Regularized fieldmap estimation in MRI

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Introduction

- Focus: fast single-shot MR imaging,
  such as echo-planar imaging (EPI) or spiral imaging for fMRI
- Long readout times
  $\Longrightarrow$ sensitive to B_0 field inhomogeniety / magnetic susceptibility
- Accurate correction for off-resonance effects requires a field map
- Field-corrected MR image reconstruction
  - Pixel-shifting for EPI (Sekihara *et al.*, 1985, IEEE T-MI)
  - Conjugate phase (Macovski, 1985, MRM; Noll *et al.*, 2005, IEEE T-MI)
  - Iterative (Sutton *et al.*, 2003, IEEE T-MI)

Measurement model

Two reconstructed images with slightly different echo times:

$$\begin{aligned} y_j &= f_j + \varepsilon_j \\ z_j &= f_j e^{\imath x_j} + \eta_j, \end{aligned} \qquad j = 1, \ldots, n_\mathrm{p}$$

- f_j: unknown complex transverse magnetization of the jth voxel
- n_p: number of voxels
- $x_j = \omega_j \triangle_t$: accrued phase
- ω_j: off-resonance of jth voxel
- $\triangle_t$: echo-time difference
- ε_j, η_j:e (complex) noise.

Goal: estimate phase map $\boldsymbol{x} = (x_1, \ldots, x_{n_\mathrm{p}})$ from images $\boldsymbol{y}$ and $\boldsymbol{z}$.

The unknown image $\boldsymbol{f} = (f_1, \ldots, f_{n_\mathrm{p}})$ is a nuisance parameter vector.

Conventional phase estimator

Phase difference of the two images: (Sekihara *et al.*, 1985, IEEE T-MI):

$$\hat{x}_j = \angle(y_j^* z_j) = \angle z_j - \angle y_j$$

- Fieldmap estimate is simply scaled by echo-time difference:

$$\hat{\omega}_j = \hat{x}_j / \triangle_t \,.$$

- Works perfectly in the absence of noise and phase wrapping, within voxels where $|f_j| > 0$.
- Sensitive to noise in voxels where magnitude $|f_j|$ is small.
- Ignores *a priori* knowledge that fieldmaps tend to be smooth
- Post-filtering works poorly because $\hat{x}_j$ is severely corrupted in low SNR voxels.

Maximum-likelihood phase estimation

Maximum likelihood (ML) method based on a statistical model.

Assume independent zero-mean white gaussian complex noise.
Assume $\boldsymbol{y}$ and $\boldsymbol{z}$ have same variance σ^2.

Joint log-likelihood for $\boldsymbol{f}$ and $\boldsymbol{x}$ given $\boldsymbol{y}$ and $\boldsymbol{z}$ is

$$\begin{aligned}\log \mathrm{p}(\boldsymbol{y}, \boldsymbol{z}; \boldsymbol{f}, \boldsymbol{x}) &= \log \mathrm{p}(\boldsymbol{y}; \boldsymbol{f}) + \log \mathrm{p}(\boldsymbol{z}; \boldsymbol{f}, \boldsymbol{x}) \\ &\equiv \frac{-1}{2\sigma^2} \sum_{j=1}^{n_\mathrm{p}} \left(|y_j - f_j|^2 + |z_j - f_j \mathrm{e}^{\imath x_j}|^2 \right).\end{aligned}$$

Simultaneous ML estimation of image $\boldsymbol{f}$ and phase map $\boldsymbol{x}$:

$$\arg\min_{\boldsymbol{x} \in \mathbb{R}^{n_\mathrm{p}}} \arg\min_{\boldsymbol{f} \in \mathbb{C}^{n_\mathrm{p}}} \sum_{j=1}^{n_\mathrm{p}} \left\| \begin{bmatrix} y_j \\ z_j \end{bmatrix} - \begin{bmatrix} 1 \\ \mathrm{e}^{\imath x_j} \end{bmatrix} f_j \right\|^2.$$

ML solution

Quadratic cost function in f_j, leading to the following ML estimate:

$$\hat{f}_j = \frac{y_j + e^{-\imath x_j} z_j}{2}.$$

Substituting back into the cost function and simplifying yields the following minimization problem for ML estimation of $\boldsymbol{x}$:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \Psi(\boldsymbol{x}), \quad \Psi(\boldsymbol{x}) = \sum_{j=1}^{n_{\mathrm{p}}} \frac{1}{2} \left| y_j - e^{-\imath x_j} z_j \right|^2.$$

After simplifying:

$$\Psi(\boldsymbol{x}) \equiv \sum_{j=1}^{n_{\mathrm{p}}} |y_j z_j| \left[1 - \cos(\angle z_j - \angle y_j - x_j)\right].$$

One ML estimate is the minimizer:

$$\hat{x}_j = \angle z_j - \angle y_j.$$

$\therefore$ Conventional phase estimate $\equiv$ ML estimate!

Penalized likelihood phase / fieldmap estimation

ML estimate ignores *a priori* smoothness of fieldmaps.
Penalized-likelihood approach using regularization (aka MAP)

- Regularize phase map $\boldsymbol{x}$ with strong roughness penalty
- No regularization of magnetization map $\boldsymbol{f}$ (anatomical details)

Regularized cost function:

$$\Psi(\boldsymbol{x}) = \sum_{j=1}^{n_\mathrm{p}} |y_j z_j| \left[1 - \cos(\angle z_j - \angle y_j - x_j)\right] + \beta R(\boldsymbol{x})$$

Down-weights data in voxels where the magnitude $|y_j z_j|$ is small.
In such voxels the phase will be estimated from neighboring voxels, due to the spatial roughness penalty $R(\boldsymbol{x})$:

$$\begin{aligned} R(\boldsymbol{x}) \;&=\; \textstyle\sum_{n=1}^{N-1} \sum_{m=0}^{M-1} \psi(x[n,m] - x[n-1,m]) \\ &+\; \textstyle\sum_{n=0}^{N-1} \sum_{m=1}^{M-1} \psi(x[n,m] - x[n,m-1])\,. \end{aligned}$$

Quadratic potential function $\psi(t) = t^2/2$. (No “edge” preservation.)

Minimization algorithm

Optimization transfer approach leads to diagonally preconditioned gradient descent algorithm:

$$x^{(n+1)} = x^{(n)} - \text{diag}\left\{ \frac{1}{|y_j z_j| + \beta \cdot 4} \right\} \nabla \Psi\left(x^{(n)}\right)$$

- Initialize $x^{(0)}$ with conventional (aka ML) phase estimate
- Guaranteed to decrease $\Psi(x)$ monotonically
- Ψ nonconvex $\Longrightarrow$ convergence to a local minimizer of $\Psi(x)$

(movie in pdf)

PWLS fieldmap estimator

Echo time difference $\triangle_t$ usually small enough to prevent phase wrap.

Taylor series approximation $1 - \cos(t) \approx t^2/2$.

Penalized weighted-least squares (PWLS) cost function:

$$\Psi(\boldsymbol{x}) = \sum_{j=1}^{n_{\mathrm{p}}} w_j \frac{1}{2} (\angle z_j - \angle y_j - x_j)^2 + \beta R(\boldsymbol{x}),$$

where we define a magnitude weighting function: $w_j \triangleq |y_j z_j|$.

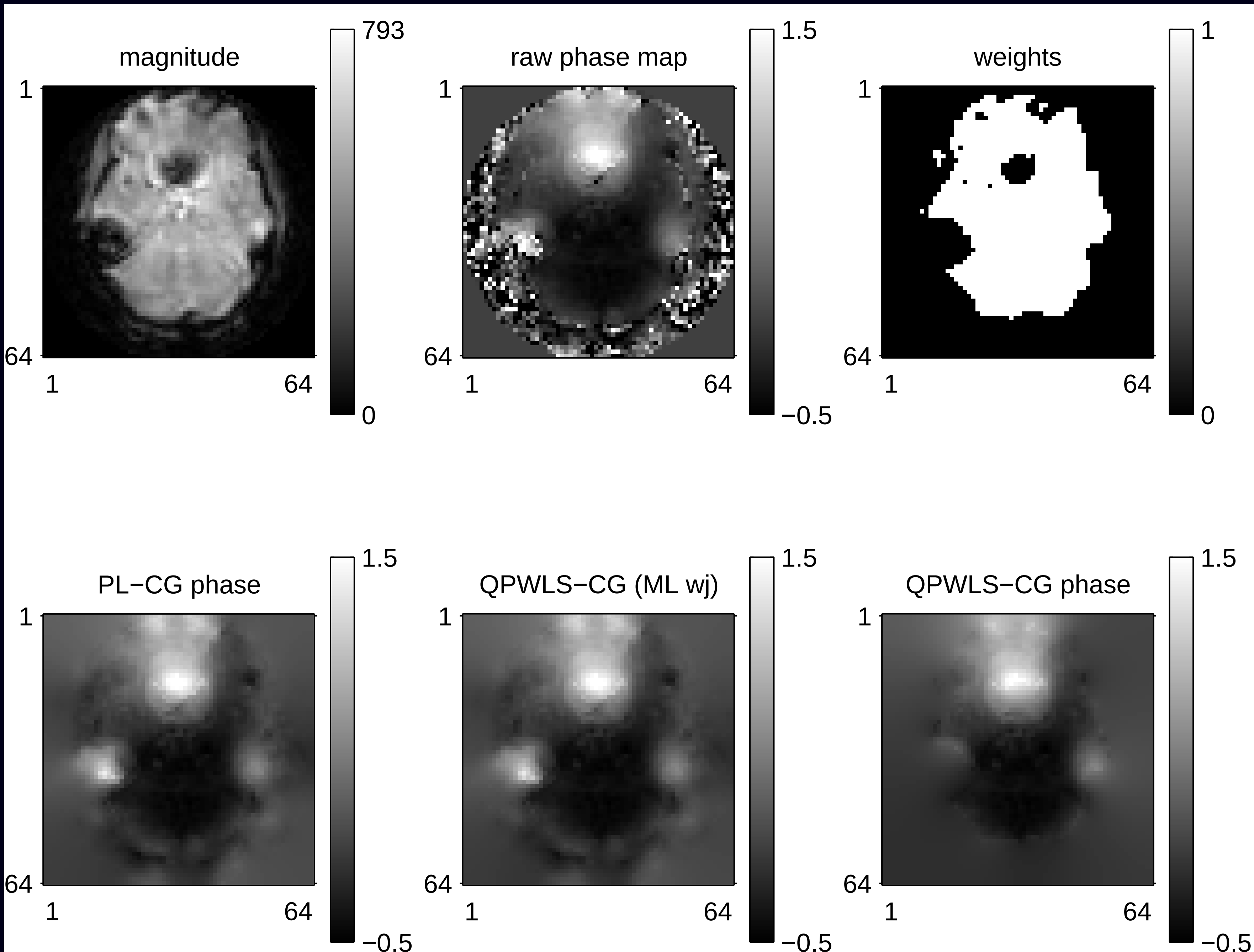
PWLS estimators give more weight to the “good data”
and use regularization to control noise.

Alternative: binarize the weights w_j using a threshold:

$$w_j \triangleq \begin{cases} 1, & |y_j z_j| > \gamma \\ 0, & \text{otherwise}, \end{cases} \qquad \textit{e.g.}, \qquad \gamma = 0.4 \max_j |y_j z_j|.$$

Use conjugate gradient (CG) algorithm for minimization.

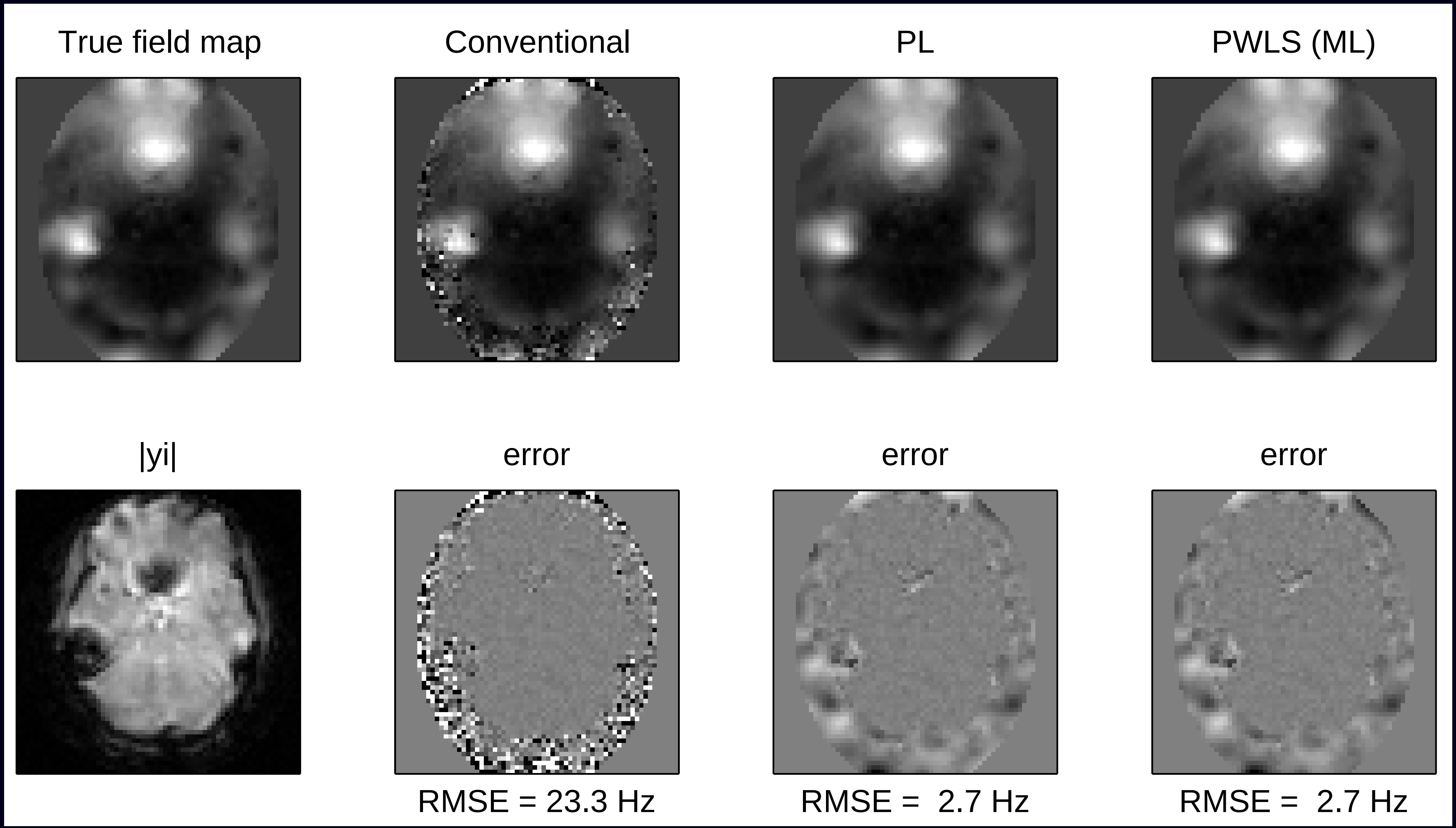
Example (3T MR scan)



Results

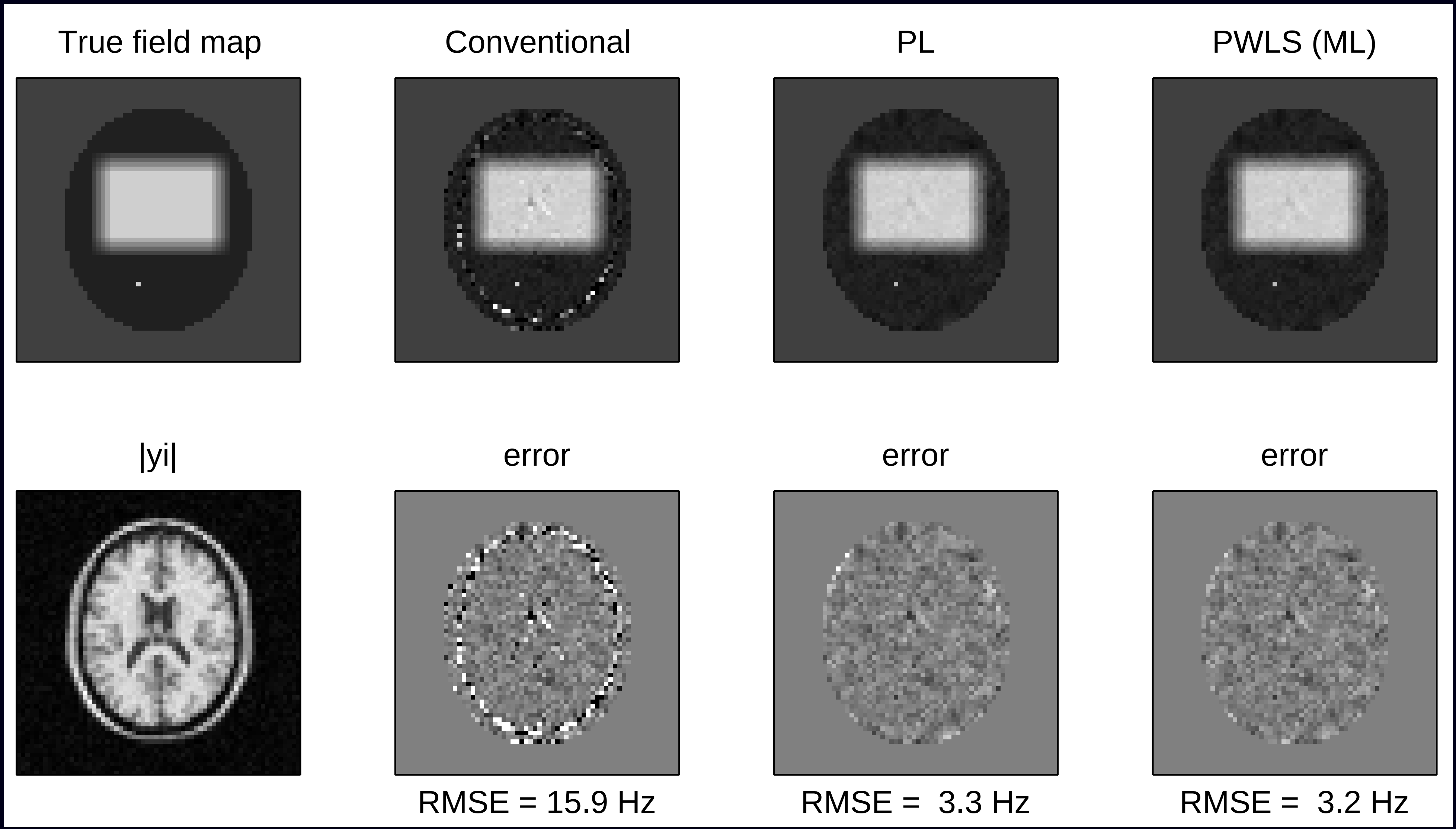
- Real data from a 3T MR scanner.
- 150 iterations for PL: 4.4 s (Matlab on G5)
- 150 iterations for PWLS: 2.2 s
- 3% normalized RMS difference
  for PL method vs PWLS approximation with ML weights.
- 42% normalized RMS difference
  for PWLS with binary weights vs PWLS with ML weights.

Simulation 1



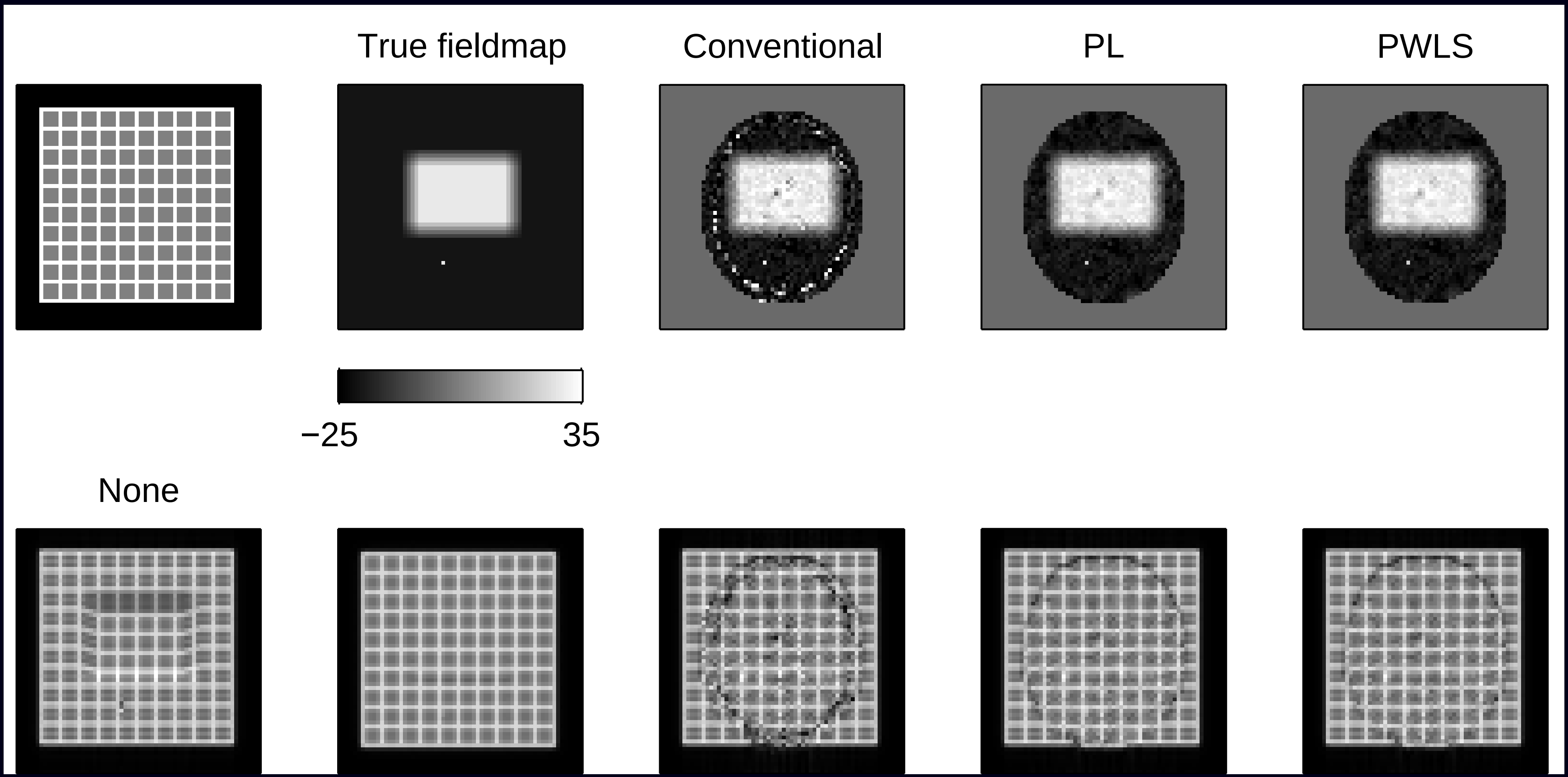
Field map display range: -40 to 120 Hz
Error map display range: -20 to 20 Hz.

Simulation 2



Field map display range: -40 to 120 Hz
Error map display range: -20 to 20 Hz.

Effect on EPI



30 msec EPI readout.

20 iterations of PWLS-CG field-corrected image reconstruction

Choosing regularization parameter β

Spatial resolution analysis of fieldmap estimator (Fessler *et al.*, IEEE T-MI, 1996):

$$\mathsf{E}[\hat{\boldsymbol{x}}] \approx \underbrace{\left[\boldsymbol{I} + \beta\,\mathrm{diag}\{w_j\}^{-1}\boldsymbol{C}'\boldsymbol{C}\right]}_{\text{“filter”}} \boldsymbol{x},$$

where $\boldsymbol{C}$ is the 1st-order or 2nd-order differencing matrix.

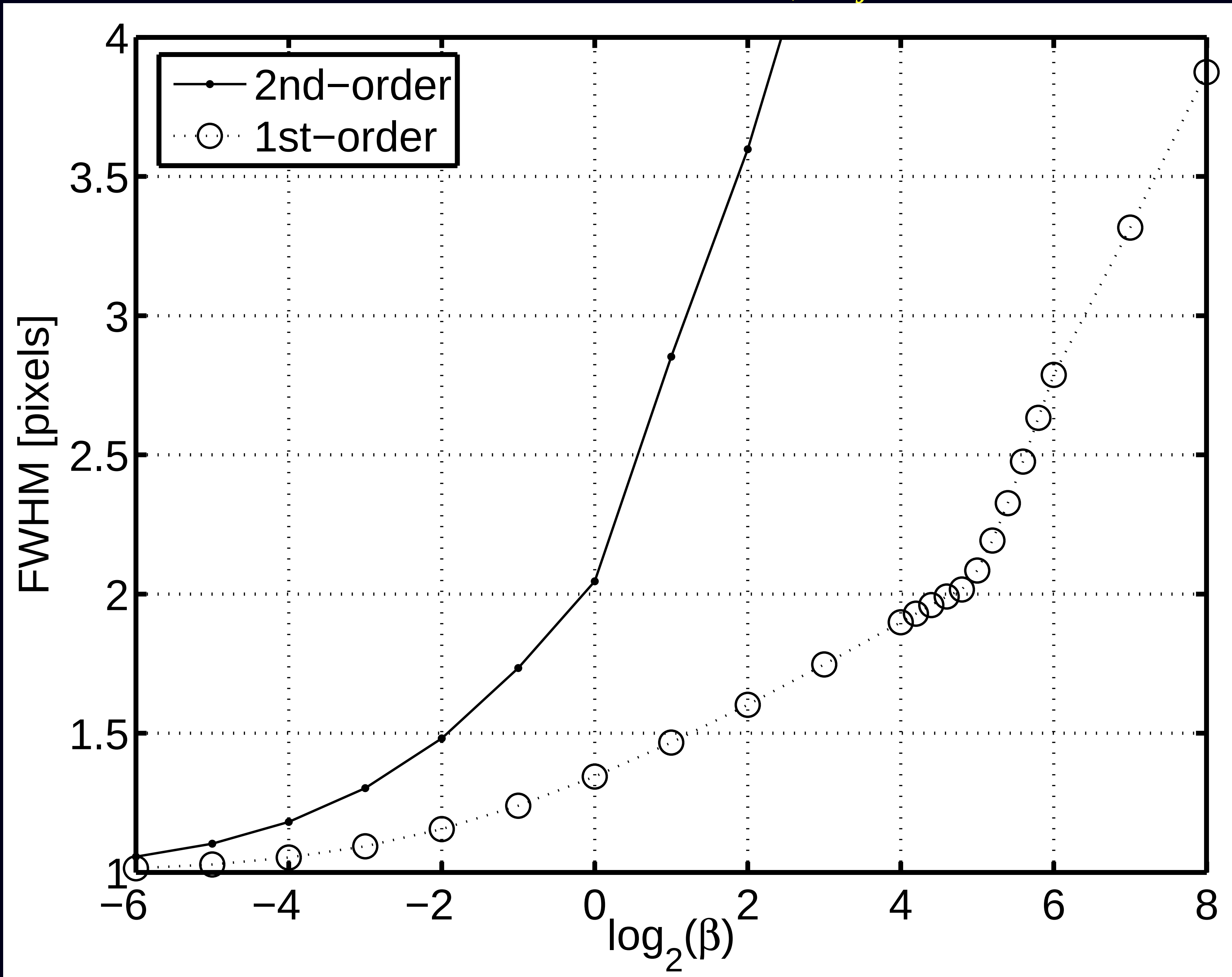
Local frequency response of the “filter” is: (Unser *et al.* IEEE T-SP 1991)

$$H(\omega_1, \omega_2) \approx \frac{1}{1 + (\beta / w_j)(\omega_1^2 + \omega_2^2)^p},$$

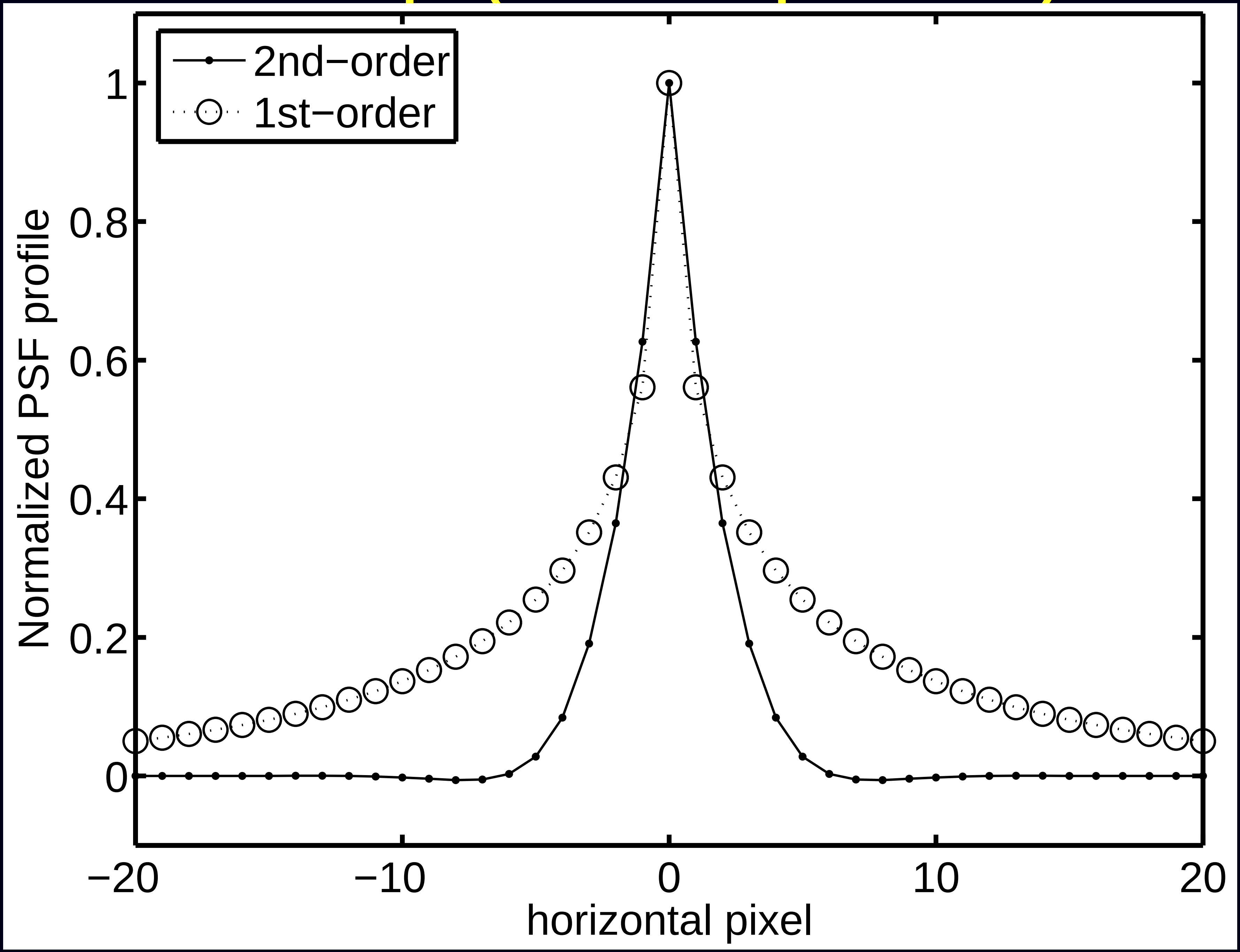
for regularization based on pth-order differences, where $p = 1$ or 2.

- Inverse 2D DSFT yields PSF $h[n, m]$.
- Tabulate FWHM of PSF vs β / w_j.
- Select required β based on desired spatial resolution (FWHM).

PSF FWHM vs β / w_j



PSF Shape (2nd-order preferable)



Summary

- Conventional phase estimate equivalent to joint ML estimate
- Penalized-likelihood estimation reduces fieldmap error
- PWLS estimator performs similarly:
  preferable due to simplicity in absence of phase wrap

Future work

- Accelerate by preconditioning or multi-resolution
- Regularized estimation from k-space data
- Illustrate effects of phase errors on real EPI and spiral scans