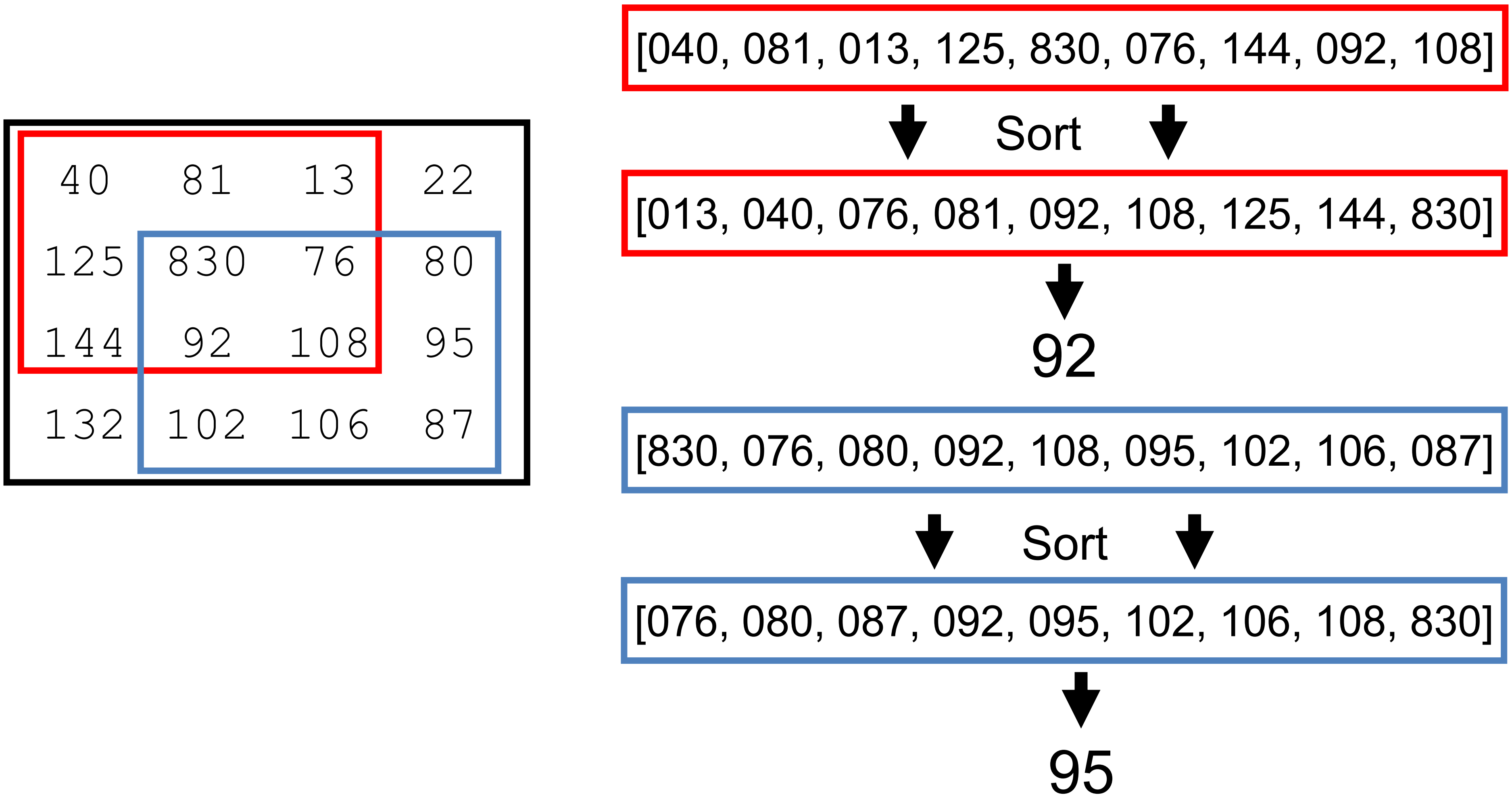
Why Does This Fail?

Means can be arbitrarily distorted by outliers

Signal	10	12	9	8	1000	11	10	12
Filter	0.1	0.8	0.1					
Output	11.5	9.2	107.3	801.9	109.8	10.3		

What else is an “average” other than a mean?

Non-linear Filters (2D)



Applying Median Filter

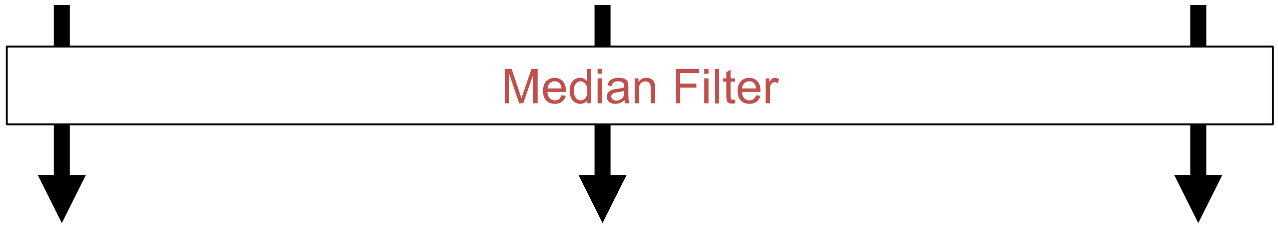
Median Filter (size=3)

Applying Median Filter

Median
Filter
(size = 7)

Is Median Filtering Linear?

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$



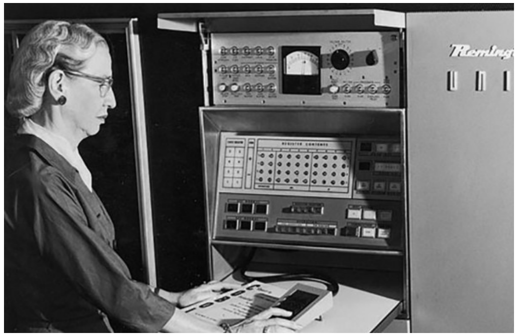
$$1 \quad + \quad 0 \quad = \quad 2$$

Example from (I believe): Kristen Grauman

Some Examples of Filtering

Filtering – Sharpening

Image

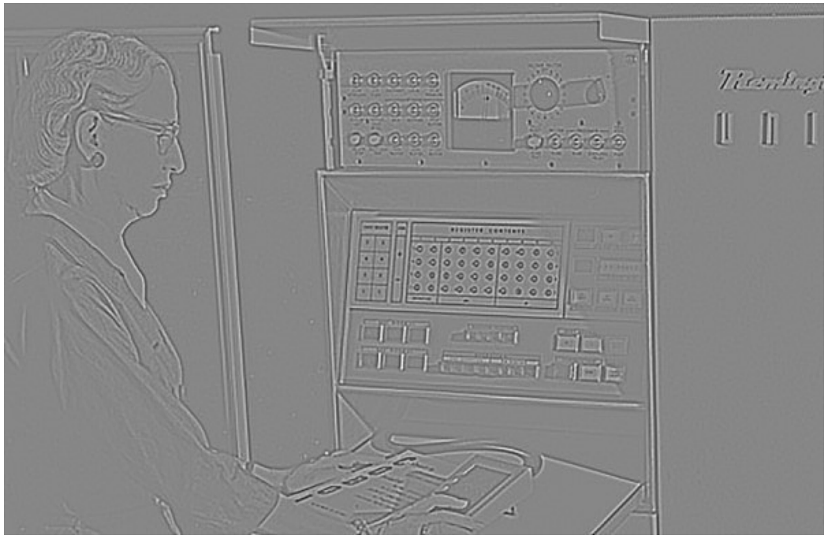


-

Smoothed

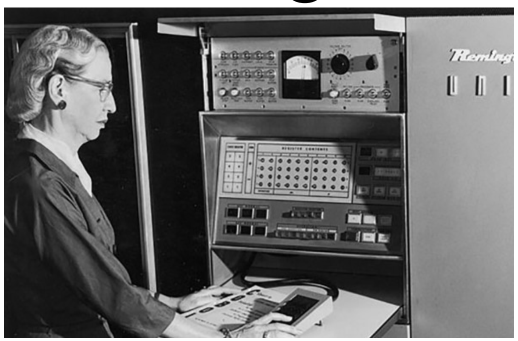
Details

=



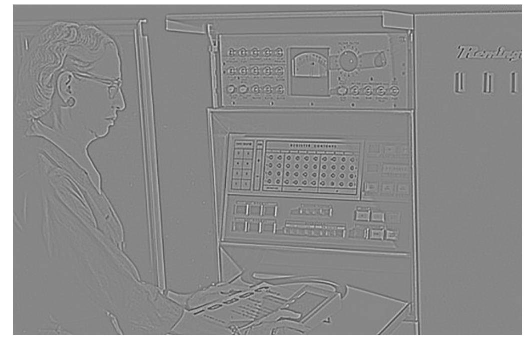
Filtering – Sharpening

Image



+α

Details

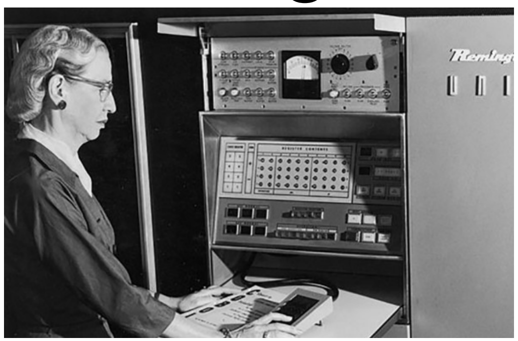


“Sharpened” α=1

=

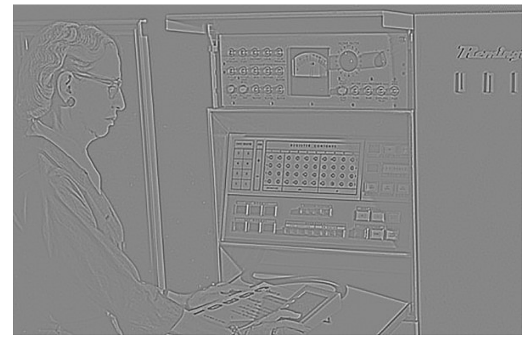
Filtering – Sharpening

Image



+α

Details

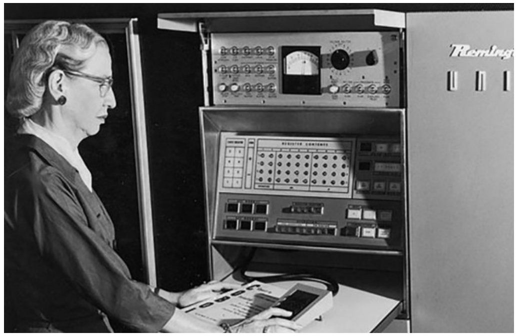


“Sharpened” α=0

=

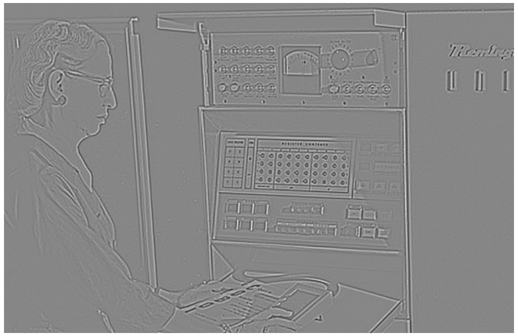
Filtering – Sharpening

Image



+α

Details

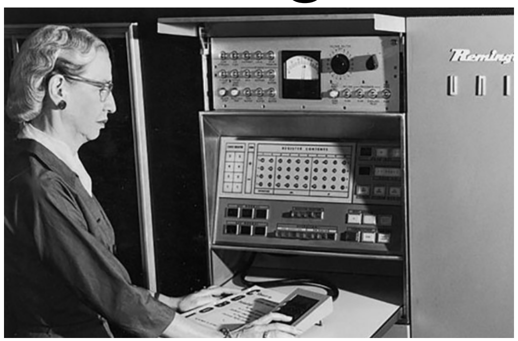


“Sharpened” α=2

=

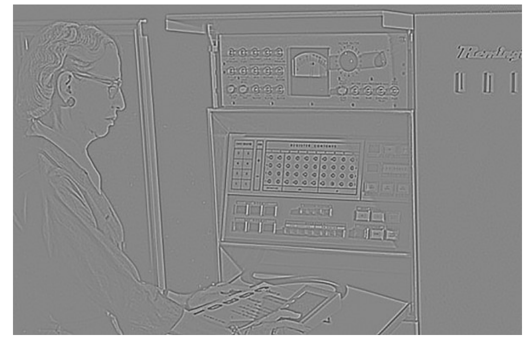
Filtering – Sharpening

Image



+α

Details

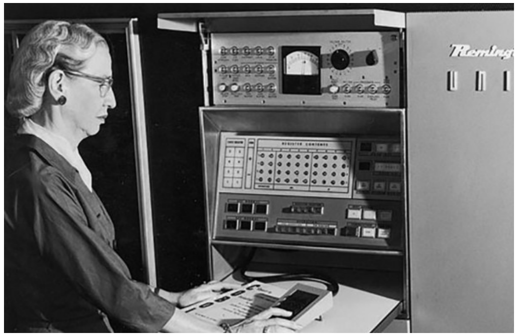


"Sharpened" α=0

=

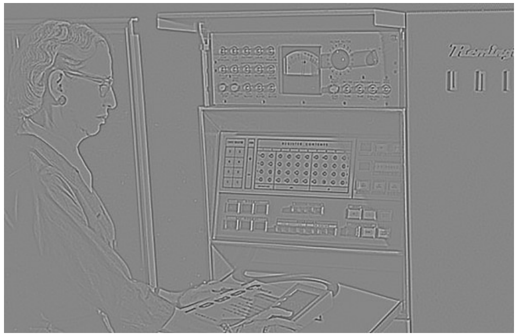
Filtering – Extreme Sharpening

Image



+α

Details



“Sharpened” α=10

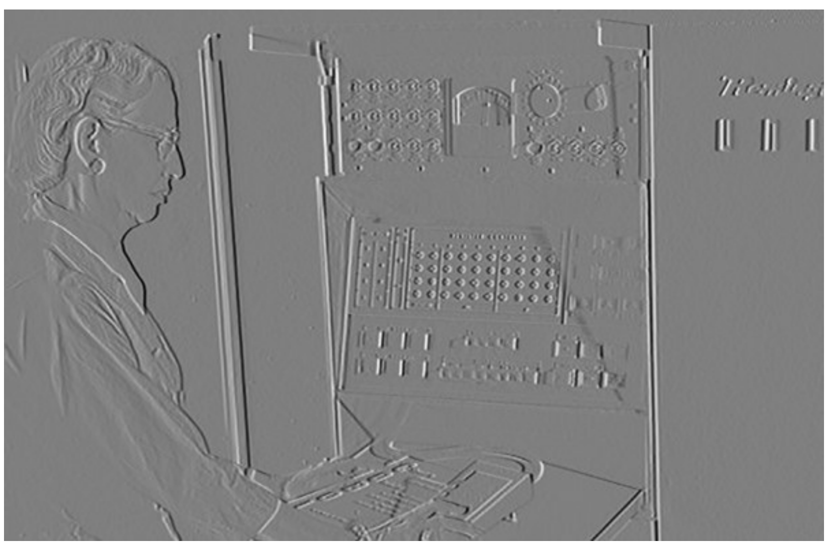
=

Filtering

What's this Filter?

-1	0	1

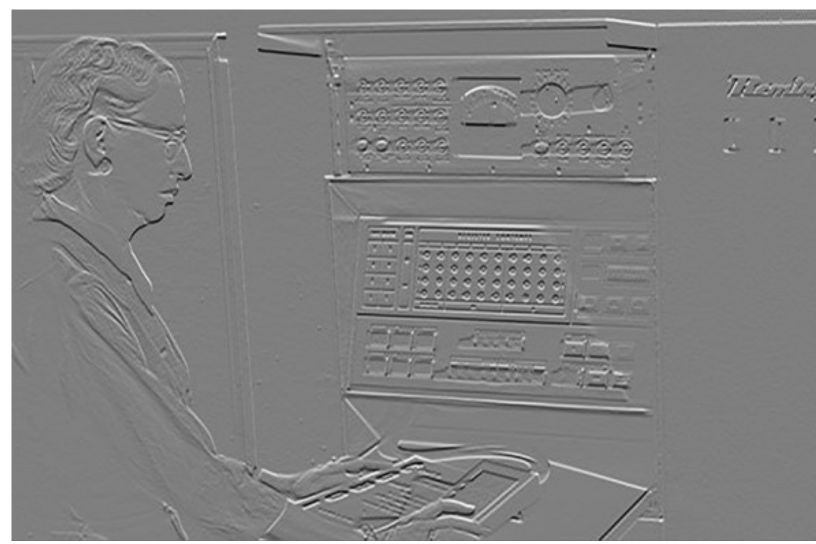
Dx



-1	0	1

T

Dy



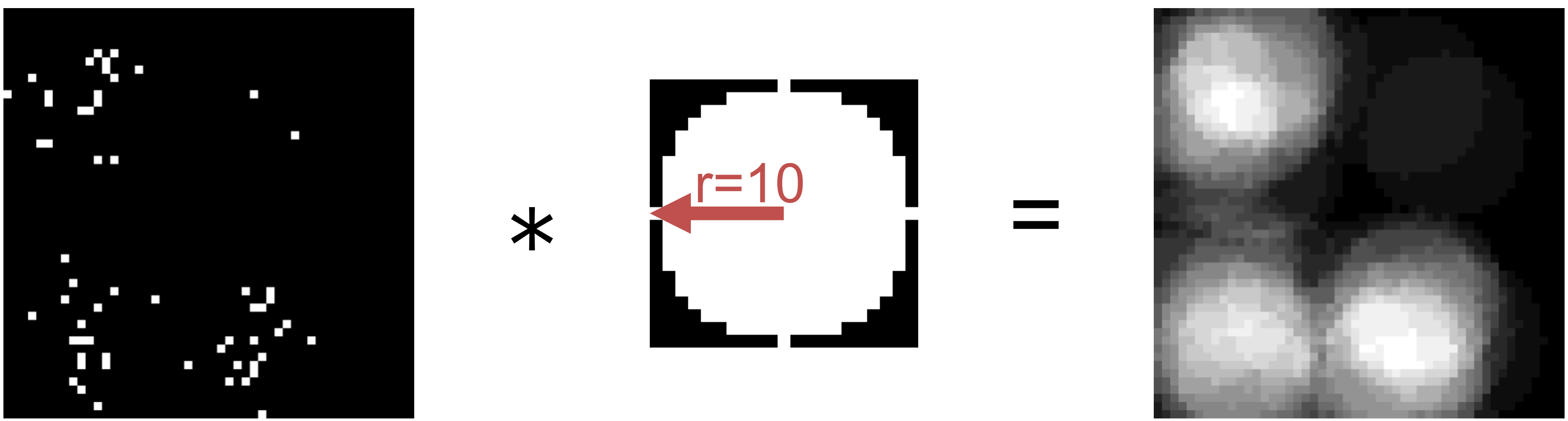
Filtering – Derivatives

$$(Dx^2 + Dy^2)^{1/2}$$

Filtering – Counting

How many “on” pixels have
10+ neighbors within 10 pixels?

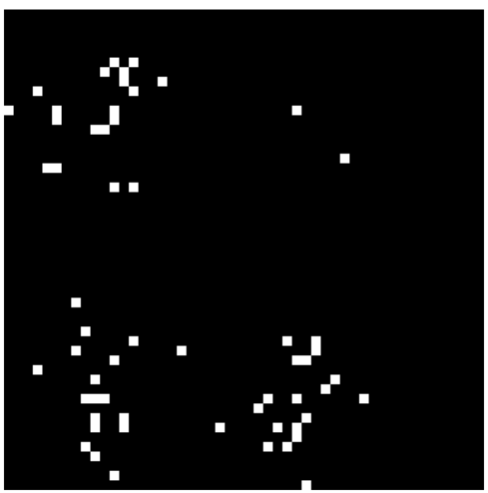
Pixels Disk ???



Filtering – Counting

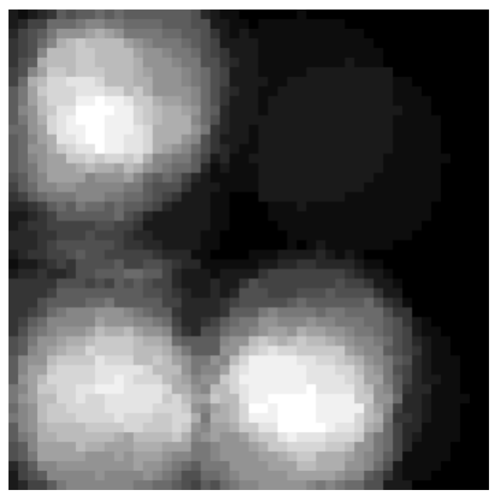
How many “on” pixels have
10+ neighbors within 10 pixels?

Pixels



x

Density

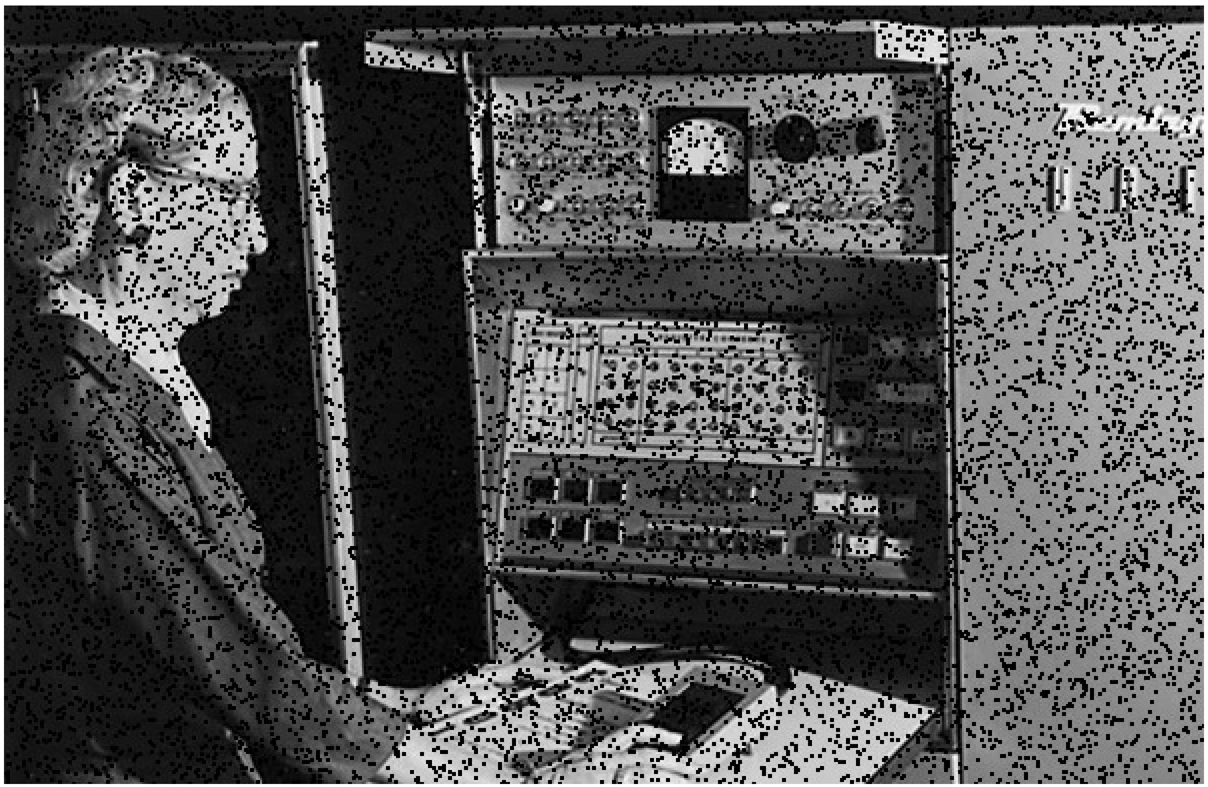


=

Answer

Filtering – Missing Data

Oh no! Missing data!
(and we know where)



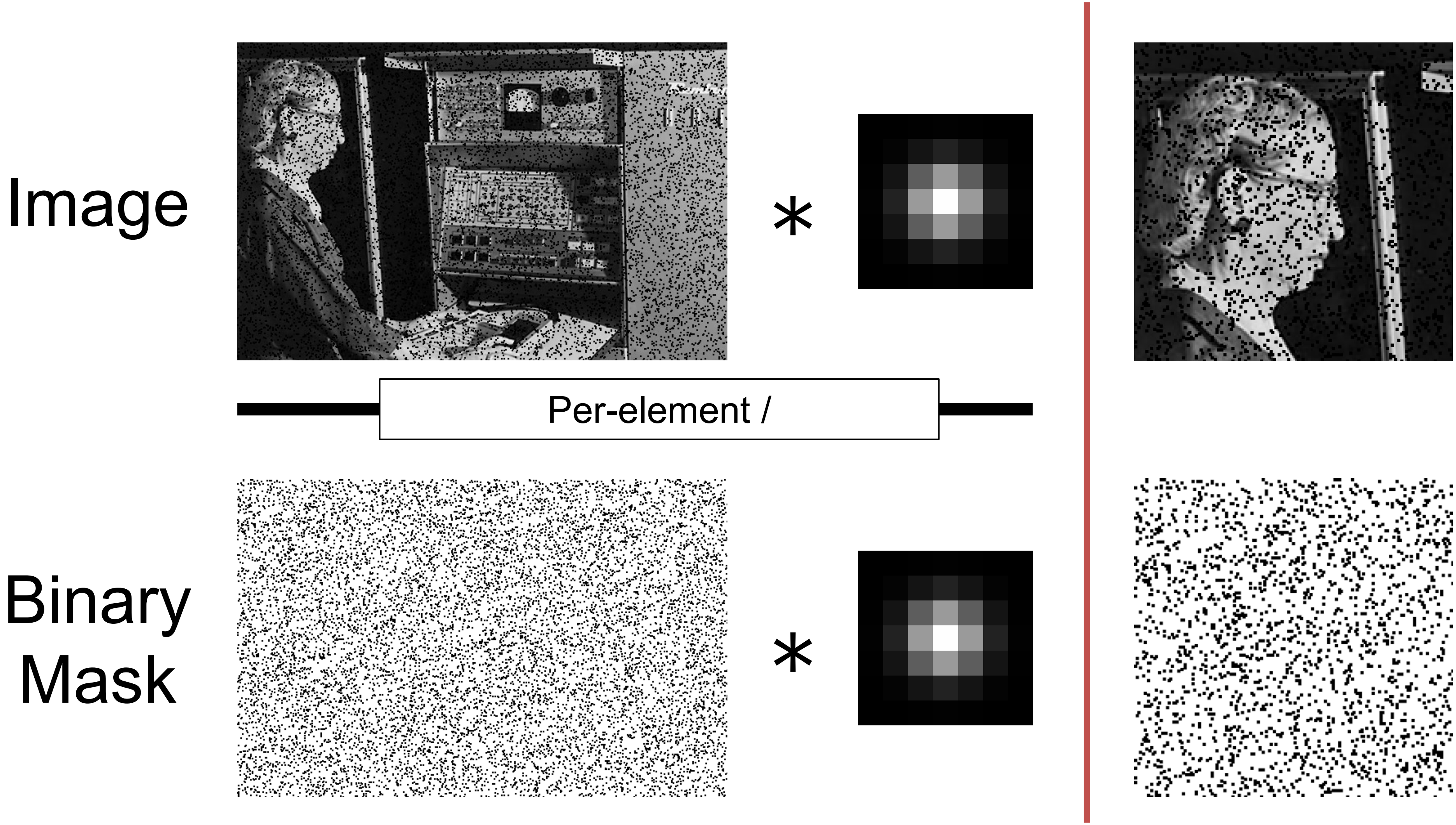
Common with many non-normal cameras (e.g., depth cameras)

Aside (Added after class)

- Element-wise operations on matrices A,B:
- Addition (same as normal):
  - $Out_{ij} = A_{ij} + B_{ij}$
- Division:
  - $Out_{ij} = A_{ij} / B_{ij}$
- Multiplication (aka Hadamard Product):
  - $Out_{ij} = A_{ij} * B_{ij}$

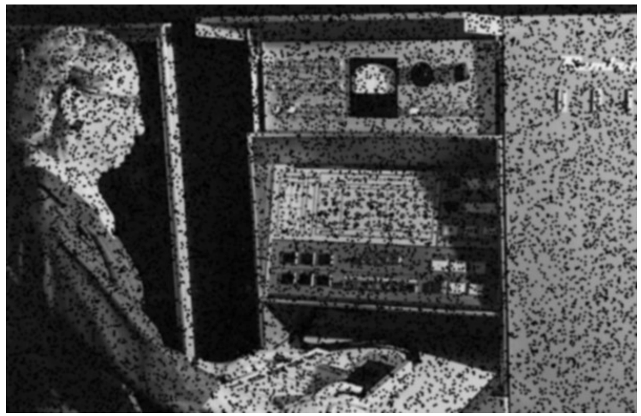
Not typically taught in entry-level linear algebra. Common when working with real matrix data.

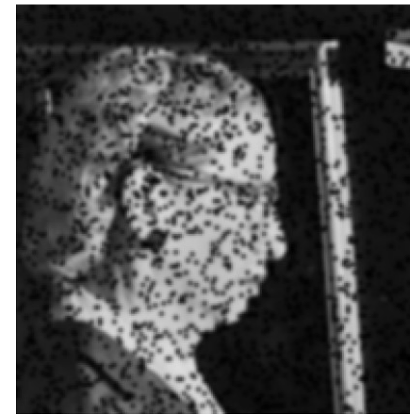
Filtering – Missing Data



Filtering – Missing Data

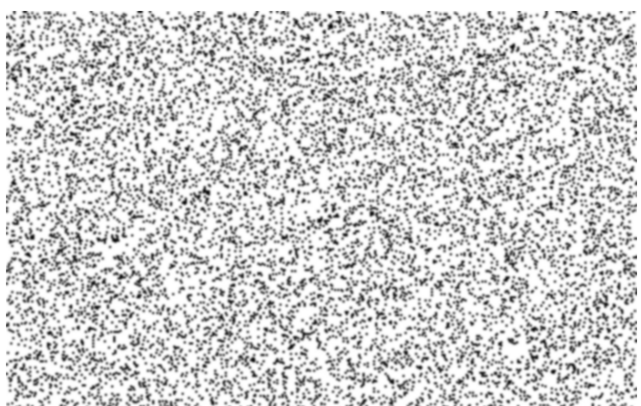
Image

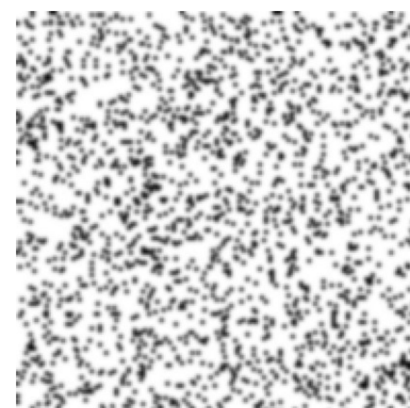




Per-element /

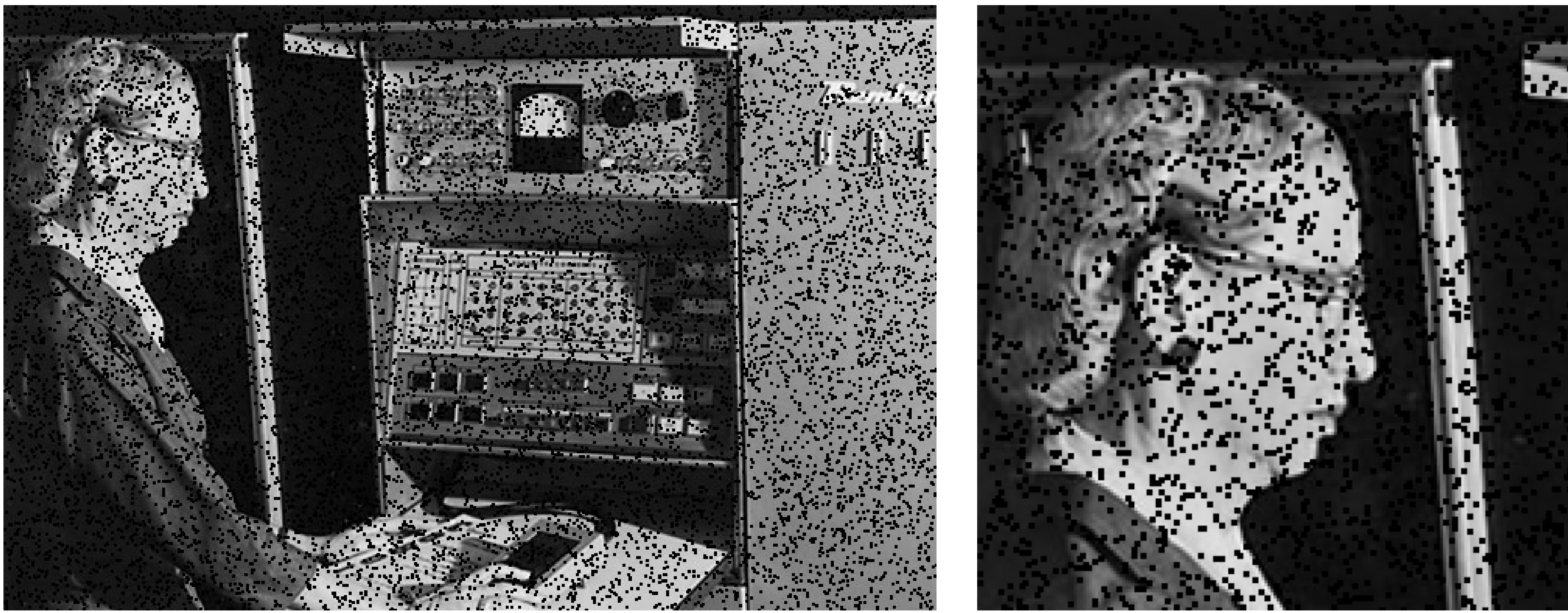
Binary Mask





Filtering – Missing Data

Before



Filtering – Missing Data

After

Filtering – Missing Data

After (without missing data)