

# Joint optimization of learning-based image reconstruction and k-space trajectories for MRI

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Joint Opt



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## Introduction

- Machine learning in imaging
- MRI k-space sampling

## Deep-learning approaches for image reconstruction

## Supervised learning of k-space sampling

## Joint optimization of k-space sampling and image reconstruction

- Problem formulation
- Results

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Most obvious place for machine learning is in post-processing (image analysis). Numerous special issues and surveys in medical imaging journals, e.g., [1–9].



Machine learning for scan design

Choose best k-space phase encoding locations based on training images

Hot topic in MRI recently [10–15].

Precursor by Yue Cao and David Levin, MRM Sep. 1993 [16–18].



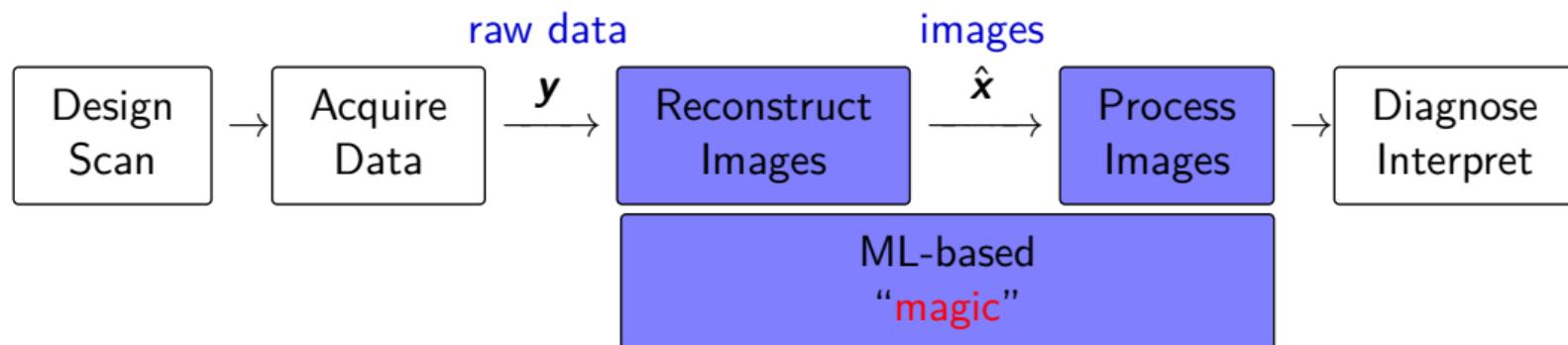
Machine learning in medical image reconstruction

June 2018 special issue of IEEE Trans. on Medical Imaging [19].

Surveys: [20–27]

Possibly easier than diagnosis due to lower bar:

- current reconstruction methods based on simplistic image models;
- human eyes are better at detection than at solving inverse problems.



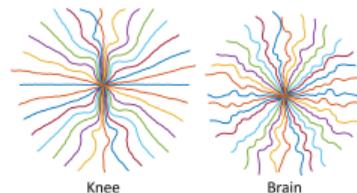
A holy grail for machine learning in medical imaging?

- ▶ CT sinogram to vessel diameter [28, 29]
- ▶ k-space to ???



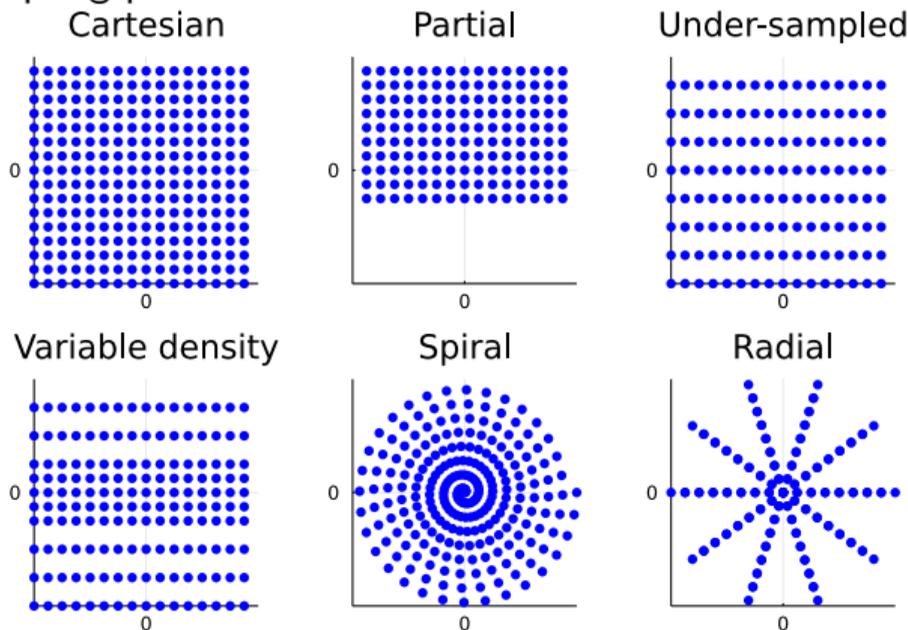
“B-spline parameterized joint optimization of reconstruction and k-space trajectories (BJORK) for accelerated 2D MRI,” arXiv 2101.11369 [30]  
Guanhua Wang, T. Luo, J.-F. Nielsen, D. Noll, J. Fessler

Preview:



Related work: “PILOT” by Weiss et al. [31]; J-MoDL work of Aggarwal et al. [14]

“Hand-crafted” sampling patterns:



Reducing k-space sampling  $\implies$  reduced scan time

Under-sampled data benefits from advanced reconstruction methods

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## Overview:

- ▶ image-domain learning [32–34]...
- ▶ k-space or data-domain learning  
e.g., [35], [36], [37]
- ▶ transform learning (direct from k-space to image)  
e.g., AUTOMAP [38], [39–41]
- ▶ hybrid-domain learning (unrolled loop, e.g., variational network)  
alternate between denoising/dealiasing and reconstruction from k-space  
e.g., [36, 42–46] ...

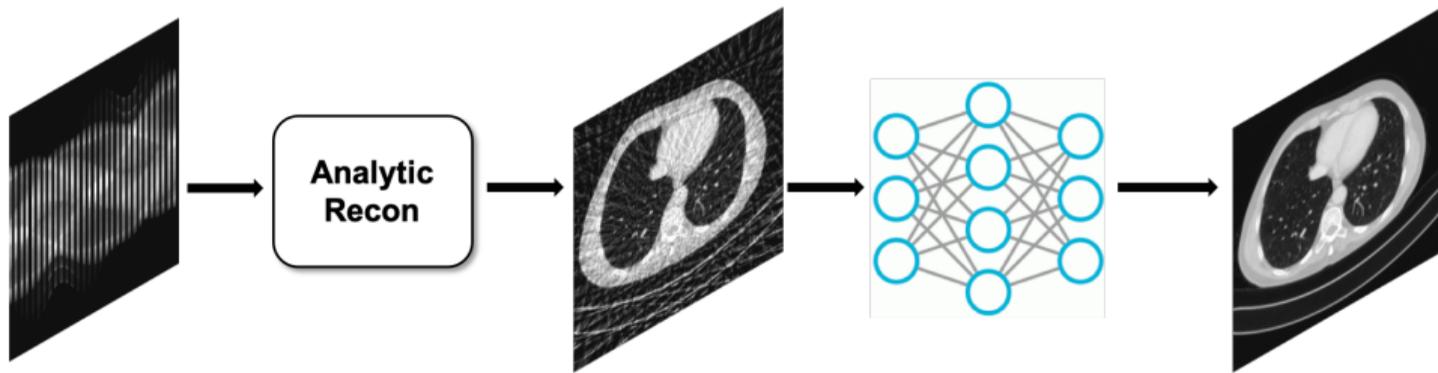


Figure courtesy of Jong Chul Ye, KAIST University.

- + simple and fast
- aliasing is spatially widespread, requires deep network

## Investigating Robustness to Unseen Pathologies in Model-Free Deep Multicoil Reconstruction

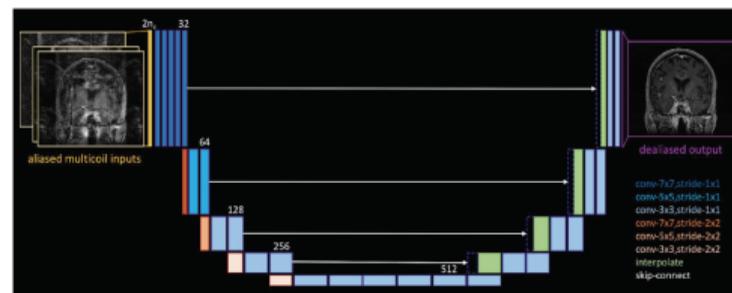
Gopal Nataraj<sup>1</sup> and Ricardo Otazo<sup>1,2</sup>

<sup>1</sup>Dept. of Medical Physics, Memorial Sloan Kettering Cancer Center

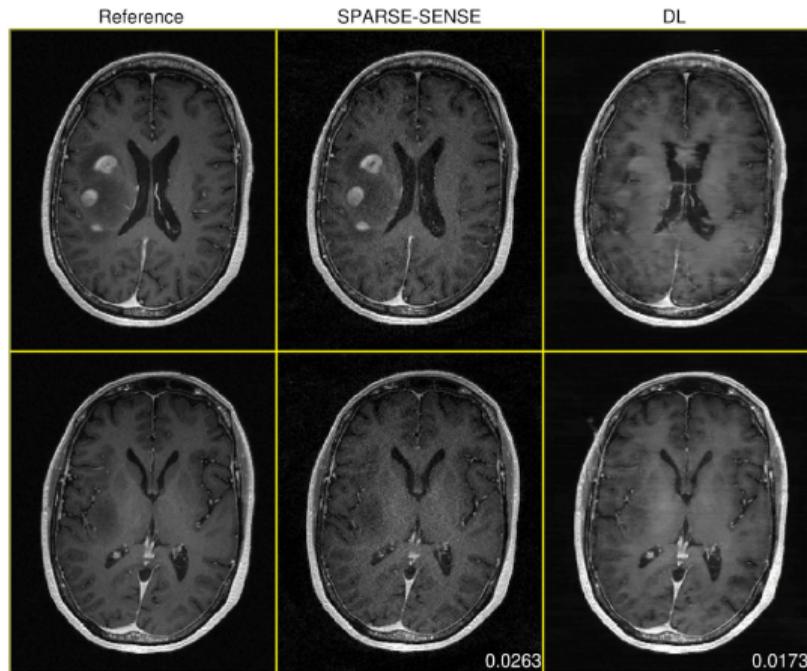
<sup>2</sup>Dept. of Radiology, Memorial Sloan Kettering Cancer Center

### Introduction

Speed is often claimed as a key advantage of deep learning (DL) for undersampled parallel MRI reconstruction [1]. However, the only DL approach that to our knowledge has studied generalizability to pathologies unseen in training [2] requires repeated application of the MR acquisition model and its adjoint, just as in iterative methods. In contrast, model-free DL reconstruction has the potential to be much faster. Prior model-free DL work [3] proposes to learn a mapping directly from k-space, but with



[47] ISMRM 2020 Workshop on Data Sampling & Image Reconstruction



**Figure 3:** Reconstructions in a case of anaplastic astrocytoma, a rare malignant brain tumor. SPARSE-SENSE and DL reconstructions are from the same 4x-accelerated retrospectively undersampled acquisition. DL achieves lower whole-volume MAE than SPARSE-SENSE, but fails to properly reconstruct regions near the tumor.

- ▶ Use NN output as a “prior” for iterative reconstruction [32, 48]:

$$\hat{\mathbf{x}}_{\beta} = \arg \min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \beta \|\mathbf{x} - \mathbf{x}_{\text{NN}}\|_2^2 = (\mathbf{A}'\mathbf{A} + \beta\mathbf{I})^{-1}(\mathbf{A}'\mathbf{y} + \beta\mathbf{x}_{\text{NN}})$$

- ▶ For single-coil Cartesian case:
  - no iterations are needed (solve with FFTs)
  - $\lim_{\beta \rightarrow 0} \hat{\mathbf{x}}_{\beta}$  replaces missing k-space data with FFT of  $\mathbf{x}_{\text{NN}}$
- ▶ Iterations needed for parallel MRI and/or non-Cartesian sampling (PCG)
  
- ▶ Learn residual (aliasing artifacts), then subtract [49, 50]

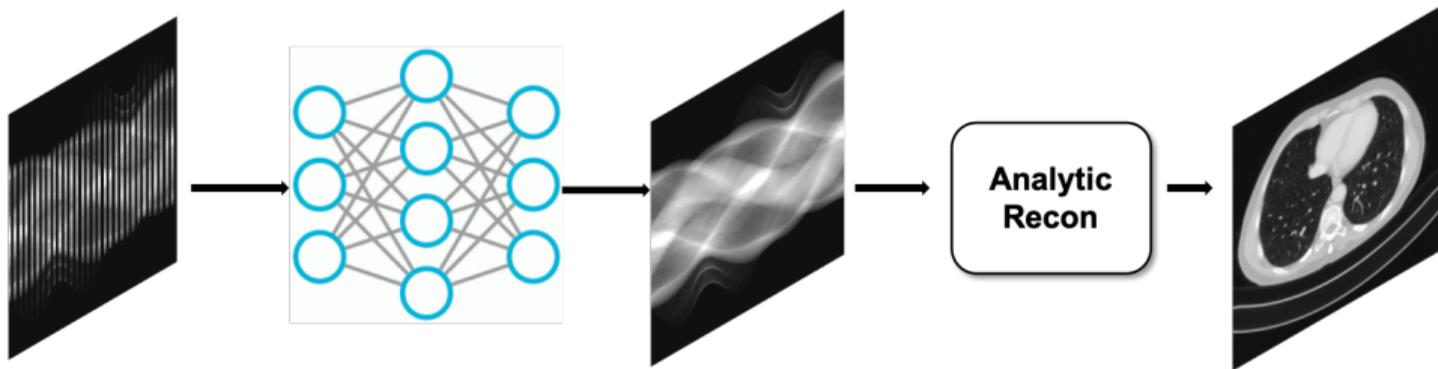


Figure courtesy of Jong Chul Ye, KAIST University.

- + simple and fast (“nonlinear GRAPPA”)
- + “database-free” : learn from auto-calibration data
- perhaps harder to represent local image features?

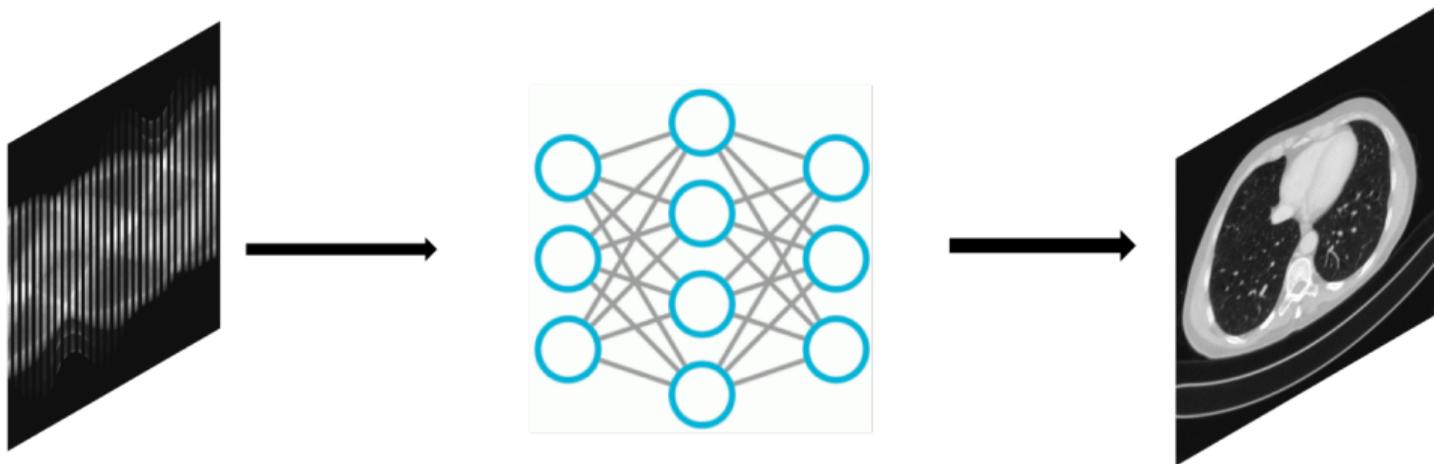


Figure courtesy of Jong Chul Ye, KAIST University.

- + in principle, purely data driven; potential to avoid model mismatch
- high memory requirement for fully connected layers

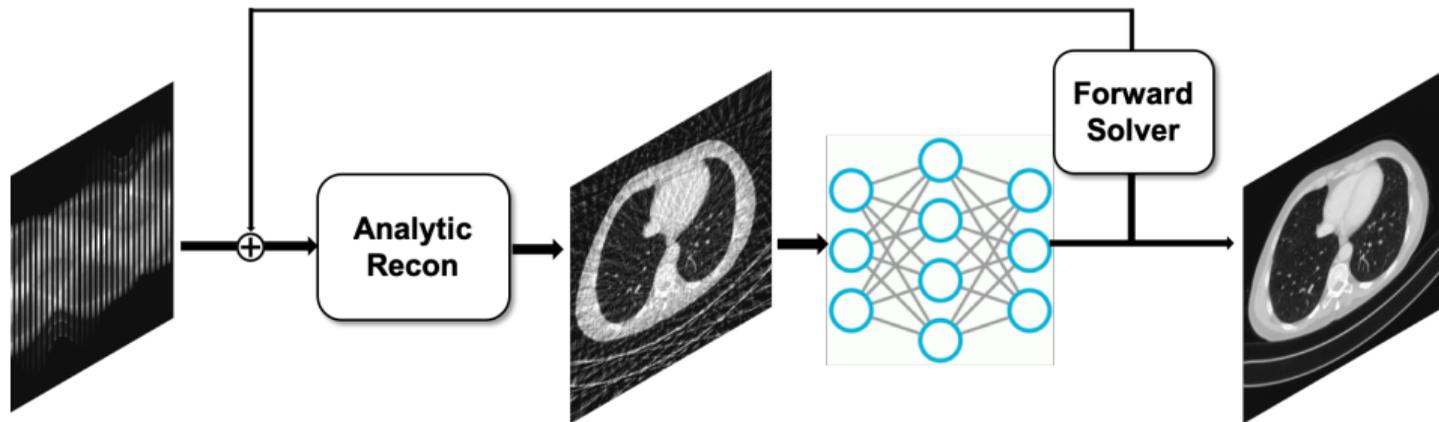


Figure courtesy of Jong Chul Ye, KAIST University.

- + physics-based use of k-space data & image-domain priors
- + interpretable connections to optimization approaches
- more computation to due to “iterations” (layers) and repeated  $\mathbf{Ax}$ ,  $\mathbf{A}'r$

Introduction

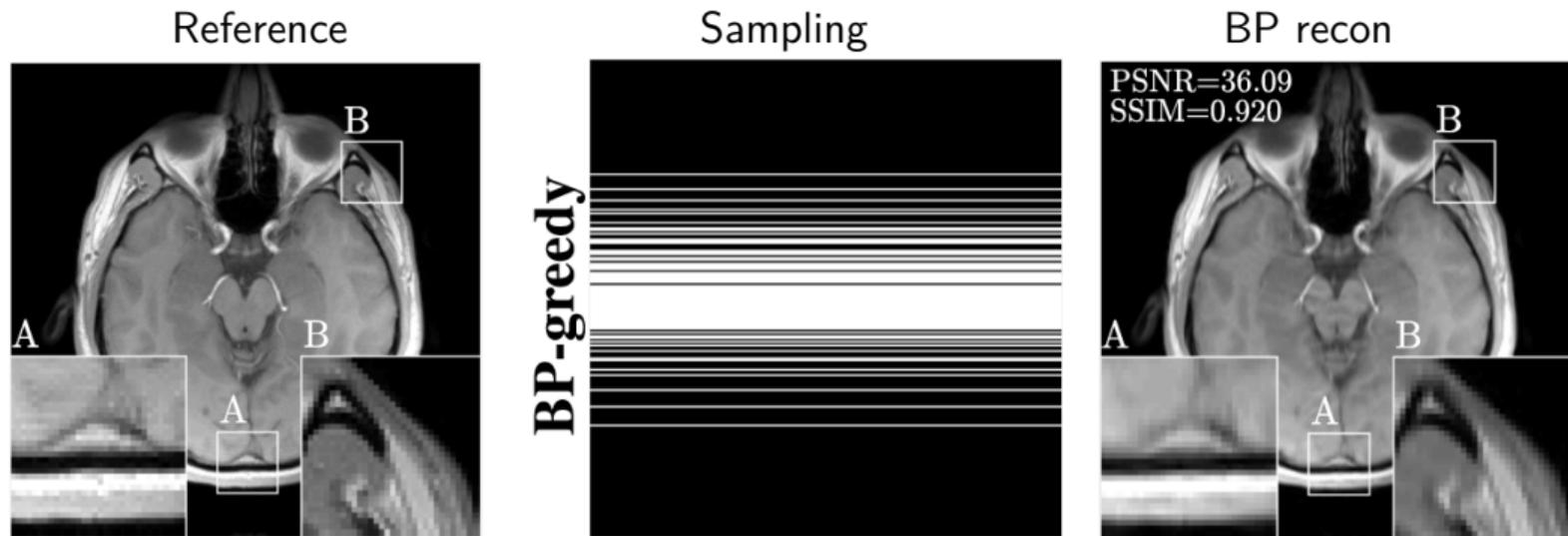
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Sampling designed to optimize PSNR for basis pursuit (BP) reconstruction using shearlet transform, at 25% sampling rate.

Sampling design considers both the training data and the reconstruction method.  
No high spatial frequencies!?

(Images from Gözcü et al. [12].)

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- ▶ MRI measurement model:

$$\mathbf{y} = \mathbf{A}(\boldsymbol{\omega})\mathbf{x} + \boldsymbol{\varepsilon}$$

$\mathbf{y}$ : k-space data

$\boldsymbol{\omega}$ : k-space sampling pattern (“trajectory”)

$\mathbf{A}(\boldsymbol{\omega})$ : encoding matrix (coil sensitivity, etc.)

$\boldsymbol{\varepsilon}$ : sensor noise

$\mathbf{x}$ : unknown true image

- ▶ Reconstruction method:

$$\hat{\mathbf{x}} = f(\mathbf{y}; \boldsymbol{\omega}, \boldsymbol{\theta})$$

$\boldsymbol{\theta}$ : model parameters of reconstruction method (e.g., CNN weights)

- ▶ Image quality goal:

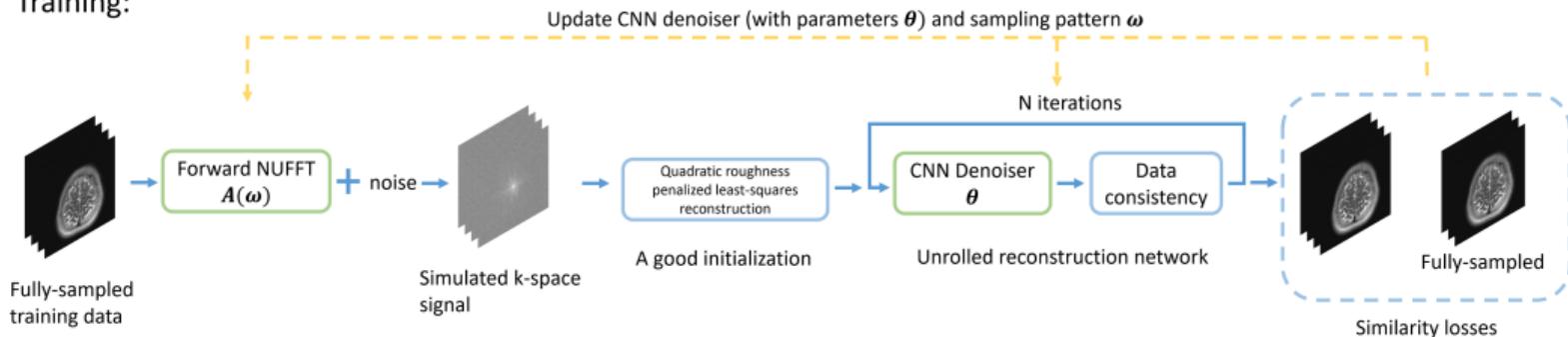
$$\hat{\mathbf{x}} = f(\mathbf{y}; \boldsymbol{\omega}, \boldsymbol{\theta}) = f(\mathbf{A}(\boldsymbol{\omega})\mathbf{x} + \boldsymbol{\varepsilon}; \boldsymbol{\omega}, \boldsymbol{\theta}) \approx \mathbf{x}$$

- ▶ Define training loss function such as  $\ell(\hat{\mathbf{x}}, \mathbf{x}) = \|\hat{\mathbf{x}} - \mathbf{x}\|_1 + \alpha \|\hat{\mathbf{x}} - \hat{\mathbf{x}}\|_2$
- ▶ Select  $N_{\text{train}}$  fully sampled training images  $\mathbf{x}_1, \mathbf{x}_2, \dots$
- ▶ Jointly optimize k-space trajectory  $\omega$  and image reconstruction method  $\theta$

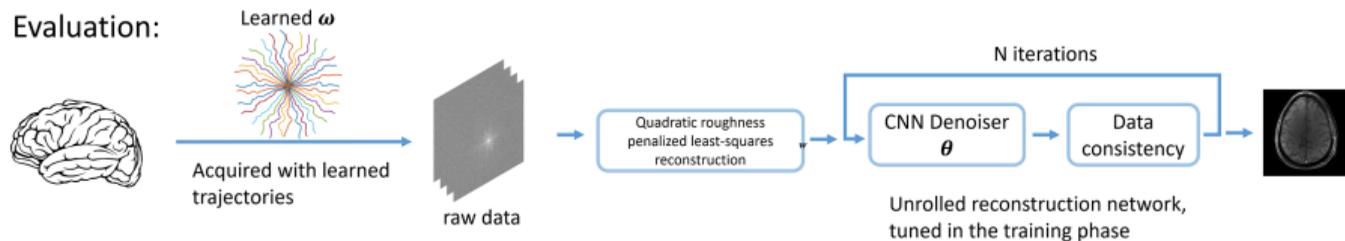
$$(\hat{\omega}, \hat{\theta}) = \arg \min_{\omega, \theta} \frac{1}{N_{\text{train}}} \sum_{n=1}^{N_{\text{train}}} \ell(f(\mathbf{A}(\omega)\mathbf{x}_n + \varepsilon; \omega, \theta), \mathbf{x}_n)$$

- ▶ Details:
  - Reconstruction using MoDL method [51]
  - Can use multiple noise realizations  $\varepsilon$
  - Enforce gradient amplitude and slew-rate limits for  $\omega$
  - Use B-spline parameterization of k-space trajectory
  - Coarse-to-fine search of trajectory to avoid poor local minimizers
  - Eddy current correction
  - Fast NUFFT Jacobian approximation [52]

## Training:



## Evaluation:



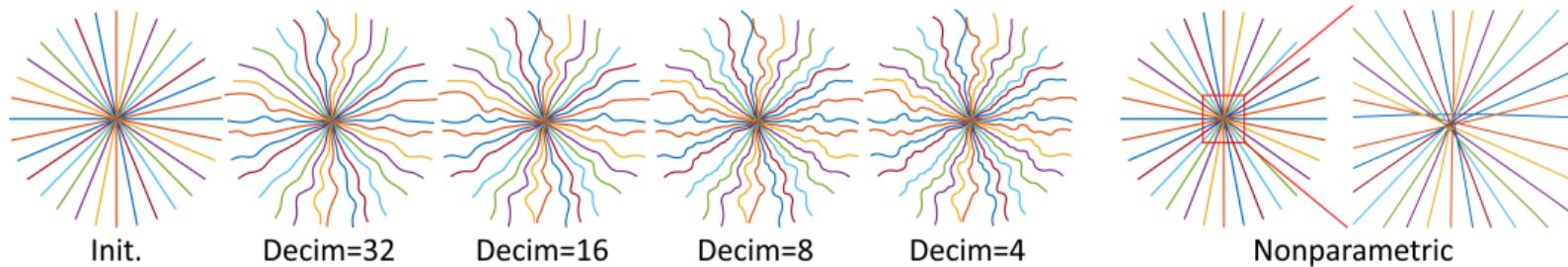
NYU/FAIR fastMRI brain and knee data

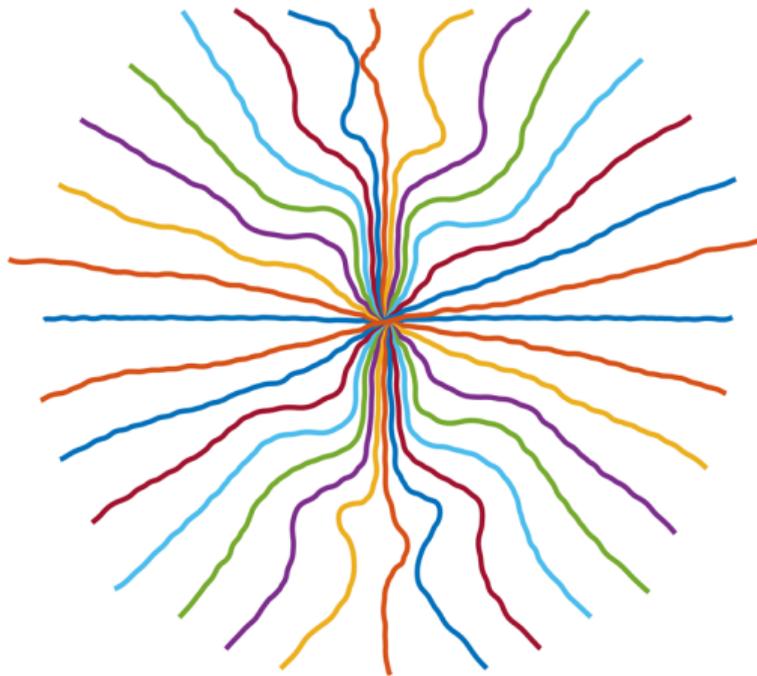
16 radial spokes of 640 points for initialization

22cm FOV,  $G_{\max} = 5$  Gauss/cm, slew rate  $\leq 15$  Gauss/cm/ms

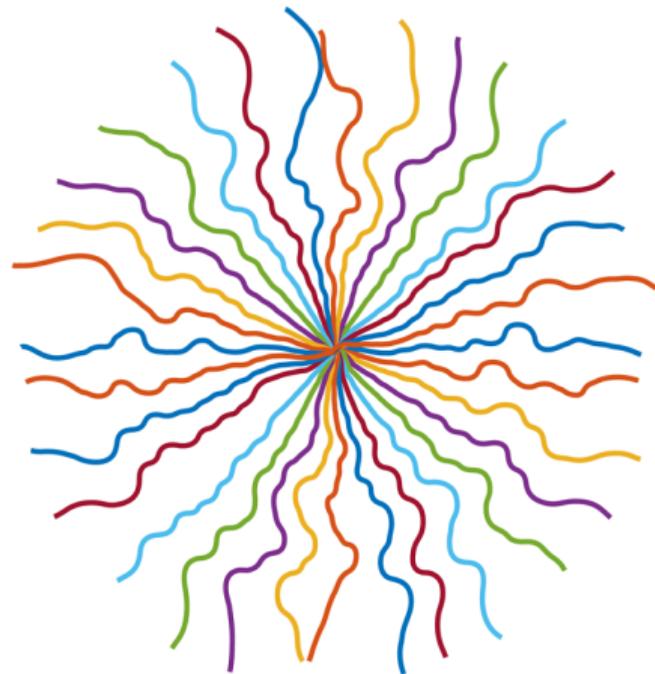
2.5ms readout duration radial, 16ms spiral

Comparison with SPARKLING approach of [53] using its default density

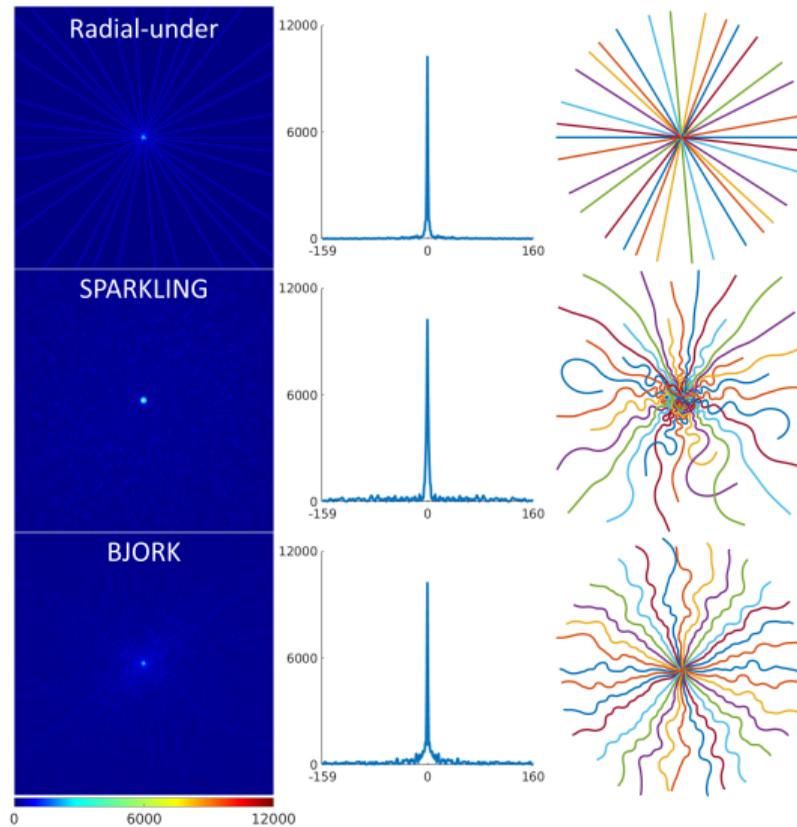


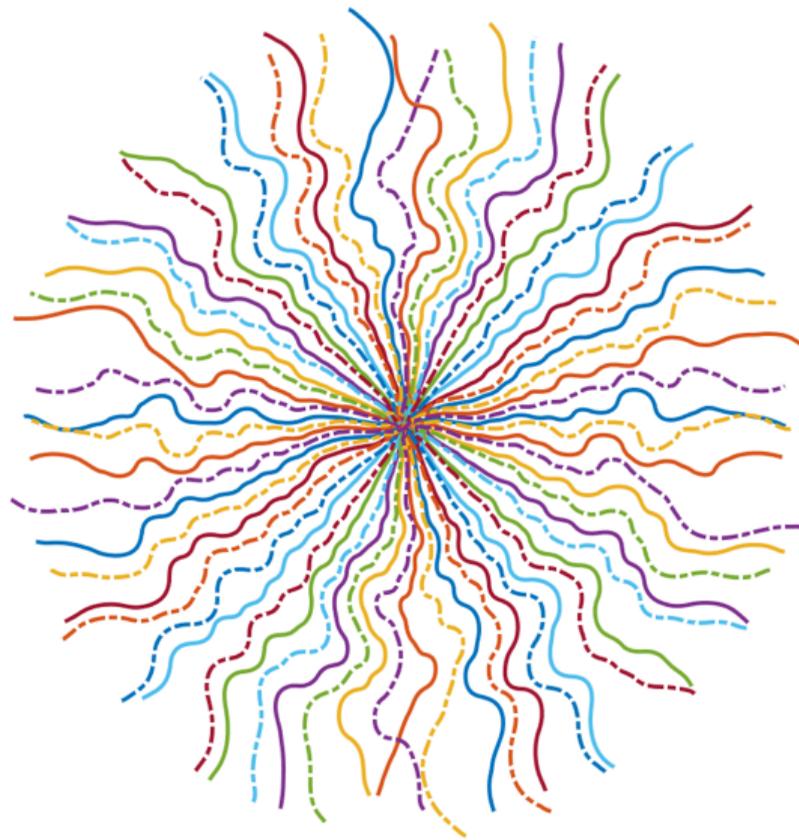


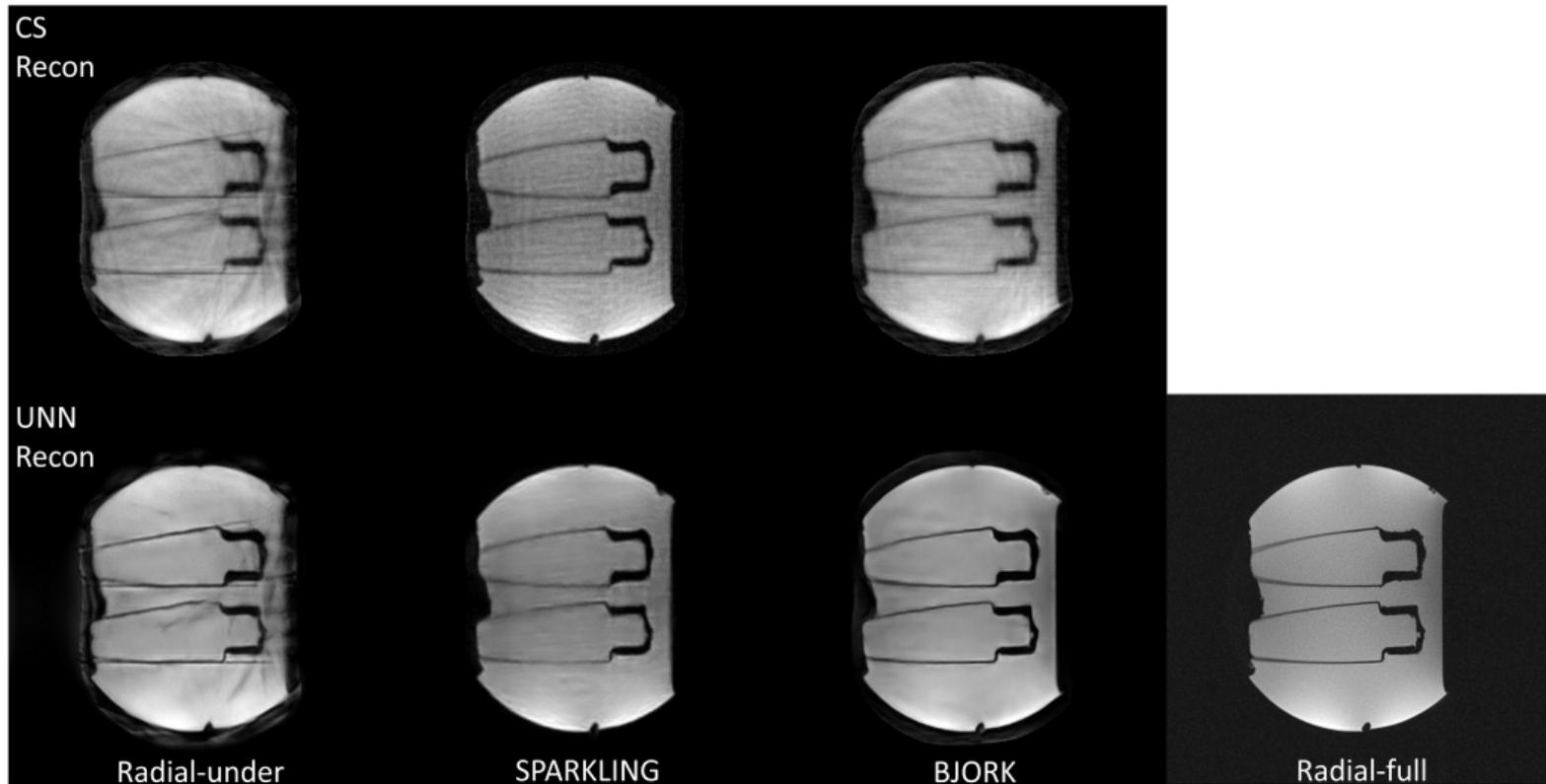
Knee



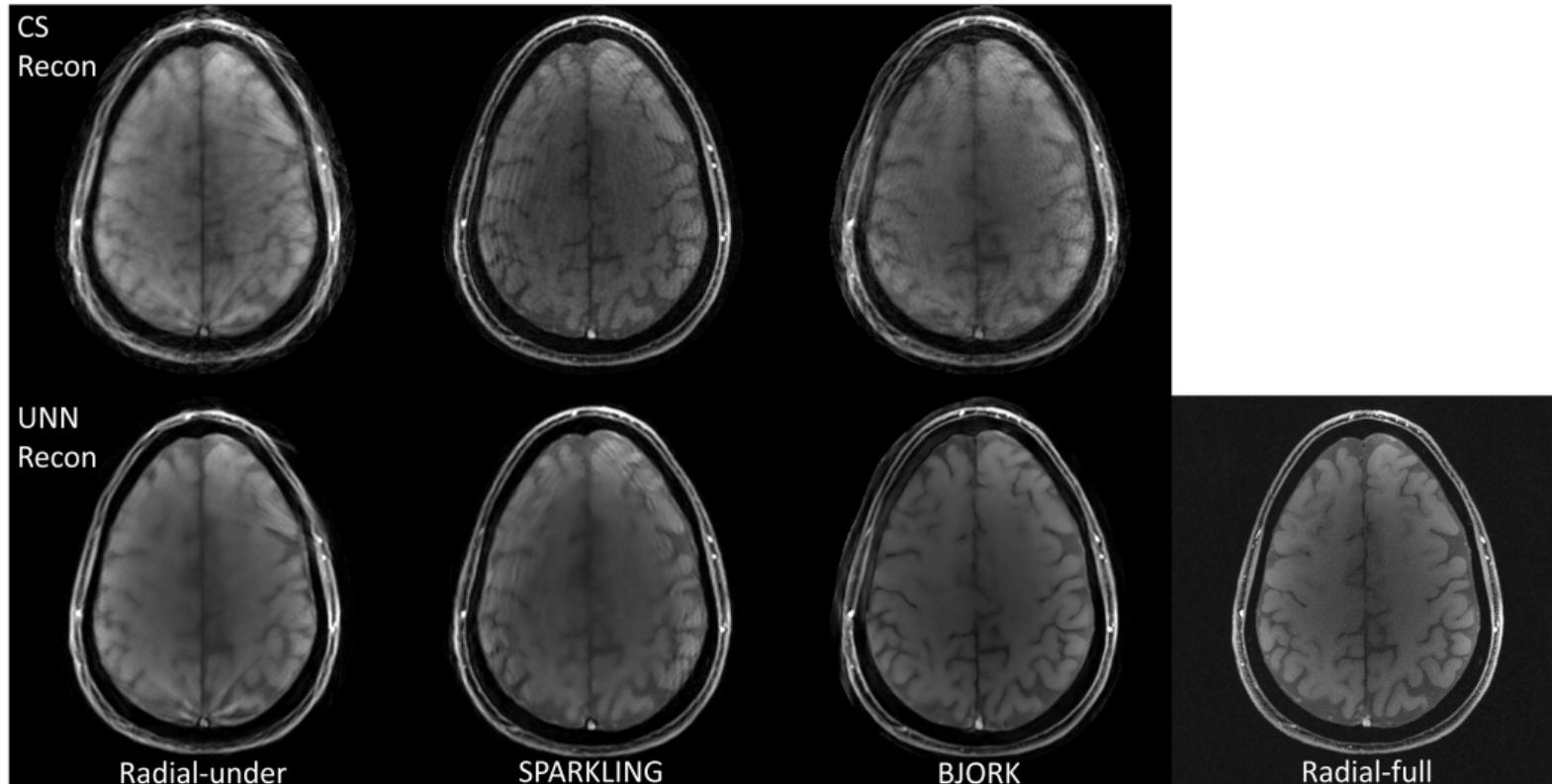
Brain







(never seen in training!)



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- ▶ Machine learning methods have much potential for both scan design and image reconstruction
- ▶ Quantitative results in paper demonstrate synergy of jointly optimizing both
- ▶ Anatomy specific trajectories: pro or con?
- ▶ Self-supervised methods when training data unavailable
- ▶ Extensions to 3D and 3D+time are planned, and challenging

Talk and code available online at  
<http://web.eecs.umich.edu/~fessler>



- [1] H. Greenspan, B. van Ginneken, and R. M. Summers. "Guest editorial deep learning in medical imaging: overview and future promise of an exciting new technique." In: *IEEE Trans. Med. Imag.* 35.5 (May 2016), 1153–9.
- [2] G. Litjens, T. Kooi, B. E. Bejnordi, A. A. A. Setio, F. Ciompi, M. Ghafoorian, J. A. W. M. . . Laak, B. . Ginneken, and C. I. Sanchez. "A survey on deep learning in medical image analysis." In: *Med. Im. Anal.* 42.C (Dec. 2017), 60–88.
- [3] G. Wang, M. Kalra, and C. G. Orton. "Machine learning will transform radiology significantly within the next five years." In: *Med. Phys.* 44.6 (June 2017), 2041–4.
- [4] V. Cheplygina, M. . Bruijine, and J. P. W. Pluim. "Not-so-supervised: A survey of semi-supervised, multi-instance, and transfer learning in medical image analysis." In: *Med. Im. Anal.* 54 (May 2019), 280–96.
- [5] A. Esteva, A. Robicquet, B. Ramsundar, V. Kuleshov, M. DePristo, K. Chou, C. Cui, G. Corrado, S. Thrun, and J. Dean. "A guide to deep learning in healthcare." In: *Nature Medicine* 25.1 (Jan. 2019), 24–9.
- [6] X. Yi, E. Walia, and P. Babyn. "Generative adversarial network in medical imaging: A review." In: *Med. Im. Anal.* 58 (Dec. 2019), p. 101552.
- [7] J. Bruna, E. Haber, G. Kutyniok, T. Pock, and Rene Vidal. "Special issue on the mathematical foundations of deep learning in imaging science." In: *J. Math. Im. Vision* 62.3 (2020), 277–8.
- [8] D. Rueckert and J. A. Schnabel. "Model-based and data-driven strategies in medical image computing." In: *Proc. IEEE* 108.1 (Jan. 2020), 110–24.
- [9] A. Maier, C. Syben, T. Lasser, and C. Riess. "A gentle introduction to deep learning in medical image processing." In: *Zeitschrift für Medizinische Physik* 29.2 (May 2019), 86–101.
- [10] S. Ravishankar and Y. Bresler. "Adaptive sampling design for compressed sensing MRI." In: *Proc. Int'l. Conf. IEEE Engr. in Med. and Biol. Soc.* 2011, 3751–5.
- [11] L. Baldassarre, Y-H. Li, J. Scarlett, B. Gozcu, I. Bogunovic, and V. Cevher. "Learning-based compressive subsampling." In: *IEEE J. Sel. Top. Sig. Proc.* 10.4 (June 2016), 809–22.

- [12] B. Gozcu, R. K. Mahabadi, Y-H. Li, E. Ilicak, T. Cukur, J. Scarlett, and V. Cevher. "Learning-based compressive MRI." In: *IEEE Trans. Med. Imag.* 37.6 (June 2018), 1394–406.
- [13] G. Godaliyadda, D. H. Ye, M. D. Uchic, M. A. Groeber, G. T. Buzzard, and C. A. Bouman. "A framework for dynamic image sampling based on supervised learning." In: *IEEE Trans. Computational Imaging* 4.1 (Mar. 2018), 1–16.
- [14] H. K. Aggarwal and M. Jacob. "J-MoDL: Joint model-based deep learning for optimized sampling and reconstruction." In: *IEEE J. Sel. Top. Sig. Proc.* 14.6 (Oct. 2020), 1151–62.
- [15] C. Bahadir, A. Wang, A. Dalca, and M. R. Sabuncu. "Deep-learning-based optimization of the under-sampling pattern in MRI." In: *IEEE Trans. Computational Imaging* 6 (2020), 1139–52.
- [16] Y. Cao and D. N. Levin. "Feature-recognizing MRI." In: *Mag. Res. Med.* 30.3 (Sept. 1993), 305–17.
- [17] Y. Cao, D. N. Levin, and L. Yao. "Locally focused MRI." In: *Mag. Res. Med.* 34.6 (Dec. 1995), 858–67.
- [18] Y. Cao and D. N. Levin. "Using an image database to constrain the acquisition and reconstruction of MR images of the human head." In: *IEEE Trans. Med. Imag.* 14.2 (June 1995), 350–61.
- [19] G. Wang, J. C. Ye, K. Mueller, and J. A. Fessler. "Image reconstruction is a new frontier of machine learning." In: *IEEE Trans. Med. Imag.* 37.6 (June 2018), 1289–96.
- [20] G. Wang. "A perspective on deep imaging." In: *IEEE Access* 4 (Nov. 2016), 8914–24.
- [21] M. T. McCann, K. H. Jin, and M. Unser. "Convolutional neural networks for inverse problems in imaging: A review." In: *IEEE Sig. Proc. Mag.* 34.6 (Nov. 2017), 85–95.
- [22] A. Lucas, M. Iliadis, R. Molina, and A. K. Katsaggelos. "Using deep neural networks for inverse problems in imaging: Beyond analytical methods." In: *IEEE Sig. Proc. Mag.* 35.1 (Jan. 2018), 20–36.
- [23] M. T. McCann and M. Unser. "Biomedical image reconstruction: from the foundations to deep neural networks." In: *Found. & Trends in Sig. Pro.* 13.3 (2019), 283–359.

- [24] S. Arridge, P. Maass, O. Oktem, and C-B. Schonlieb. "Solving inverse problems using data-driven models." In: *Acta Numerica* 28 (May 2019), 1–174.
- [25] V. Monga, Y. Li, and Y. C. Eldar. "Algorithm unrolling: interpretable, efficient deep learning for signal and image processing." In: *IEEE Sig. Proc. Mag.* 38.2 (Mar. 2021), 18–44.
- [26] S. Ravishankar, J. C. Ye, and J. A. Fessler. "Image reconstruction: from sparsity to data-adaptive methods and machine learning." In: *Proc. IEEE* 108.1 (Jan. 2020), 86–109.
- [27] G. Ongie, A. Jalal, C. A. M. R. G. Baraniuk, A. G. Dimakis, and R. Willett. "Deep learning techniques for inverse problems in imaging." In: *ieee-jsait* (2020).
- [28] E. Haneda, B. Claus, P. FitzGerald, G. Wang, and B. De Man. "CT sinogram analysis using deep learning." In: *Proc. 5th Intl. Mtg. on Image Formation in X-ray CT*. 2018, 419–22.
- [29] Q. De Man, E. Haneda, B. Claus, P. Fitzgerald, B. De Man, G. Qian, H. Shan, J. Min, M. Sabuncu, and G. Wang. "A two-dimensional feasibility study of deep learning-based feature detection and characterization directly from CT sinograms." In: *Med. Phys.* 46.12 (Dec. 2019), e790–800.
- [30] G. Wang, T. Luo, J-F. Nielsen, D. C. Noll, and J. A. Fessler. *B-spline parameterized joint optimization of reconstruction and k-space trajectories (BJORK) for accelerated 2D MRI*. 2021.
- [31] T. Weiss, O. Senouf, S. Vedula, O. Michailovich, M. Zibulevsky, and A. Bronstein. *PILOT: Physics-informed learned optimal trajectories for accelerated MRI*. 2019.
- [32] S. Wang, Z. Su, L. Ying, X. Peng, and D. Liang. "Exploiting deep convolutional neural network for fast magnetic resonance imaging." In: *Proc. Intl. Soc. Mag. Res. Med.* 2016, p. 1778.
- [33] D. Lee, J. Yoo, and J. C. Ye. *Deep artifact learning for compressed sensing and parallel MRI*. 2017.

- [34] K. H. Jin, M. T. McCann, E. Froustey, and M. Unser. "Deep convolutional neural network for inverse problems in imaging." In: *IEEE Trans. Im. Proc.* 26.9 (Sept. 2017), 4509–22.
- [35] M. Akcakaya, S. Moeller, S. Weingartner, and Kamil Ugurbil. "Scan-specific robust artificial-neural-networks for k-space interpolation (RAKI) reconstruction: Database-free deep learning for fast imaging." In: *Mag. Res. Med.* 81.1 (Jan. 2019), 439–53.
- [36] Y. Han and J. C. Ye. "K-space deep learning for accelerated MRI." In: *IEEE Trans. Med. Imag.* 39.2 (Feb. 2020), 377–86.
- [37] M. U. Ghani and W. C. Karl. *Data and image prior integration for image reconstruction using consensus equilibrium*. 2020.
- [38] B. Zhu, J. Z. Liu, S. F. Cauley, B. R. Rosen, and M. S. Rosen. "Image reconstruction by domain-transform manifold learning." In: *Nature* 555 (Mar. 2018), 487–92.
- [39] I. Haggstrom, C. R. Schmidlein, G. Campanella, and T. J. Fuchs. "DeepPET: A deep encoder-decoder network for directly solving the PET image reconstruction inverse problem." In: *Med. Im. Anal.* 54 (May 2019), 253–62.
- [40] W. Whiteley, W. K. Luk, and J. Gregor. "DirectPET: full-size neural network PET reconstruction from sinogram data." In: *J. Med. Im.* 7.3 (Feb. 2020), 1–16.
- [41] W. Whiteley, V. Panin, C. Zhou, J. Cabello, D. Bharkhada, and J. Gregor. "FastPET: near real-time reconstruction of PET histo-image data using a neural network." In: *IEEE Trans. Radiation and Plasma Med. Sci.* 5.1 (Jan. 2021), 65–77.
- [42] Y. Yang, J. Sun, H. Li, and Z. Xu. "Deep ADMM-net for compressive sensing MRI." In: *Neural Info. Proc. Sys.* 2016, 10–18.
- [43] K. Hammernik, T. Klatzer, E. Kobler, M. P. Recht, D. K. Sodickson, T. Pock, and F. Knoll. "Learning a variational network for reconstruction of accelerated MRI data." In: *Mag. Res. Med.* 79.6 (June 2018), 3055–71.
- [44] J. Schlemper, J. Caballero, J. V. Hajnal, A. N. Price, and D. Rueckert. "A deep cascade of convolutional neural networks for dynamic MR image reconstruction." In: *IEEE Trans. Med. Imag.* 37.2 (Feb. 2018), 491–503.
- [45] T. M. Quan, T. Nguyen-Duc, and W-K. Jeong. "Compressed sensing MRI reconstruction using a generative adversarial network with a cyclic loss." In: *IEEE Trans. Med. Imag.* 37.6 (June 2018), 1488–97.

- [46] D. Lee, J. Yoo, S. Tak, and J. C. Ye. "Deep residual learning for accelerated MRI using magnitude and phase networks." In: *IEEE Trans. Biomed. Engin.* 65.9 (Sept. 2018), 1985–95.
- [47] G. Nataraj and R. Otazo. "Investigating robustness to unseen pathologies in model-free deep multicoil reconstruction." In: *ISMRM Workshop on Data Sampling and Image Reconstruction*. 2020.
- [48] G. Yang, S. Yu, H. Dong, G. Slabaugh, P. L. Dragotti, X. Ye, F. Liu, S. Arridge, J. Keegan, Y. Guo, and D. Firmin. "DAGAN: Deep de-aliasing generative adversarial networks for fast compressed sensing MRI reconstruction." In: *IEEE Trans. Med. Imag.* 37.6 (June 2018), 1310–21.
- [49] K. He, X. Zhang, S. Ren, and J. Sun. "Deep residual learning for image recognition." In: *Proc. IEEE Conf. on Comp. Vision and Pattern Recognition*. 2016, 770–8.
- [50] K. Zhang, W. Zuo, Y. Chen, D. Meng, and L. Zhang. "Beyond a Gaussian denoiser: residual learning of deep CNN for image denoising." In: *IEEE Trans. Im. Proc.* 26.7 (July 2017), 3142–55.
- [51] H. K. Aggarwal, M. P. Mani, and M. Jacob. "MoDL: model-based deep learning architecture for inverse problems." In: *IEEE Trans. Med. Imag.* 38.2 (Feb. 2019), 394–405.
- [52] G. Wang, D. C. Noll, and J. A. Fessler. "Efficient NUFFT backpropagation for stochastic sampling optimization." In: *Proc. Intl. Soc. Mag. Res. Med.* 2021, p. 0913.
- [53] C. Lazarus, P. Weiss, N. Chauffert, F. Mauconduit, L. El Gueddari, C. Destrieux, I. Zemmoura, A. Vignaud, and P. Ciuciu. "SPARKLING: variable-density k-space filling curves for accelerated T2\*-weighted MRI." In: *Mag. Res. Med.* 81.6 (June 2019), 3643–61.