Optimization transfer approach to joint registration / reconstruction for motion-compensated image reconstruction

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Introduction

Image reconstruction of moving objects with unknown motion

Joint estimation of motion parameters and object

- Jacobson *et al.*, IEEE NSS 2003
- Taguchi *et al.* SPIE 2007
- Odille *et al.* MRM Jul. 2008
- Also super-resolution problems with unknown motion
  (cf starting with low-resolution images vs starting with sinograms or k-space)
- ...

Computational challenge: motion operator in forward model.

We use optimization transfer to put motion estimation step in image domain

Measurement model

M "frames" (defined generally)

$$\boldsymbol{y}_m = \boldsymbol{A}_m \boldsymbol{x}_m + \boldsymbol{\varepsilon}_m, \qquad m = 1, \ldots, M$$

- $\boldsymbol{y}_m$ measured data for mth frame
- $\boldsymbol{A}_m$ system matrix for mth frame
- $\boldsymbol{x}_m$ unknown image for mth frame
- $\boldsymbol{\varepsilon}_m$ measurement noise for mth frame

Nominal goal:
reconstruct image frames $\{\boldsymbol{x}_m\}$ from measured data $\{\boldsymbol{y}_m\}$.

Object model

Assume each frame is a spatial transformation of one base image:

$$\boldsymbol{x}_m = \boldsymbol{T}(\boldsymbol{\alpha}_m)\,\boldsymbol{c}$$

- $\boldsymbol{\alpha}_m$ motion parameters for mth frame
- $\boldsymbol{T}(\cdot)$ nonrigid warp operator
- $\boldsymbol{c}$ base image coefficient vector (*e.g.*, for B-splines)

$$\boldsymbol{x}_0 = \boldsymbol{T}(\mathbf{0})\,\boldsymbol{c}$$

Motion-compensated image reconstruction goal:
reconstruct base image coefficients $\boldsymbol{c}$ and motion parameters $\{\boldsymbol{\alpha}_m\}$ from measured data $\{\boldsymbol{y}_m\}$.

Joint registration/reconstruction

Combined measurement model / object model:

$$\boldsymbol{y}_m = \boldsymbol{A}_m \boldsymbol{T}(\boldsymbol{\alpha}_m)\,\boldsymbol{c} + \boldsymbol{\varepsilon}_m, \qquad m = 1, \ldots, M$$

Stacking, where $\boldsymbol{\alpha} \triangleq (\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_M)$:

$$\boldsymbol{y} = \boldsymbol{A}\,\boldsymbol{T}(\boldsymbol{\alpha})\,\boldsymbol{c} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{y} \triangleq \begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{y}_M \end{bmatrix} \quad \boldsymbol{A} \triangleq \begin{bmatrix} \boldsymbol{A}_1 & & \\ & \ddots & \\ & & \boldsymbol{A}_M \end{bmatrix} \quad \boldsymbol{T}(\boldsymbol{\alpha}) \triangleq \begin{bmatrix} \boldsymbol{T}(\boldsymbol{\alpha}_1) \\ \vdots \\ \boldsymbol{T}(\boldsymbol{\alpha}_M) \end{bmatrix} \quad \boldsymbol{\varepsilon} \triangleq \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \vdots \\ \boldsymbol{\varepsilon}_M \end{bmatrix}$$

Penalized weighted least-squares (PWLS) estimation:

$$(\hat{\boldsymbol{c}}, \hat{\boldsymbol{\alpha}}) = \arg\min_{\boldsymbol{c},\boldsymbol{\alpha}} \Psi(\boldsymbol{c}, \boldsymbol{\alpha})$$

$$\Psi(\boldsymbol{c}, \boldsymbol{\alpha}) = \frac{1}{2} \|\boldsymbol{y} - \boldsymbol{A}\,\boldsymbol{T}(\boldsymbol{\alpha})\,\boldsymbol{c}\|_{\boldsymbol{W}}^2 + R_1(\boldsymbol{c}) + R_2(\boldsymbol{\alpha})$$

Optimization by alternation

Initialize base image $\boldsymbol{c}^0$ and motion parameters $\boldsymbol{\alpha}^0$.

Alternating updates:

$$\begin{aligned}\boldsymbol{c}^{n+1} &= \arg\min_{\boldsymbol{c}} \Psi(\boldsymbol{\alpha}^n, \boldsymbol{c}) \\ \boldsymbol{\alpha}^{n+1} &= \arg\min_{\boldsymbol{\alpha}} \Psi\left(\boldsymbol{\alpha}, \boldsymbol{c}^{n+1}\right)\end{aligned}$$

Update $\boldsymbol{c}$ using standard image reconstruction methods:

$$\boldsymbol{c}^{n+1} = \arg\min_{\boldsymbol{c}} \frac{1}{2} \|\boldsymbol{y} - \boldsymbol{A}\,\boldsymbol{T}(\boldsymbol{\alpha}^n)\,\boldsymbol{c}\|_{\boldsymbol{W}}^2 + R_1(\boldsymbol{c})$$

Updating motion parameters $\boldsymbol{\alpha}$ is challenging:

$$\boldsymbol{\alpha}^{n+1} = \arg\min_{\boldsymbol{\alpha}} \psi(\boldsymbol{\alpha}), \quad \psi(\boldsymbol{\alpha}) = \frac{1}{2} \left\|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{T}(\boldsymbol{\alpha})\boldsymbol{c}^{n+1}\right\|_{\boldsymbol{W}}^2 + R_2(\boldsymbol{\alpha})$$

Optimization transfer / Majorize-minimize

Alternative to minimizing cost function $\psi(\boldsymbol{\alpha})$ directly:

- S-step: find a surrogate function (majorizer) $\phi(\boldsymbol{\alpha}; \boldsymbol{\alpha}^n)$ s.t.

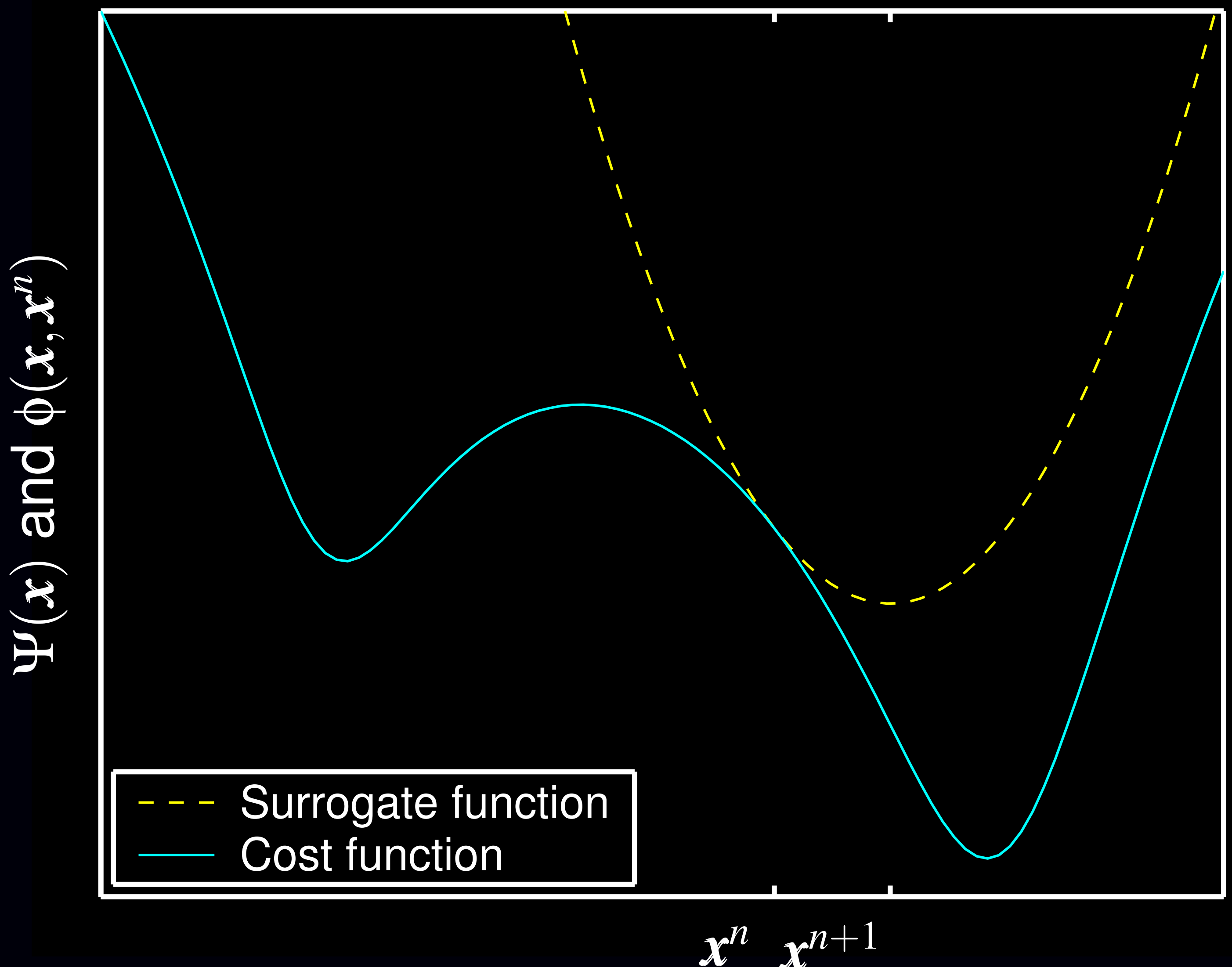
$$\begin{aligned} \phi(\boldsymbol{\alpha}; \boldsymbol{\alpha}^n) &\geq \psi(\boldsymbol{\alpha}), \qquad \forall \boldsymbol{\alpha} \\ \phi(\boldsymbol{\alpha}^n; \boldsymbol{\alpha}^n) &= \psi(\boldsymbol{\alpha}^n) \end{aligned}$$

- M-step: minimize surrogate function:

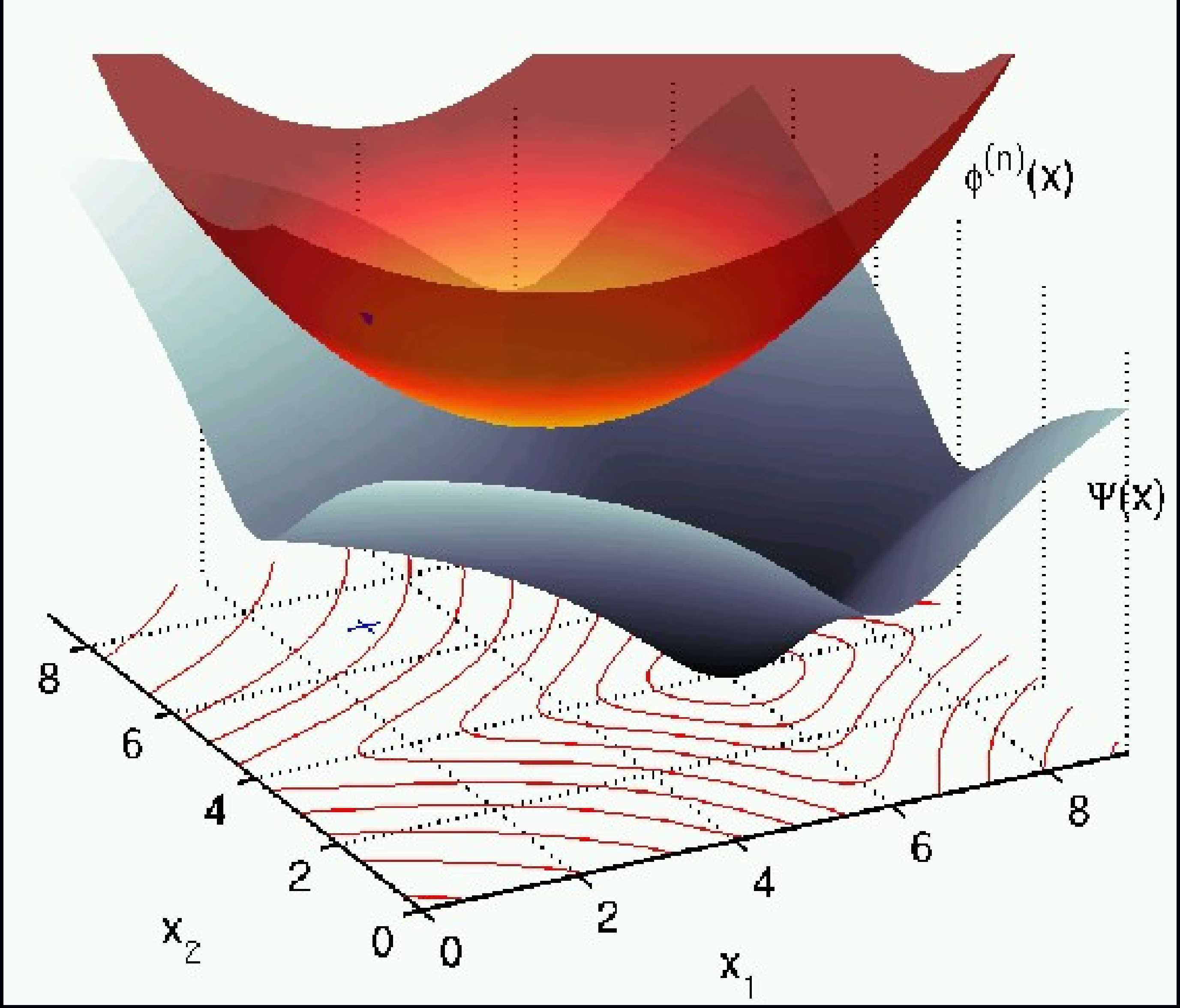
$$\boldsymbol{\alpha}^{n+1} = \arg\min_{\boldsymbol{\alpha}} \phi(\boldsymbol{\alpha}; \boldsymbol{\alpha}^n).$$

Guaranteed to decrease cost function $\psi(\boldsymbol{\alpha}^n)$ *monotonically*.

Optimization Transfer Illustrated



Optimization Transfer in 2d



Separable quadratic surrogate for WLS

$$
\begin{aligned}
L(\boldsymbol{x}) &= \frac{1}{2}\|\boldsymbol{y}-\boldsymbol{A}\boldsymbol{x}\|_{\boldsymbol{W}}^2 = \sum_i \frac{w_i}{2}(y_i - [\boldsymbol{A}\boldsymbol{x}]_i)^2 \\
&= L(\boldsymbol{x}^n) + (\boldsymbol{x}-\boldsymbol{x}^n)'\boldsymbol{g}^n + \frac{1}{2}(\boldsymbol{x}-\boldsymbol{x}^n)'\boldsymbol{A}'\boldsymbol{W}\boldsymbol{A}(\boldsymbol{x}-\boldsymbol{x}^n) \\
&\leq L(\boldsymbol{x}^n) + (\boldsymbol{x}-\boldsymbol{x}^n)'\boldsymbol{g}^n + \frac{1}{2}(\boldsymbol{x}-\boldsymbol{x}^n)'\boldsymbol{D}(\boldsymbol{x}-\boldsymbol{x}^n) \\
&\equiv \frac{1}{2}\left\|\boldsymbol{D}^{1/2}\left[\boldsymbol{x}-\left(\boldsymbol{x}^n+\boldsymbol{D}^{-1}\boldsymbol{g}^n\right)\right]\right\|^2 \triangleq Q(\boldsymbol{x};\boldsymbol{x}^n),
\end{aligned}
$$

when $\boldsymbol{D}$ is any matrix that satisfies $\boldsymbol{D} \succeq \boldsymbol{A}'\boldsymbol{W}\boldsymbol{A}$, where

$$
\boldsymbol{g}^n \triangleq -\nabla L(\boldsymbol{x}^n) = \boldsymbol{A}'\boldsymbol{W}\left(\boldsymbol{y}-\boldsymbol{A}\boldsymbol{x}^n\right).
$$

Useful choice (Erdoğan and Fessler, PMB 1999):

$$
\boldsymbol{D} = \operatorname{diag}\{d_j\}, \quad d_j \triangleq \sum_i w_i |a_{ij}| \left(\sum_k |a_{ik}|\right).
$$

Surrogate for motion parameters

Recall: $\psi(\boldsymbol{\alpha}) = \frac{1}{2}\left\| \boldsymbol{y} - \boldsymbol{A}\,\underbrace{\boldsymbol{T}(\boldsymbol{\alpha})\boldsymbol{c}^{n+1}}_{\text{“}\boldsymbol{x}\text{”}} \right\|_{\boldsymbol{W}}^2 + R_2(\boldsymbol{\alpha})$

Using result for WLS, the following majorizes ψ:

$$\begin{aligned}
\phi(\boldsymbol{\alpha}, \boldsymbol{\alpha}^n) &\triangleq Q(\boldsymbol{T}(\boldsymbol{\alpha})\,\boldsymbol{c}^n, \boldsymbol{T}(\boldsymbol{\alpha}^n)\,\boldsymbol{c}^n) + R_2(\boldsymbol{\alpha}) \\
&= \frac{1}{2}\left\| \boldsymbol{D}^{1/2}\left[\boldsymbol{T}(\boldsymbol{\alpha})\,\boldsymbol{c}^n - \left(\boldsymbol{T}(\boldsymbol{\alpha}^n)\,\boldsymbol{c}^n + \boldsymbol{D}^{-1}\boldsymbol{g}^n\right)\right] \right\|^2 + R_2(\boldsymbol{\alpha}),
\end{aligned}$$

where the gradient depends on *previous* estimates:

$$\boldsymbol{g}^n \triangleq -\nabla L(\boldsymbol{x}^n) = \boldsymbol{A}'\boldsymbol{W}\left(\boldsymbol{y} - \boldsymbol{A}\,\boldsymbol{T}(\boldsymbol{\alpha}^n)\,\boldsymbol{c}^n\right).$$

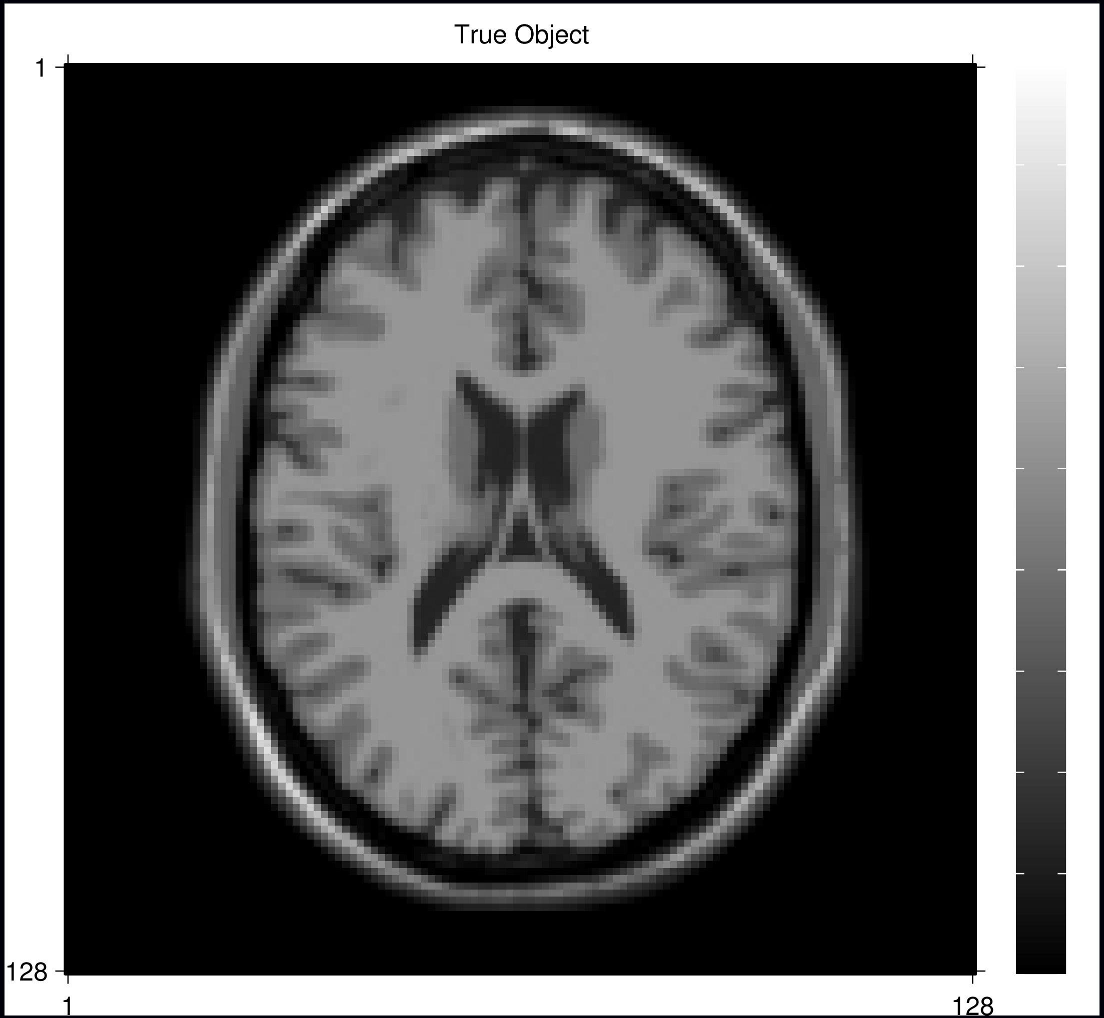
M-step is entirely image-domain operations:

$$\boldsymbol{\alpha}^{n+1} = \arg\min_{\boldsymbol{\alpha}} \phi(\boldsymbol{\alpha}, \boldsymbol{\alpha}^n).$$

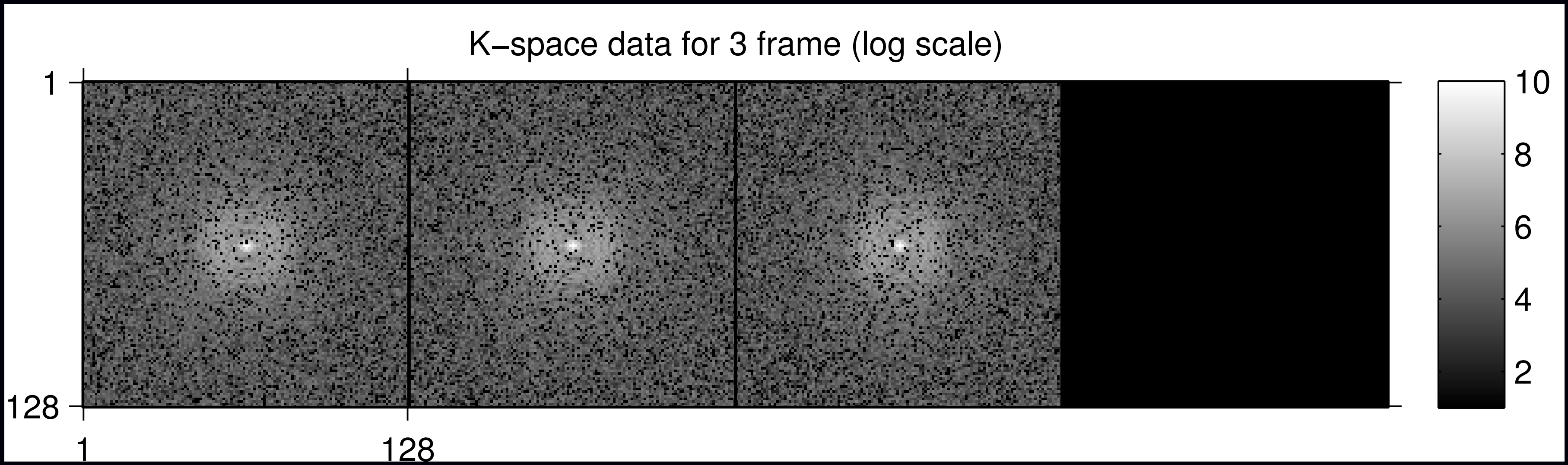
(Guobao Wang & Jinyi Qi did something similar for PET kinetics in ISBI 2008)

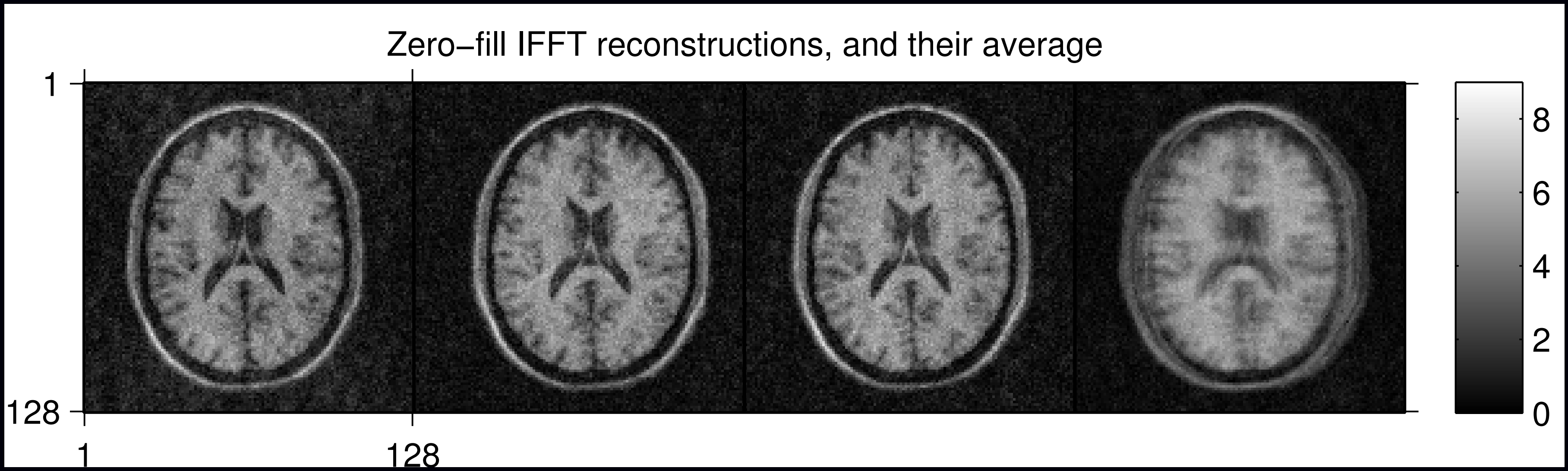
Simulation Example

MRI with randomly ordered k_y,k_z phase encodes



Simulation Example



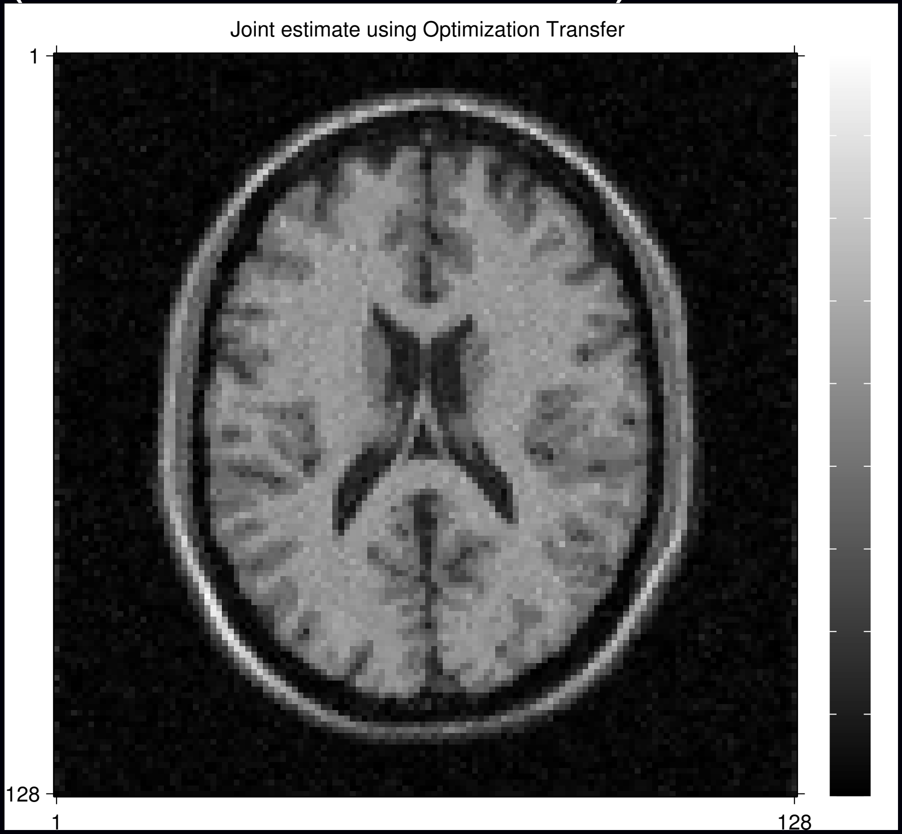


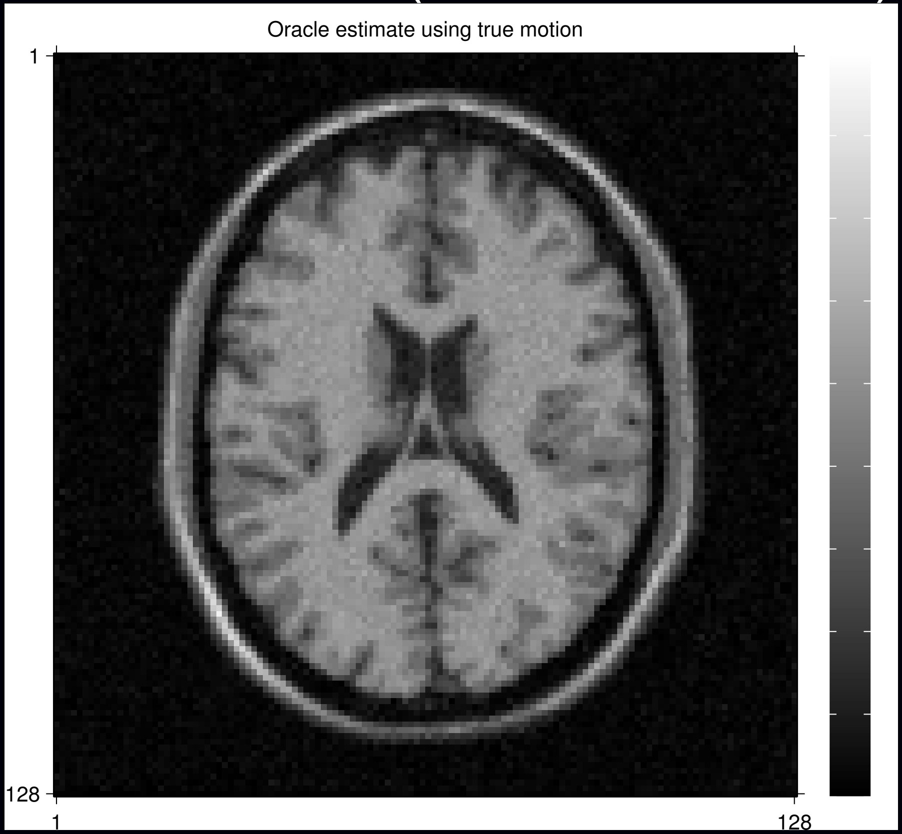
True motion: horizontal translation: -2, 4, 0

Joint motion estimation / reconstruction

Joint estimate:
(3 alternations, 9 CG)

Oracle reconstruction
("known" motion)





Estimated horizontal translation: -1.4, 4.4, 0.4
True horizontal translation: -2, 4, 0

Discussion

- Proposed optimization transfer approach to joint image reconstruction and motion estimation
- Puts motion estimation step in image domain
  - akin to image registration
  - avoids expensive forward projections in registration step
  - promotes software modularity

    (*e.g.*, importance sampling, Bhagalia *et al.*, IEEE T-MI, Aug. 2009)
- Optimization transfer approach may require “more” iterations
- Usual nonconvexity issues (multi-resolution...)
- Generalizes readily to non-quadratic log-likelihood functions (*e.g.*, PET, polyenergetic CT, ...)
- Awaits evaluation with real data...
- Awaits comparisons with nonlinear CG
- Open problem: using ordered-subsets approaches efficiently