

# Cameras

EECS 442 – Prof. David Fouhey  
Winter 2019, University of Michigan

[http://web.eecs.umich.edu/~fouhey/teaching/EECS442\\_W19/](http://web.eecs.umich.edu/~fouhey/teaching/EECS442_W19/)

# Next Few Classes

- Tuesday: Cameras (Projective Geometry)
- Thursday: Cameras (Light, Lenses)
- Next Tuesday: Light and Image Formation

Discussion This Week:

- Linear algebra

# Administrivia

- HW 1 is out Thursday Jan 17, due Jan 31:
  - Linear algebra
  - Projective geometry
  - Image alignment
- You have 3 late days, and it's 1% off for every hour late.
- **Any administrative questions?**

**SIGGRAPH Talks 2011**

# **KinectFusion:**

**Real-Time Dynamic 3D Surface  
Reconstruction and Interaction**

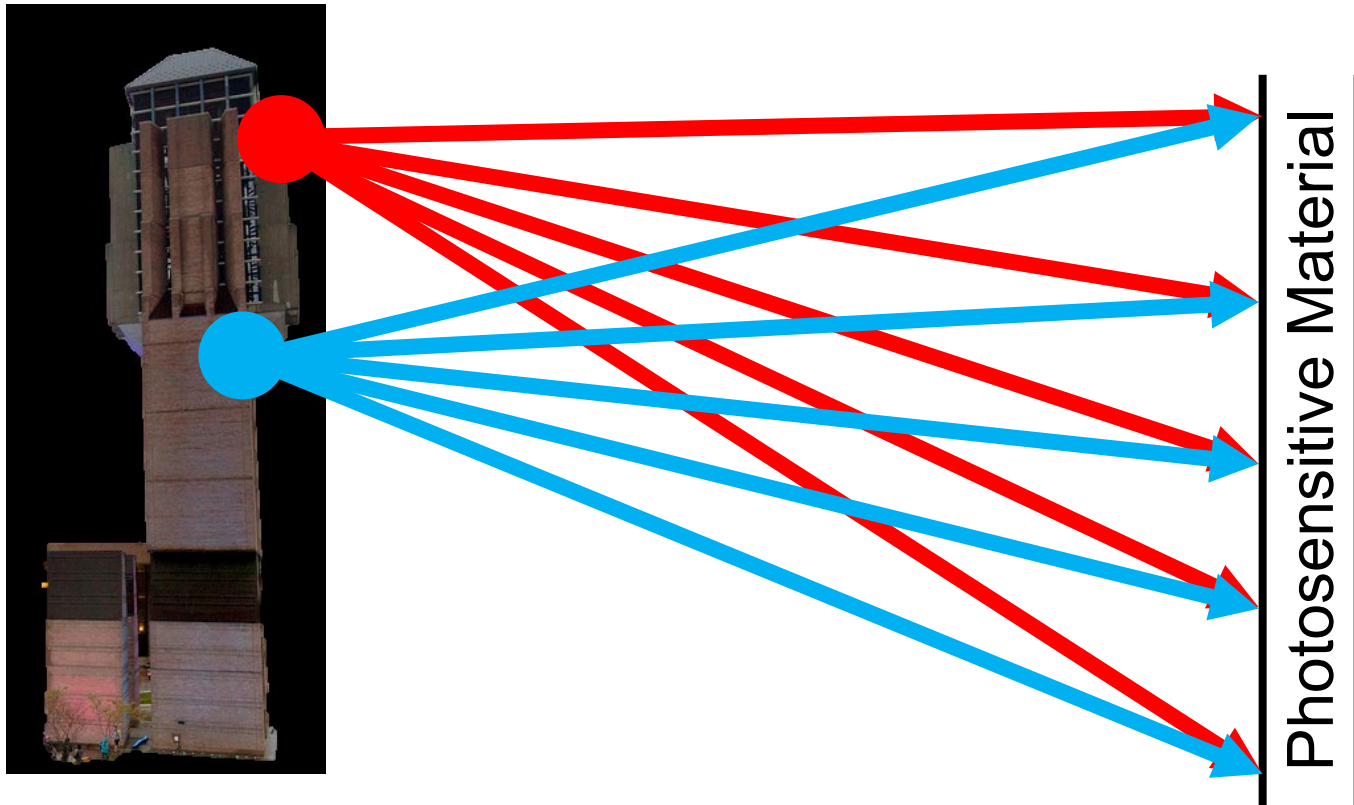
**Shahram Izadi 1, Richard Newcombe 2, David Kim 1,3, Otmar Hilliges 1,  
David Molyneaux 1,4, Pushmeet Kohli 1, Jamie Shotton 1,  
Steve Hodges 1, Dustin Freeman 5, Andrew Davison 2, Andrew Fitzgibbon 1**

1 Microsoft Research Cambridge 2 Imperial College London

3 Newcastle University 4 Lancaster University

5 University of Toronto

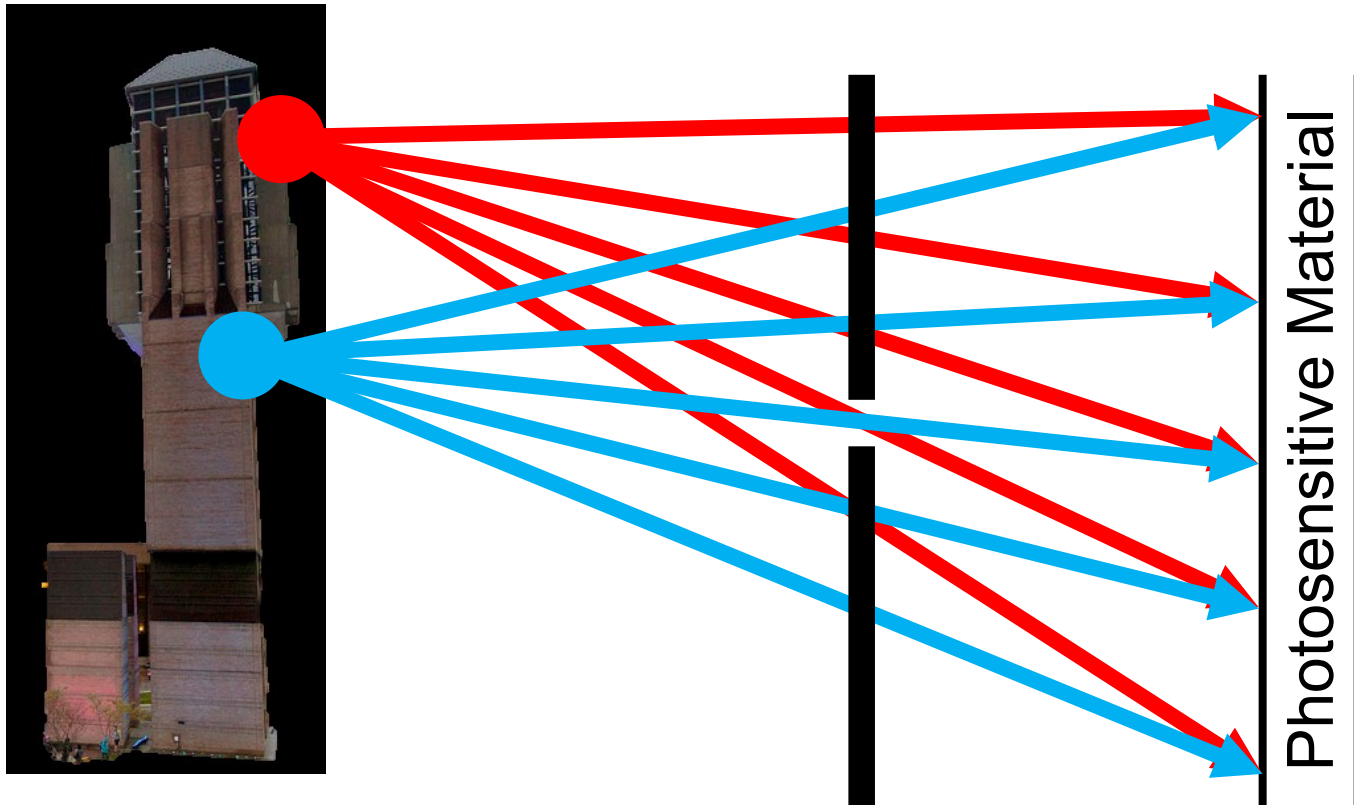
# Let's Take a Picture!



Idea 1: Just use film

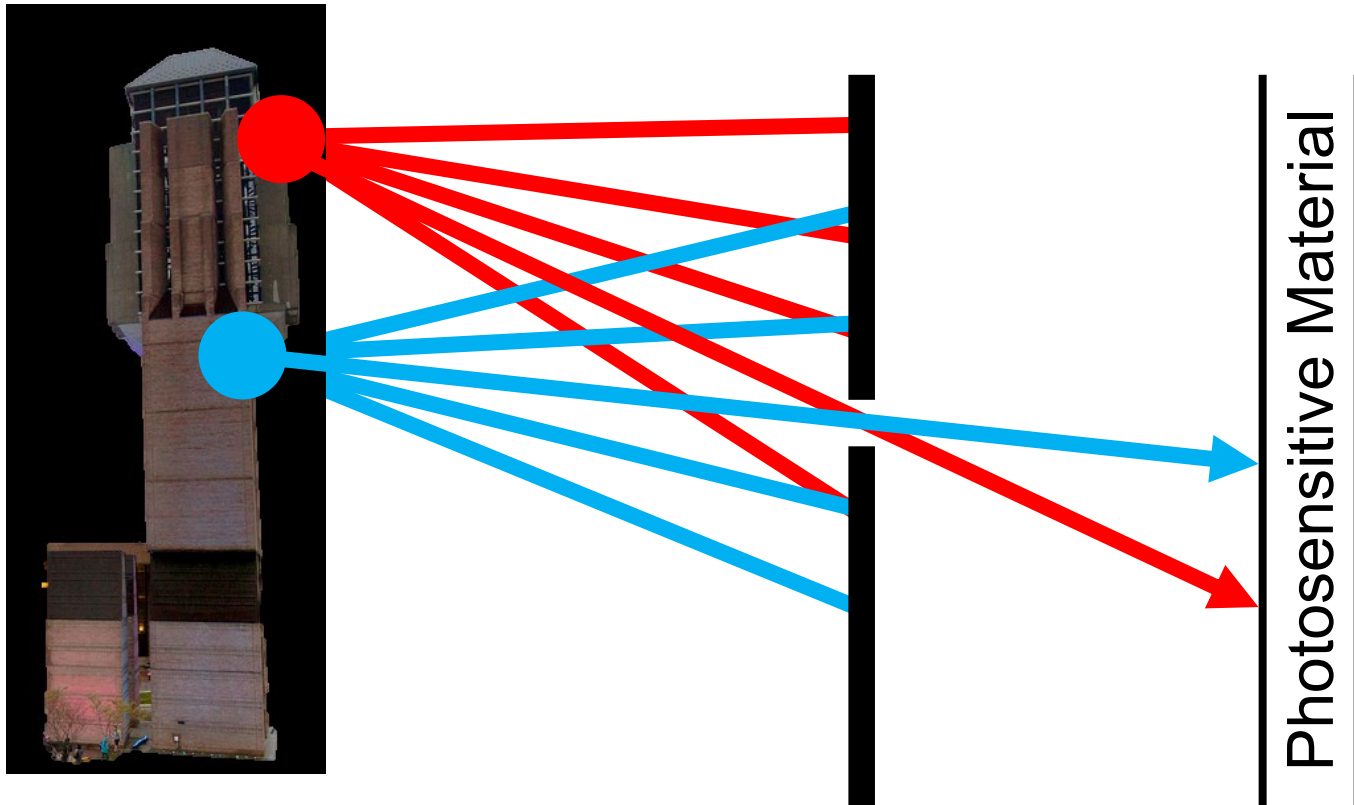
Result: **Junk**

# Let's Take a Picture!



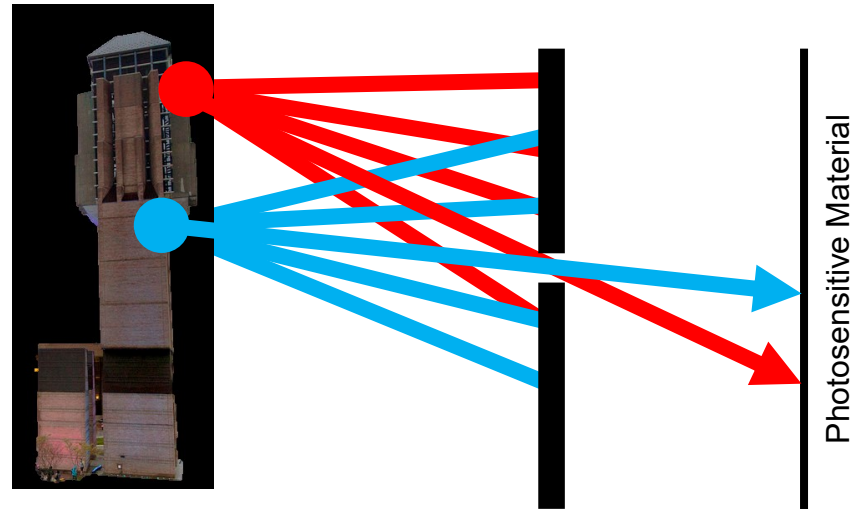
Idea 2: add a barrier

# Let's Take a Picture!



Idea 2: add a barrier

# Let's Take a Picture!

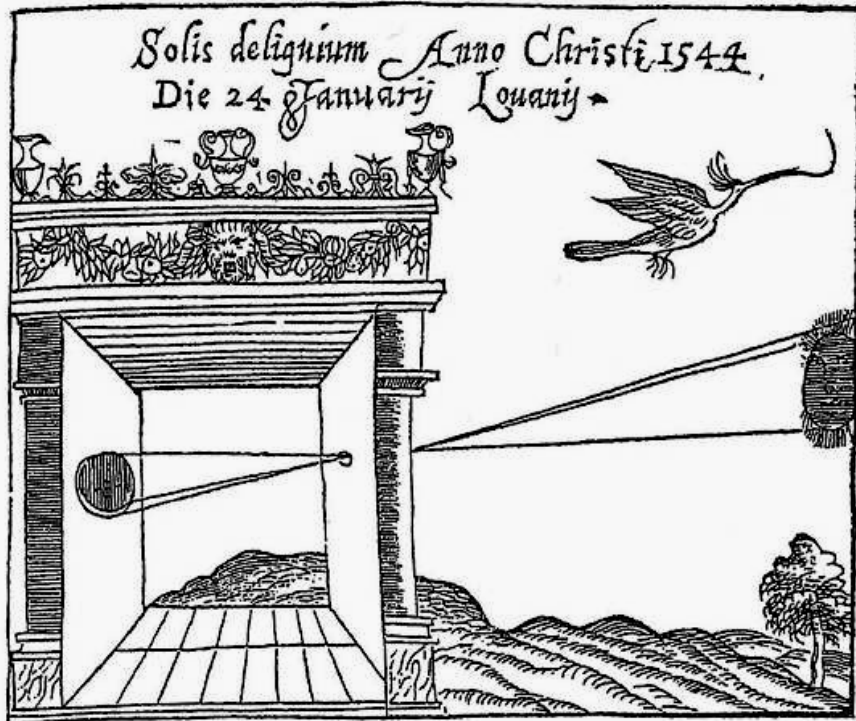


Film captures all the rays going through a point (a *pencil of rays*).

Result: good in theory!



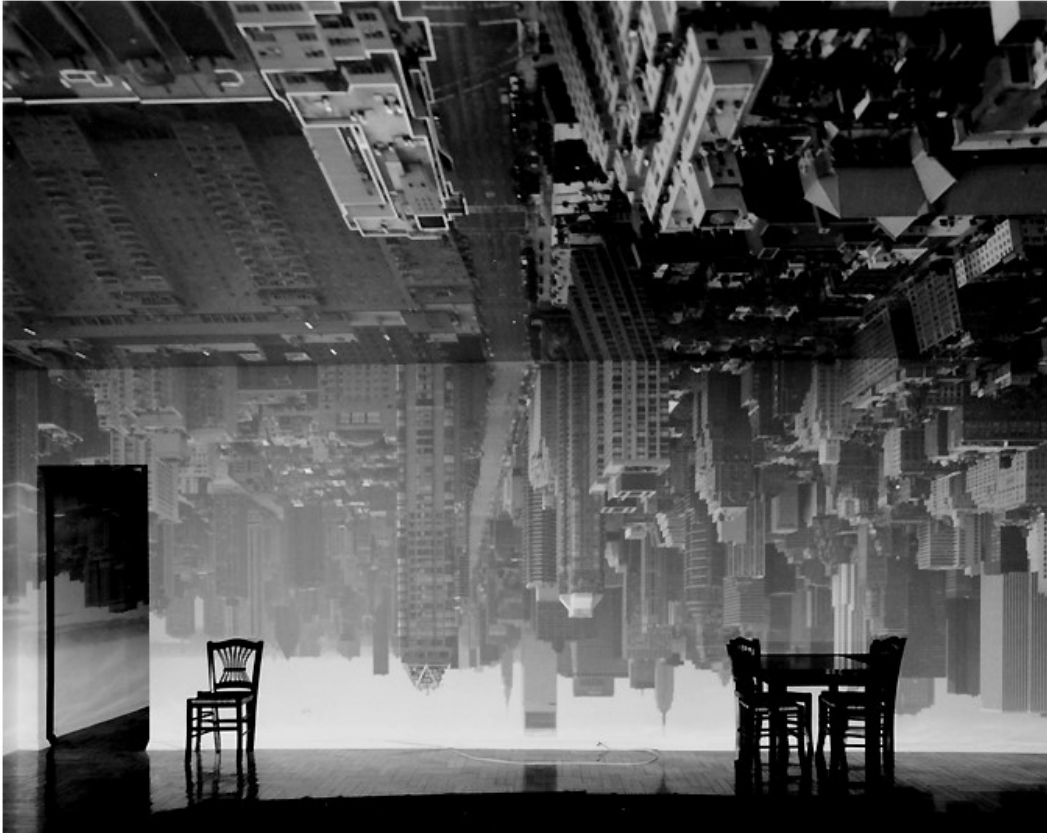
# Camera Obscura



Gemma Frisius, 1558

- Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)

# Camera Obscura



Abelardo Morell, Camera Obscura Image of Manhattan View Looking South in Large Room, 1996

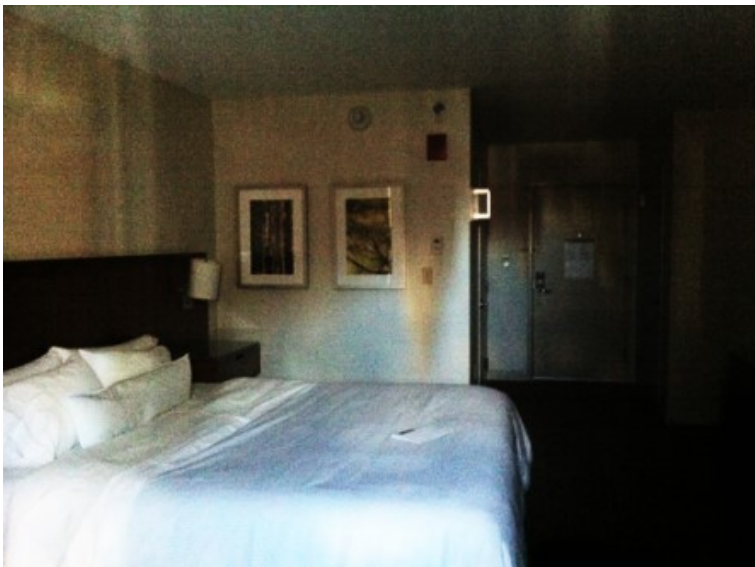
After scouting rooms and reserving one for at least a day, Morell masks the windows except for the aperture. He controls three elements: the size of the hole, with a smaller one yielding a sharper but dimmer image; the length of the exposure, usually eight hours; and the distance from the hole to the surface on which the outside image falls and which he will photograph. He used 4 x 5 and 8 x 10 view cameras and lenses ranging from 75 to 150 mm.

After he's done inside, it gets harder. "I leave the room and I am constantly checking the weather, I'm hoping the maid reads my note not to come in, I'm worrying that the sun will hit the plastic masking and it will fall down, or that I didn't trigger the lens."

From *Grand Images Through a Tiny Opening*, **Photo District News**, February 2005

# Camera Obscura

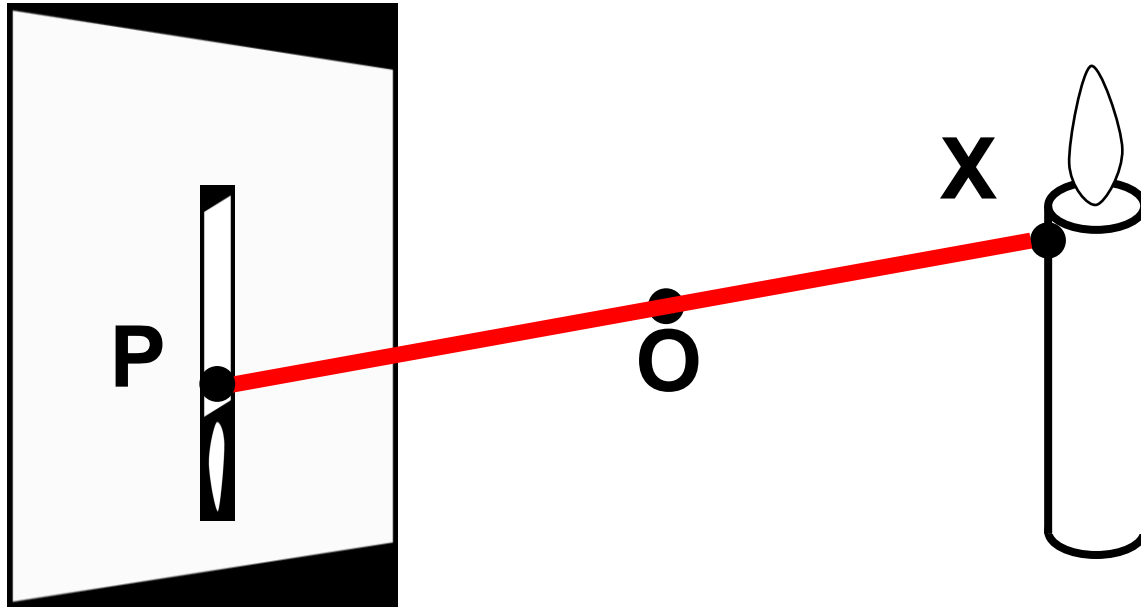
Hotel room  
contrast enhanced



View Out of  
Hotel Room Window



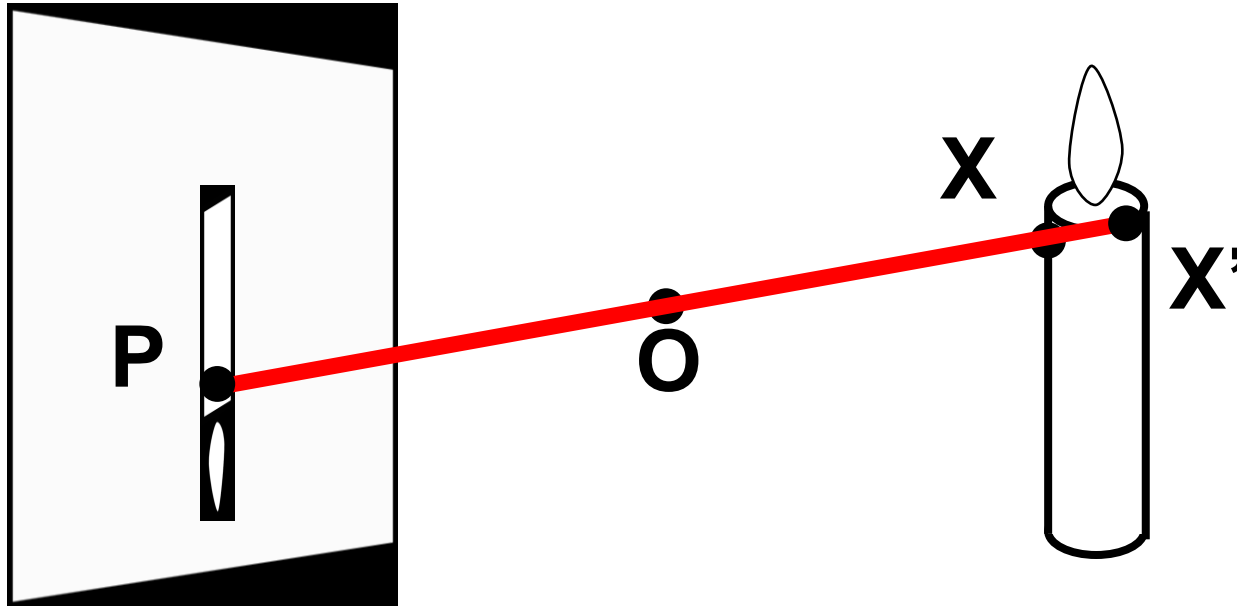
# Projection



**How do we find the projection P of a point X?**

Form visual ray from X to camera center and intersect it with camera plane

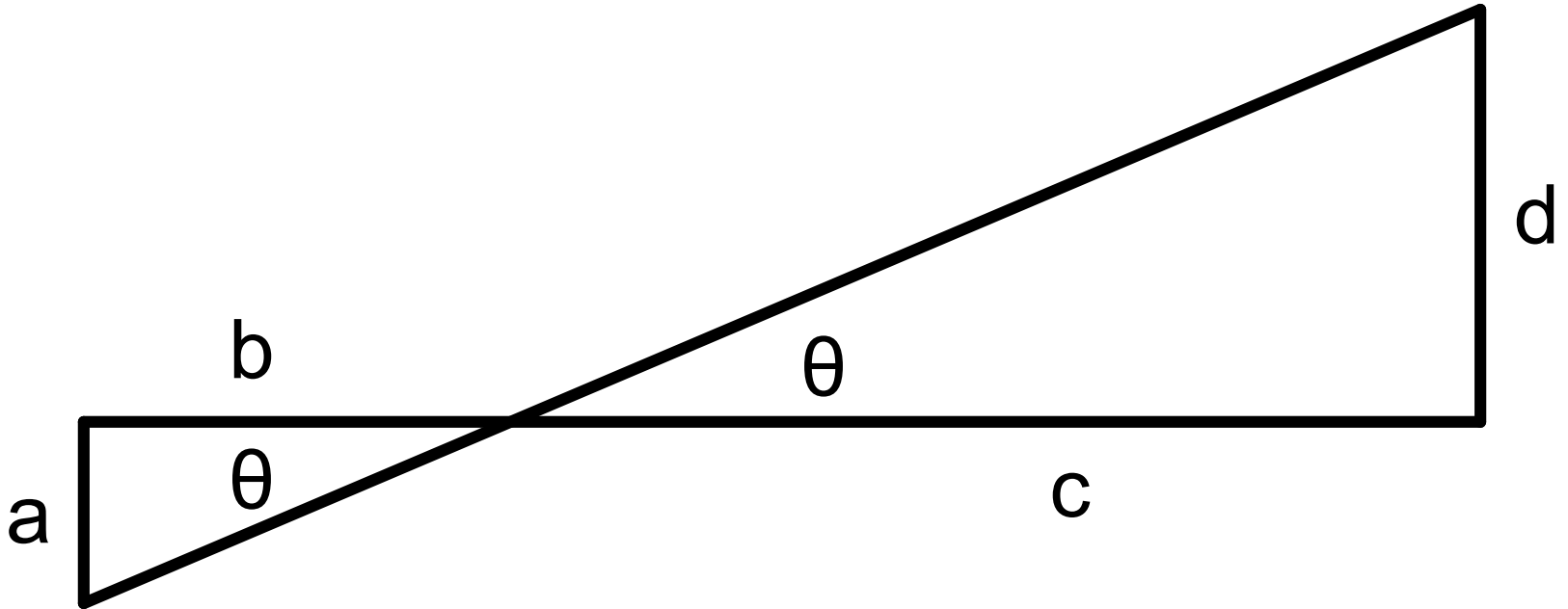
# Projection



**Both  $X$  and  $X'$  project to  $P$ . Which appears in the image?**

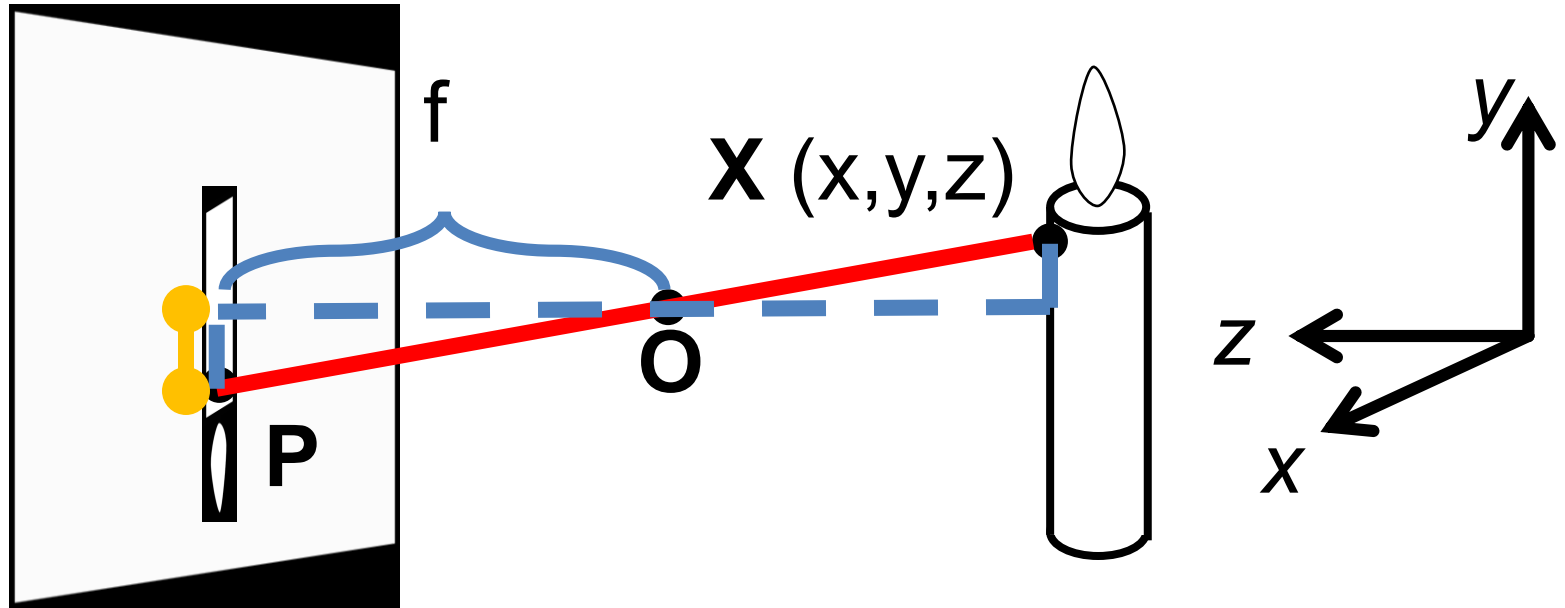
**Are there points for which projection is undefined?**

Quick Aside: Remember This?



$$\frac{a}{b} = \frac{d}{c} \longrightarrow a = \frac{bd}{c}$$

# Projection Equations



Coordinate system:  $O$  is origin,  $XY$  in image,  $Z$  sticks out.  
 $XY$  is image plane,  $Z$  is optical axis.

$(x, y, z)$  projects to  $(fx/z, fy/z)$  via similar triangles

# Some Facts About Projection



3D lines project to 2D lines

The projection of any 3D parallel lines converge at a vanishing point

Distant objects are smaller

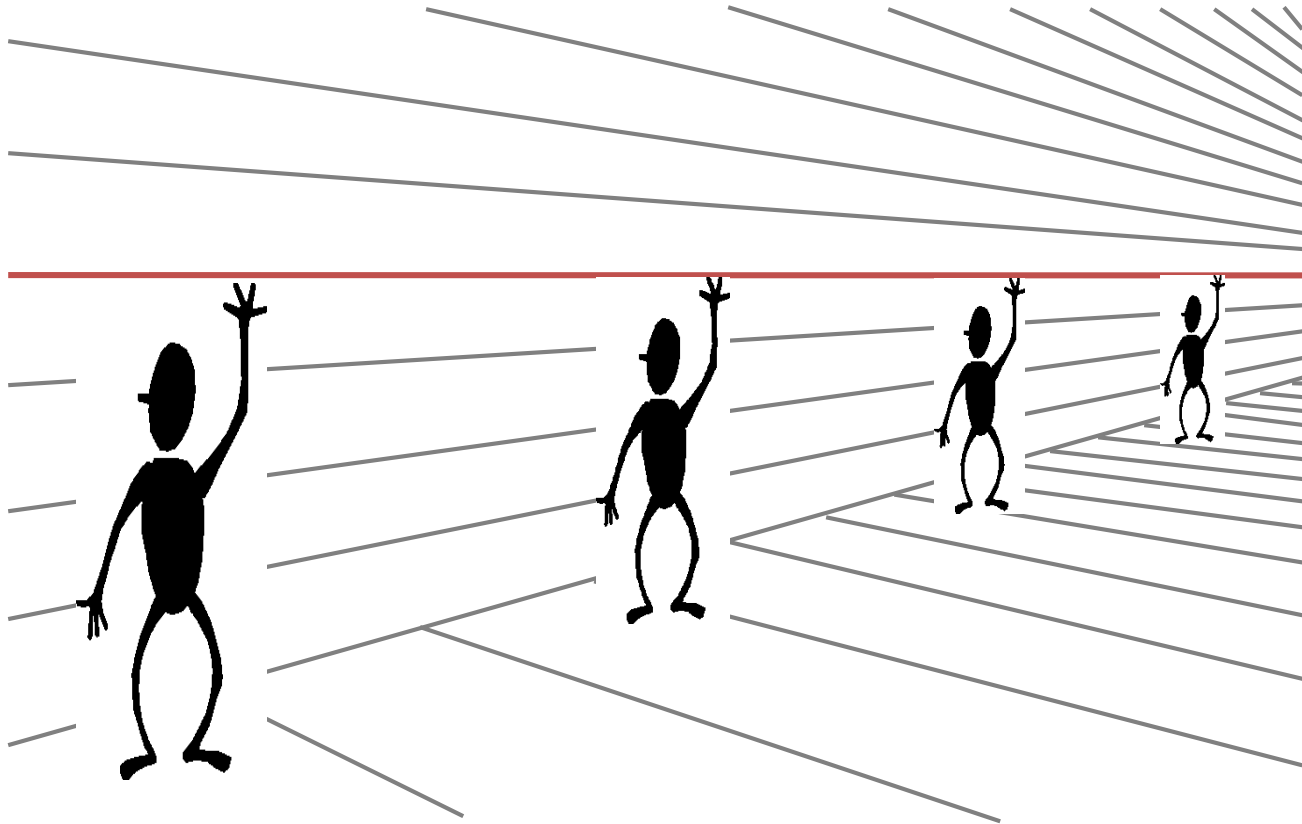




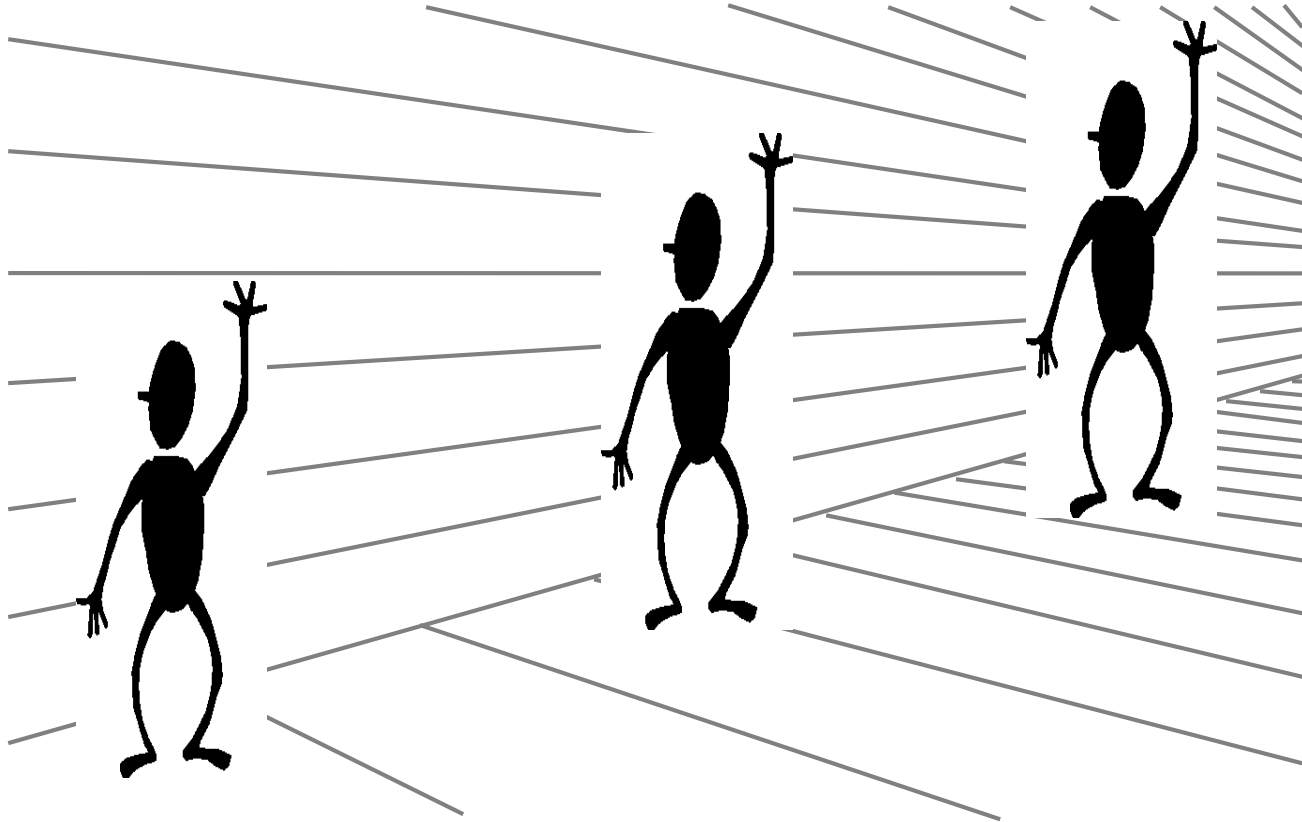
# Some Facts About Projection

Let's try some fake images

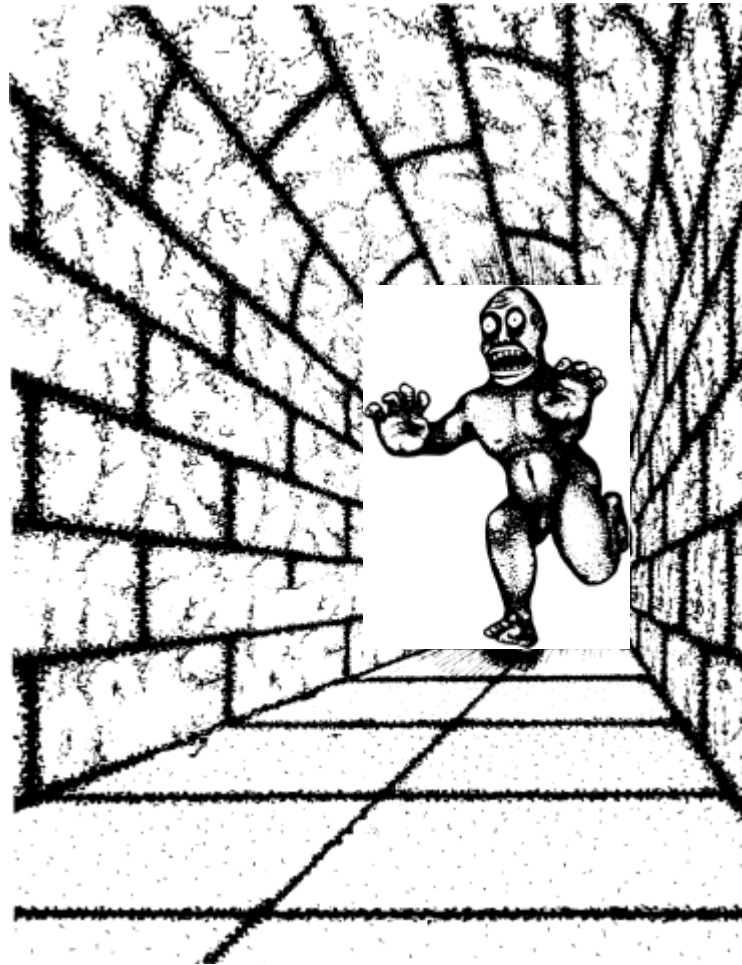
# Some Facts About Projection



# Some Facts About Projection



# Some Facts About Projection



Illusion Credit: RN Shepard, Mind Sights: Original Visual Illusions, Ambiguities, and other Anomalies

# What's Lost?



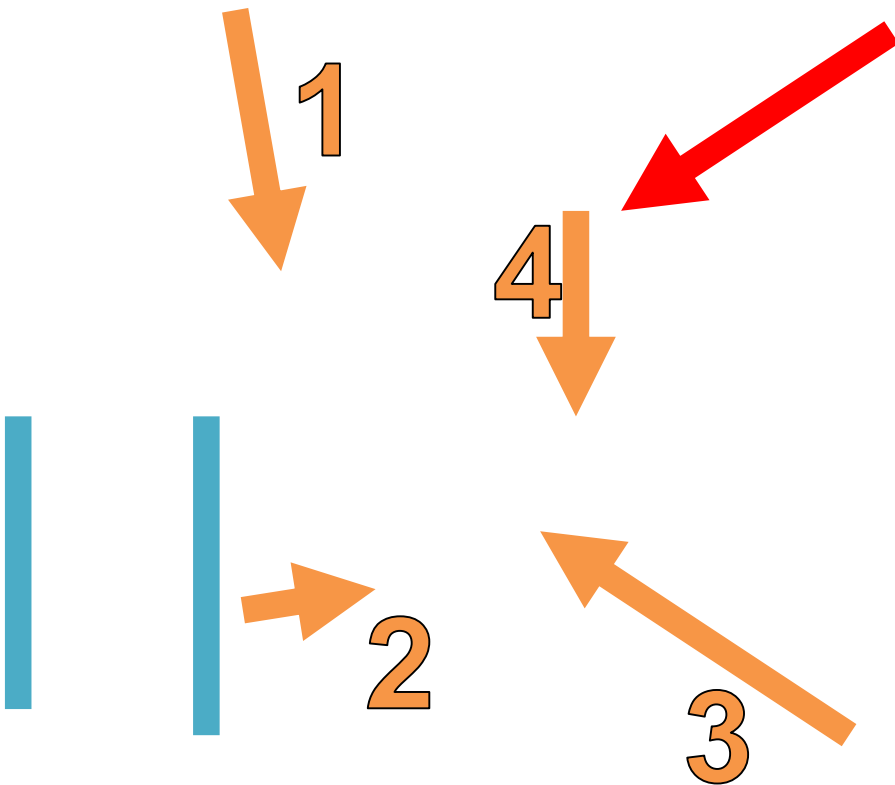
Is she shorter or further away?

Are the orange lines we see parallel / perpendicular / neither to the red line?

# What's Lost?

Is she shorter or further away?

Are the **orange lines** we see parallel / perpendicular / neither to the **red line**?

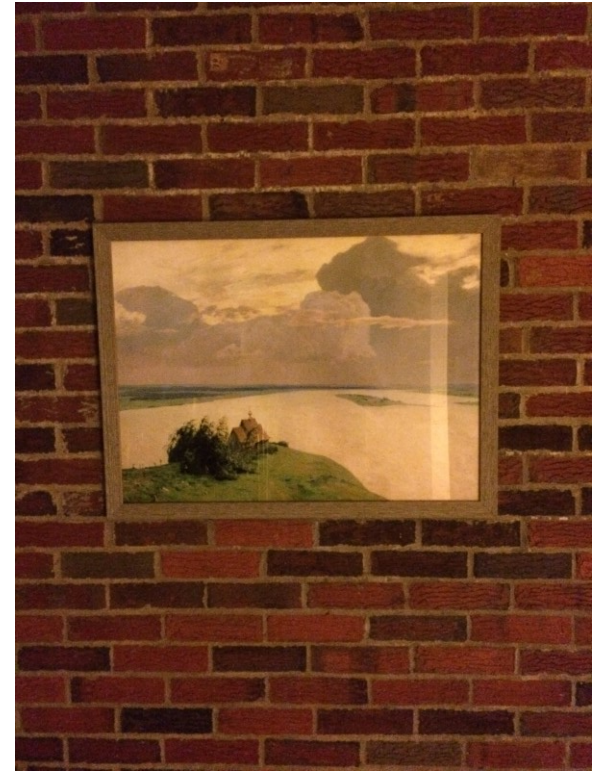


# What's Lost?

Be careful of drawing conclusions:

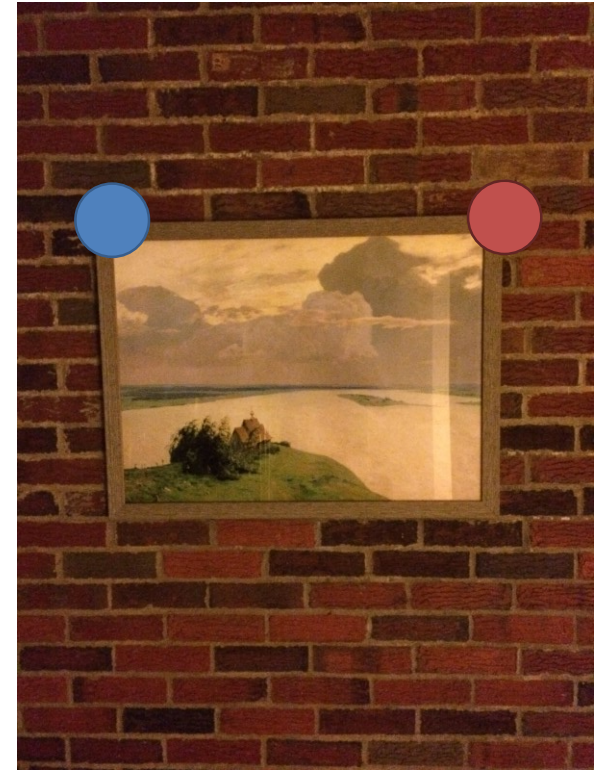
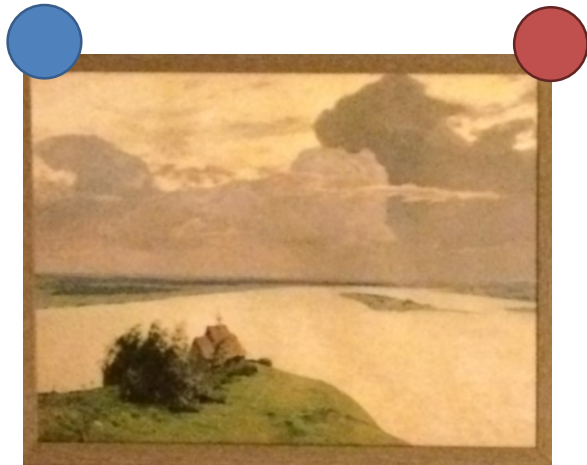
- Projection of 3D line is 2D line; NOT 2D line is 3D line.
- **Can you think of a counter-example (a 2D line that is not a 3D line)?**
- Projections of parallel 3D lines converge at VP; NOT any pair of lines that converge are parallel in 3D.
- **Can you think of a counter-example?**

# Do You Always Get Perspective?





# Do You Always Get Perspective?



Y location of  
blue and red  
dots in image:

$$\frac{fy}{z_2}$$

$$\frac{fy}{z_1}$$

$$\frac{fy}{z}$$

$$\frac{fy}{z}$$

# Do You Always Get Perspective?



When plane is fronto-parallel  
(parallel to camera plane),  
everything is:

- scaled by  $f/z$
- otherwise is preserved.



# What's This Useful For?



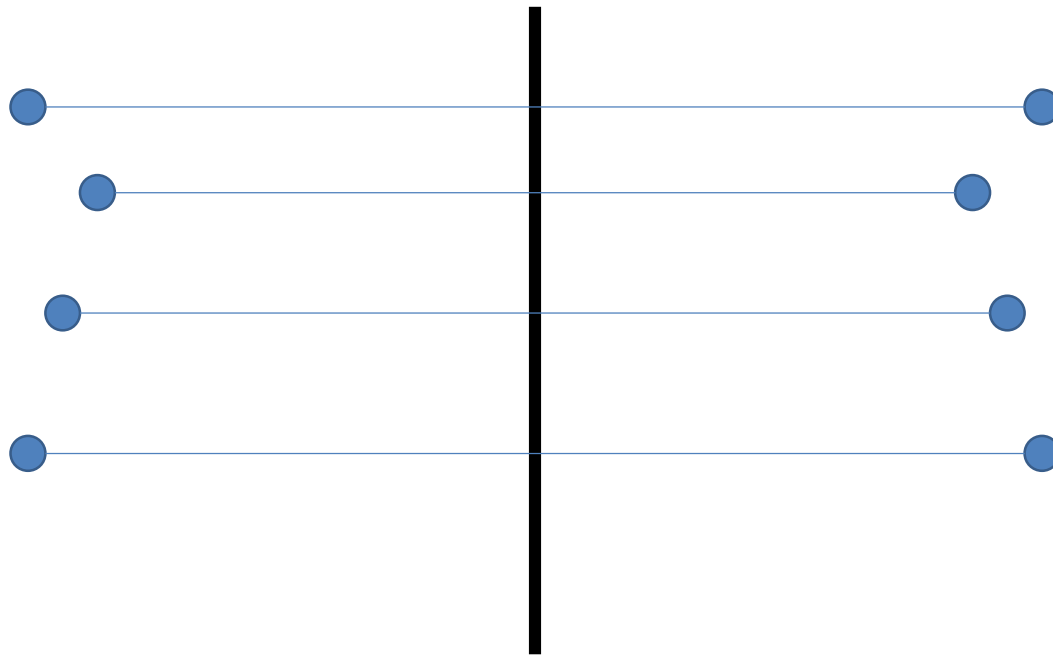
Things looking different when viewed from different angles seems like a nuisance. It's also a cue. **Why?**



# What's This Useful For?

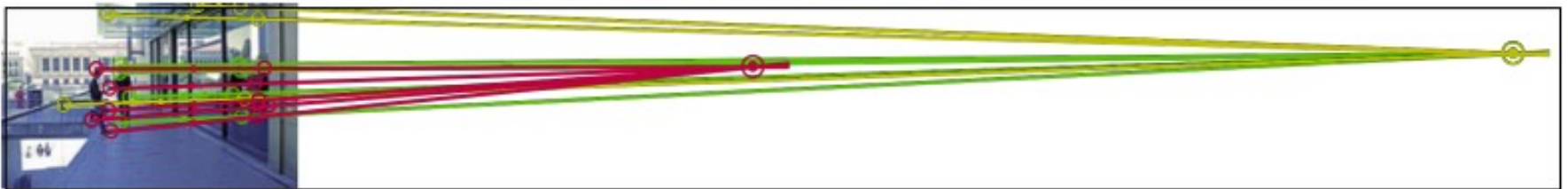
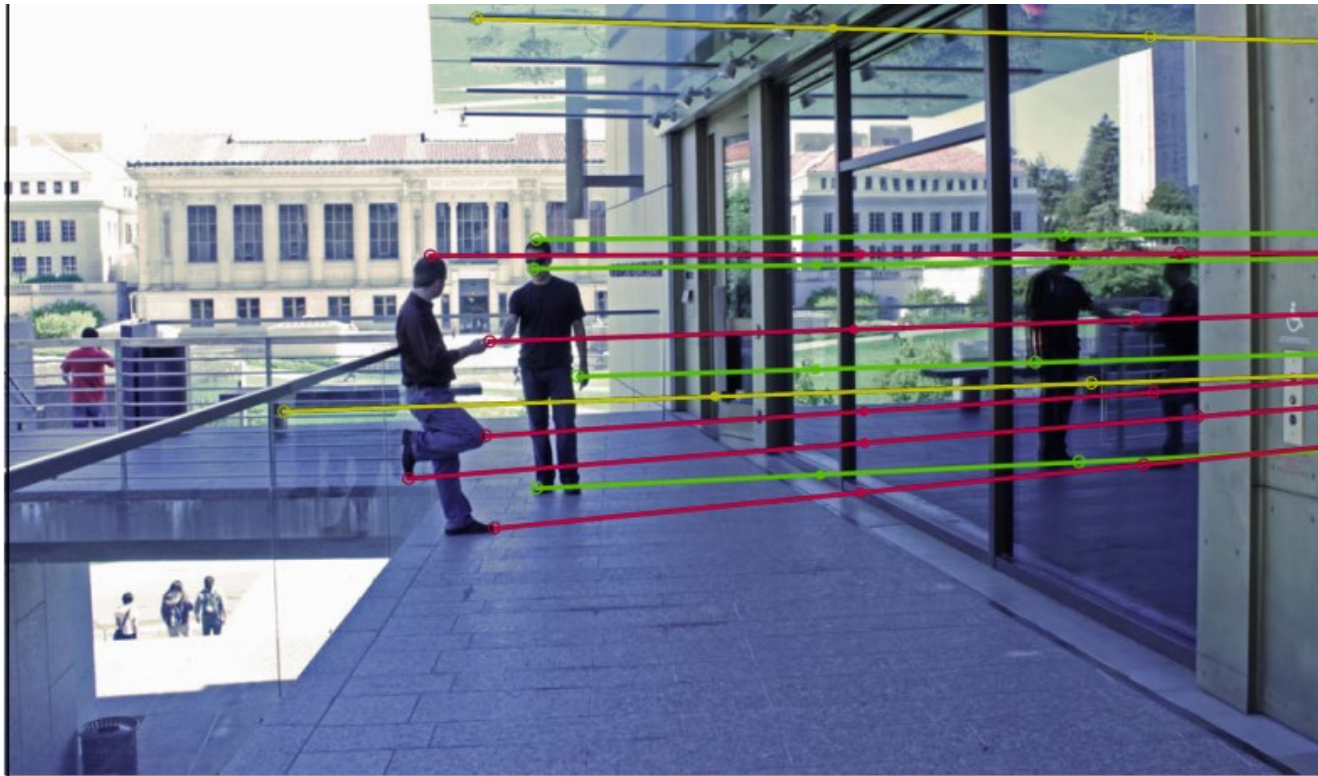


# What's This Useful For?

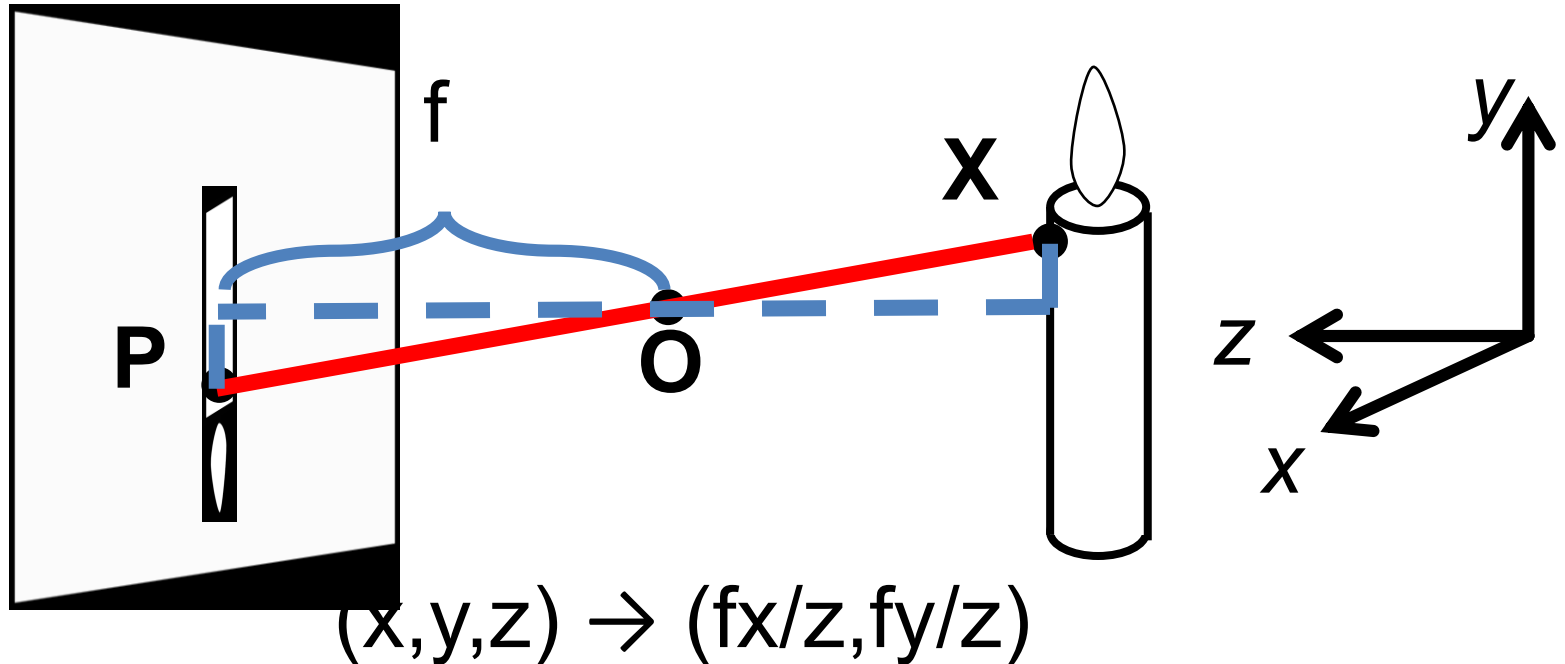


**Mirror**

# What's This Useful For?



# Projection Equation



**I promised you linear algebra: is this linear?**

**Nope:** division by  $z$  is non-linear  
(and risks division by 0)

# Homogeneous Coordinates (2D)

Trick: add a dimension!

*This also clears up lots of nasty special cases*

Physical  
Point

$$\begin{bmatrix} x \\ y \end{bmatrix}$$



Concat  
 $w=1$

Homogeneous  
Point

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix}$$



Divide  
by  $w$

Physical  
Point

$$\begin{bmatrix} u/w \\ v/w \end{bmatrix}$$

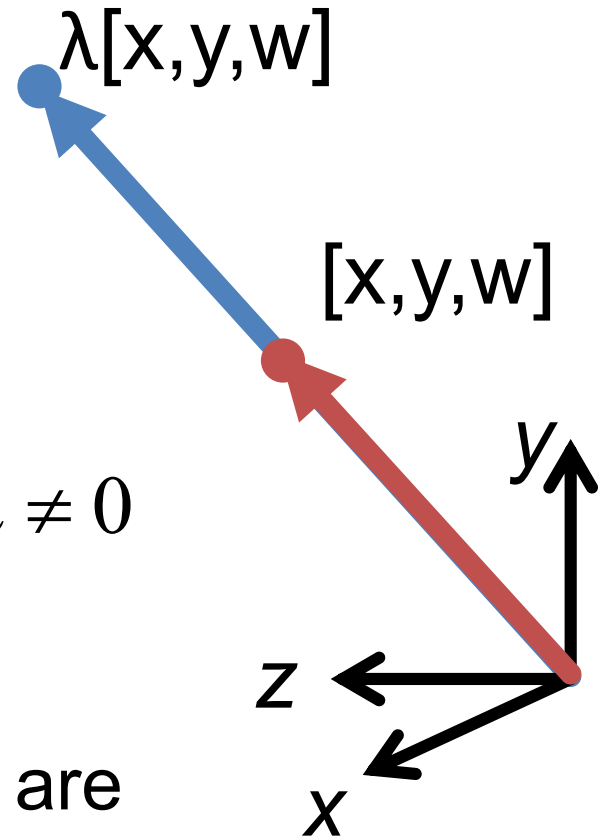
**What if  $w = 0$ ?**



# Homogeneous Coordinates

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \equiv \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} \Leftrightarrow \begin{bmatrix} u \\ v \\ w \end{bmatrix} \equiv \lambda \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} \quad \lambda \neq 0$$

Two homogeneous coordinates are **equivalent** if they are proportional to each other. **Not = !**



# Benefits of Homogeneous Coords

General equation of 2D line:

$$ax + by + c = 0$$

Homogeneous Coordinates

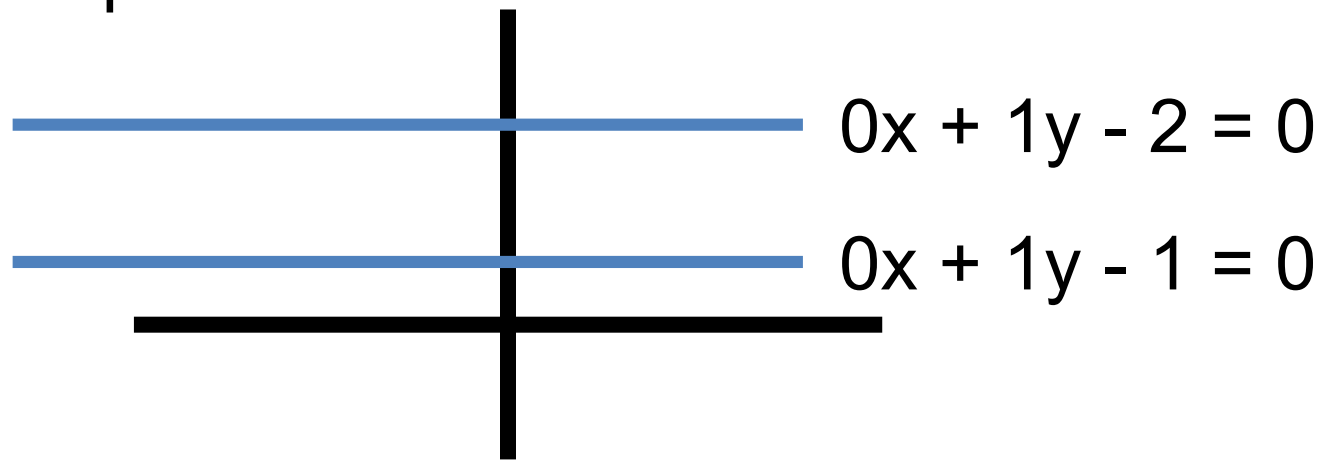
$$l^T p = 0, \quad l = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad p = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Benefits of Homogeneous Coords

- Lines (3D) and points (2D  $\rightarrow$  3D) are now the same dimension.
- Use the cross ( $\times$ ) and dot product for:
  - Intersection of lines  $\mathbf{l}$  and  $\mathbf{m}$ :  $\mathbf{l} \times \mathbf{m}$
  - Line through two points  $\mathbf{p}$  and  $\mathbf{q}$ :  $\mathbf{p} \times \mathbf{q}$
  - Point  $\mathbf{p}$  on line  $\mathbf{l}$ :  $\mathbf{l}^T \mathbf{p}$
- **But what about parallel lines?**

# Benefits of Homogeneous Coords

What about parallel lines?

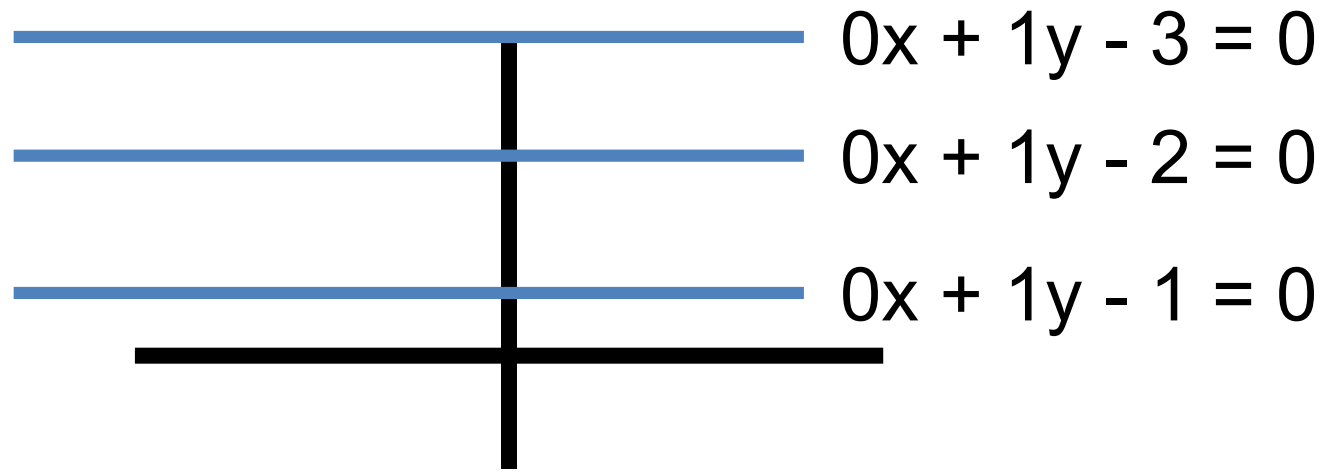


$$[0, 1-2] \times [0, 1, -1] = [1, 0, 0]$$

Any point  $[x, y, 0]$  is at infinity

All operations generate valid results!

# Benefits of Homogeneous Coords



Intersection of  $y=2$ ,  $y=1$

$$[0, 1, -2] \times [0, 1, -1] = [1, 0, 0]$$

**Does it lie on  $y=3$ ? Intuitively?**

$$[0, 1, -3]^T [1, 0, 0] = 0$$

# 3D Homogeneous Coordinates

- Same story: add a coordinate, things are equivalent if they're proportional

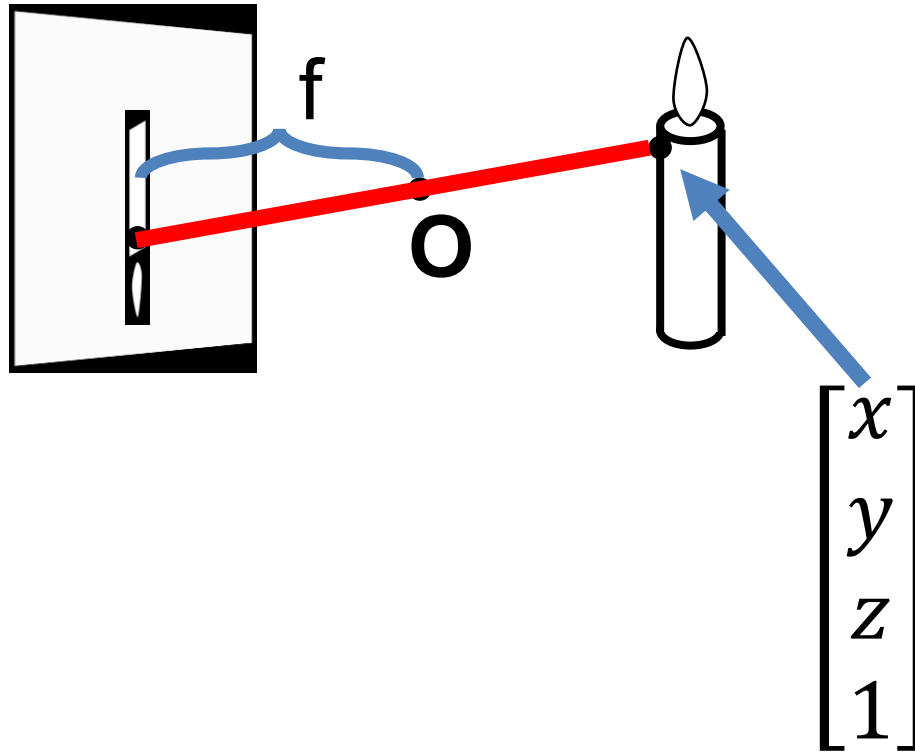
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \longrightarrow \begin{bmatrix} u \\ v \\ w \\ t \end{bmatrix} \longrightarrow \begin{bmatrix} u/t \\ v/t \\ w/t \end{bmatrix}$$

# 3D Homogeneous Coordinates

	Vector Coordinates	Homogeneous Coordinates
Translation	$\mathbf{y} = \mathbf{x} + \mathbf{t}$	$\mathbf{y} = \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{t} \\ 0 & 1 \end{bmatrix} \mathbf{x}$
Rotation	$\mathbf{y} = \mathbf{R}\mathbf{x}$ $\mathbf{R}^T \mathbf{R} = \mathbf{R}\mathbf{R}^T = \mathbf{I}$ $\det(\mathbf{R}) = 1$	$\mathbf{y} = \begin{bmatrix} \mathbf{R}_{3 \times 3} & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x}$
Rigid Body	$\mathbf{y} = \mathbf{R}\mathbf{x} + \mathbf{t}$	$\mathbf{y} = \begin{bmatrix} \mathbf{R}_{3 \times 3} & \mathbf{t} \\ 0 & 1 \end{bmatrix} \mathbf{x}$
Affine	$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{t}$	$\mathbf{y} = \begin{bmatrix} \mathbf{A}_{3 \times 3} & \mathbf{t} \\ 0 & 1 \end{bmatrix} \mathbf{x}$

# Projection Matrix

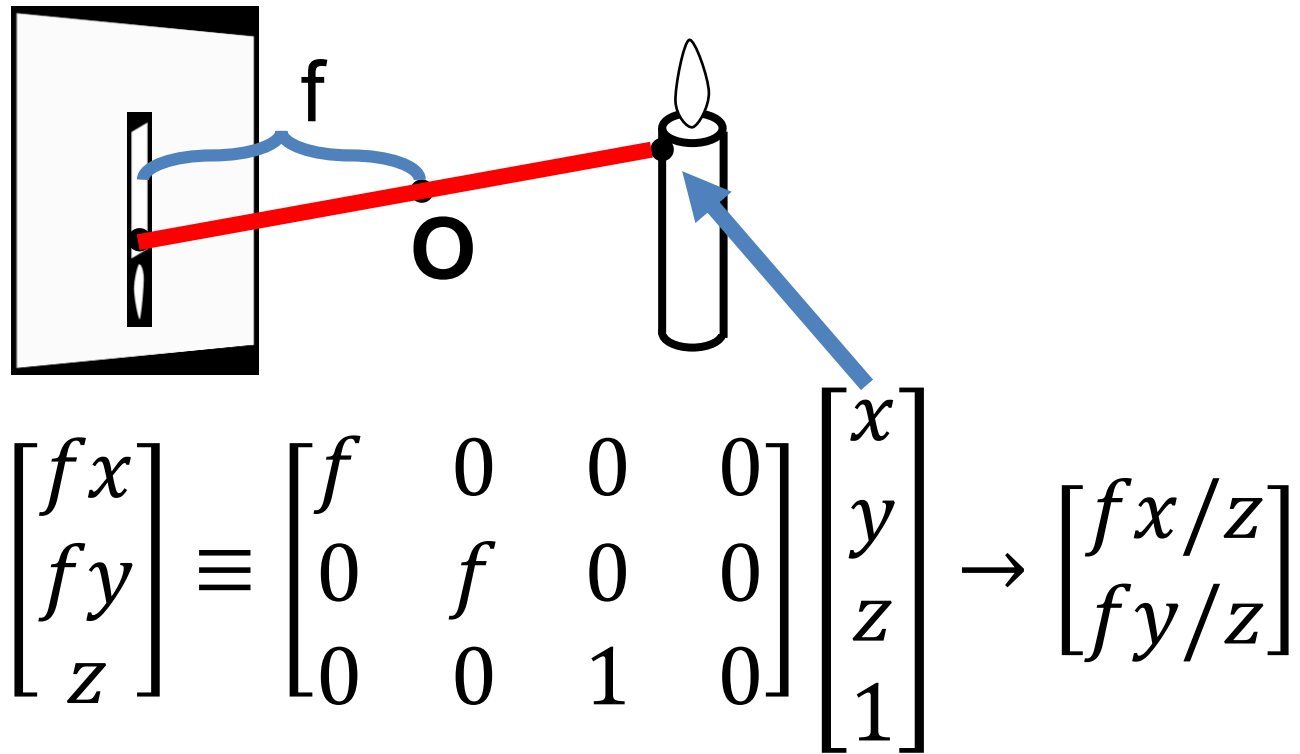
Projection  $(f_x/z, f_y/z)$  is matrix multiplication



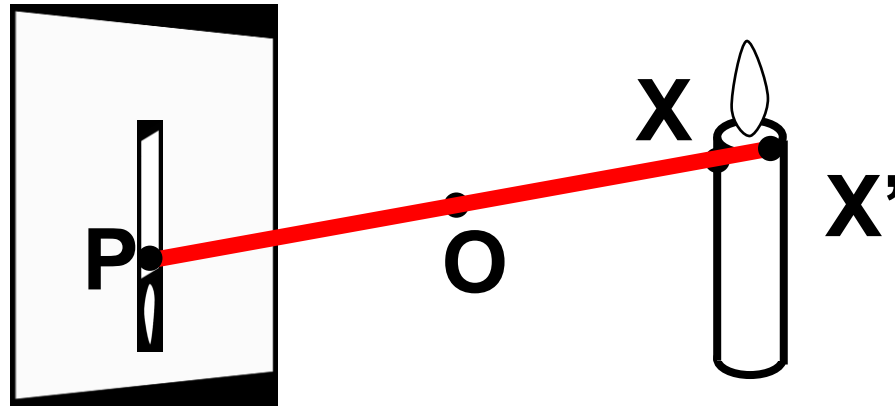


# Projection Matrix

Projection  $(fx/z, fy/z)$  is matrix multiplication



Why  $\equiv \neq =$



Project X and X' to the image and compare them

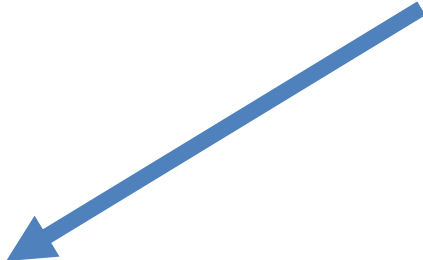
**YES**  $\begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \equiv \begin{bmatrix} fx' \\ fy' \\ z' \end{bmatrix}$

**NO**  $\begin{bmatrix} fx \\ fy \\ z \end{bmatrix} = \begin{bmatrix} fx' \\ fy' \\ z' \end{bmatrix}$

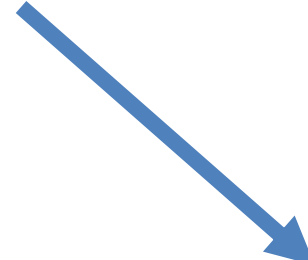
# Standard Full Perspective Model

**P**: 2D homogeneous  
point (3D)

**X**: 3d homogeneous  
point (4D)



***P*** ≡



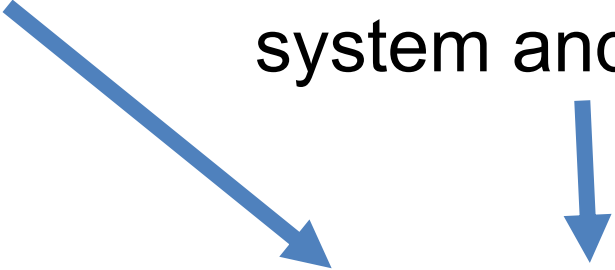
***X***<sub>4x1</sub>

# Standard Full Perspective Model

**R**: rotation between world system and camera

**t**: translation between world system and camera

$P \equiv$

$$[R_{3 \times 3} \quad t_{3 \times 1}] X_{4 \times 1}$$


# Standard Full Perspective Model

$\alpha$  scale between world  
and image x coords

$\theta$  skew of camera axes

$u_0, v_0$ : principal  
point (image coords  
of camera origin on  
retina)

$$\mathbf{P} \equiv \begin{bmatrix} \alpha & -\alpha \cot \theta & u_0 \\ 0 & \frac{\beta}{\sin \theta} & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\mathbf{R}_{3 \times 3} \quad \mathbf{t}_{3 \times 1}] \quad \mathbf{X}_{4 \times 1}$$

$\beta$  scale between world  
and image y coords

# Standard Full Perspective Model

3x1



$\mathbf{P}$

$$\equiv \begin{bmatrix} \alpha & -\alpha \cot \theta & u_0 \\ 0 & \frac{\beta}{\sin \theta} & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

3x3



$[\mathbf{R}_{3 \times 3}$

3x4



$\mathbf{t}_{3 \times 1}]$

4x1



$\mathbf{X}_{4 \times 1}$

# Typical Perspective Model

f focal length

$u_0, v_0$ : principal point (image coords of camera origin on retina)

$$\mathbf{P} \equiv \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\mathbf{R}_{3 \times 3} \quad \mathbf{t}_{3 \times 1}] \quad \mathbf{X}_{4 \times 1}$$

# Typical Perspective Model

**Intrinsic  
Matrix  $K$**

**Extrinsic  
Matrix  $[R, t]$**

$$P \equiv \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{3 \times 3} & t_{3 \times 1} \end{bmatrix} X_{4 \times 1}$$

$$P \equiv K[R, t]X \equiv M_{3 \times 4}X_{4 \times 1}$$



# Other Cameras – Orthographic

Orthographic Camera (z infinite)

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{X}_{3 \times 1}$$



# Other Cameras – Orthographic

Why does this make things easy and why is this popular in old games?

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

# Cameras

EECS 442 – Prof. David Fouhey  
Winter 2019, University of Michigan

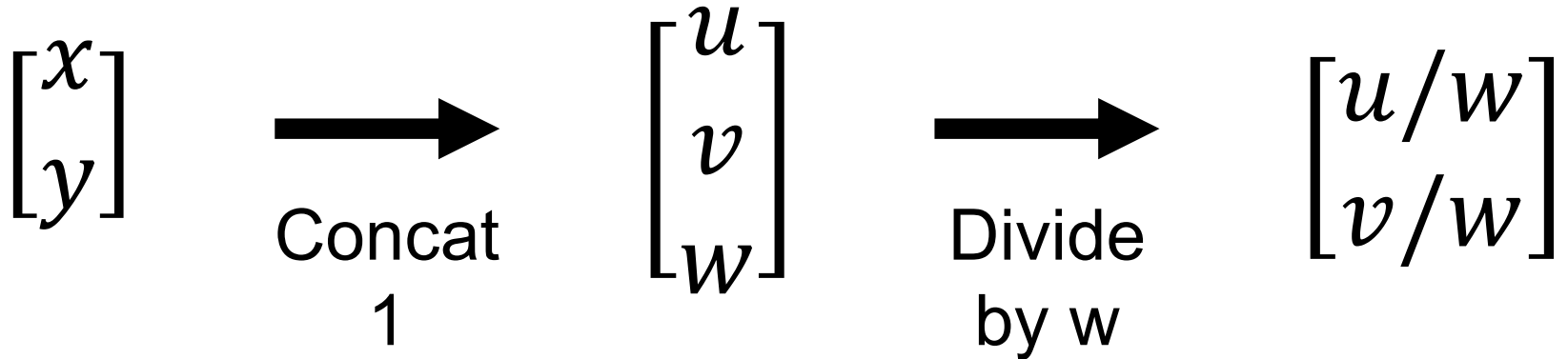
[http://web.eecs.umich.edu/~fouhey/teaching/EECS442\\_W19/](http://web.eecs.umich.edu/~fouhey/teaching/EECS442_W19/)

# Recap – Homog. Coordinates

Normal

Homogeneous

Normal



**$w = 0 \Rightarrow$  point is at infinity**

$$x \equiv y$$

$$\exists \lambda \neq 0 \ x = \lambda y$$

$$x = y$$

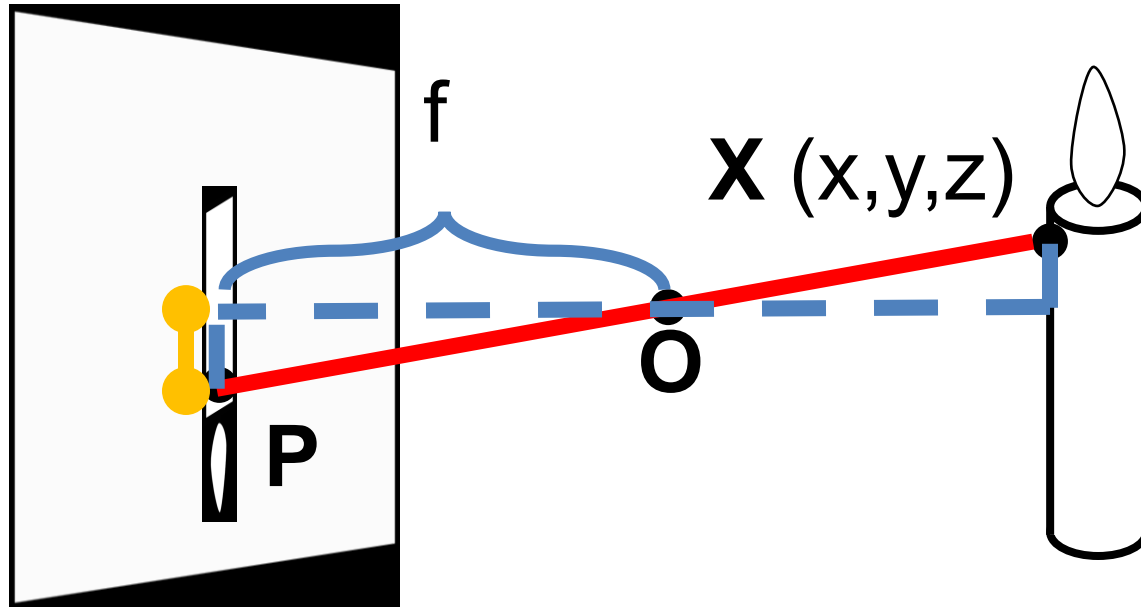
**YES!**

**NO!**

# Recap – Homog. Coordinates

- Lots of transformations become linear:
  - Rotation, translation, affine transformations are now all matrix multiplications
- Lots of special cases go away (useful for both computation *and proofs*):
  - Try defining two vertical lines
  - Try finding their intersection

# Recap – Projection



$$(x,y,z) \rightarrow (fx/z, fy/z)$$

*Your location in the image depends on depth!*

# Recap – Projection

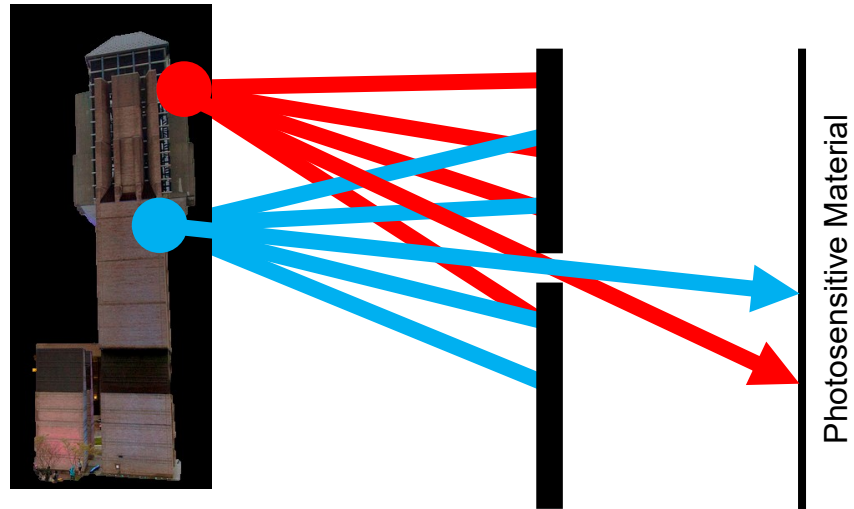
**f**: focal length      **u0, v0**: principal point (image coords of camera origin on retina)

**R**: camera rotation

**t**: camera translation

$$\mathbf{P}_{3 \times 1} \equiv \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_{3 \times 3} & \mathbf{t}_{3 \times 1} \end{bmatrix} \mathbf{X}_{4 \times 1}$$

# The Big Issue



Film captures all the rays going through a **point** (a *pencil of rays*).

**How big is a point?**



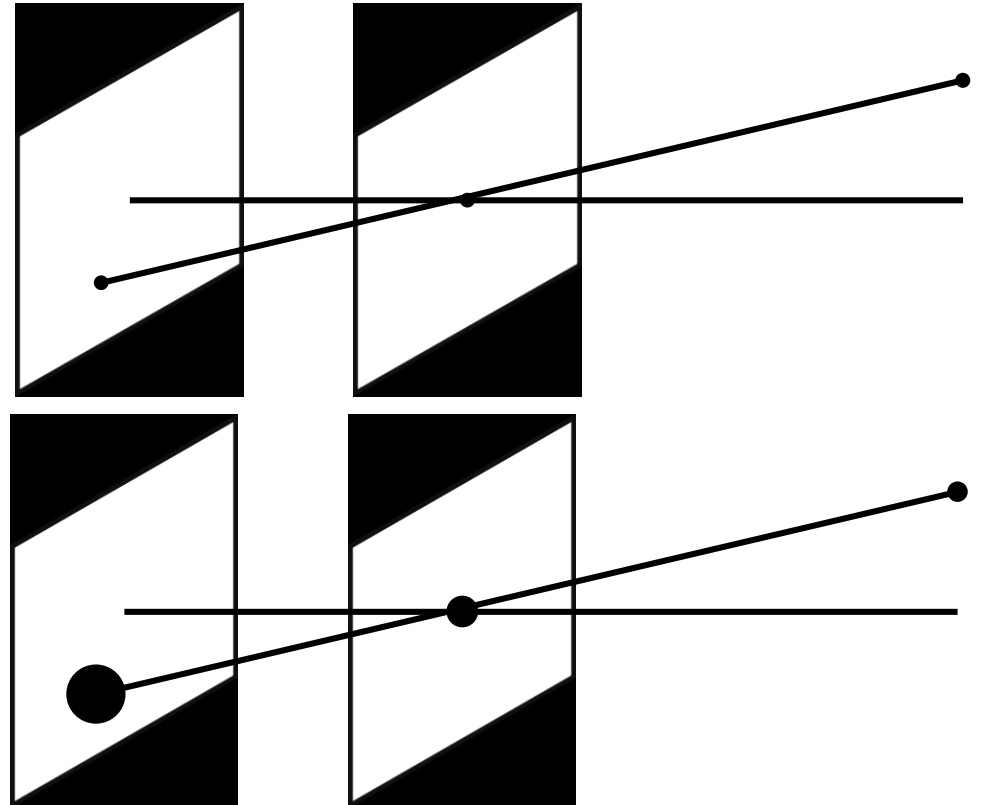
# Math vs. Reality

- Math: Any point projects to one point
- Reality (as pointed out by the class)
  - Don't image points behind the camera / objects
  - Don't have an infinite amount of sensor material
- Other issues
  - Light is limited
  - Spooky stuff happens with infinitely small holes

# Limitations of Pinhole Model

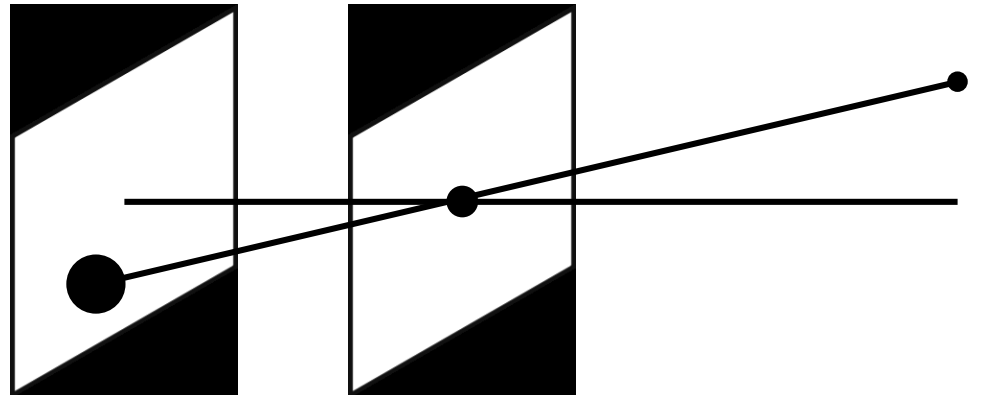
## Ideal Pinhole

- 1 point generates 1 image
- Low-light levels**

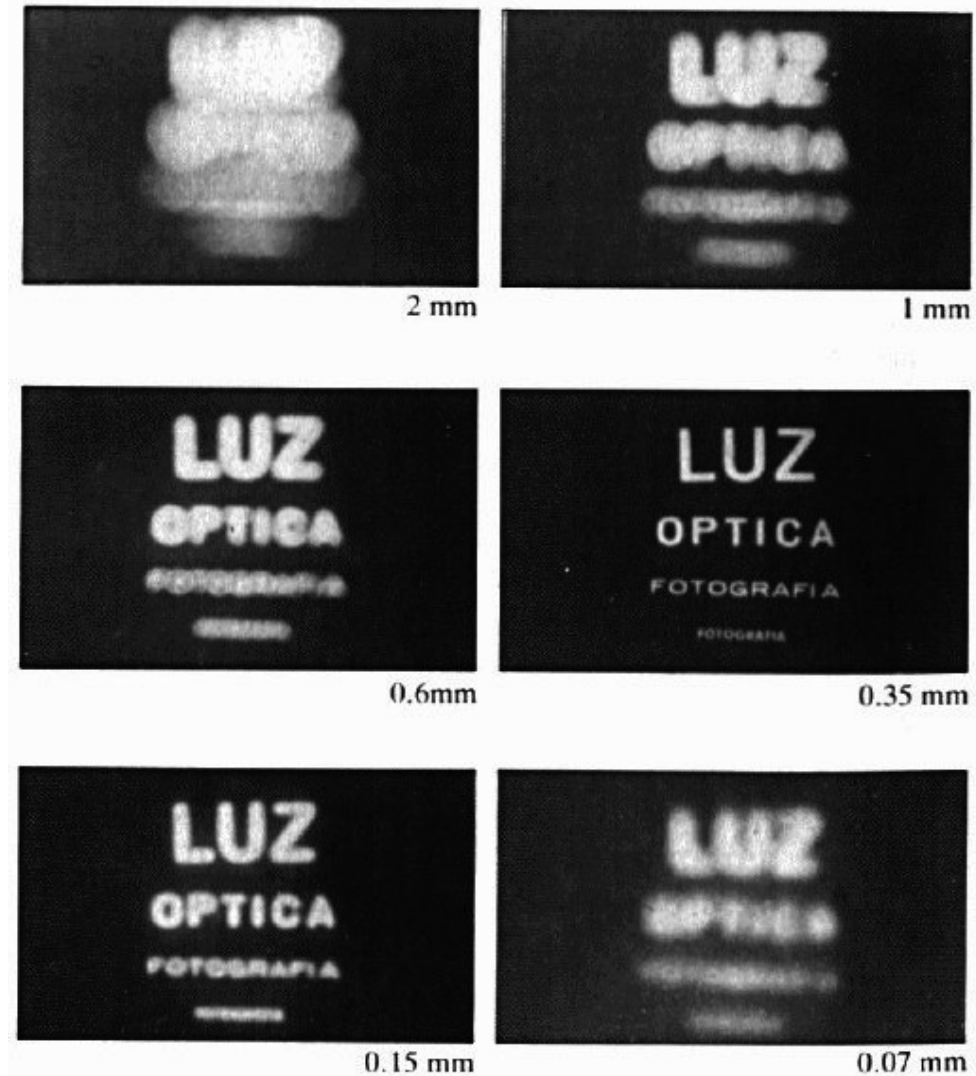


## Finite Pinhole

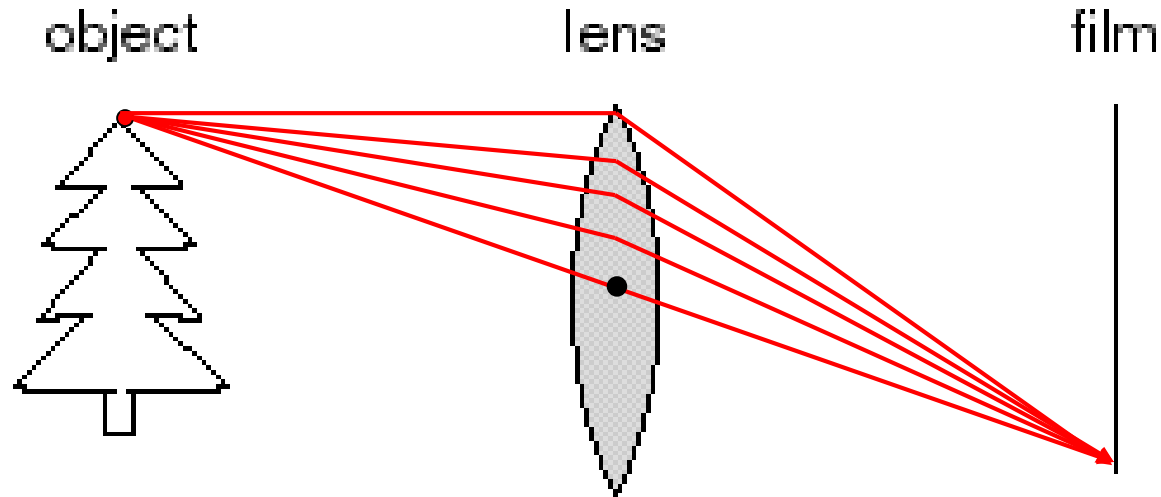
- 1 point generates region
  - Blurry.**
- Why is it blurry?**



# Limitations of Pinhole Model

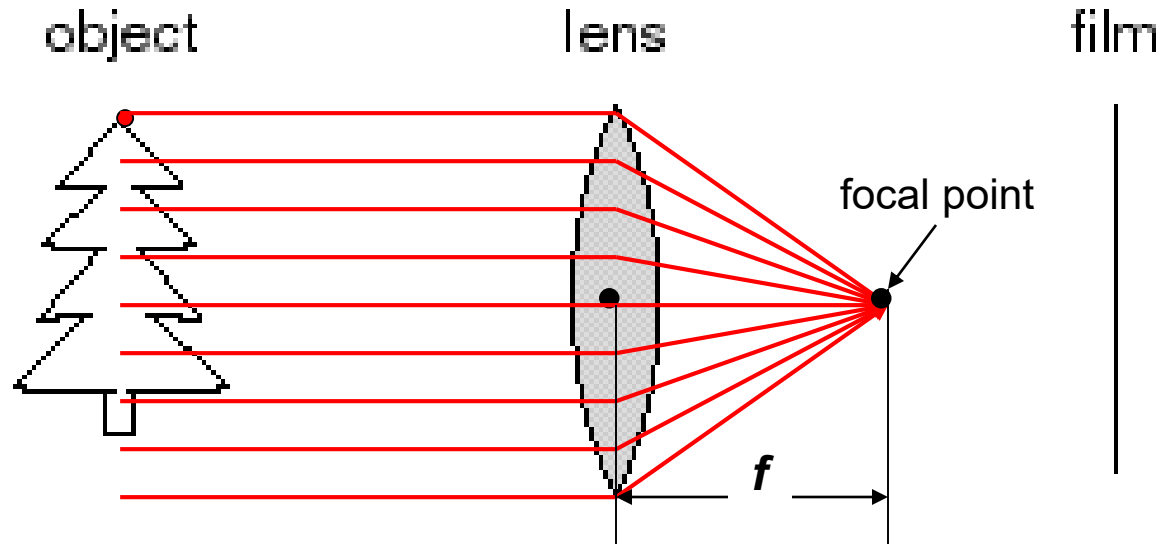


# Adding a Lens



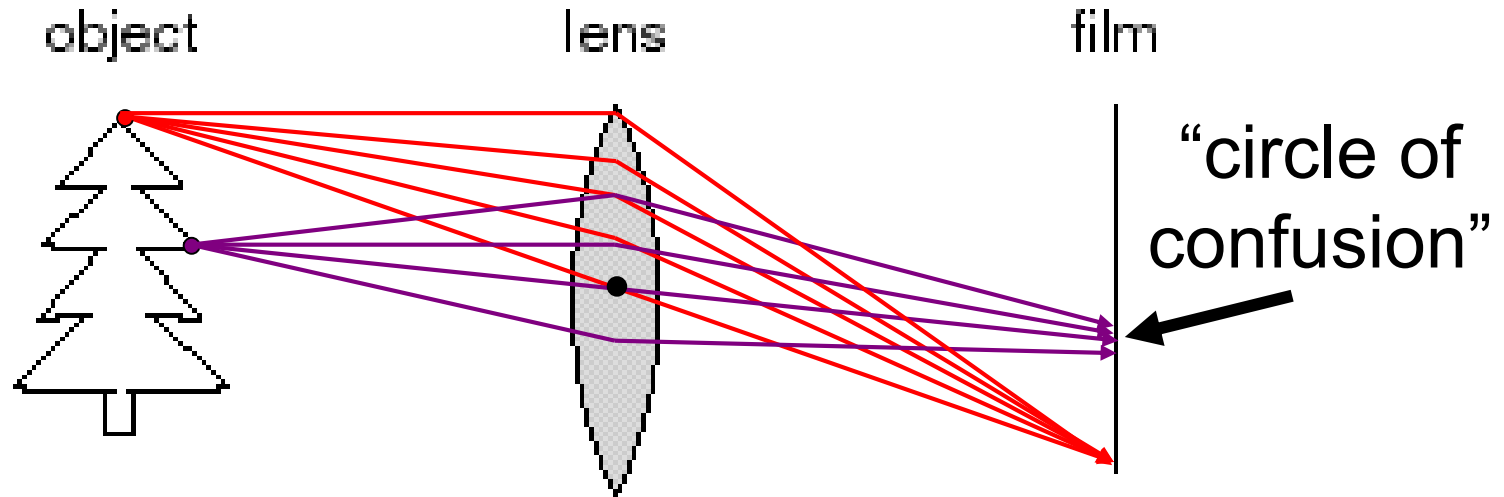
- A lens focuses light onto the film
- Thin lens model: rays passing through the center are not deviated (pinhole projection model still holds)

# Adding a Lens



- All rays parallel to the optical axis pass through the *focal point*

# What's The Catch?



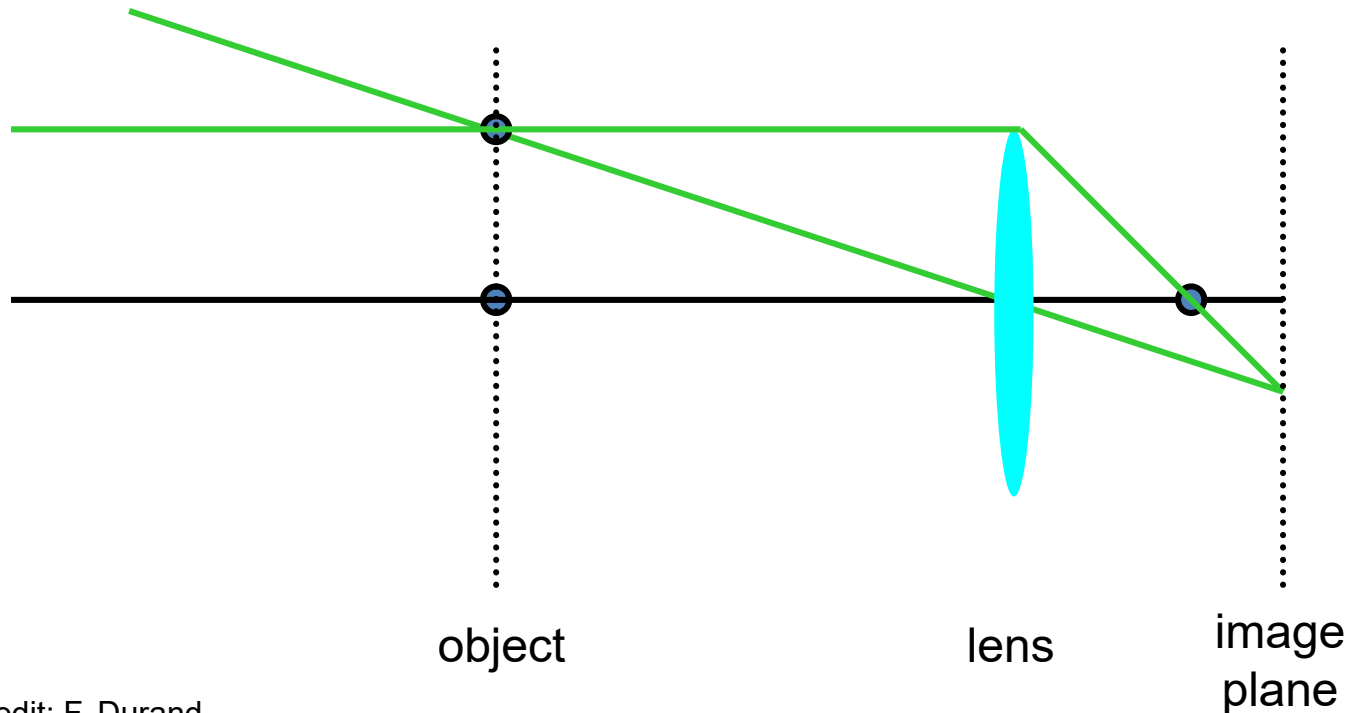
- There's a distance where objects are "in focus"
- Other points project to a "circle of confusion"

# Thin Lens Formula

We care about images that are in focus.

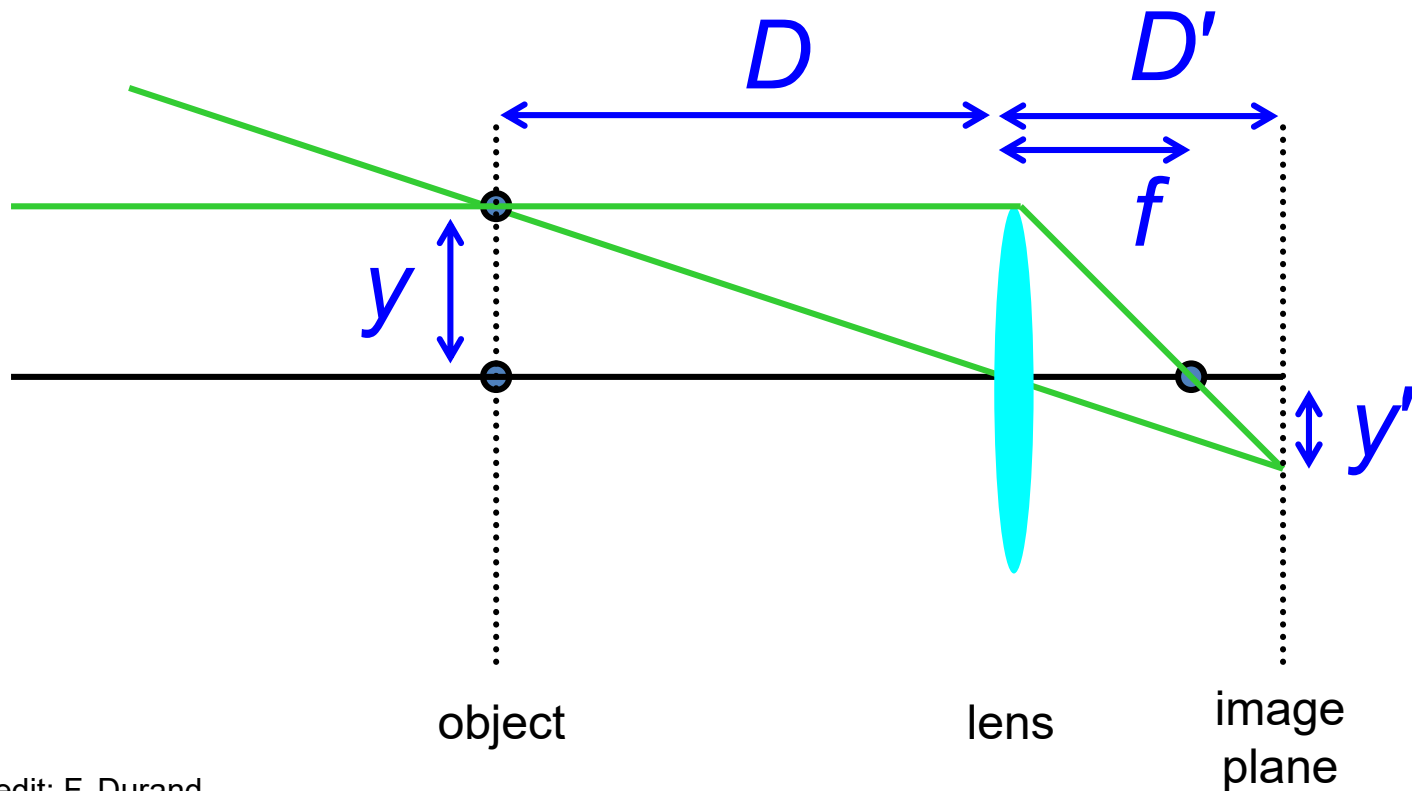
**When is this true? Discuss with your neighbor.**

When two paths from a point hit the same image location.



# Thin Lens Formula

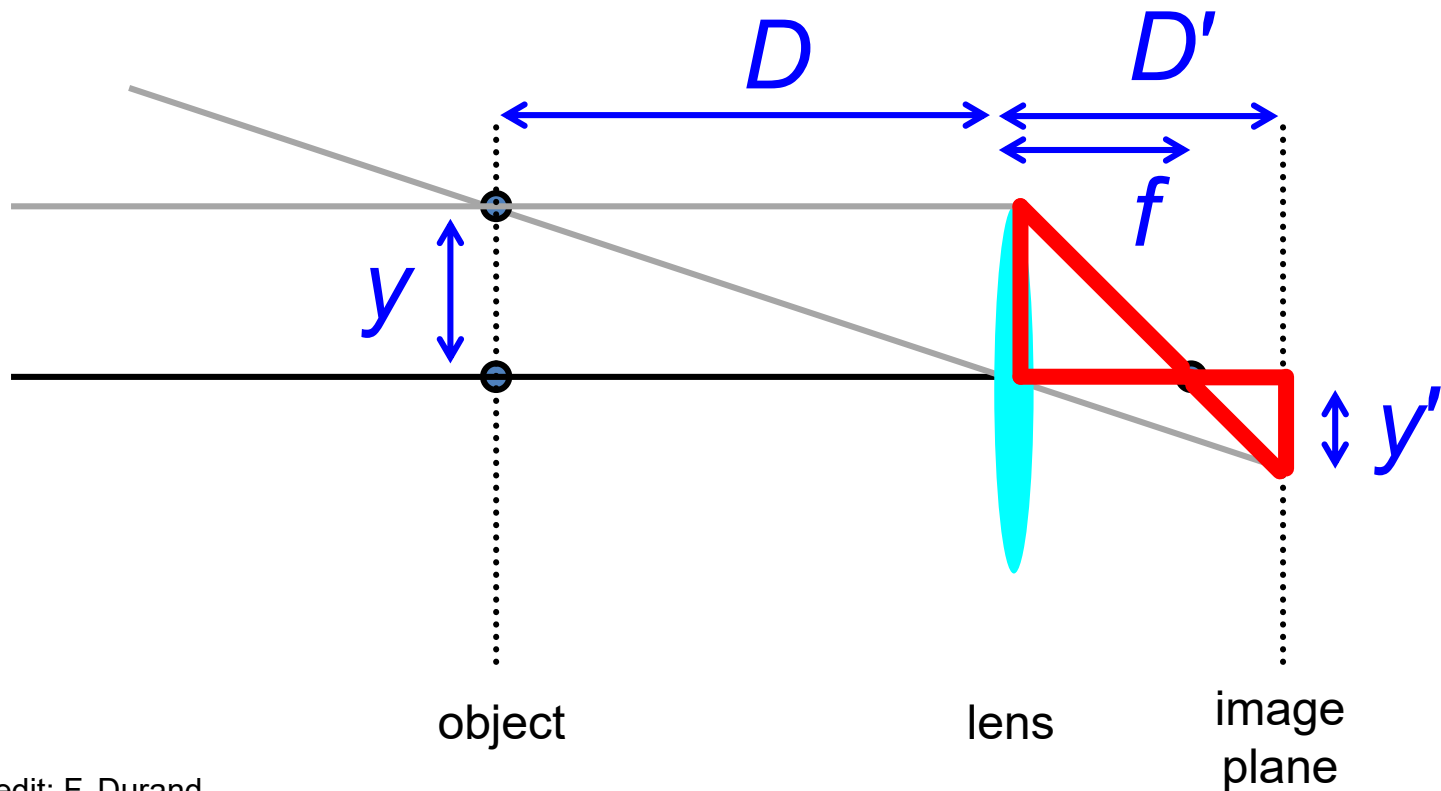
Let's derive the relationship between object distance  $D$ , image plane distance  $D'$ , and focal length  $f$ .





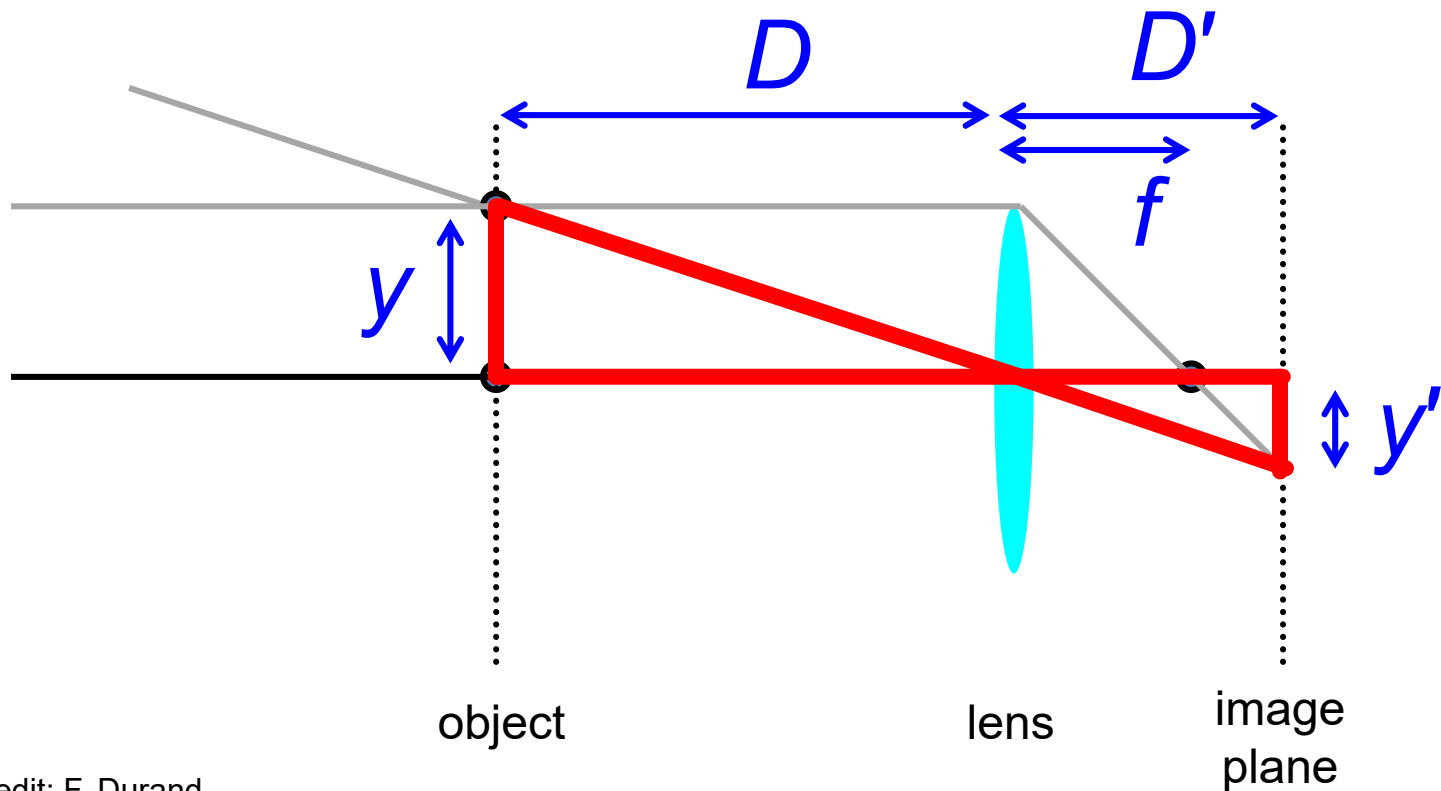
# Thin Lens Formula

One set of similar triangles:  $\frac{y'}{D' - f} = \frac{y}{f} \rightarrow \frac{y'}{y} = \frac{D' - f}{f}$



# Thin Lens Formula

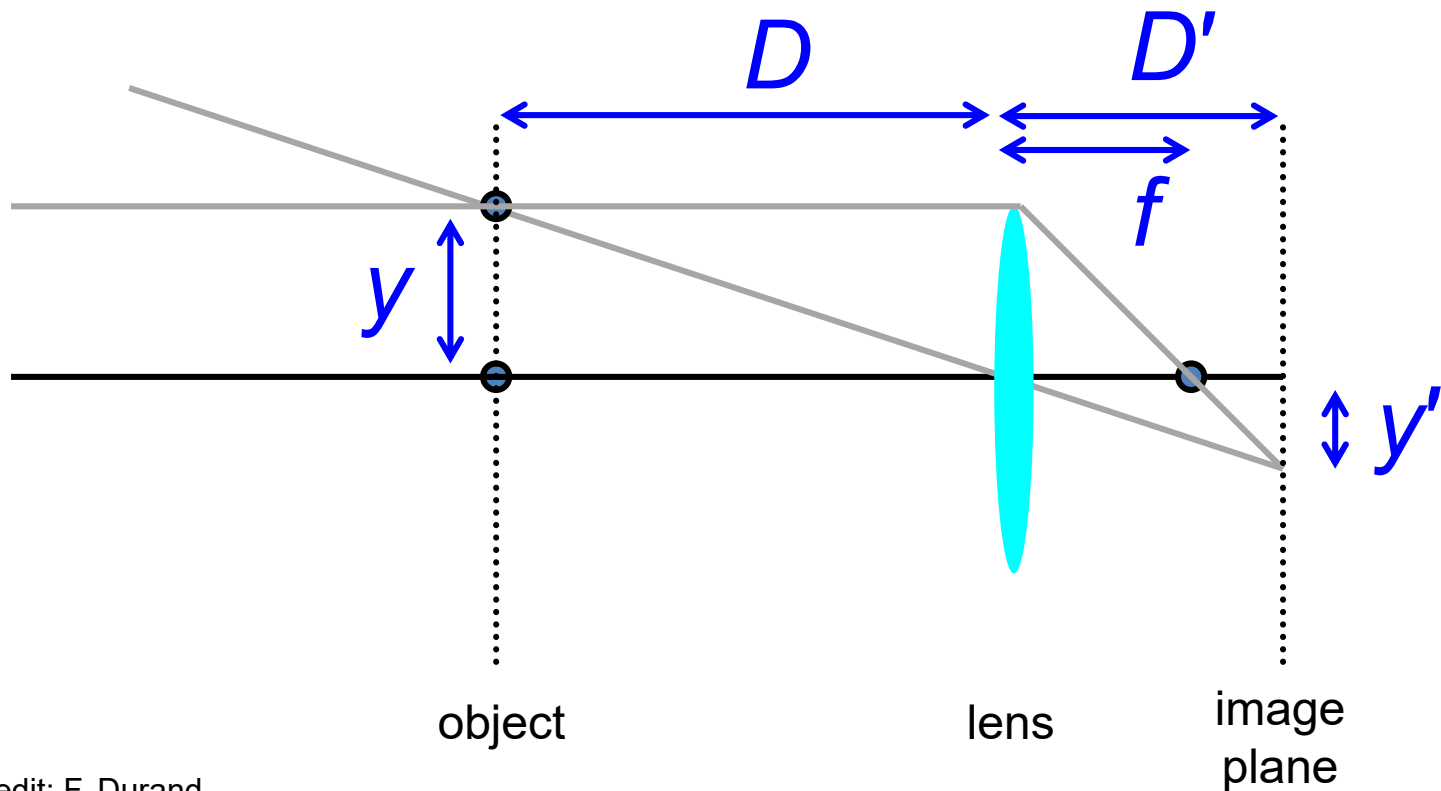
Another set of similar triangles:  $\frac{y'}{D'} = \frac{y}{D} \rightarrow \frac{y'}{y} = \frac{D'}{D}$



# Thin Lens Formula

Set them  
equal:

$$\frac{D'}{D} = \frac{D - f}{f} \quad \longrightarrow \quad \frac{1}{D} + \frac{1}{D'} = \frac{1}{f}$$

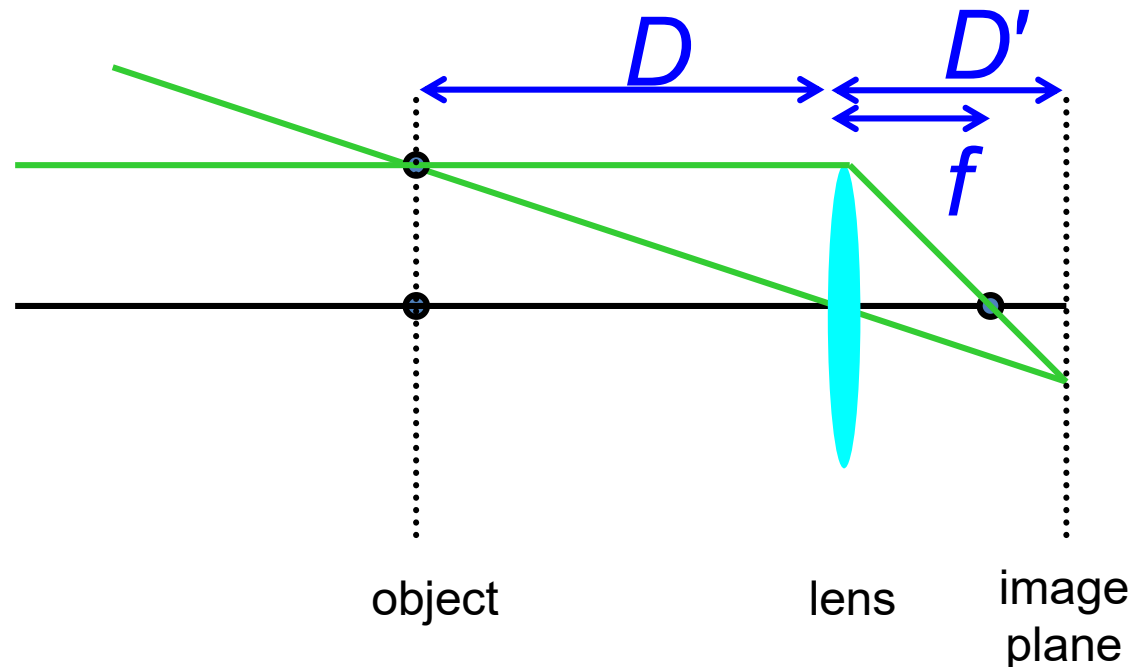


# Thin Lens Formula

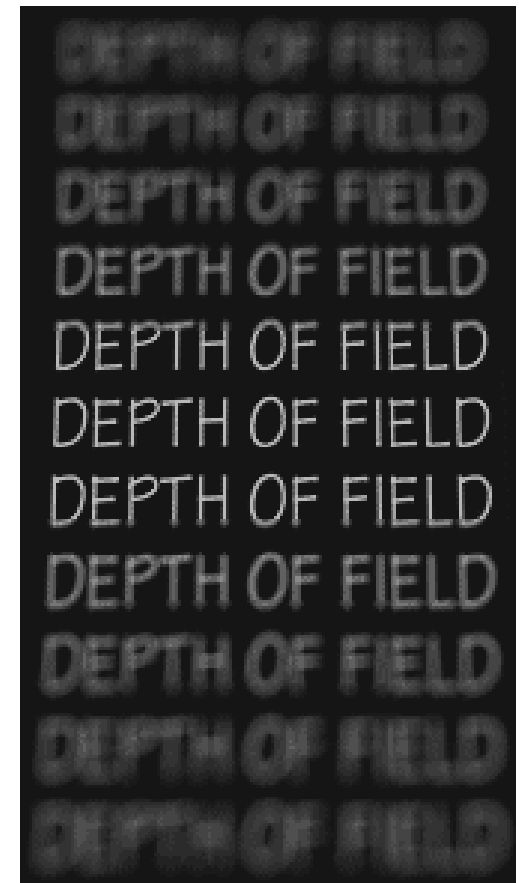
Suppose I want to take a picture of a lion.  
Which of  $D$ ,  $D'$ ,  $f$  are fixed?

How do we take pictures of things at different distances?

$$\frac{1}{D} + \frac{1}{D'} = \frac{1}{f}$$

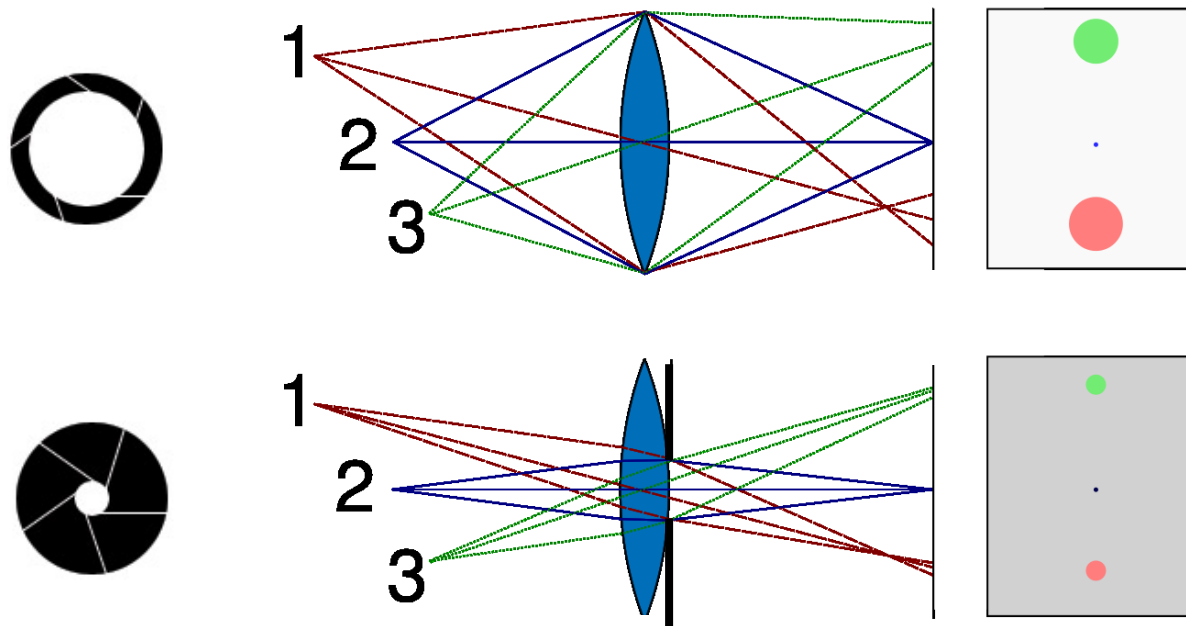


# Depth of Field



<http://www.cambridgeincolour.com/tutorials/depth-of-field.htm>

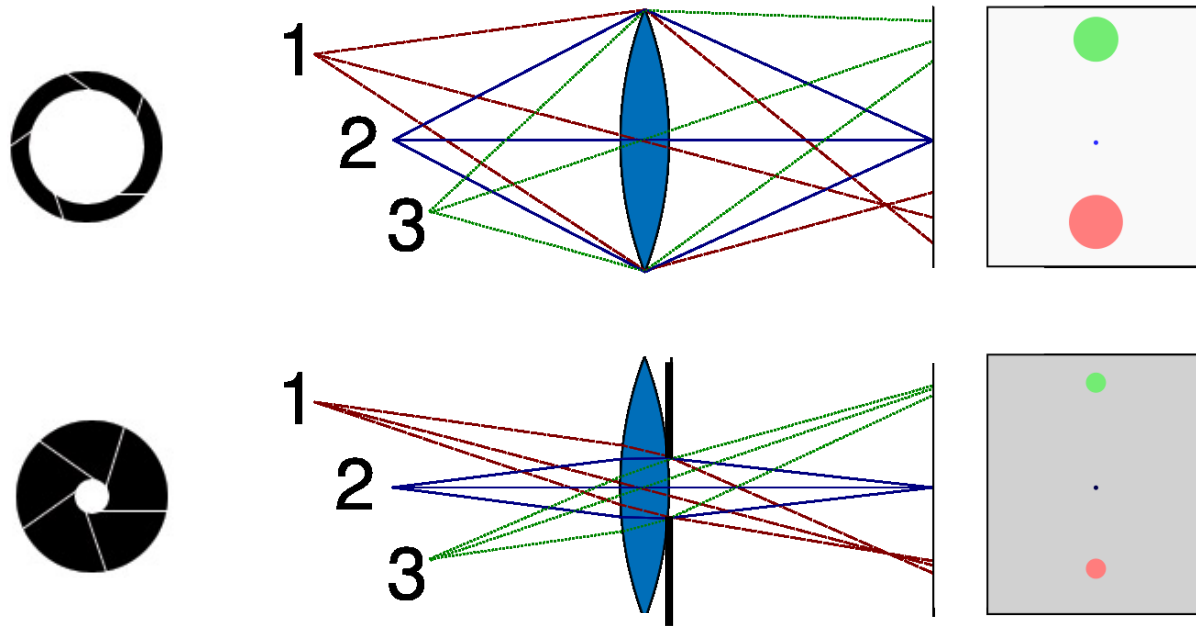
# Controlling Depth of Field



Changing the aperture size affects depth of field

A smaller aperture increases the range in which the object is approximately in focus

# Controlling Depth of Field



**If a smaller aperture makes everything focused, why don't we just always use it?**

# Varying the Aperture



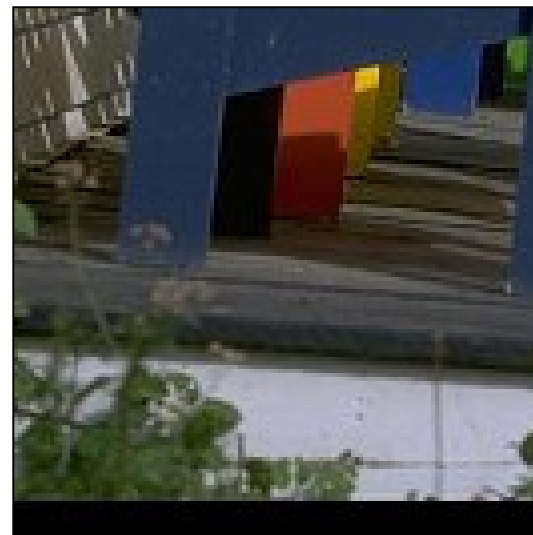
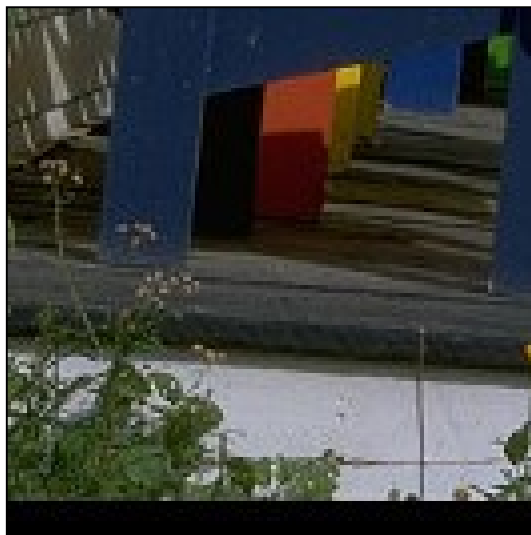
Small aperture = large DOF



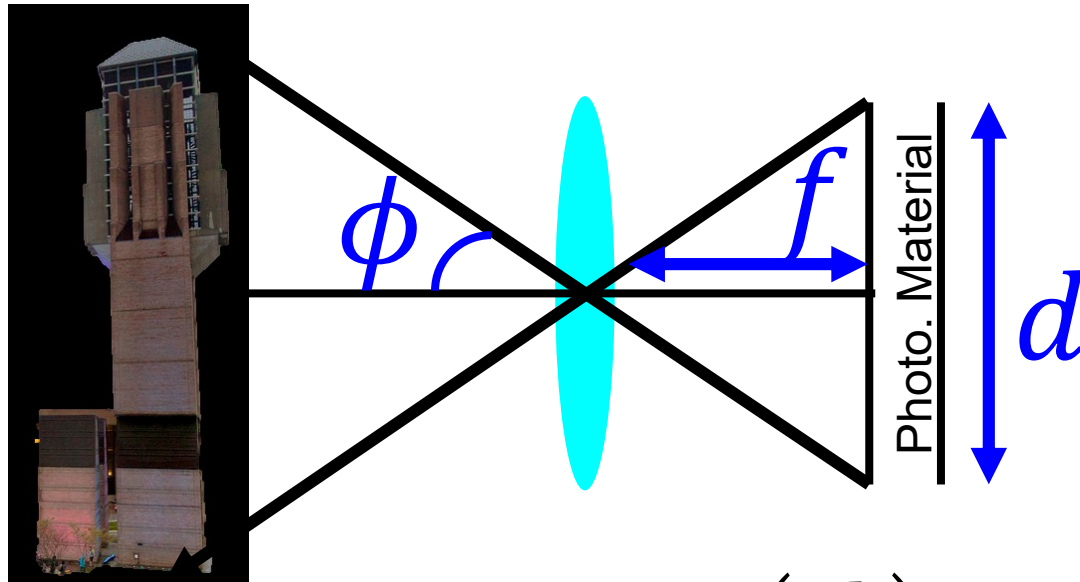
Large aperture = small DOF



# Varying the Aperture



# Field of View (FOV)

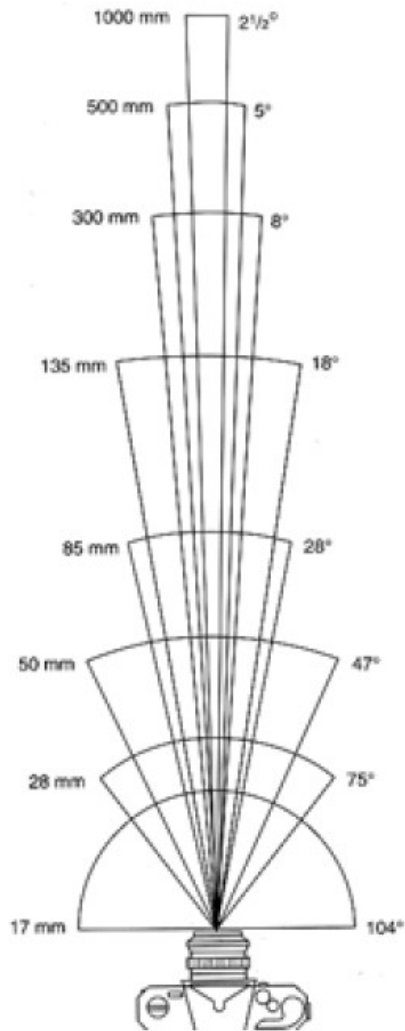


$$\phi = \tan^{-1} \left( \frac{d}{2f} \right)$$

$\tan^{-1}$  is monotonic increasing.

**How can I get the FOV bigger?**

# Field of View



17mm



28mm

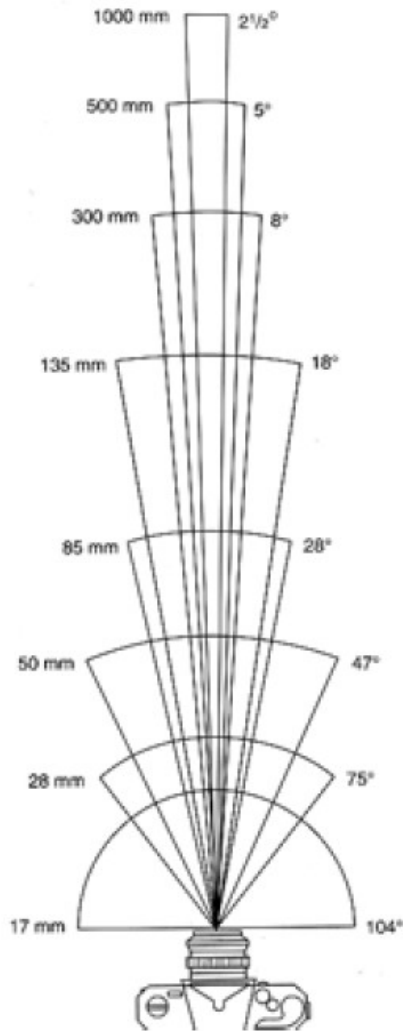


50mm

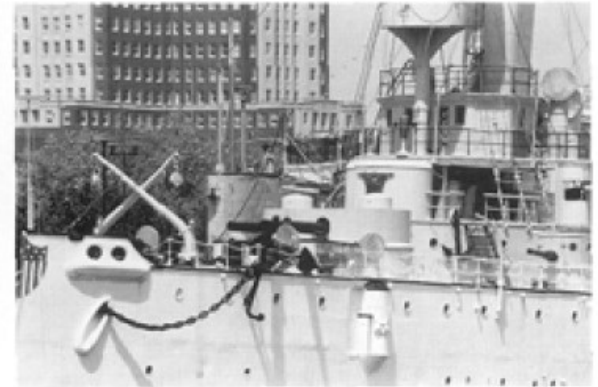


85mm

# Field of View



135mm



300mm

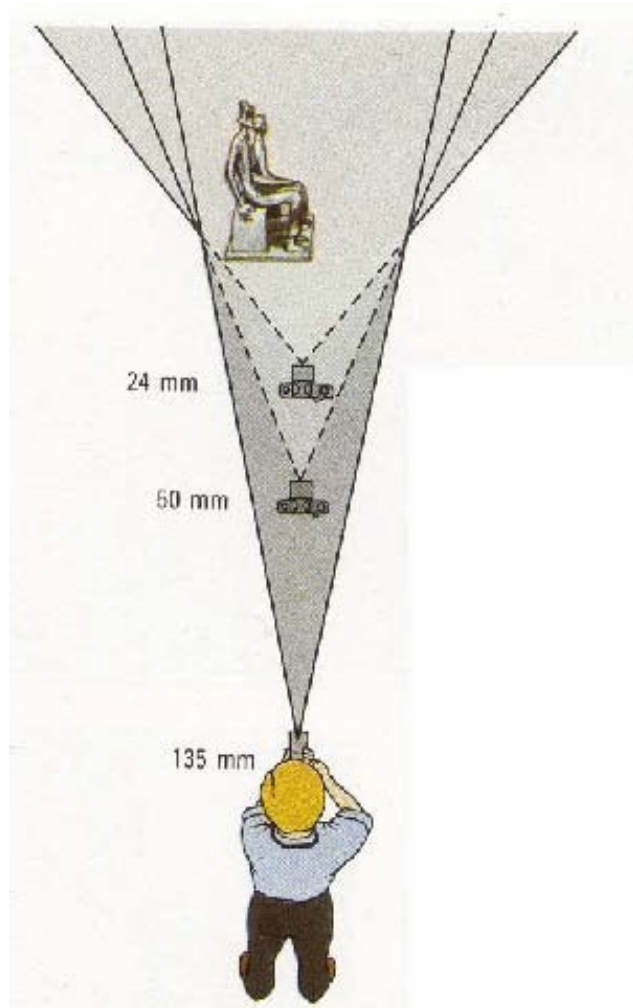


500mm



1000mm

# Field of View and Focal Length



Large FOV, small  $f$   
Camera close to car



Small FOV, large  $f$   
Camera far from the car

# Field of View and Focal Length



wide-angle



standard



telephoto



# Dolly Zoom

Change  $f$  and distance at the same time



# More Bad News!

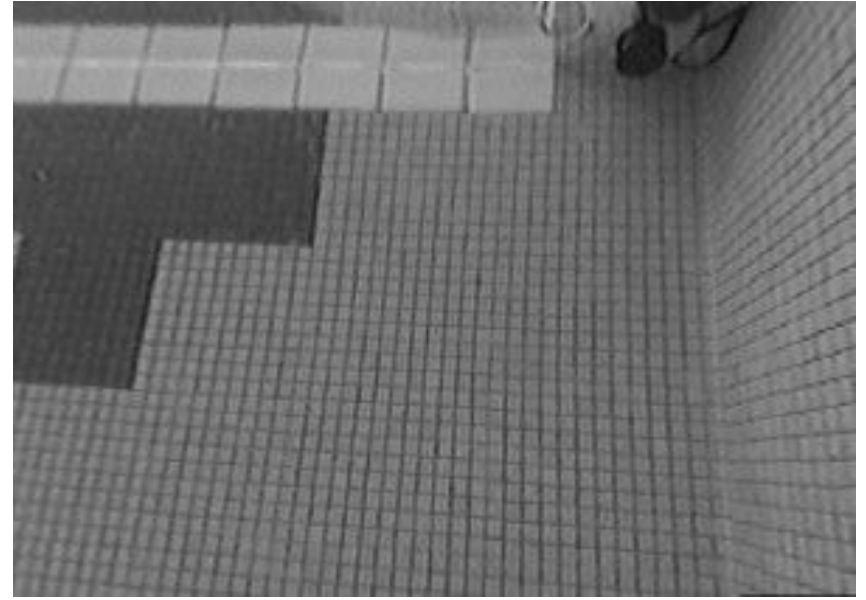
- First a pinhole...
- Then a thin lens model....





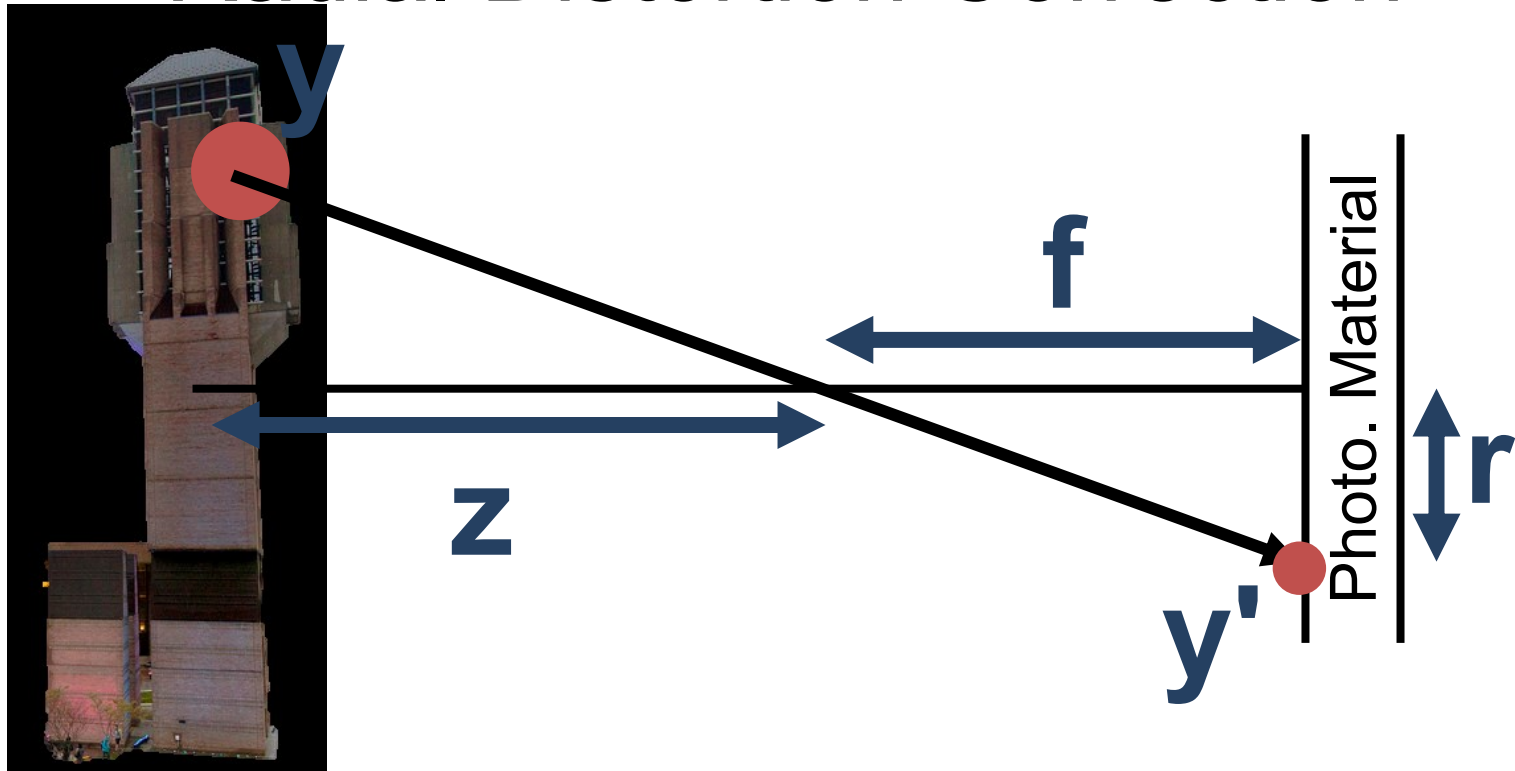
# Lens Flaws: Radial Distortion

Lens imperfections cause distortions as a function of distance from optical axis



Less common these days in consumer devices

# Radial Distortion Correction



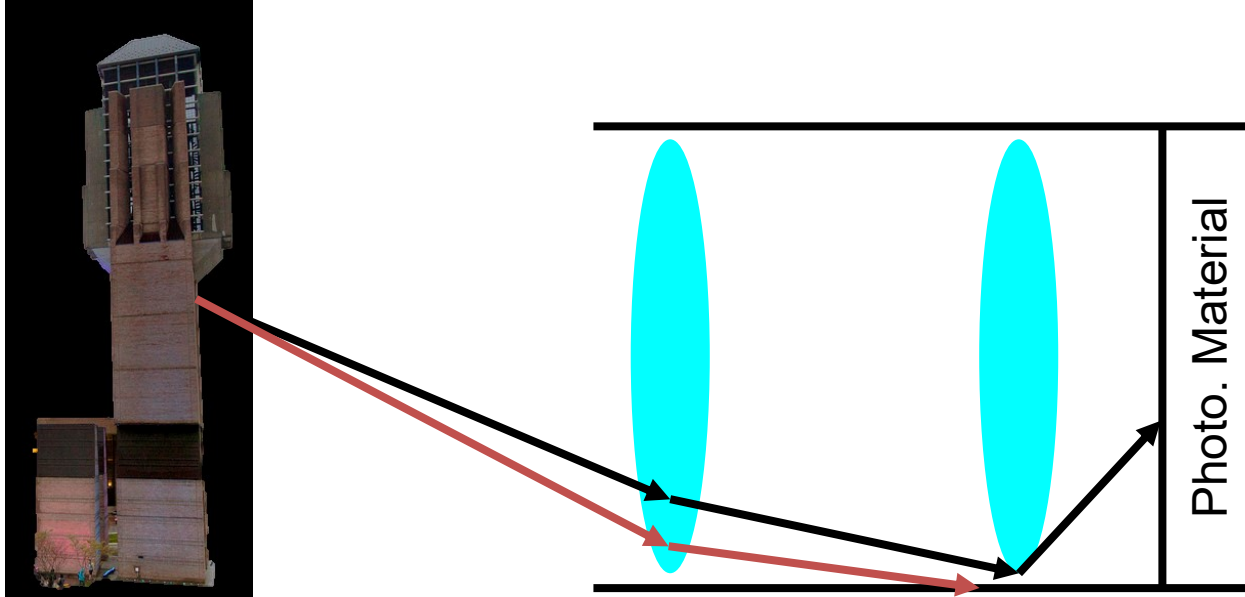
Ideal

$$y' = f \frac{y}{z}$$

Distorted

$$y' = (1 + k_1 r^2 + \dots) \frac{y}{z}$$

# Vignetting



**What happens to the light between the black and red lines?**

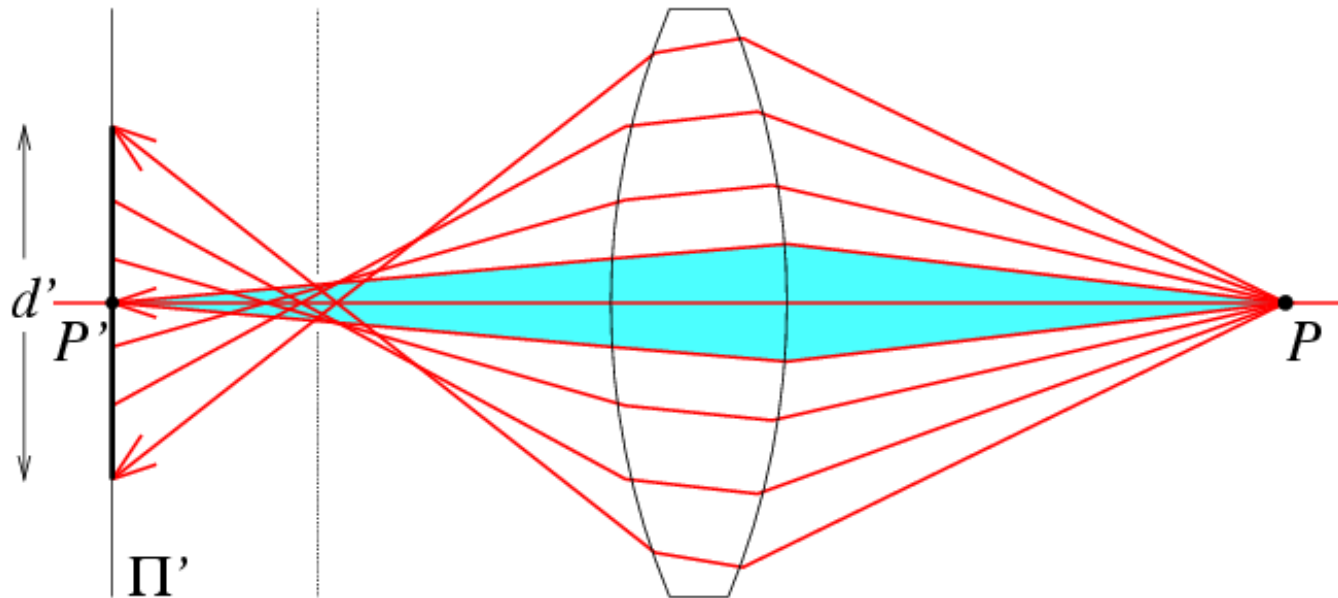
# Vignetting



Photo credit: Wikipedia (<https://en.wikipedia.org/wiki/Vignetting>)

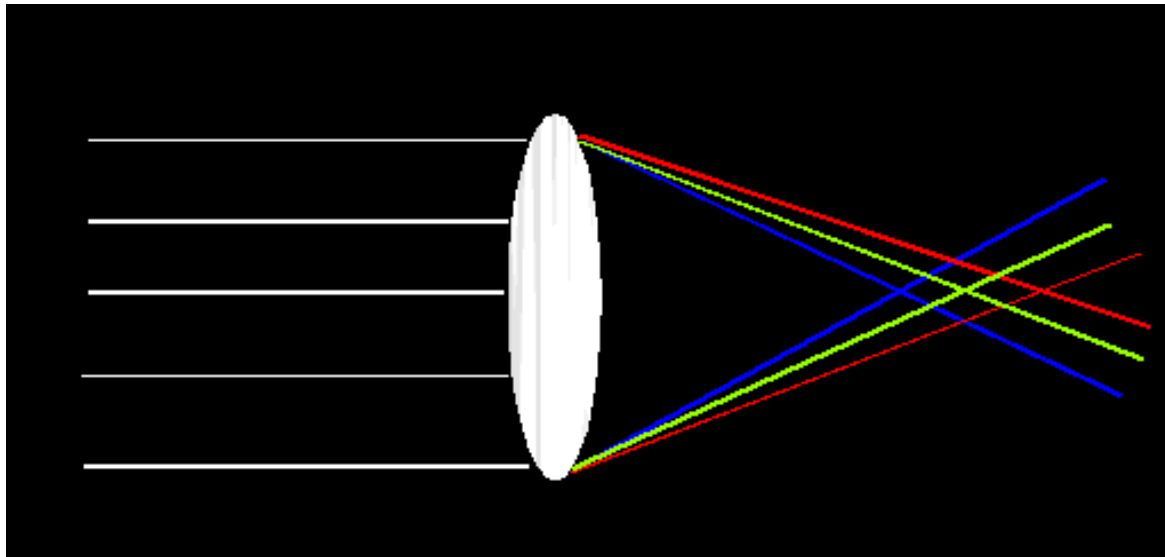
# Lens Flaws: Spherical Abberation

Lenses don't focus light perfectly!  
Rays farther from the optical axis focus closer



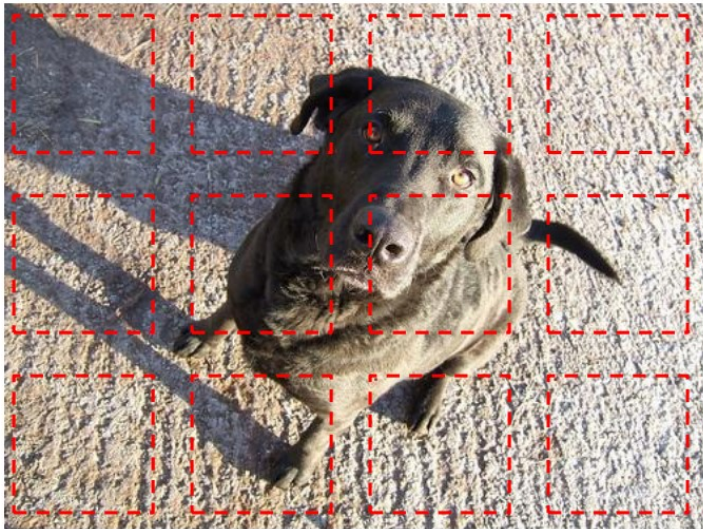
# Lens Flaws: Chromatic Abberation

Lens refraction index is a function of the wavelength. Colors “fringe” or bleed



# Lens Flaws: Chromatic Abberation

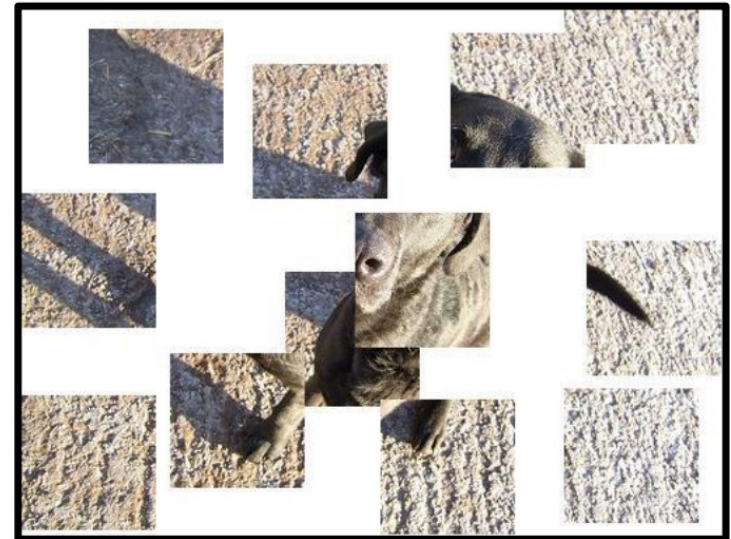
Researchers tried teaching a network about objects by forcing it to assemble jigsaws.



Initial layout, with sampled patches in red



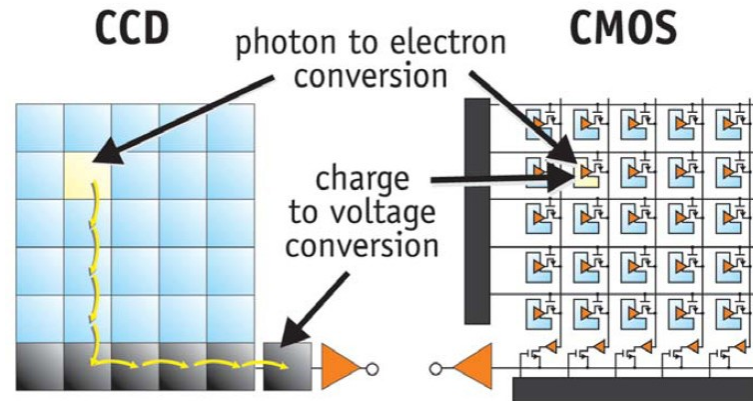
Image layout



We can recover image layout automatically



# From Photon to Photo



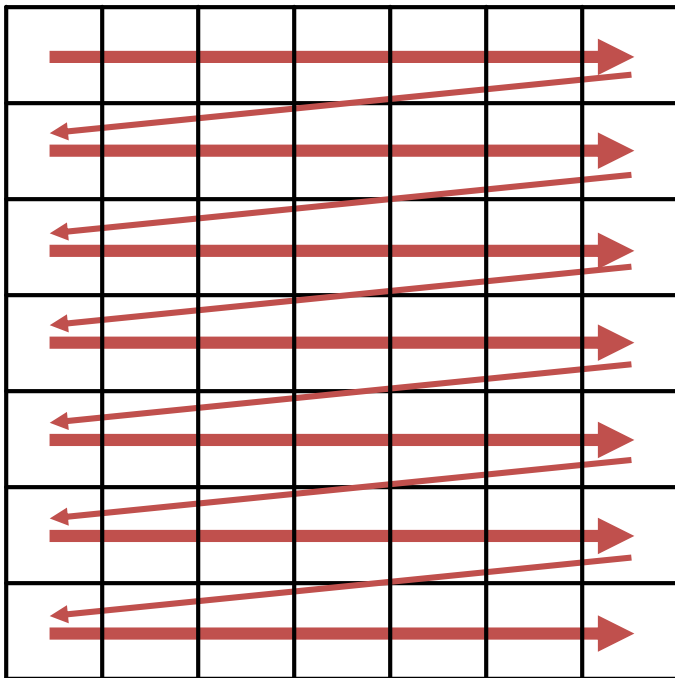
*CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.*

- Each cell in a sensor array is a light-sensitive diode that converts photons to electrons
  - Dominant in the past: **Charge Coupled Device (CCD)**
  - Dominant now: **Complementary Metal Oxide Semiconductor (CMOS)**



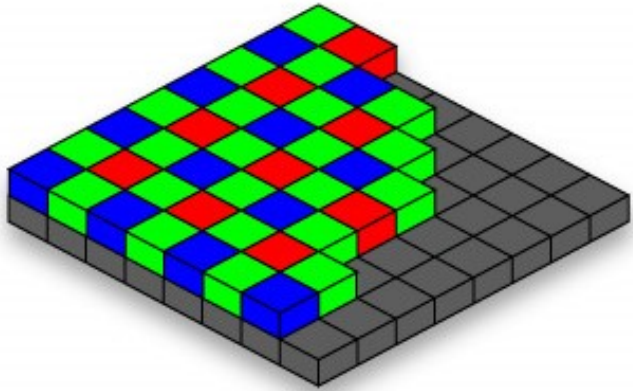
# From Photon to Photo

Rolling Shutter: pixels read in sequence  
Can get global reading, but \$\$\$



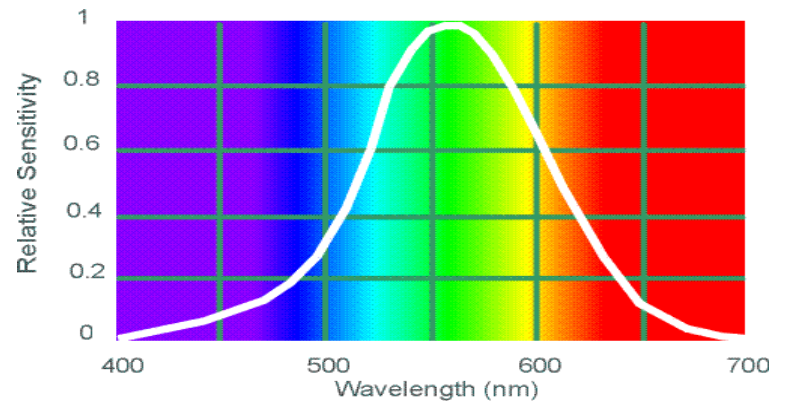
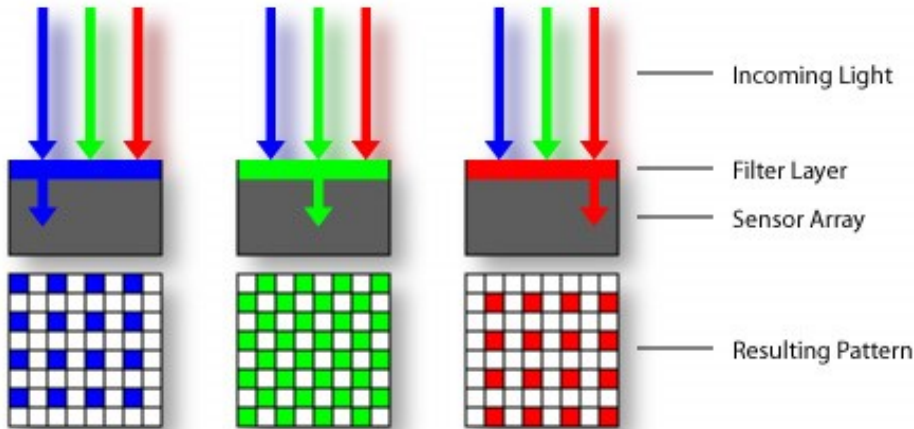
# Preview of What's Next

Bayer grid



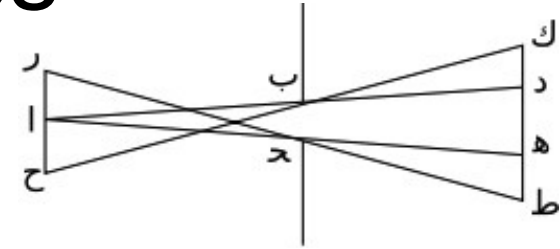
## Demosaicing:

Estimation of missing components from neighboring values

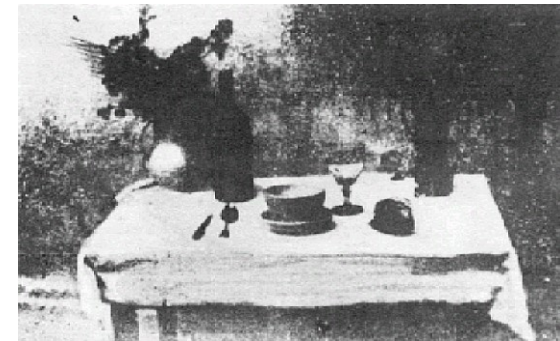


# Historic milestones

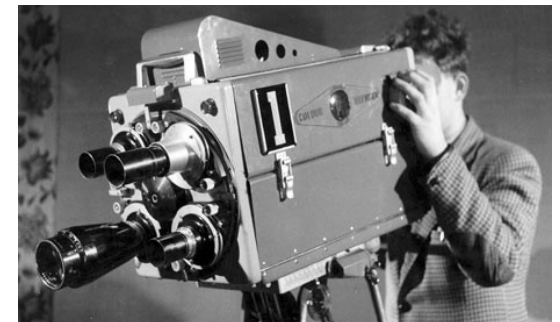
- **Pinhole model:** Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses):** Alhacen (965-1039 CE)
- **Camera obscura:** Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo:** Joseph Nicephore Niepce (1822)
- **Daguerréotypes** (1839)
- **Photographic film** (Eastman, 1889)
- **Cinema** (Lumière Brothers, 1895)
- **Color Photography** (Lumière Brothers, 1908)
- **Television** (Baird, Farnsworth, Zworykin, 1920s)
- **First consumer camera with CCD** Sony Mavica (1981)
- **First fully digital camera:** Kodak DCS100 (1990)



Alhacen's notes



Niepce, "La Table Servie," 1822



Old television camera

# First digitally scanned photograph

- 1957, 176x176 pixels





# Historic Milestone

Sergey Prokudin-Gorskii (1863-1944)

Photographs of the Russian empire (1909-1916)

**Blue  
Filter  
(B)**



**Green  
Filter  
(G)**



**Red  
Filter  
(R)**



# Historic Milestone



# Future Milestone

Your job in homework 1:  
turn the left result into the right result.

