Joint estimation of attenuation and emission images from PET scans

Hakan Erdoğan and Jeffrey A. Fessler

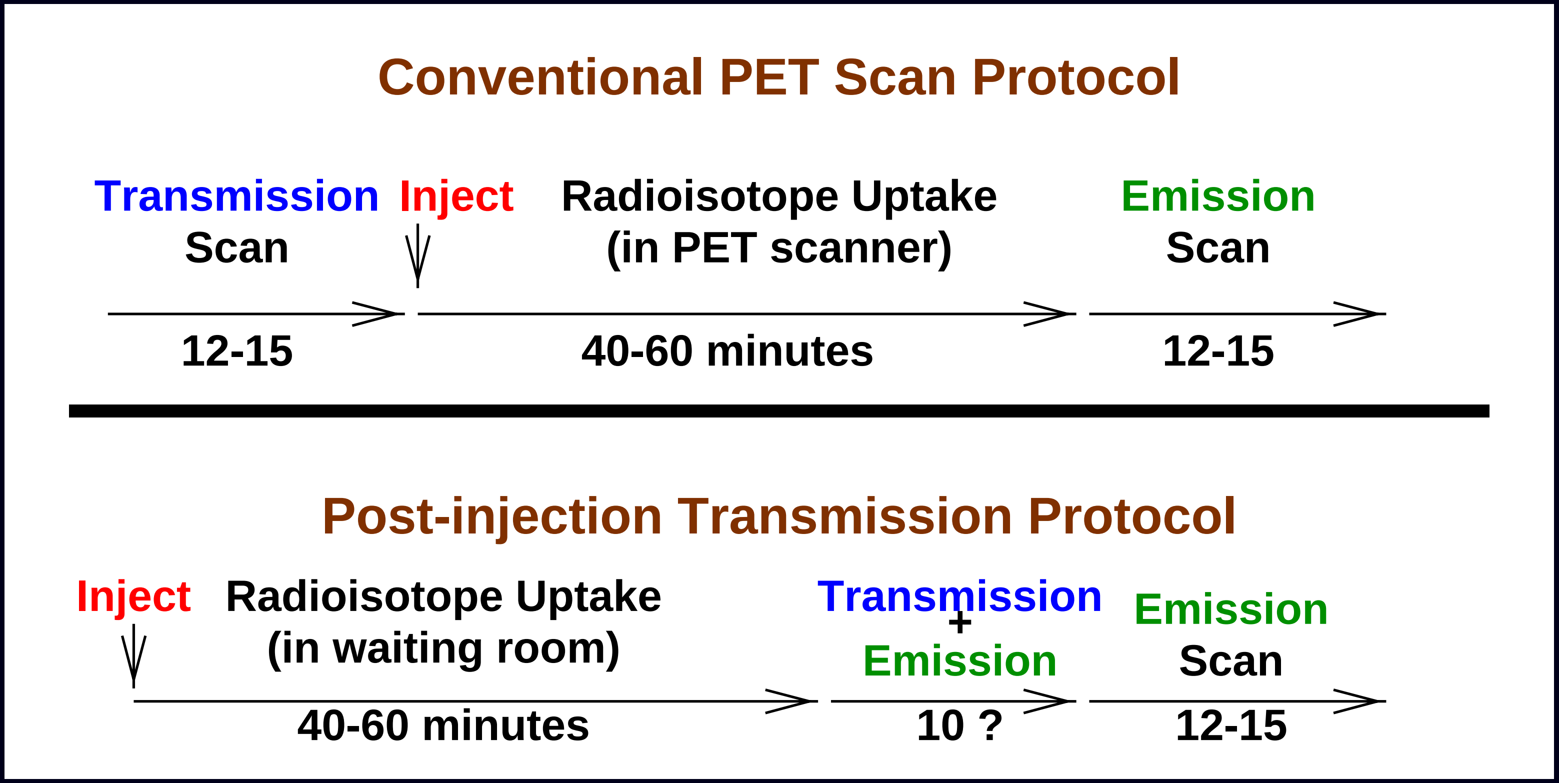
EECS Department, BME Department, and
Nuclear Medicine Division of Dept. of Internal Medicine
The University of Michigan

Nuclear Science Symposium and Medical Imaging Conference

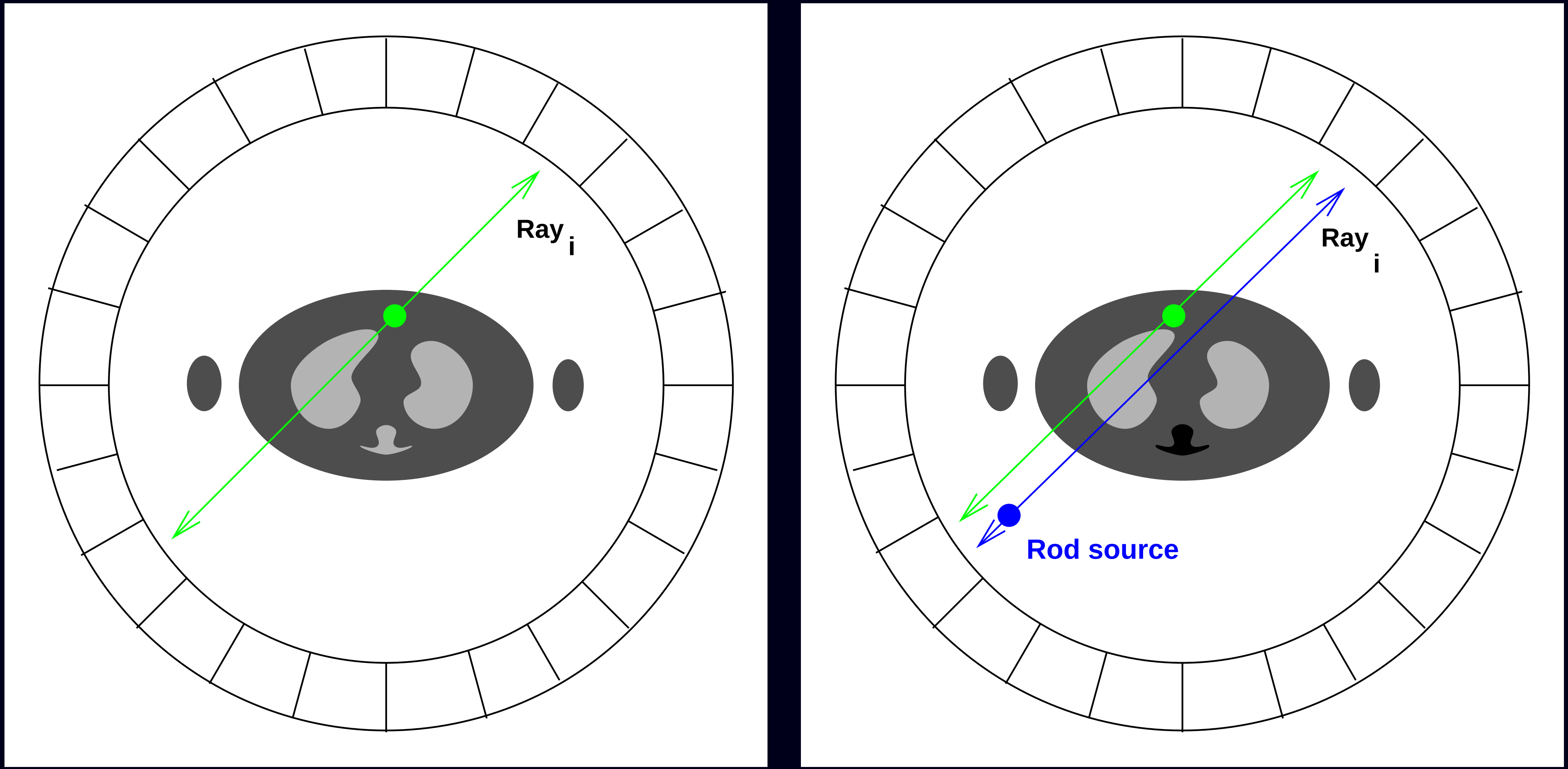
October 30, 1999

Problem Motivation

- Attenuation correction needed for quantitatively accurate PET
- Post-injection transmission scans necessitated by whole-body PET



Chicken/Egg Problem



PET Emission Scan
(Nonuniform attenuation)

Post-injection Transmission Scan
(Emission contamination)

Measurement Statistical Model

Emission scan:

$$Y_i^{\mathrm{E}} \sim \mathrm{Poisson}\{\bar{y}_i^{\mathrm{E}}(\lambda,\mu)\}, \quad \bar{y}_i^{\mathrm{E}}(\lambda,\mu) = e^{-l_i^{\mathrm{T}}(\mu)} l_i^{\mathrm{E}}(\lambda) + r_i^{\mathrm{E}}$$

Transmission scan:

$$Y_i^{\mathrm{T}} \sim \mathrm{Poisson}\{\bar{y}_i^{\mathrm{T}}(\lambda,\mu)\}, \quad \bar{y}_i^{\mathrm{T}}(\lambda,\mu) = b_i e^{-l_i^{\mathrm{T}}(\mu)} + \kappa_i e^{-l_i^{\mathrm{T}}(\mu)} l_i^{\mathrm{E}}(\lambda) + r_i^{\mathrm{T}}$$

- λ $[\lambda_1,\ldots,\lambda_p]$ unknown emission pixel values
- Y_i^{E} recorded emission counts by ith detector pair, $i = 1,\ldots,N$
- Y_i^{T} recorded transmission counts by ith detector pair, $i = 1,\ldots,N$
- l_i^{E} $= \sum_{j=1}^{p} a_{ij}^{\mathrm{E}} \lambda_j$. reprojection of emission distribution (including efficiency)
- l_i^{T} $= \sum_{j=1}^{p} a_{ij}^{\mathrm{T}} \mu_j$. reprojection of attenuation map $\mu = [\mu_1,\ldots,\mu_p]$
- r_i^{E} contribution of randoms and scatter to emission scan
- r_i^{T} contribution of randoms and scatter to transmission scan
- κ_i loss of emission coincidences due to rod windowing/absorption
- b_i blank scan value for ith detector element

Goal: reconstruct emission image λ and attenuation map μ from $\{Y_i^{\mathrm{E}}, Y_i^{\mathrm{T}}\}_{i=1}^{N}$

Conventional Sequential Approach

- Carson, JNM, 1988. Daube-Witherspoon, T-NS, 1988. (Brain imaging)
- Subtract (scaled) emission sinogram from transmission scan
- Scaling accounts for deadtime, scan durations, decay, rod windowing, etc.
- Reconstruct attenuation map from “corrected” sinogram
- Form attenuation correction factors
- Apply to emission sinogram and reconstruct emission image $\hat{\lambda}$

- Method of moments: disregards measurement noise statistics
- Subtraction (further) destroys Poisson statistics of transmission sinogram
- Emission contamination high where transmission scan values are low
- Negatives in “corrected” transmission scan problematic
- Smoothing reduces spatial resolution, can induce artifacts
- May require unreasonably long transmission scans for whole-body studies

Joint Maximum-Likelihood Reconstruction

$$(\hat{\lambda}, \hat{\mu}) \triangleq \arg \max_{\lambda \geq \underline{0},\ \mu \geq \underline{0}} L(\lambda, \mu)$$ (Log-likelihood)

Statistical independence of emission and transmission scans:

$$L(\lambda, \mu) = L^{\mathrm{E}}(\lambda, \mu) + L^{\mathrm{T}}(\lambda, \mu)$$

$$L^{\mathrm{E}}(\lambda, \mu) = \sum_{i=1}^{N} Y_i^{\mathrm{E}} \log \left[e^{-l_i^{\mathrm{T}}(\mu)} l_i^{\mathrm{E}}(\lambda) + r_i^{\mathrm{E}} \right] - \bar{y}_i^{\mathrm{E}}(\lambda, \mu)$$

$$L^{\mathrm{T}}(\lambda, \mu) = \sum_{i=1}^{N} Y_i^{\mathrm{T}} \log \left[b_i e^{-l_i^{\mathrm{T}}(\mu)} + \kappa_i e^{-l_i^{\mathrm{T}}(\mu)} l_i^{\mathrm{E}}(\lambda) + r_i^{\mathrm{T}} \right] - \bar{y}_i^{\mathrm{T}}(\lambda, \mu)$$

- Joint log-likelihood is non-concave
- Use paraboloidal surrogates to form a monotonic algorithm
- Convergence to local maximum

Alternating-Maximization Approach

- Form initial conventional emission image / attenuation map estimates
- Update emission image using most recent attenuation map

$$\lambda^{n+1} = \arg\max_{\lambda \geq \underline{0}} L(\lambda, \mu^n)$$

- Update attenuation map using most recent emission image

$$\mu^{n+1} = \arg\max_{\mu \geq \underline{0}} L(\lambda^{n+1}, \mu)$$

- Repeat as necessary

- In practice, we replace “max” with “increase”
- Guaranteed to monotonically increase the joint log-likelihood

- For fixed μ^n, $L(\lambda, \mu^n)$ has the usual form of emission log-likelihood.
- For fixed λ^n, $L(\lambda^n, \mu)$ is very similar to usual transmission log-likelihood.
  $\therefore$ Apply EM, CG, SAGE, PSCA, ...

Alternative Pairwise Maximization Approach

- Form appropriate surrogate function (paraboloids?)
- Sequentially update (λ_1, μ_1), (λ_2, μ_2), ... (λ_p, μ_p)

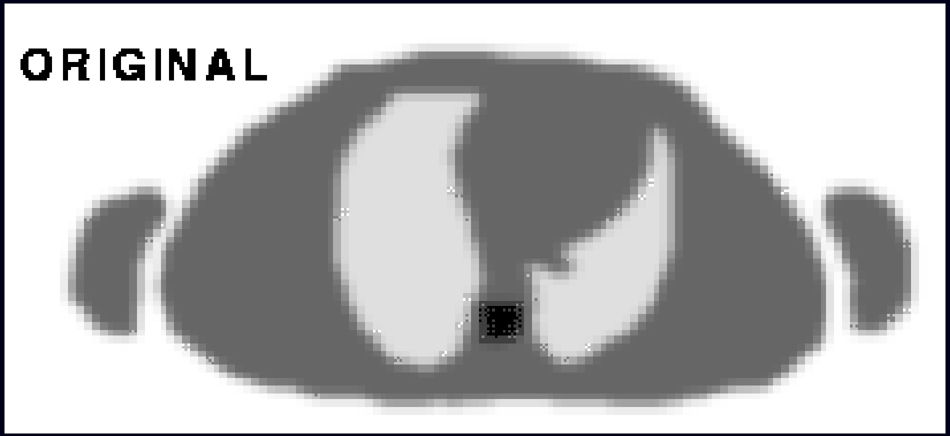
- More complicated to derive/implement
- May converge faster due to inherent coupling between $\hat{\lambda}_j$ and $\hat{\mu}_j$
- Regularization is essential!

- Better conditioned than “sourceless” attenuation correction...

Challenges

- Precorrected random coincidences
- Dynamics of emission distribution
- Determining κ_i: rod windowing factors, deadtime, etc.
- Obtaining good initial estimates
- Matching (?) the spatial resolutions of attenuation correction factors and emission measurements
- Demonstrating convincingly that joint estimation outperforms a “good” sequential approach based on approximate statistical models
- One “iteration” of the alternating-maximization method works well

Simulation



Attenuation map

Emission distribution

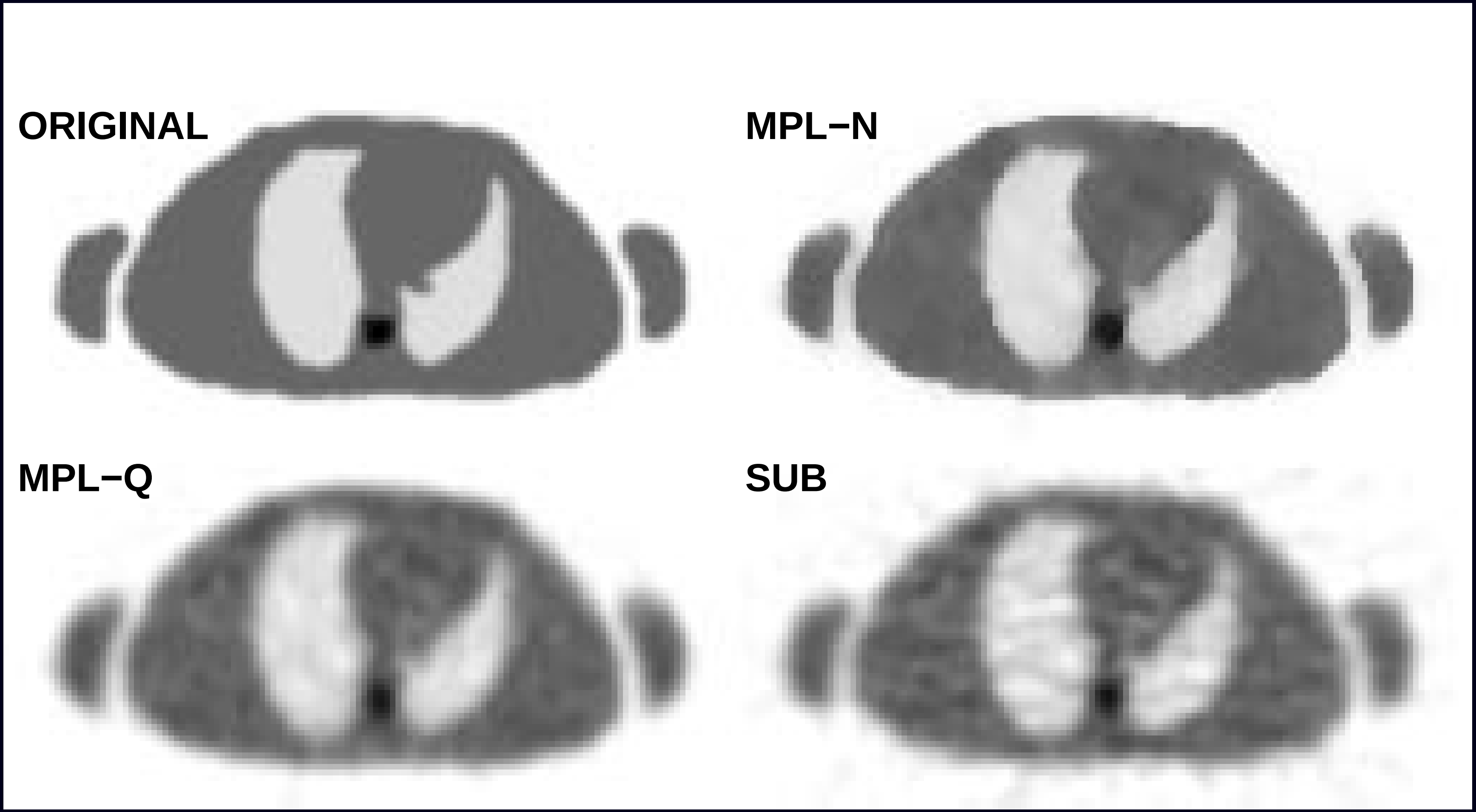
- 128×64 4.22mm pixels
- 192×160 sinogram (CTI ECAT EXACT)
- 1M emission counts, 10% random coincidences
- 2M transmission counts, 10% emission contamination
- 100 pseudo-random Poisson realizations

Reconstruction Methods

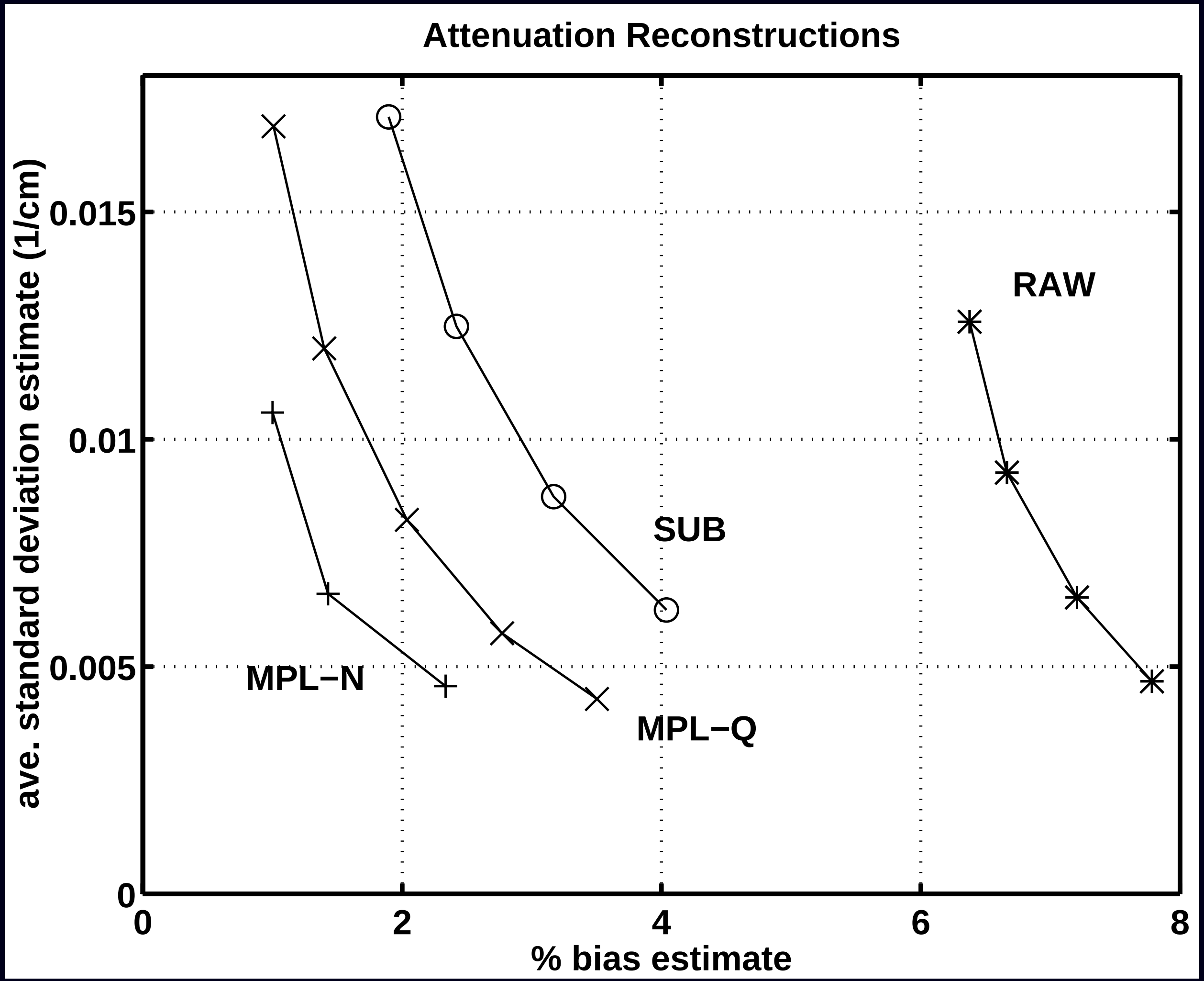
- RAW: no correction for of emission contamination
- SUB: simple subtraction of emission contamination, FBP reconstruction
- MPL-Q
  - model emission contamination (estimated from emission scan)
  - reconstruction μ-map using quadratic penalty
  - reproject to form ACFs
  - PL emission reconstruction with quadratic penalty
- MPL-N
  - same except for nonquadratic penalty for attenuation map

MPL-Q and MPL-N correspond to one iteration of joint estimation

Simulation Results: Attenuation Maps

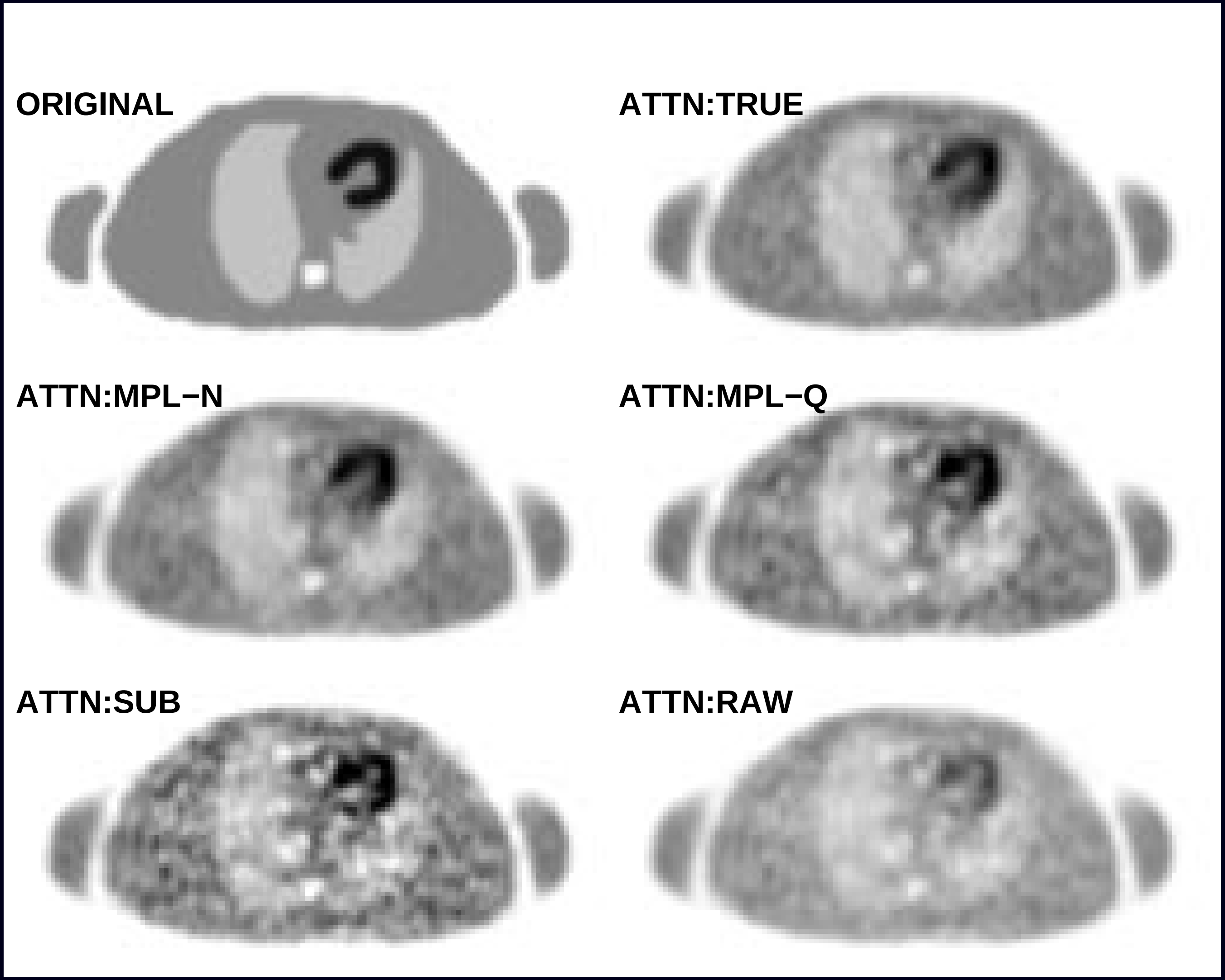


Simulation Results: Attenuation Maps

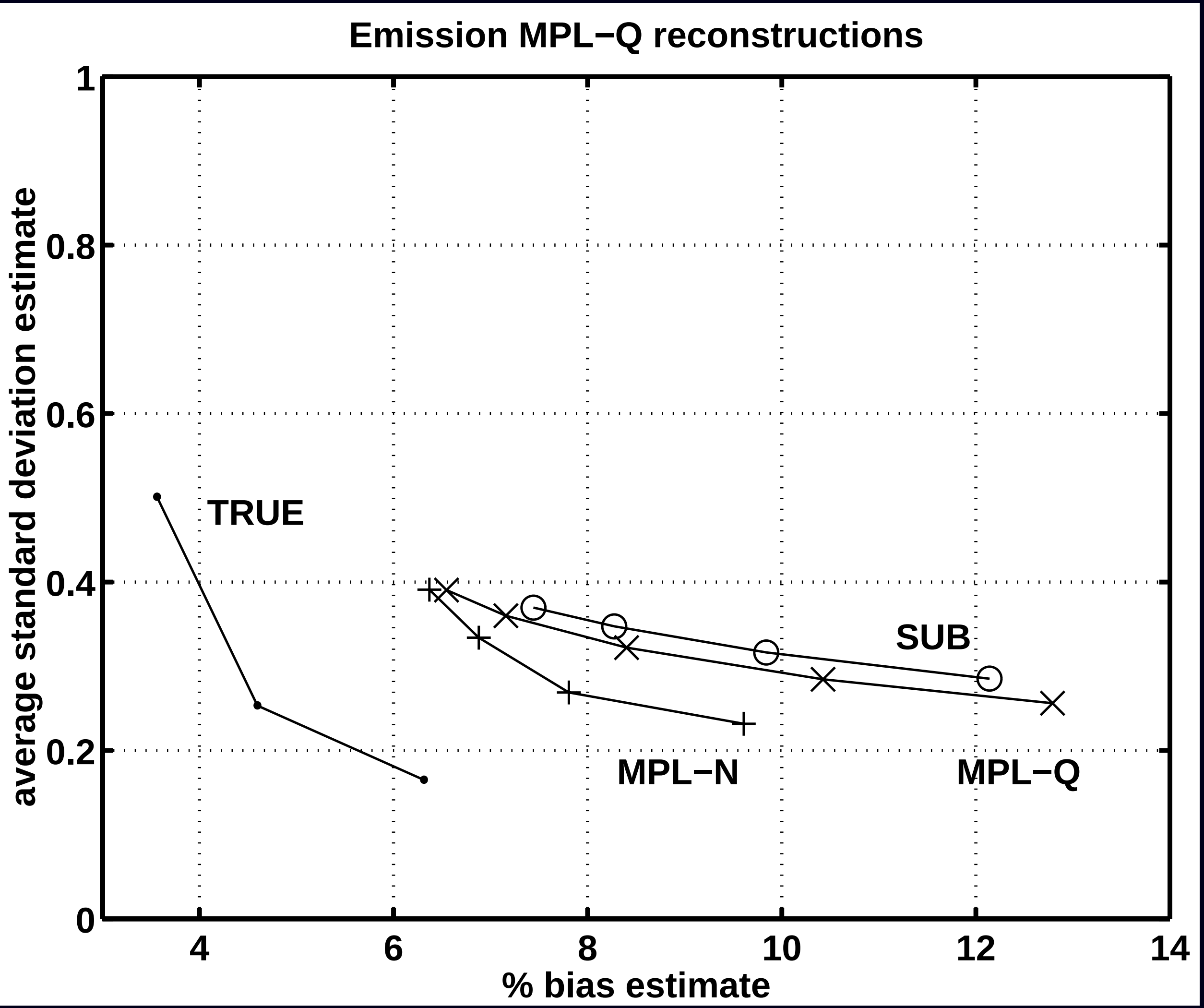


normalized average absolute bias over large rectangular ROI

Simulation Results: Emission Images



Simulation Results: Emission Images



Summary

- Method for jointly estimating attenuation map and emission distribution from emission scan and post-injection transmission scan
- Intrinsically monotonic increase in joint log-likelihood
- Simple one-step version yields improved bias/variance tradeoffs over conventional approaches