

# Joint optimization of learning-based image reconstruction and k-space trajectories for MRI

J. Fessler  
Joint Opt



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Brain



Knee

Acknowledgments:

**Guanhua Wang**, Tianrui Luo, Jon Nielsen, Doug Noll

## Introduction

- Machine learning in imaging
- MRI k-space sampling

## Deep-learning approaches for image reconstruction

## Supervised learning of k-space sampling

## Joint optimization of k-space sampling and image reconstruction

- Problem formulation
- Results

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Most obvious place for machine learning is in post-processing (image analysis). Numerous special issues and surveys in medical imaging journals, e.g., [1–9].



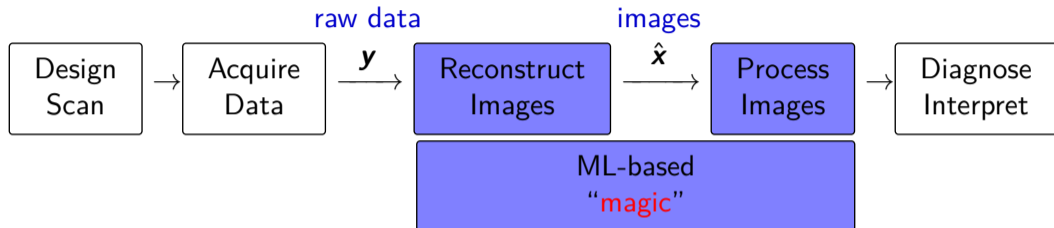
Machine learning for scan design (k-space sampling):

- ▶ Choose best k-space phase encoding locations (usually Cartesian sampling)
- ▶ Uses “ground truth” (fully sampled) training images
- ▶ Hot topic in MRI research recently, e.g., [10–15]
- ▶ Precursor by Yue Cao and David Levin, MRM Sep. 1993 [16–18]



Machine learning in medical image reconstruction:

- ▶ June 2018 special issue of IEEE Trans. on Medical Imaging [19].
- ▶ Surveys: [20–27]
- ▶ Possibly easier than diagnosis due to lower bar:
  - current reconstruction methods based on simplistic image models;
  - human eyes are better at detection than at solving inverse problems.



A holy grail for machine learning in medical imaging?

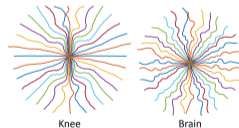
- ▶ CT sinogram to vessel diameter [28, 29]
- ▶ k-space to ???





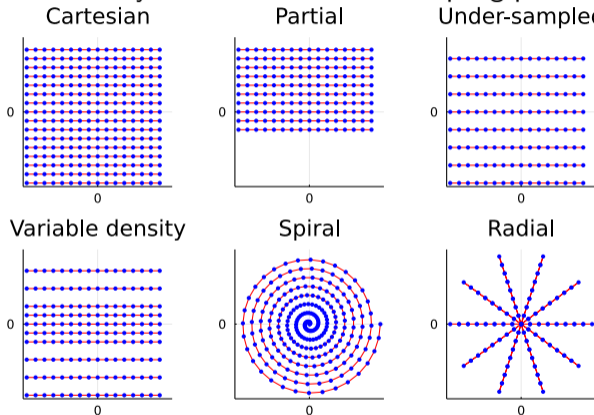
“B-spline parameterized joint optimization of reconstruction and k-space trajectories (BJORK) for accelerated 2D MRI,” arXiv 2101.11369 [30] IEEE T-MI 2022 [31]  
Guanhua Wang, T. Luo, J.-F. Nielsen, D. Noll, J. Fessler

Preview:



Related work: “PILOT” by Weiss et al. [32]; J-MoDL work of Aggarwal et al. [14].

All clinical MRI scans currently use “hand-crafted” sampling patterns:



- ▶ Reducing k-space sampling  $\implies$  reduced scan time / improved temporal resolution
- ▶ Under-sampled data benefits from advanced reconstruction methods

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## Overview:

- ▶ image-domain learning [33–35]...
- ▶ k-space or data-domain learning  
e.g., [36], [37], [38]
- ▶ transform learning (direct from k-space to image)  
e.g., AUTOMAP [39], [40–42]
- ▶ hybrid-domain learning (unrolled loop, e.g., variational network)  
alternate between denoising/dealiasing and reconstruction from k-space  
e.g., [37, 43–47] ...

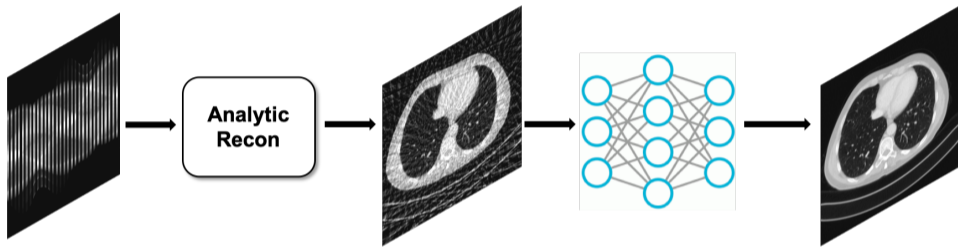


Figure courtesy of Jong Chul Ye, KAIST University.

- + simple and fast
- aliasing is spatially widespread, requires deep network

## Investigating Robustness to Unseen Pathologies in Model-Free Deep Multicoil Reconstruction

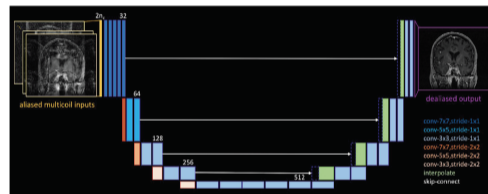
Gopal Nataraj<sup>1</sup> and Ricardo Otazo<sup>1,2</sup>

<sup>1</sup>Dept. of Medical Physics, Memorial Sloan Kettering Cancer Center

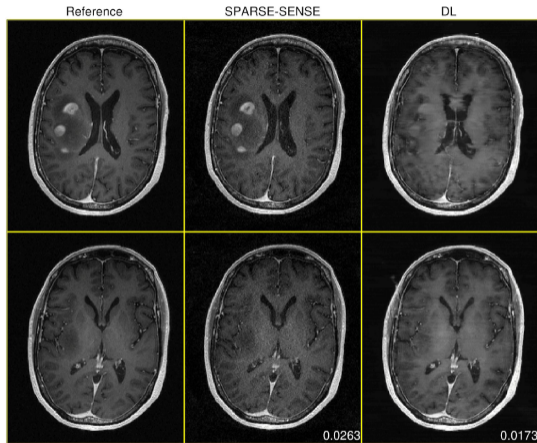
<sup>2</sup>Dept. of Radiology, Memorial Sloan Kettering Cancer Center

### Introduction

Speed is often claimed as a key advantage of deep learning (DL) for undersampled parallel MRI reconstruction [1]. However, the only DL approach that to our knowledge has studied generalizability to pathologies unseen in training [2] requires repeated application of the MR acquisition model and its adjoint, just as in iterative methods. In contrast, model-free DL reconstruction has the potential to be much faster. Prior model-free DL work [3] proposes to learn a mapping directly from k-space, but with



[48] ISMRM 2020 Workshop on Data Sampling & Image Reconstruction



**Figure 3:** Reconstructions in a case of anaplastic astrocytoma, a rare malignant brain tumor. SPARSE-SENSE and DL reconstructions are from the same 4x-accelerated retrospectively undersampled acquisition. DL achieves lower whole-volume MAE than SPARSE-SENSE, but fails to properly reconstruct regions near the tumor.

- ▶ Use NN output as a “prior” for iterative reconstruction [33, 49]:

$$\hat{\mathbf{x}}_{\beta} = \arg \min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \beta \|\mathbf{x} - \mathbf{x}_{\text{NN}}\|_2^2 = (\mathbf{A}'\mathbf{A} + \beta\mathbf{I})^{-1}(\mathbf{A}'\mathbf{y} + \beta\mathbf{x}_{\text{NN}})$$

- ▶ For single-coil Cartesian case:
  - no iterations are needed (solve with FFTs)
  - $\lim_{\beta \rightarrow 0} \hat{\mathbf{x}}_{\beta}$  replaces missing k-space data with FFT of  $\mathbf{x}_{\text{NN}}$
- ▶ Iterations needed for parallel MRI and/or non-Cartesian sampling (PCG)
  
- ▶ Learn residual (aliasing artifacts), then subtract [50, 51]



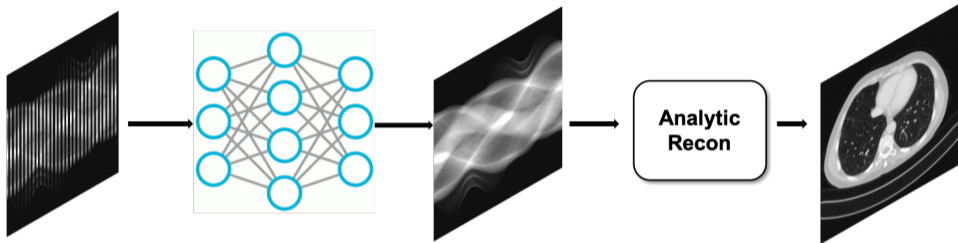


Figure courtesy of Jong Chul Ye, KAIST University.

- + simple and fast (“nonlinear GRAPPA”)
- + “database-free” : learn from auto-calibration data [36], [37], [38]
- perhaps harder to represent local image features?

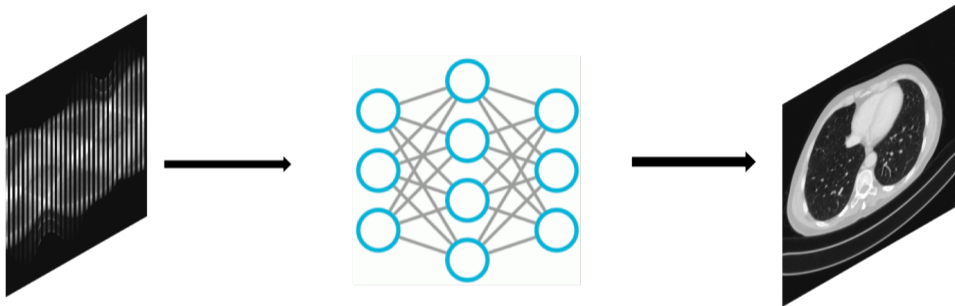


Figure courtesy of Jong Chul Ye, KAIST University.

- + in principle, purely data driven; potential to avoid model mismatch
- high memory requirement for fully connected layers [39]

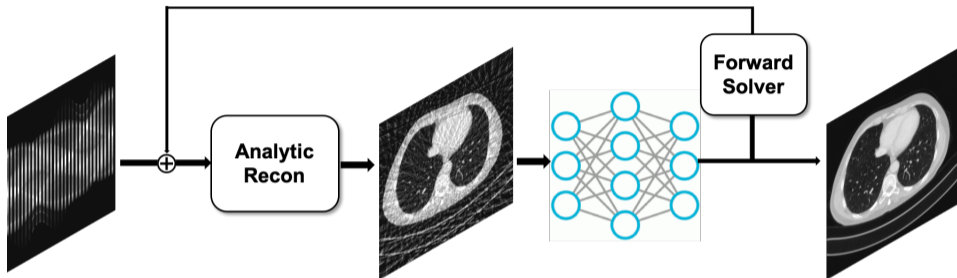
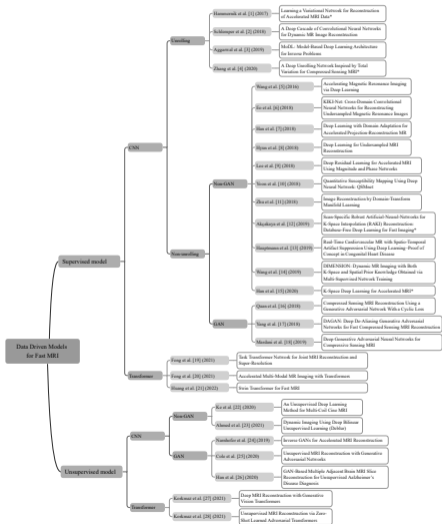


Figure courtesy of Jong Chul Ye, KAIST University.

- + physics-based use of k-space data & image-domain priors, e.g., [37, 43–47, 52, 53] ...
- + interpretable connections to optimization approaches
- + best results in MRI recon challenges [54–56]
- more computation to due to “iterations” (hyper-layers) and repeated  $\mathbf{Ax}$ ,  $\mathbf{A}'r$

Huang et al., arXiv 2204.01706,  
Apr. 2022 [57]



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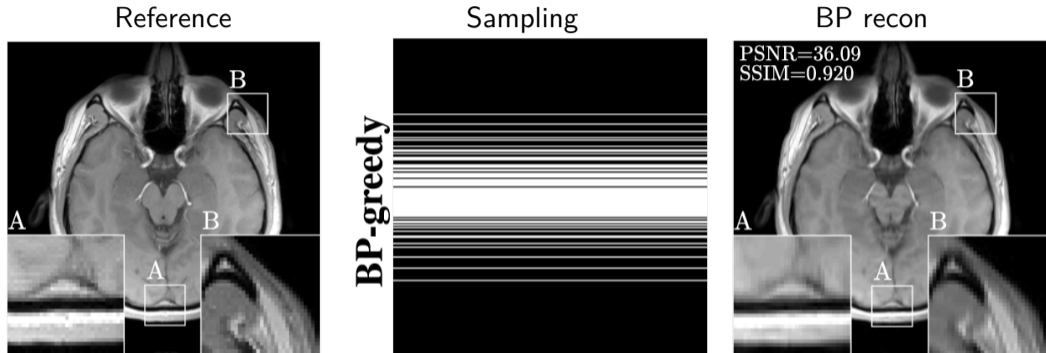
## Pre-specified image reconstruction methods

### Cartesian sampling pattern optimization

- ▶ Yue Cao & David Levin, MRM Sep. 1993 [16–18]  
Feature recognizing MRI
- ▶ Seeger et al., MRM 2010 [58]  
Single coil, 1D Cartesian, 2D spiral angles, CS-type recon, Bayesian information gain
- ▶ Ravishankar & Bresler, EMBS 2011 [10]  
Single coil, 1D & 2D sampling, DLMRI recon (DL = dictionary learning), weighted k-space loss
- ▶ Baldassarre . . . Cevher, IEEE J-STSP 2016 [11]  
Single coil, 2D sampling, energy preserving criterion

## Joint learning of sampling and reconstruction

- ▶ Gözcü . . . Cevher, IEEE T-MI 2018 [12]  
Single coil, 1D sampling, several fixed recon methods (TV, BP, BM3D, image-domain NN), image-domain training loss
- ▶ Aggarwal & Jacob IEEE J-STSP 2020 [14] (J-MoDL)  
Multi-coil, 1D (horizontal and vertical) sampling, MoDL recon
- ▶ Bahadir . . . Sabuncu, IEEE T-CI 2020 [15] (LOUPE)  
Single coil, 1D & 2D sampling, IFFT/U-Net recon
- ▶ Weiss et al., arXiv 1909.05773 (2019, 2020, 2021) (PILOT) [32]  
Single coil, Non-Cartesian sampling, IFFT/U-Net recon
- ▶ Wang . . . Fessler, ISMRM 2021, arXiv 2021 (BJORK) [30, 59]  
Multi-coil, Non-Cartesian sampling, MoDL-type recon  
Fast and efficient DFT Jacobian approximations [60, 61]



- ▶ Sampling designed to optimize PSNR for basis pursuit (BP) reconstruction using shearlet transform, at 25% sampling rate.
- ▶ Sampling design considers both the training data and the reconstruction method.
- ▶ No high spatial frequencies!?

(Images from Gözcü et al. [12].)



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► MRI measurement model:

$$\mathbf{y} = \mathbf{A}(\boldsymbol{\omega})\mathbf{x} + \boldsymbol{\varepsilon}$$

- $\mathbf{y} \in \mathbb{C}^M$  : k-space data;  $M \sim 10 - 30\text{K}$
- $\boldsymbol{\omega} \in \mathbb{R}^{M \times 2}$  : k-space sampling pattern (“trajectory”): 2D in this work
- $\mathbf{x} \in \mathbb{C}^N$  : unknown true image,  $N \sim 100\text{K}$
- $\mathbf{A}(\boldsymbol{\omega}) \in \mathbb{C}^{M \times N}$  : encoding matrix (coil sensitivity, etc.)
- $\boldsymbol{\varepsilon} \in \mathbb{C}^M$  : measurement noise

► Reconstruction method:

$$\hat{\mathbf{x}} = f(\mathbf{y}; \boldsymbol{\omega}, \boldsymbol{\theta})$$

- $\boldsymbol{\theta}$ : model parameters of reconstruction method (e.g., CNN weights)
- Deep iterative down-up CNN (DIDN) has  $\sim 165\text{M}$  learned parameters [62]

► Image quality goal:

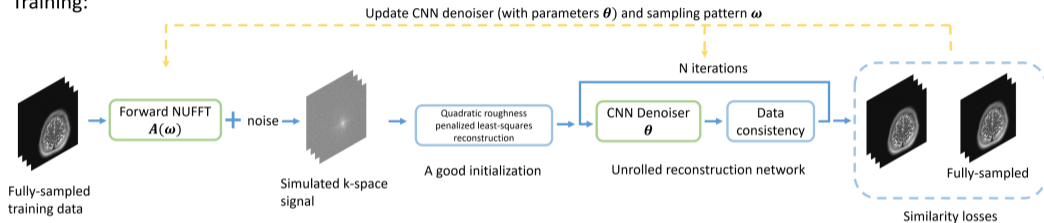
$$\hat{\mathbf{x}} = f(\mathbf{y}; \boldsymbol{\omega}, \boldsymbol{\theta}) = f(\mathbf{A}(\boldsymbol{\omega})\mathbf{x} + \boldsymbol{\varepsilon}; \boldsymbol{\omega}, \boldsymbol{\theta}) \approx \mathbf{x}$$

- ▶ Define training loss function such as  $\ell(\hat{\mathbf{x}}, \mathbf{x}) = \|\hat{\mathbf{x}} - \mathbf{x}\|_1 + \alpha \|\hat{\mathbf{x}} - \hat{\mathbf{x}}\|_2$
- ▶ Select  $N_{\text{train}}$  fully sampled training images  $\mathbf{x}_1, \mathbf{x}_2, \dots$
- ▶ Jointly optimize k-space trajectory  $\omega$  and image reconstruction method  $\theta$

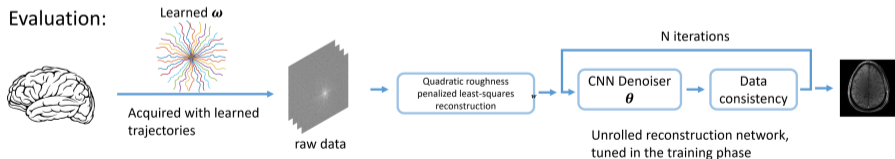
$$(\hat{\omega}, \hat{\theta}) = \arg \min_{\omega, \theta} \frac{1}{N_{\text{train}}} \sum_{n=1}^{N_{\text{train}}} \ell(f(\mathbf{A}(\omega)\mathbf{x}_n + \varepsilon_{ni}, \omega, \theta), \mathbf{x}_n)$$

- ▶ Details:
  - Reconstruction using MoDL method [52]
  - Can use multiple noise realizations  $\varepsilon$  per training image
  - Enforce gradient amplitude and slew-rate limits for  $\omega$
  - Use B-spline parameterization of k-space trajectory
  - Coarse-to-fine search of trajectory to avoid poor local minimizers
  - Eddy current correction
  - Fast NUFFT Jacobian approximation [60, 61, 63]

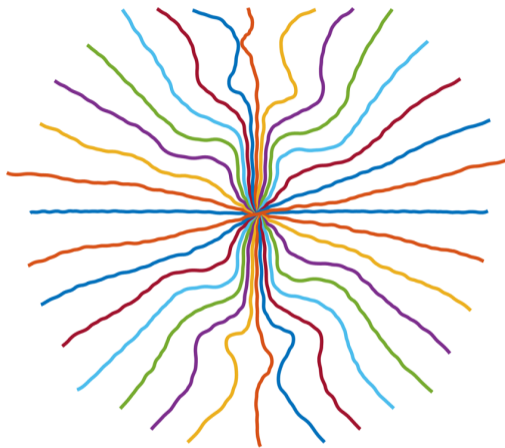
## Training:



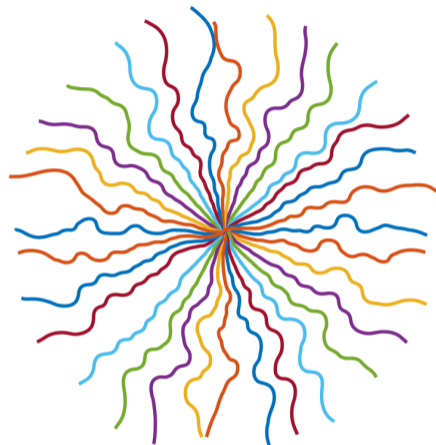
## Evaluation:



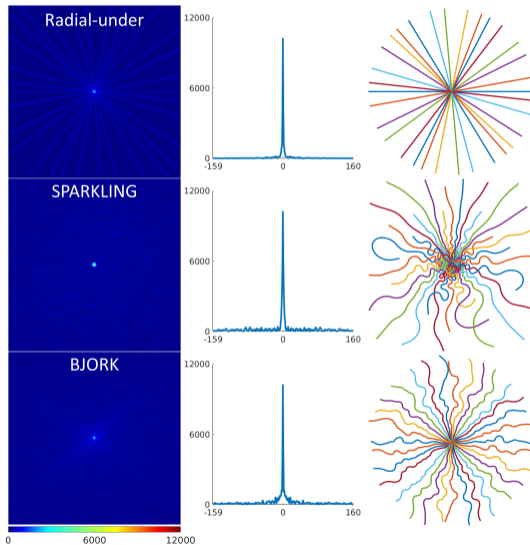
- ▶ NYU/FAIR fastMRI brain and knee data
- ▶ 16 or 32 radial spokes of 640 points for trajectory initialization ( $\approx 20$  or  $10\times$  acceleration for  $320 \times 320$  image)
- ▶ 22cm FOV,  $G_{\max} = 5$  Gauss/cm, slew rate  $\leq 15$  Gauss/cm/ms
- ▶ 2.5ms readout duration radial, 16ms spiral
- ▶ Comparison with SPARKLING approach of [64] using its default density



Knee



Brain



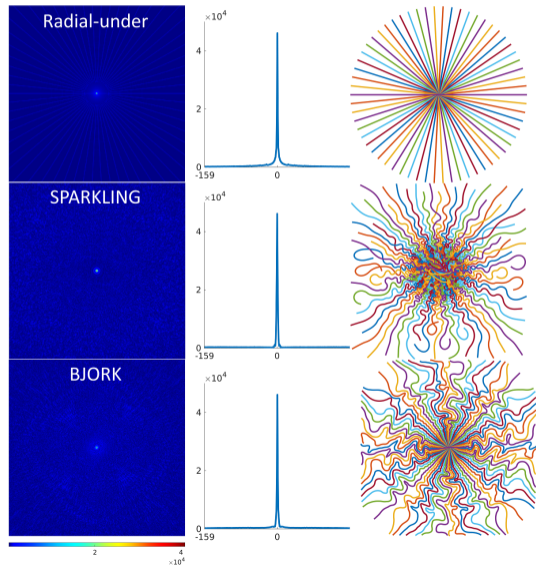
32-spoke results

FWHM (pixels):

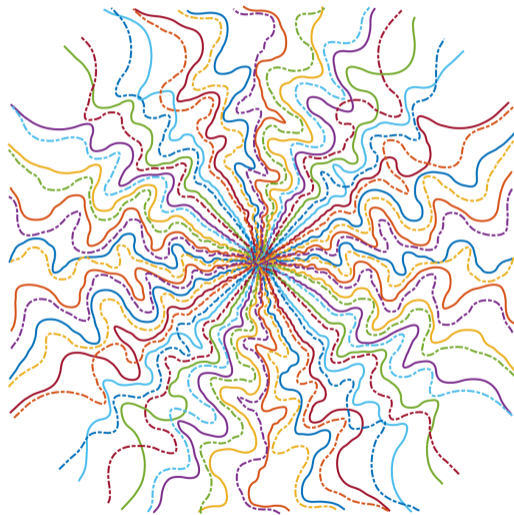
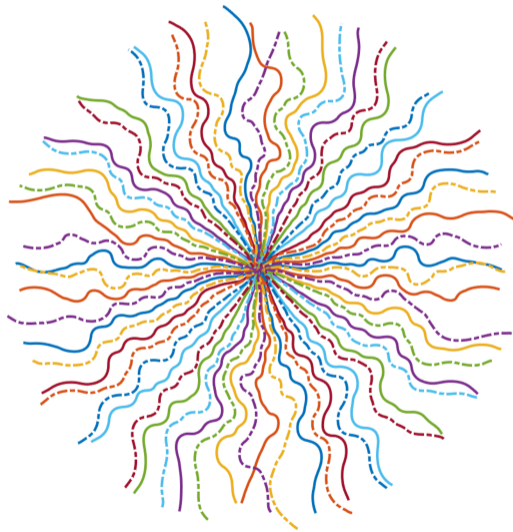
1.5

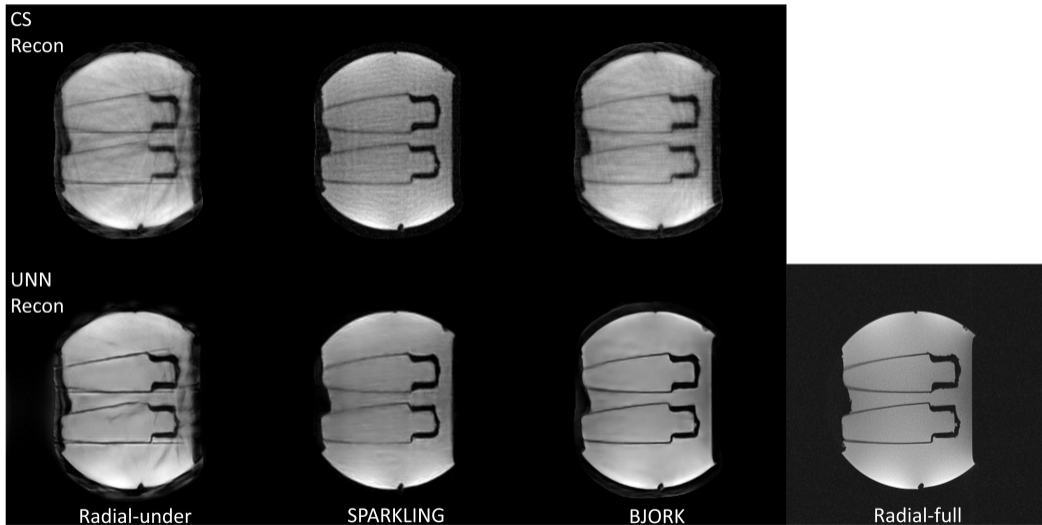
2.1

1.6

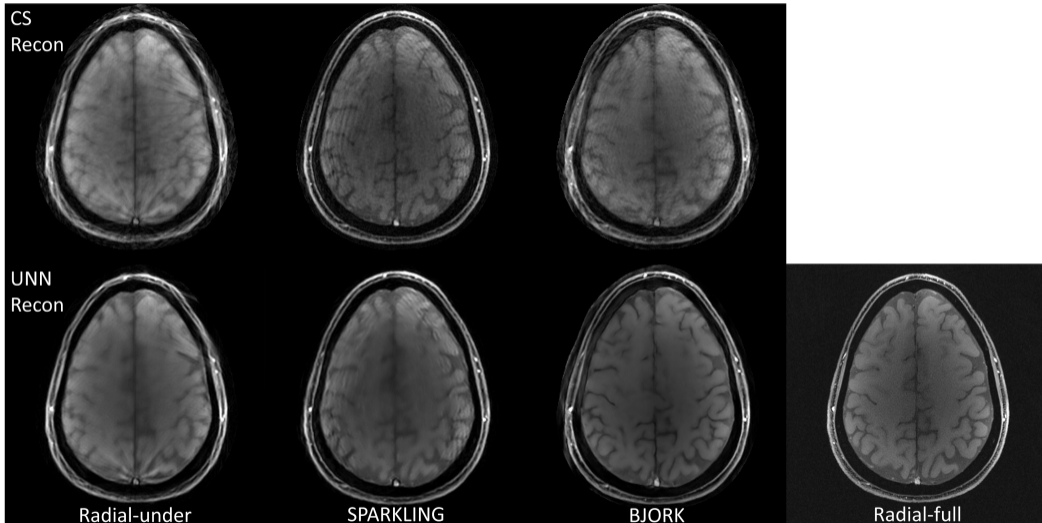




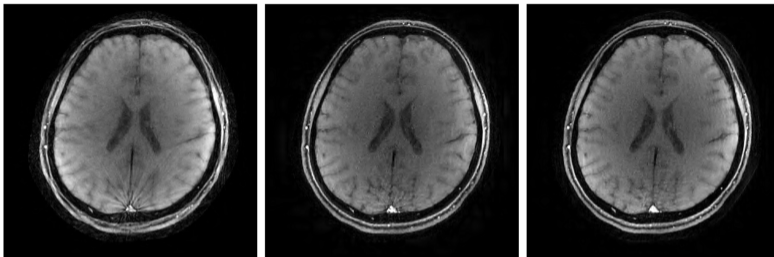




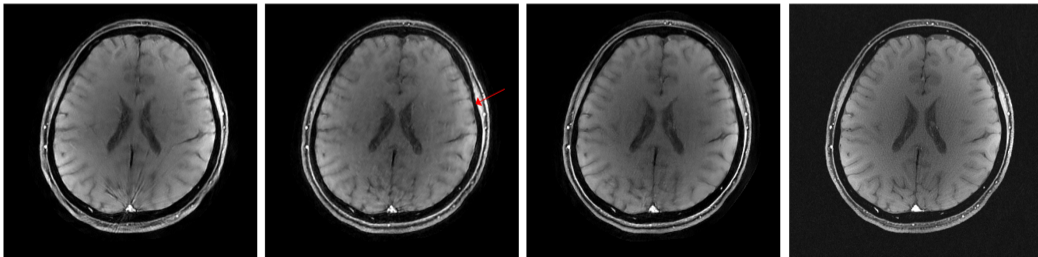
(no phantoms in training data!) (no CS recon in training)



CS Recon



UNN Recon



Radial-under

SPARKLING

RIOBK

Radial-full

SSIM:

		Standard	SPARKLING	BJORK
radial-like	UNN	0.957	0.963	<b>0.968</b>
	CS	0.932	0.946	<b>0.956</b>
spiral-like	UNN	0.986	0.989	<b>0.990</b>
	CS	0.976	0.978	<b>0.981</b>

PSNR (in dB):

		Standard	SPARKLING	BJORK
radial-like	UNN	35.0	36.0	<b>36.9</b>
	CS	33.9	35.7	<b>36.3</b>
spiral-like	UNN	40.9	41.7	<b>41.9</b>
	CS	39.9	40.4	<b>40.7</b>

MRI gradient amplifiers have maximum amplitude and slew rate

- ▶ gradient amplitude is 1st derivative of k-space trajectory:

$$\|D_1\omega_d\|_\infty \leq g_{\max}$$

- ▶ slew rate is 2nd derivative of k-space trajectory:

$$\|D_2\omega_d\|_\infty \leq s_{\max}$$

- ▶ Box constraints relaxed to penalty functions that rise rapidly above 1, on:

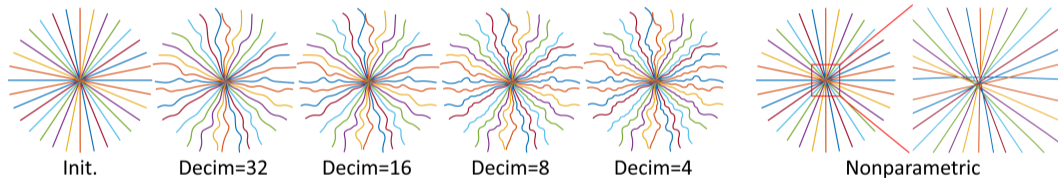
$$\|D_1\omega_d\|_\infty / g_{\max} \text{ and } \|D_2\omega_d\|_\infty / s_{\max}$$



Facilitates (sub)gradient-based optimization using Adam

Quadratic B-spline kernels for non-Cartesian k-space trajectory:

$$\omega_d = \mathbf{B}\mathbf{c}_d, \quad d = 1, 2, \mathbf{c}_d \in \mathbb{R}^{M/\text{Decim}}$$



Highly non-convex problem in  $\omega$ .

- Coarse-to-fine search may find better local minimizers
- However, parameterization/decimation does not save much computation

Motivated by model-based image reconstruction with variable splitting of the form

$$\begin{aligned}\hat{\mathbf{x}} &= \arg \min_{\mathbf{x}} \|\mathbf{A}(\boldsymbol{\omega})\mathbf{x} - \mathbf{y}\|_2^2 + R(\mathbf{x}) \\ &= \arg \min_{\mathbf{x}} \min_{\mathbf{z}} \|\mathbf{A}(\boldsymbol{\omega})\mathbf{x} - \mathbf{y}\|_2^2 + R(\mathbf{z}), \quad \text{s.t. } \mathbf{z} = \mathbf{x}\end{aligned}$$

Alternating minimization:

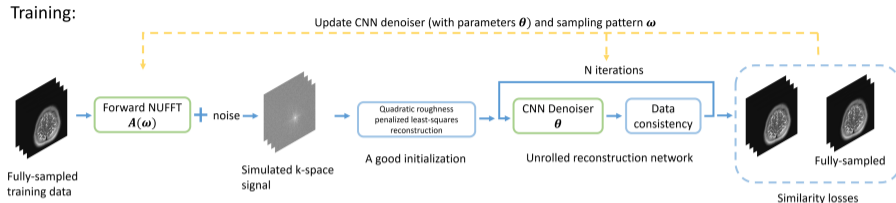
$$\mathbf{x}_{t+1} = \arg \min_{\mathbf{x}} \|\mathbf{A}(\boldsymbol{\omega})\mathbf{x} - \mathbf{y}\|_2^2 + \mu \|\mathbf{x} - \mathbf{z}_t\|_2^2 \quad (\text{data consistency, solved via CG})$$

$$\mathbf{z}_{t+1} = \arg \min_{\mathbf{z}} R(\mathbf{z}) + \mu \|\mathbf{x}_{t+1} - \mathbf{z}\|_2^2 \quad (\text{denoising})$$

$$\text{"} = \mathcal{D}_{\boldsymbol{\theta}}(\mathbf{x}_{t+1}) \quad (\text{CNN denoiser})$$

- CNN weights  $\boldsymbol{\theta}$  shared across iterations, per MODL [52]
- 6 outer iterations for results shown, with augmented Lagrangian parameter  $\mu = 2$





Data consistency block has steps like

$$\mathbf{x}_{t+1} = \mathbf{x}_t + \alpha (\mathbf{A}'(\boldsymbol{\omega}) (\mathbf{A}(\boldsymbol{\omega})\mathbf{x} - \mathbf{y}) + \mu(\mathbf{x} - \mathbf{z}_t))$$

$\mathbf{A}(\boldsymbol{\omega})$  is dense and huge:

$$a_{ij} = e^{-i\vec{\omega}_i \cdot \vec{r}_j} \quad (1)$$

- ▶ Fast approach to  $\mathbf{A}(\boldsymbol{\omega})\mathbf{x}$  uses NUFFT approximation: zero-padding, over-sampled FFT, interpolation [65, 66].
- ▶ Backpropagation for  $\boldsymbol{\omega}$  update through NUFFT steps via autodifferentiation is slow.

Derive Jacobian matrix for exact form (1):

$$\frac{\partial}{\partial \omega_d} \mathbf{A}(\omega) \mathbf{x} = -i \text{Diag}\{\mathbf{A}(\omega)(\mathbf{x} \odot \mathbf{r}_d)\}.$$

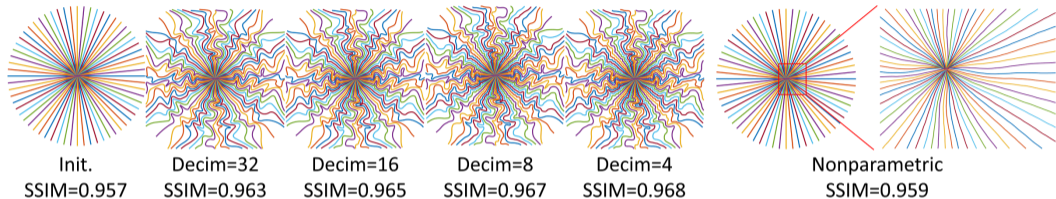
Applying this Jacobian to a vector  $\mathbf{v} \in \mathbb{C}^M$  during backpropagation yields

$$\left( \frac{\partial}{\partial \omega_d} \mathbf{A}(\omega) \mathbf{x} \right) \mathbf{v} = -i \text{Diag}\{\mathbf{A}(\omega)(\mathbf{x} \odot \mathbf{r}_d)\} \mathbf{v} = -i (\mathbf{A}(\omega)(\mathbf{x} \odot \mathbf{r}_d)) \odot \mathbf{v}.$$

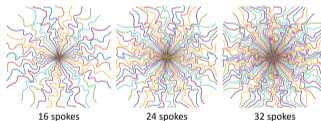
Implemented efficiently using NUFFT applied to  $\mathbf{x} \odot \mathbf{r}_d$

Similar idea for Jacobian of adjoint of  $\mathbf{A}$ .

Even more important is accurately approximating Jacobian of CG solve of  $(\mathbf{A}'\mathbf{A} + \mu\mathbf{I})^{-1}$



Different acceleration factors:



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- ▶ Machine learning methods have much potential for both scan design and image reconstruction
- ▶ Quantitative results in paper demonstrate synergy of jointly optimizing both
- ▶ Anatomy specific trajectories: pro or con?
- ▶ Self-supervised methods when training data unavailable
- ▶ Extension to 3D is in progress  
Also control of peripheral nerve stimulation (PNS)
- ▶ Extension to 3D+time is planned (and challenging)

2D code for BJORK: <https://github.com/guanhuaw/Bjork>

Talk and code available online at  
<http://web.eecs.umich.edu/~fessler>



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