Next Few Classes

• Tuesday: Cameras (Projective Geometry)
• Thursday: Cameras (Light, Lenses)
• Next Tuesday: Light and Image Formation

Discussion This Week:
• Linear algebra
Administrivia

• HW 1 is out Thursday Jan 17, due Jan 31:
  • Linear algebra
  • Projective geometry
  • Image alignment
• You have 3 late days, and it’s 1% off for every hour late.
• Any administrative questions?
SIGGRAPH Talks 2011

KinectFusion:
Real-Time Dynamic 3D Surface Reconstruction and Interaction

Shahram Izadi 1, Richard Newcombe 2, David Kim 1,3, Otmar Hilliges 1,
David Molyneaux 1,4, Pushmeet Kohli 1, Jamie Shotton 1,
Steve Hodges 1, Dustin Freeman 5, Andrew Davison 2, Andrew Fitzgibbon 1

1 Microsoft Research Cambridge 2 Imperial College London
3 Newcastle University  4 Lancaster University
5 University of Toronto
Let’s Take a Picture!

Idea 1: Just use film  
Result: Junk

Slide inspired by S. Seitz; image from Michigan Engineering
Let’s Take a Picture!

Idea 2: add a barrier

Slide inspired by S. Seitz; image from Michigan Engineering
Let’s Take a Picture!

Idea 2: add a barrier

Photosensitive Material

Slide inspired by S. Seitz; image from Michigan Engineering
Let’s Take a Picture!

Film captures all the rays going through a point (a pencil of rays).
Result: good in theory!
Camera Obscura

• Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)

• Drawing aid for artists: described by Leonardo da Vinci (1452-1519)

Source: A. Efros
Camera Obscura

After scouting rooms and reserving one for at least a day, Morell masks the windows except for the aperture. He controls three elements: the size of the hole, with a smaller one yielding a sharper but dimmer image; the length of the exposure, usually eight hours; and the distance from the hole to the surface on which the outside image falls and which he will photograph. He used 4 x 5 and 8 x 10 view cameras and lenses ranging from 75 to 150 mm.

After he’s done inside, it gets harder. “I leave the room and I am constantly checking the weather, I’m hoping the maid reads my note not to come in, I’m worrying that the sun will hit the plastic masking and it will fall down, or that I didn’t trigger the lens.”

From Grand Images Through a Tiny Opening, Photo District News, February 2005

Abelardo Morell, Camera Obscura Image of Manhattan View Looking South in Large Room, 1996

http://www.abelardomorell.net/project/camera-obscura/
Camera Obscura

Hotel room
contact enhanced

View Out of
Hotel Room Window

Source: A. Torralba, W. Freeman Accidental Pinhole and Pinspeck Cameras, CVPR 2012
How do we find the projection $P$ of a point $X$?
Form visual ray from $X$ to camera center and intersect it with camera plane
Both $X$ and $X'$ project to $P$. Which appears in the image?

Are there points for which projection is undefined?

Source: L Lazebnik
Quick Aside: Remember This?

\[
\frac{a}{b} = \frac{d}{c} \quad \Rightarrow \quad a = \frac{bd}{c}
\]
Projection Equations

Coordinate system: \( O \) is origin, \( XY \) in image, \( Z \) sticks out. \( XY \) is image plane, \( Z \) is optical axis.

\( (x,y,z) \) projects to \( (fx/z, fy/z) \) via similar triangles

Source: L Lazebnik
Some Facts About Projection

3D lines project to 2D lines

The projection of any 3D parallel lines converge at a vanishing point

Distant objects are smaller

List of properties from M. Hebert
Some Facts About Projection

Let’s try some fake images
Some Facts About Projection

Slide by Steve Seitz
Some Facts About Projection
Some Facts About Projection

Illusion Credit: RN Shepard, Mind Sights: Original Visual Illusions, Ambiguities, and other Anomalies
What’s Lost?

Is she shorter or further away?

Are the orange lines we see parallel / perpendicular / neither to the red line?

Inspired by D. Hoiem slide
What’s Lost?

Is she shorter or further away?

Are the orange lines we see parallel / perpendicular / neither to the red line?

Adapted from D. Hoiem slide
What’s Lost?

Be careful of drawing conclusions:

• Projection of 3D line is 2D line; NOT 2D line is 3D line.

• Can you think of a counter-example (a 2D line that is not a 3D line)?

• Projections of parallel 3D lines converge at VP; NOT any pair of lines that converge are parallel in 3D.

• Can you think of a counter-example?
Do You Always Get Perspective?
Do You Always Get Perspective?

Y location of blue and red dots in image:

\[
\frac{fy}{z_2} \quad \frac{fy}{z_1} \quad \frac{fy}{z} \quad \frac{fy}{z}
\]
Do You Always Get Perspective?

When plane is fronto-parallel (parallel to camera plane), everything is:

- scaled by f/z
- otherwise is preserved.
What’s This Useful For?

Things looking different when viewed from different angles seems like a nuisance. It’s also a cue. Why?
What’s This Useful For?
What’s This Useful For?

Mirror
What’s This Useful For?

O’Brien and Farid, SIGGRAPH 2012, Exposing Photo Manipulation with Inconsistent Reflections
Projection Equation

\[(x, y, z) \rightarrow \left(\frac{fx}{z}, \frac{fy}{z}\right)\]

I promised you linear algebra: is this linear?

**Nope:** division by \(z\) is non-linear
(and risks division by 0)

Adapted from S. Seitz slide
Homogeneous Coordinates (2D)

Trick: add a dimension!

This also clears up lots of nasty special cases

What if $w = 0$?

Adapted from M. Hebert slide
Homogeneous Coordinates

Two homogeneous coordinates are equivalent if they are proportional to each other. Not = !
Benefits of Homogeneous Coords

General equation of 2D line:

\[ ax + by + c = 0 \]

Homogeneous Coordinates

\[ \mathbf{l}^T \mathbf{p} = 0, \quad \mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \]
Benefits of Homogeneous Coords

• Lines (3D) and points (2D → 3D) are now the same dimension.

• Use the cross (x) and dot product for:
  • Intersection of lines \( l \) and \( m \): \( l \times m \)
  • Line through two points \( p \) and \( q \): \( p \times q \)
  • Point \( p \) on line \( l \): \( l^T p \)

• But what about parallel lines?
Benefits of Homogeneous Coords

What about parallel lines?

\[ 0x + 1y - 2 = 0 \]
\[ 0x + 1y - 1 = 0 \]

\[ [0,1,-2] \times [0,1,-1] = [1,0,0] \]

Any point \([x,y,0]\) is at infinity

All operations generate valid results!
Benefits of Homogeneous Coords

Intersection of $y=2$, $y=1$

$[0,1,-2] \times [0,1,-1] = [1,0,0]$

Does it lie on $y=3$? Intuitively?

$[0,1,-3]^T[1,0,0] = 0$
3D Homogeneous Coordinates

• Same story: add a coordinate, things are equivalent if they’re proportional

\[
\begin{bmatrix}
  x \\
  y \\
  z \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  u \\
  v \\
  w \\
  t \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  u/t \\
  v/t \\
  w/t \\
\end{bmatrix}
\]
<table>
<thead>
<tr>
<th></th>
<th>Vector Coordinates</th>
<th>Homogeneous Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Translation</strong></td>
<td>( y = x + t )</td>
<td>( y = \begin{bmatrix} I_{3 \times 3} &amp; t \ 0 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix} x )</td>
</tr>
<tr>
<td><strong>Rotation</strong></td>
<td>( y = Rx )</td>
<td>( y = \begin{bmatrix} R_{3 \times 3} &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix} x )</td>
</tr>
<tr>
<td></td>
<td>( R^T R = R R^T = I )</td>
<td>( \text{det}(R) = 1 )</td>
</tr>
<tr>
<td><strong>Rigid Body</strong></td>
<td>( y = Rx + t )</td>
<td>( y = \begin{bmatrix} R_{3 \times 3} &amp; t \ 0 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix} x )</td>
</tr>
<tr>
<td><strong>Affine</strong></td>
<td>( y = Ax + t )</td>
<td>( y = \begin{bmatrix} A_{3 \times 3} &amp; t \ 0 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix} x )</td>
</tr>
</tbody>
</table>
Projection Matrix

Projection \((fx/z, fy/z)\) is matrix multiplication

\[
\begin{bmatrix}
0 & 0 & f_x & 0 \\
0 & 0 & 0 & f_y \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
\end{bmatrix}
\]

O
Projection Matrix

Projection \((fx/z, fy/z)\) is matrix multiplication

\[
\begin{bmatrix}
f x \\
f y \\
f z
\end{bmatrix} \equiv \begin{bmatrix}
f & 0 & 0 & 0 & 0 \\
0 & f & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z
\end{bmatrix} \rightarrow \begin{bmatrix}
fx/z \\
fy/z
\end{bmatrix}
\]

Slide inspired from L. Lazebnik
Why $\equiv \neq =$

Project $X$ and $X'$ to the image and compare them

**YES**

\[
\begin{bmatrix}
fx \\
fy \\
z
\end{bmatrix}
\equiv
\begin{bmatrix}
fx' \\
fy' \\
z'
\end{bmatrix}
\]

**NO**

\[
\begin{bmatrix}
fx \\
fy \\
z
\end{bmatrix}
\neq
\begin{bmatrix}
fx' \\
fy' \\
z'
\end{bmatrix}
\]
Standard Full Perspective Model

\[ P \equiv \alpha - \alpha \cot \theta u_0 \]

\[ \beta \sin \theta v_0 \]

\[ R_{3 \times 3} t_{3 \times 1} X_{4 \times 1} \]

\( P \): 2D homogeneous point (3D)

\( X \): 3d homogeneous point (4D)
Standard Full Perspective Model

\[ P \equiv [R_{3x3} \quad t_{3x1}] \quad X_{4x1} \]

**R**: rotation between world system and camera

**t**: translation between world system and camera
**Standard Full Perspective Model**

\[ P \equiv \begin{bmatrix} \alpha & -\alpha \cot \theta & u_0 \\ 0 & \frac{\beta}{\sin \theta} & v_0 \\ 0 & 0 & 1 \end{bmatrix} \]

- \( \alpha \) scale between world and image x coords
- \( \beta \) scale between world and image y coords
- \( \theta \) skew of camera axes
- \( u_0, v_0 \): principal point (image coords of camera origin on retina)
- \( R_{3\times3}, t_{3\times1} \)
- \( X_{4\times1} \)
\[ P \equiv \begin{bmatrix} \alpha & -\alpha \cot \theta & u_0 \\ 0 & \frac{\beta}{\sin \theta} & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{3x3} & t_{3x1} \end{bmatrix} X_{4x1} \]
Typical Perspective Model

\[ P \equiv \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{3\times3} & t_{3\times1} \end{bmatrix} X_{4\times1} \]

- \( f \) focal length
- \( u_0, v_0 \): principal point (image coords of camera origin on retina)
Typical Perspective Model

\[ P \equiv K[R, t]X \equiv M_{3 \times 4}X_{4 \times 1} \]
Other Cameras – Orthographic

Orthographic Camera (z infinite)

\[ P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times 3x1 \]

Other Cameras – Orthographic

Why does this make things easy and why is this popular in old games?

\[ P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \]
Recap – Homog. Coordinates

Normal

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
\]

Concat

Homogeneous

\[
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix}
\]

Divide by w

Normal

\[
\begin{bmatrix}
u/w \\
v/w
\end{bmatrix}
\]

w = 0 => point is at infinity

YES!  NO!

\[
x \equiv y \quad \exists \lambda \neq 0 \quad x = \lambda y
\]

x = y
Recap – Homog. Coordinates

• Lots of transformations become linear:
  • Rotation, translation, affine transformations are now all matrix multiplications

• Lots of special cases go away (useful for both computation \textit{and} proofs):
  • Try defining two vertical lines
  • Try finding their intersection
Recap – Projection

\[(x,y,z) \rightarrow \left(\frac{fx}{z}, \frac{fy}{z}\right)\]

*Your location in the image depends on depth!*
Recap – Projection

\( f: \) focal length \hspace{1cm} \( u_0, v_0: \) principal point (image coords of camera origin on retina)

\( \mathbf{R}: \) camera rotation \hspace{1cm} \( \mathbf{t}: \) camera translation

\[
P_{3 \times 1} \equiv \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_{3 \times 3} \\ \mathbf{t}_{3 \times 1} \end{bmatrix} \mathbf{X}_{4 \times 1}
\]
The Big Issue

Film captures all the rays going through a **point** (a pencil of rays).

**How big is a point?**
Math vs. Reality

- Math: Any point projects to one point

- Reality (as pointed out by the class)
  - Don’t image points behind the camera / objects
  - Don’t have an infinite amount of sensor material

- Other issues
  - Light is limited
  - Spooky stuff happens with infinitely small holes
Limitations of Pinhole Model

**Ideal Pinhole**
- 1 point generates 1 image
- Low-light levels

**Finite Pinhole**
- 1 point generates region
- Blurry.

Why is it blurry?

Slide inspired by M. Hebert
Limitations of Pinhole Model
Adding a Lens

- A lens focuses light onto the film
- Thin lens model: rays passing through the center are not deviated (pinhole projection model still holds)

Slide Credit: S. Seitz
Adding a Lens

- All rays parallel to the optical axis pass through the *focal point*
What’s The Catch?

• There’s a distance where objects are “in focus”
• Other points project to a “circle of confusion”
Thin Lens Formula

We care about images that are in focus. When is this true? Discuss with your neighbor.
When two paths from a point hit the same image location.

Diagram credit: F. Durand
Let’s derive the relationship between object distance $D$, image plane distance $D'$, and focal length $f$. 

Diagram credit: F. Durand
Thin Lens Formula

One set of similar triangles:

\[ \frac{y'}{D' - f} = \frac{y}{f} \quad \Rightarrow \quad \frac{y'}{y} = \frac{D' - f}{f} \]

Diagram credit: F. Durand
Thin Lens Formula

Another set of similar triangles:

\[
\frac{y'}{D'} = \frac{y}{D} \quad \Rightarrow \quad \frac{y'}{y} = \frac{D'}{D}
\]
Thin Lens Formula

Set them equal:

\[
\frac{D'}{D} = \frac{D - f}{f} \quad \rightarrow \quad \frac{1}{D} + \frac{1}{D'} = \frac{1}{f}
\]

Diagram credit: F. Durand
Thin Lens Formula

Suppose I want to take a picture of a lion. Which of $D$, $D'$, $f$ are fixed?

How do we take pictures of things at different distances?

$$\frac{1}{D} + \frac{1}{D'} = \frac{1}{f}$$

Diagram credit: F. Durand
Depth of Field

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Slide Credit: A. Efros
Controlling Depth of Field

Changing the aperture size affects depth of field. A smaller aperture increases the range in which the object is approximately in focus.

If a smaller aperture makes everything focused, why don’t we just always use it?
Varying the Aperture

Small aperture  = large DOF  
Large aperture  = small DOF

Slide Credit: A. Efros, Photo: Philip Greenspun
Varying the Aperture
Field of View (FOV)

\[ \phi = \tan^{-1} \left( \frac{d}{2f} \right) \]

\( \tan^{-1} \) is monotonic increasing.

How can I get the FOV bigger?
Field of View

Slide Credit: A. Efros
Field of View
Field of View and Focal Length

Large FOV, small $f$
Camera close to car

Small FOV, large $f$
Camera far from the car

Slide Credit: A. Efros, F. Durand
Field of View and Focal Length

wide-angle  standard  telephoto

Slide Credit: F. Durand
Dolly Zoom

Change f and distance at the same time

Video Credit: Goodfellas 1990
More Bad News!

- First a pinhole…
- Then a thin lens model….
Lens Flaws: Radial Distortion

Lens imperfections cause distortions as a function of distance from optical axis

Less common these days in consumer devices

Photo: Mark Fiala, U. Alberta
Radial Distortion Correction

Ideal

\[ y' = f \frac{y}{z} \]

Distorted

\[ y' = (1 + k_1 r^2 + \cdots) \frac{y}{z} \]
What happens to the light between the black and red lines?
Lens Flaws: Spherical Abberation

Lenses don’t focus light perfectly!
Rays farther from the optical axis focus closer
Lens Flaws: Chromatic Abberation

Lens refraction index is a function of the wavelength. Colors “fringe” or bleed.
Lens Flaws: Chromatic Abberation

Researchers tried teaching a network about objects by forcing it to assemble jigsaws.
From Photon to Photo

- Each cell in a sensor array is a light-sensitive diode that converts photons to electrons
  - Dominant in the past: **Charge Coupled Device (CCD)**
  - Dominant now: **Complementary Metal Oxide Semiconductor (CMOS)**

Slide Credit: L. Lazebnik, Photo Credit: Wikipedia, Stefano Meroli
From Photon to Photo

Rolling Shutter: pixels read in sequence
Can get global reading, but $$$
Preview of What’s Next

Bayer grid

Demosaicing:
Estimation of missing components from neighboring values

Slide Credit: S. Seitz

Human Luminance Sensitivity Function
Historic milestones

- **Pinhole model:** Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses):** Alhacen (965-1039 CE)
- **Camera obscura:** Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo:** Joseph Nicephore Niepce (1822)
- **Daguerréotypes** (1839)
- **Photographic film:** (Eastman, 1889)
- **Cinema:** (Lumière Brothers, 1895)
- **Color Photography** (Lumière Brothers, 1908)
- **Television:** (Baird, Farnsworth, Zworykin, 1920s)
- **First consumer camera with CCD:** Sony Mavica (1981)
- **First fully digital camera:** Kodak DCS100 (1990)
First digitally scanned photograph

- 1957, 176x176 pixels

Slide Credit: http://listverse.com/history/top-10-incredible-early-firsts-in-photography/
Historic Milestone

Sergey Prokudin-Gorskii (1863-1944)
Photographs of the Russian empire (1909-1916)

Slide Credit: S. Maji
Historic Milestone
Future Milestone

Your job in homework 1: turn the left result into the right result.