Nuclear Magnetic Resonance Imaging

Jeffrey A. Fessler

EECS Department
The University of Michigan

NSS-MIC: Fundamentals of Medical Imaging

Oct. 20, 2003

Outline

- Background
- Basic physics
- 4 magnetic fields
- Bloch equation
- Excitation
- Signal equation and k-space
- Pulse sequences
- Image reconstruction
- Summary

History

- 1946. NMR phenomenon discovered independently by
  - Felix Bloch (Stanford)
  - Edward Purcell (Harvard)
- 1952. Nobel prize in physics to Bloch and Purcell
- 1966. Richard Ernst and W. Anderson develop Fourier transform spectroscopy
- ... NMR spectroscopy used in physics and chemistry
- 1971. Ray Damadian discriminates malignant tumors from normal tissue by NMR spectroscopy
- 1973. Paul Lauterbur and Peter Mansfield (independently) add field gradients, to make images
- 1991. Nobel prize in chemistry to Ernst for NMR spectroscopy
- 2002. Nobel prize in chemistry to Kurt Wüthrich for NMR spectroscopy to determine 3D structure of biological macromolecules
- 2003. Nobel prize in medicine to Lauterbur and Sir Mansfield!

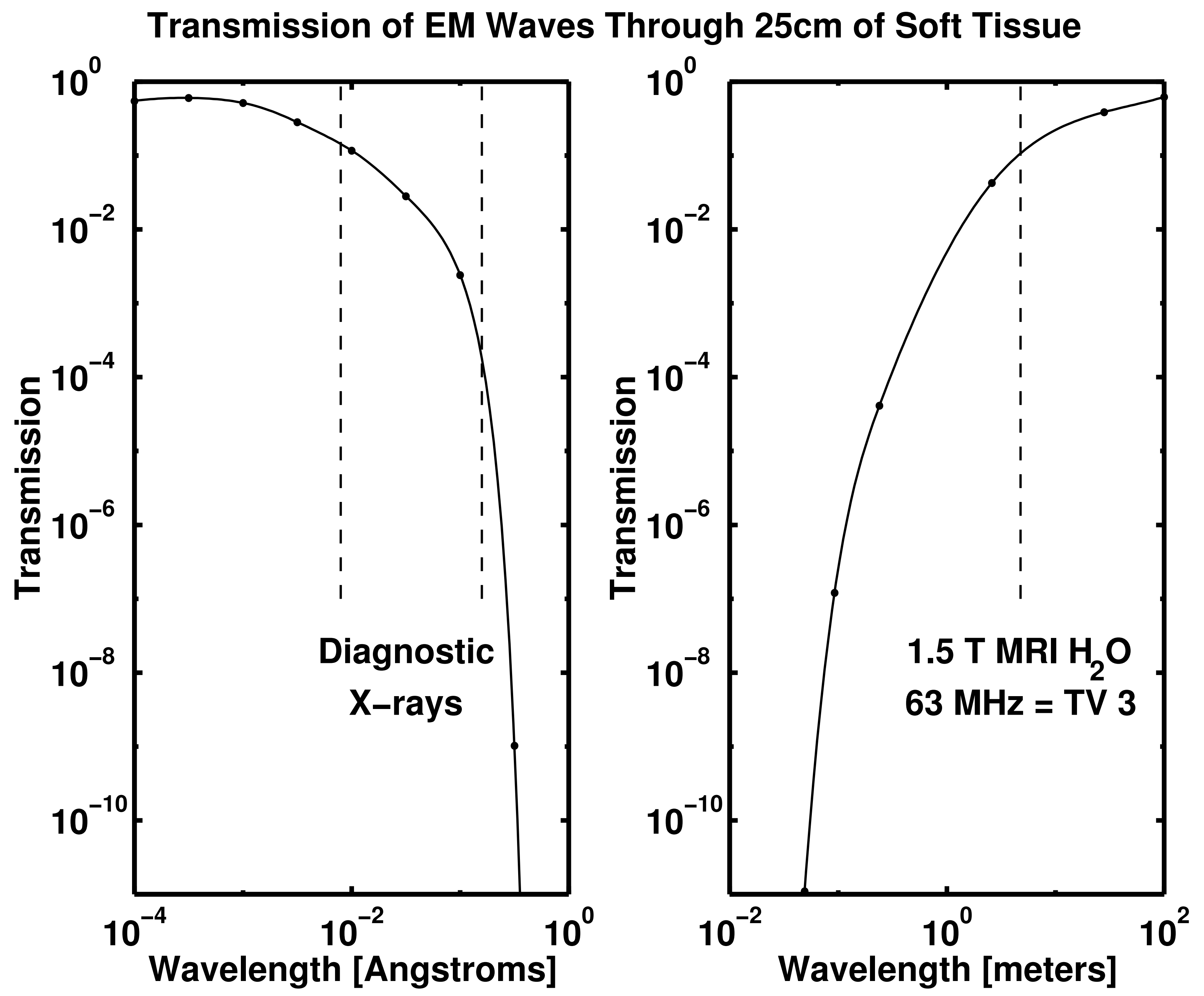
Advantages of NMR Imaging

- Nonionizing radiation
- Good soft-tissue contrast
- High spatial resolution
- Flexible user control - many acquisition parameters
- Minimal attenuation
- Image in arbitrary plane (or volume)
- Potential for chemically specified imaging
- Flow imaging

Disadvantages of NMR Imaging

- High cost
- Complicated “siting” due to large magnetic fields
- Low sensitivity
- Little signal from bone
- Scan durations

Attenuation Considerations



Nuclear Spins

The NMR phenomena is present in nuclei having an odd number of protons or neutrons and hence *nuclear spin angular momentum*.

Such nuclei often just called “spins”

Most abundant spin is ^{1}H, a single proton, in water molecules.

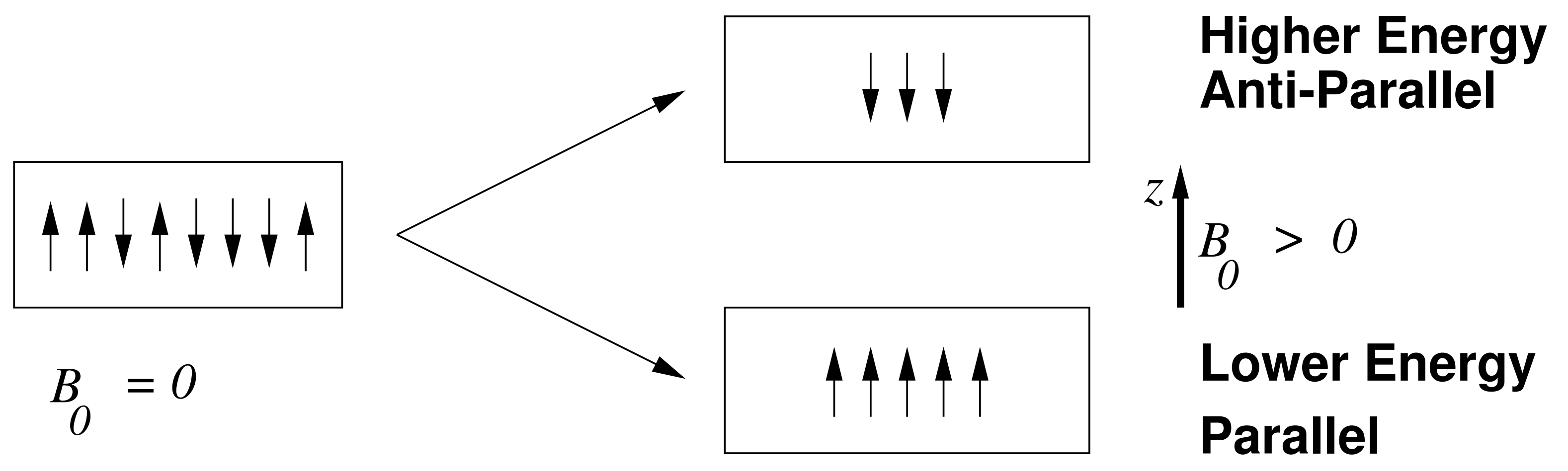
NMR concerns the interaction of these spins with magnetic fields:

1. Main field $\vec{B}_0$ (static)
2. Local field effects
   - ○ Effect of local orbiting electrons (chemical shift)
   - ○ Differences in magnetic susceptibility
3. RF field $\vec{B}_1(t)$ (user-controlled, amplitude-modulated pulse)
4. Gradient fields $\vec{G}(t)$ (user-controlled, time-varying)

1. Main Field $\vec{B}_0$

^{1}H has two energy states: parallel and anti-parallel to main field.

Zeeman Diagram:



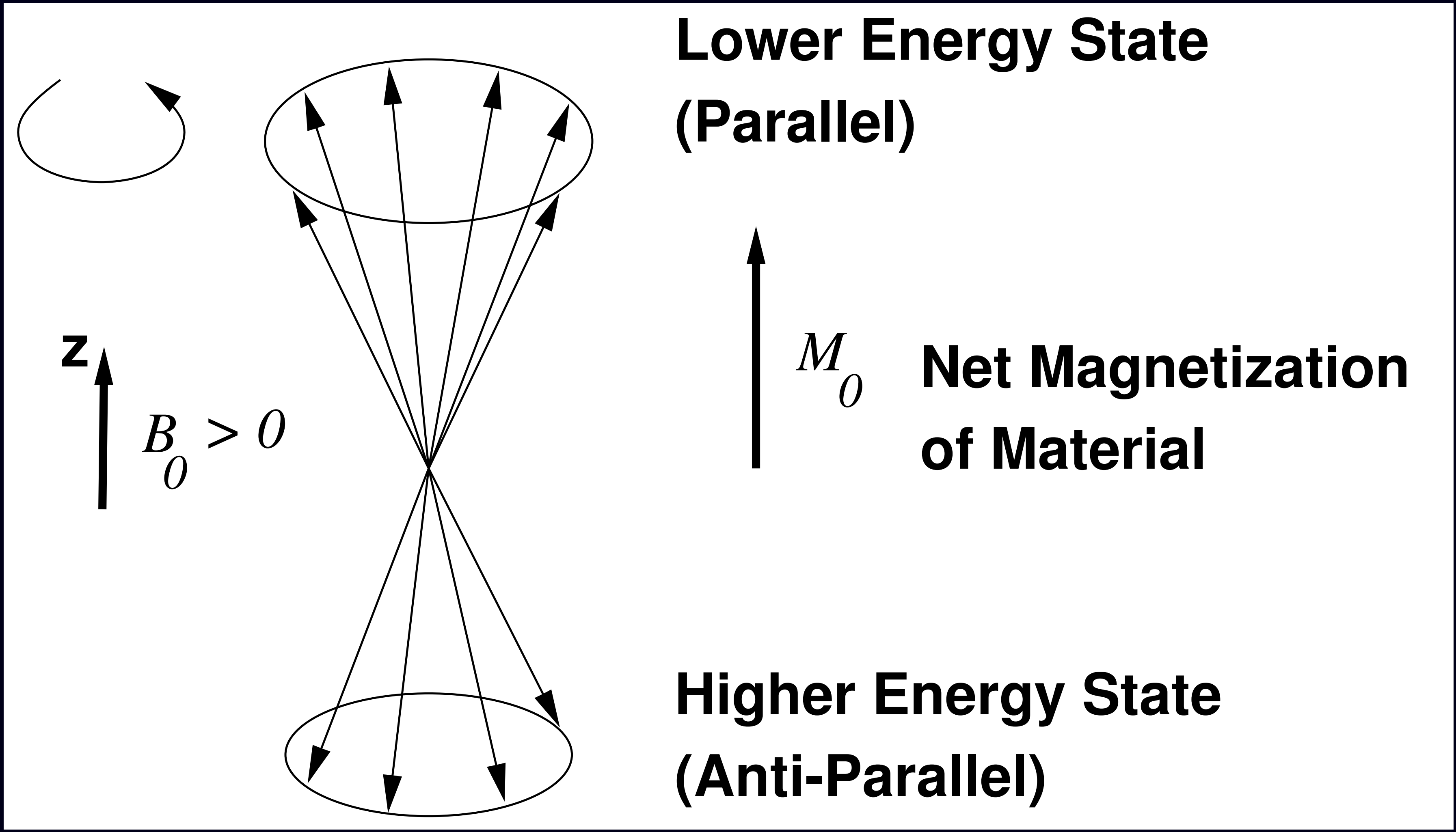
$$\updownarrow \; \Delta\mathcal{E} = h \frac{\gamma}{2\pi} B_0$$

Thermal agitation causes random transitions between states.
Equilibrium ratio of anti-parallel (n_-) to parallel (n_+) nuclei:

$$\frac{n_-}{n_+} = \mathrm{e}^{-\Delta\mathcal{E}/(k_b T)} \quad \text{(Boltzmann distribution)}$$

Precession (Classical Description)

Spins do not align exactly parallel or anti-parallel to z.
A spin's magnetic moment experiences a torque, causing precession.



Larmor Equation

Precession frequency is proportional to (local) field strength:

$$\boxed{\omega = \gamma |\vec{B}|}$$

- γ is called the *gyromagnetic ratio*
- $\gamma / 2\pi =$ 42.48 MHz/Tesla for ^{1}H
- At $B_0 =$ 1.5T, $f_0 = \frac{\gamma}{2\pi} B_0 \approx 63$ MHz (TV Channel 3)

2.1 Local Field Effect: Chemical Shift

Orbital electrons surrounding a nuclei perturb the local magnetic field:

$$B_{\text{eff}}(\vec{r}) = B_0(1 - \sigma(\vec{r})),$$

where $\sigma(\vec{r})$ depends on local electron environment.

Because of the Larmor relationship, this field shift causes a shift in the local resonant frequency:

$$\omega_{\text{eff}} = \omega_0(1 - \sigma).$$

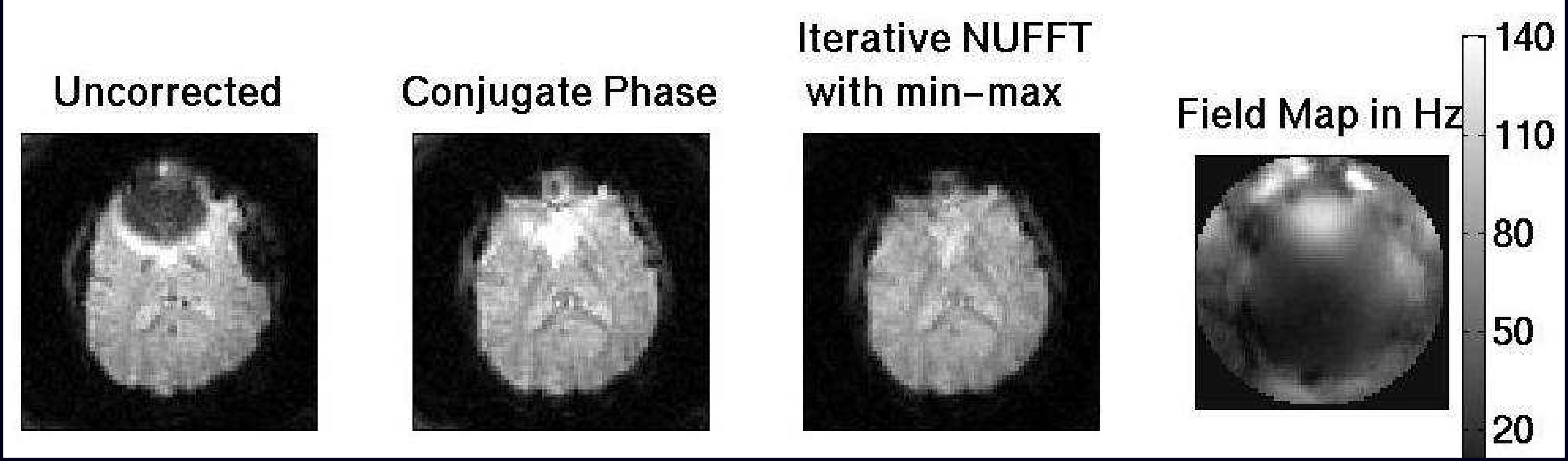
Example:
the resonant frequency of ^{1}H in fat is about 3.5 ppm lower than in water.

Chemical shift causes artifacts in usual “frequency encoded” imaging.

2.2 Local Field Effect: Susceptibility

Differences in the magnetic susceptibility of different materials, particularly air and water, cause local perturbations of the magnetic field that also cause local shifts in the resonance frequency.

This effect can cause undesirable signal loss.



It can also be a source of contrast, such as the BOLD effect in fMRI.

3. RF Field $\vec{B}_1(t)$

The first step in any NMR experiment is to perturb the spins away from equilibrium using an RF pulse.

Amplitude-modulated RF pulse circularly polarized in x, y plane:

$$\vec{B}_1(t) = B_1 \begin{bmatrix} \cos \omega_0 t \\ -\sin \omega_0 t \\ 0 \end{bmatrix} p(t).$$

Spins now precess about the *total* time-varying magnetic field:

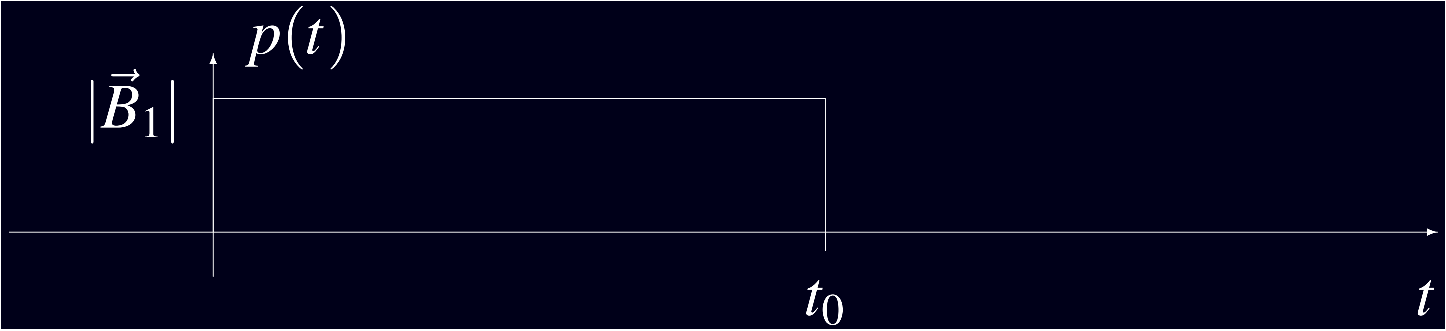
$$\vec{B}(t) = \vec{B}_0 + \vec{B}_1(t).$$

$|\vec{B}_1| \ll |\vec{B}_0|$

“RF excitation” puts energy into the system.

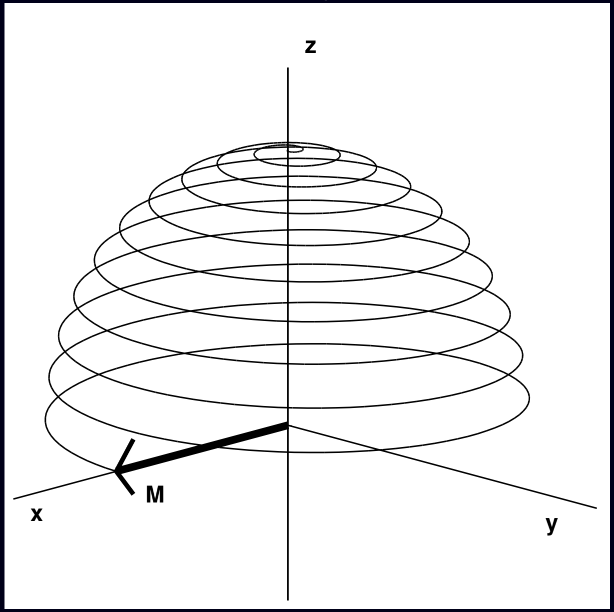
RF Excitation Example

Non-selective excitation:

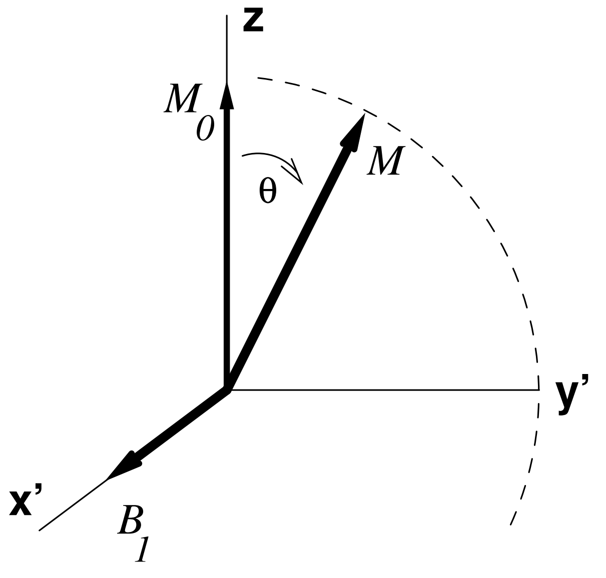


Time evolution of local magnetization $\vec{M}(t)$

Laboratory Frame

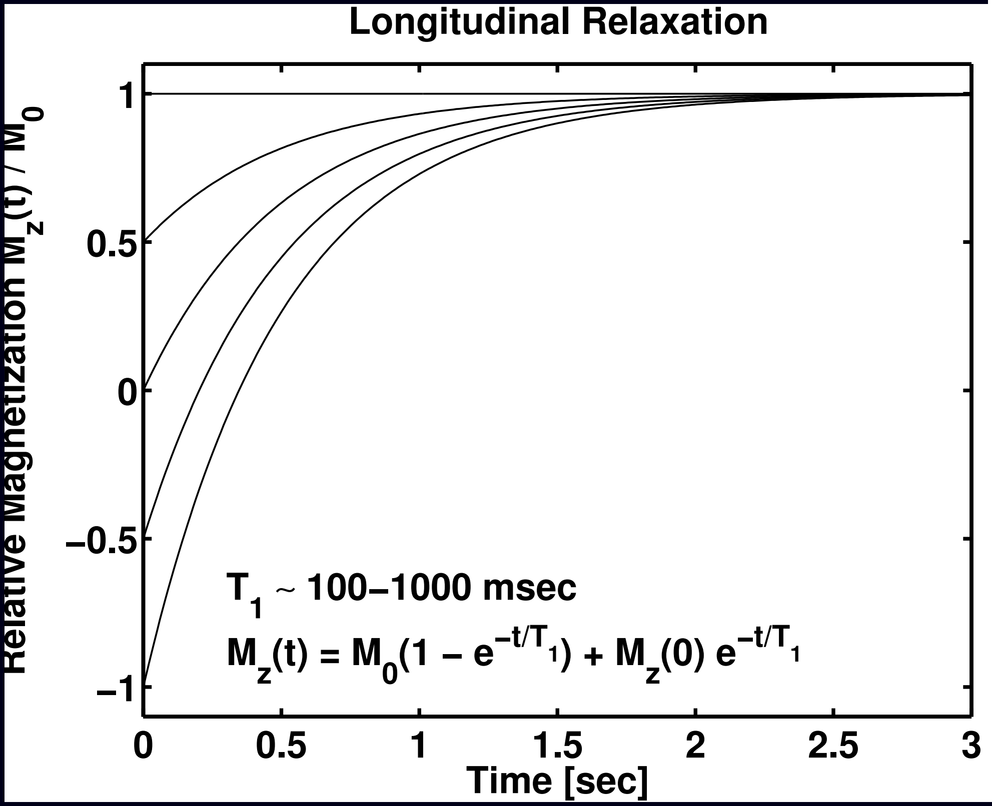


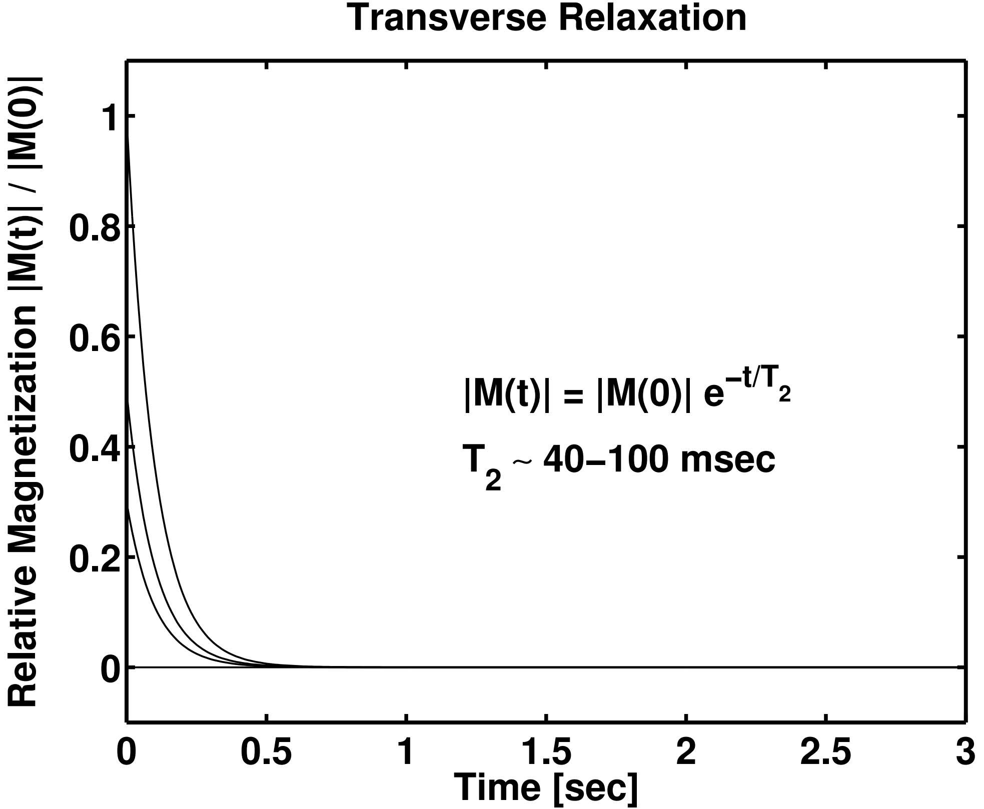
Rotating Frame



Relaxation

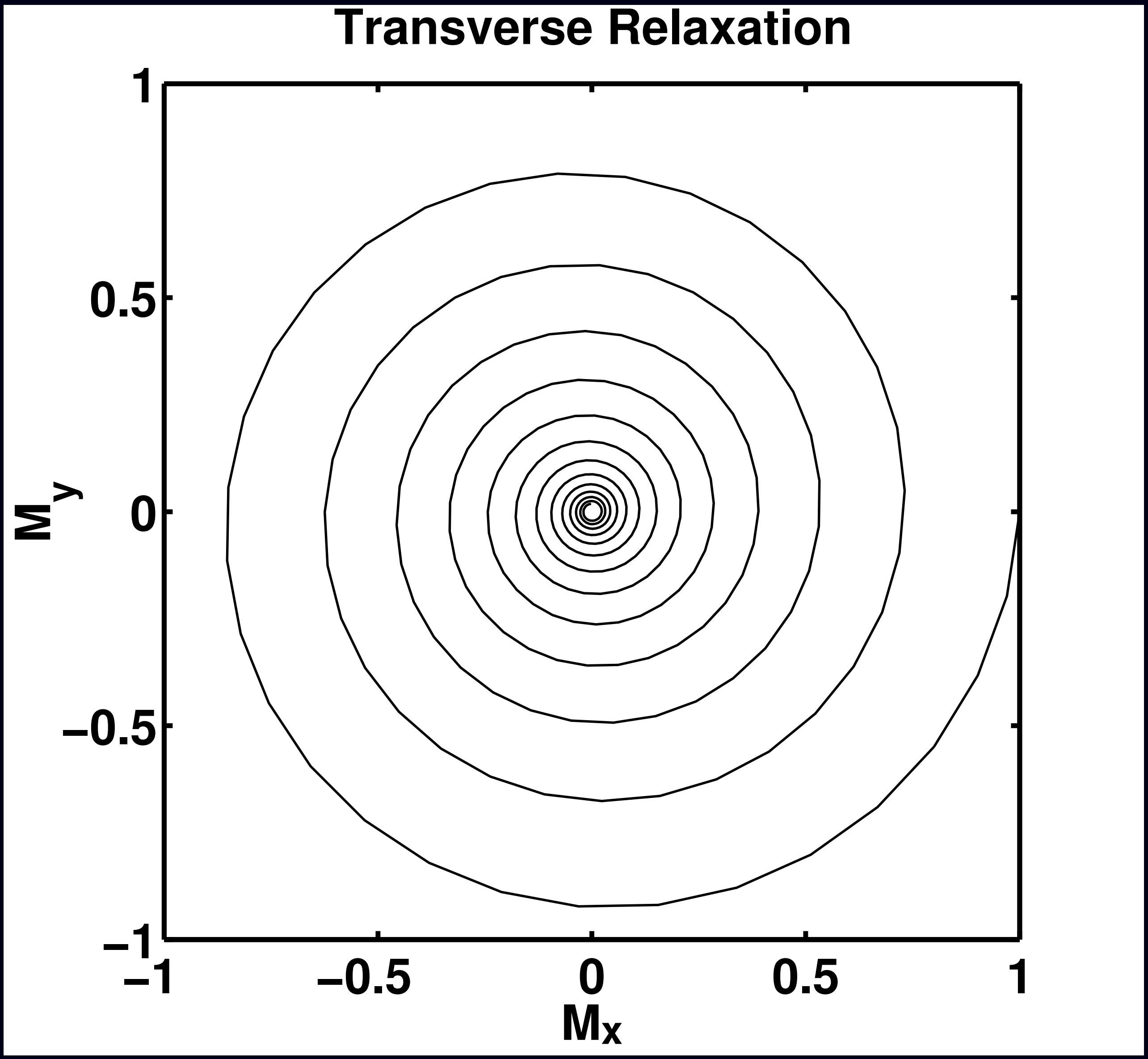
After RF excitation, the magnetization $\vec{M}(t)$ returns to its equilibrium exponentially, releasing (some of) the energy put in by the excitation.





- T_1: spin-lattice time constant. Long T_1 slows imaging :-(
- T_2: spin-spin time constant
- $T_1 \neq T_2 \Rightarrow |\vec{M}(t)|$ not constant

Relaxation of Transverse Magnetization



Signal!

As spins return to equilibrium, measurable energy is released.

By Faraday's law, precessing magnetic moment induces current in RF receive coil:

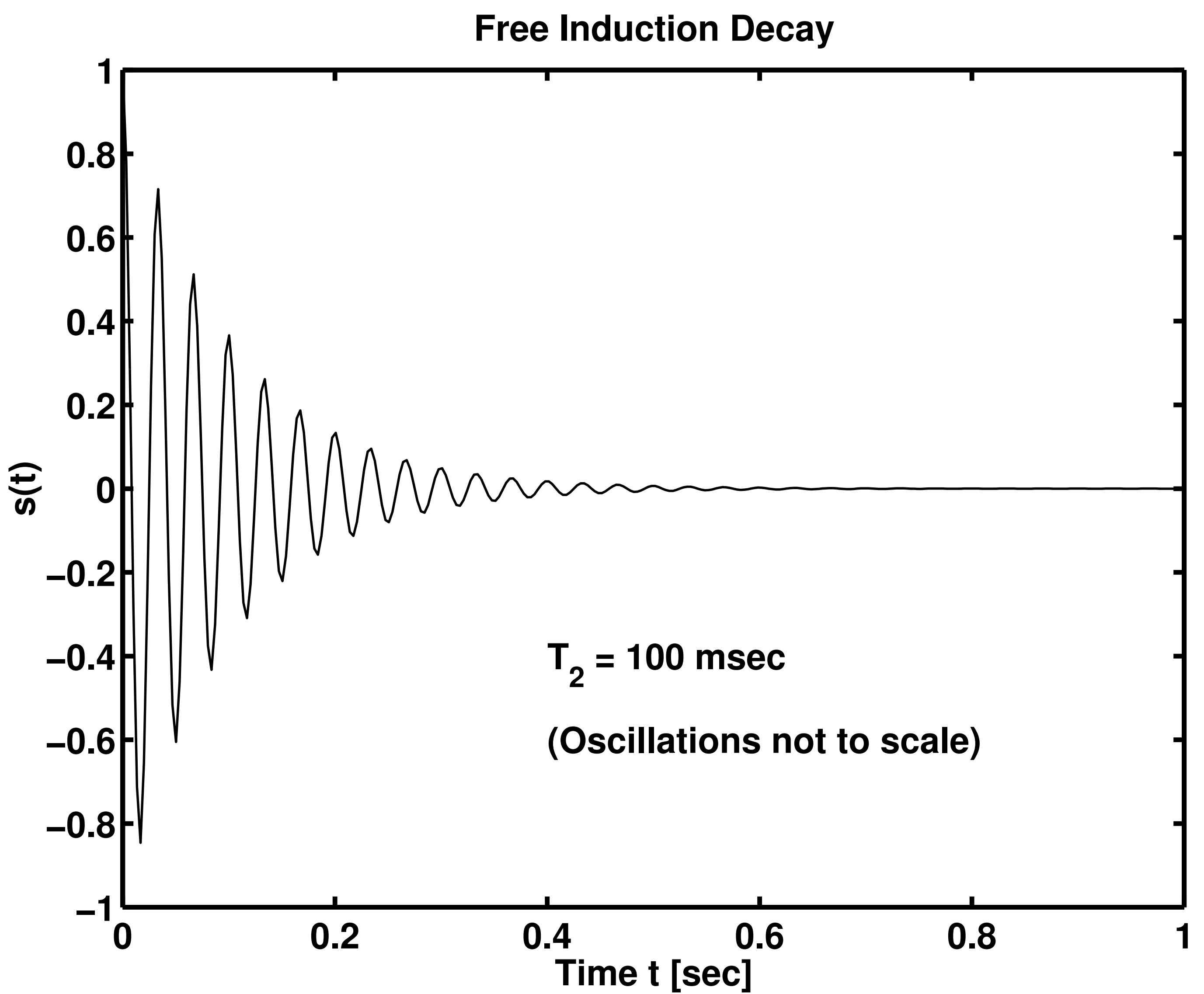
$$s_r(t) = \int_{\text{vol}} b_{\text{coil}}(\vec{r}) \frac{\partial}{\partial t} M(\vec{r}, t)\, \mathrm{d}\vec{r},$$

- *transverse* magnetization: $M(\vec{r},t) \overset{\triangle}{=} M_{\mathrm{X}}(\vec{r},t) + \imath M_{\mathrm{Y}}(\vec{r},t)$
- Transverse RF coil sensitivity pattern: $b_{\text{coil}}(\vec{r})$.
- Higher spin density or larger $\omega_0 \Rightarrow$ more signal.

Spatial localization? Not much!

Small coils have a *somewhat* localized spatial sensitivity pattern. This property has been exploited recently in "sensitivity encoded" (SENSE) imaging, to reduce scan times.

Free-Induction Decay



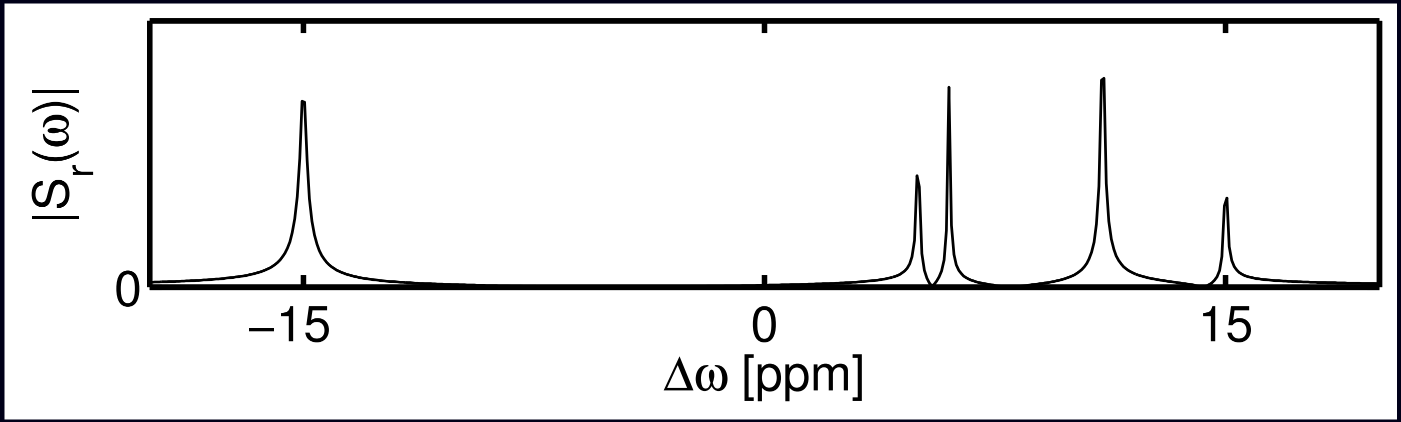
NMR Spectroscopy

Material with ^{1}H having several relaxation rates and chemical shifts.

Received FID signal: $s_r(t) = \sum_{n=1}^{N} \rho_n \mathrm{e}^{-t/T_{2,n}} \mathrm{e}^{\imath \omega_n t}, \qquad t \geq 0$

- ρ_n: spin density of nth species
- $T_{2,n}$: relaxation time of nth species
- ω_n: resonant frequency of nth species

Fourier transform: $S_r(\omega) = \sum_{n=1}^{N} \rho_n \frac{1}{1 + \imath(\omega - \omega_n) T_{2,n}}$



3. Gradient Fields

For a uniform main field $\vec{B}_0$, all spins have (almost) the same resonant frequency. For *imaging* we must “encode” spin spatial position.

Frequency encoding relates spatial location to (temporal) frequency by using gradient coils to induce a field gradient, *e.g.*, along x: a gradient along the x direction:

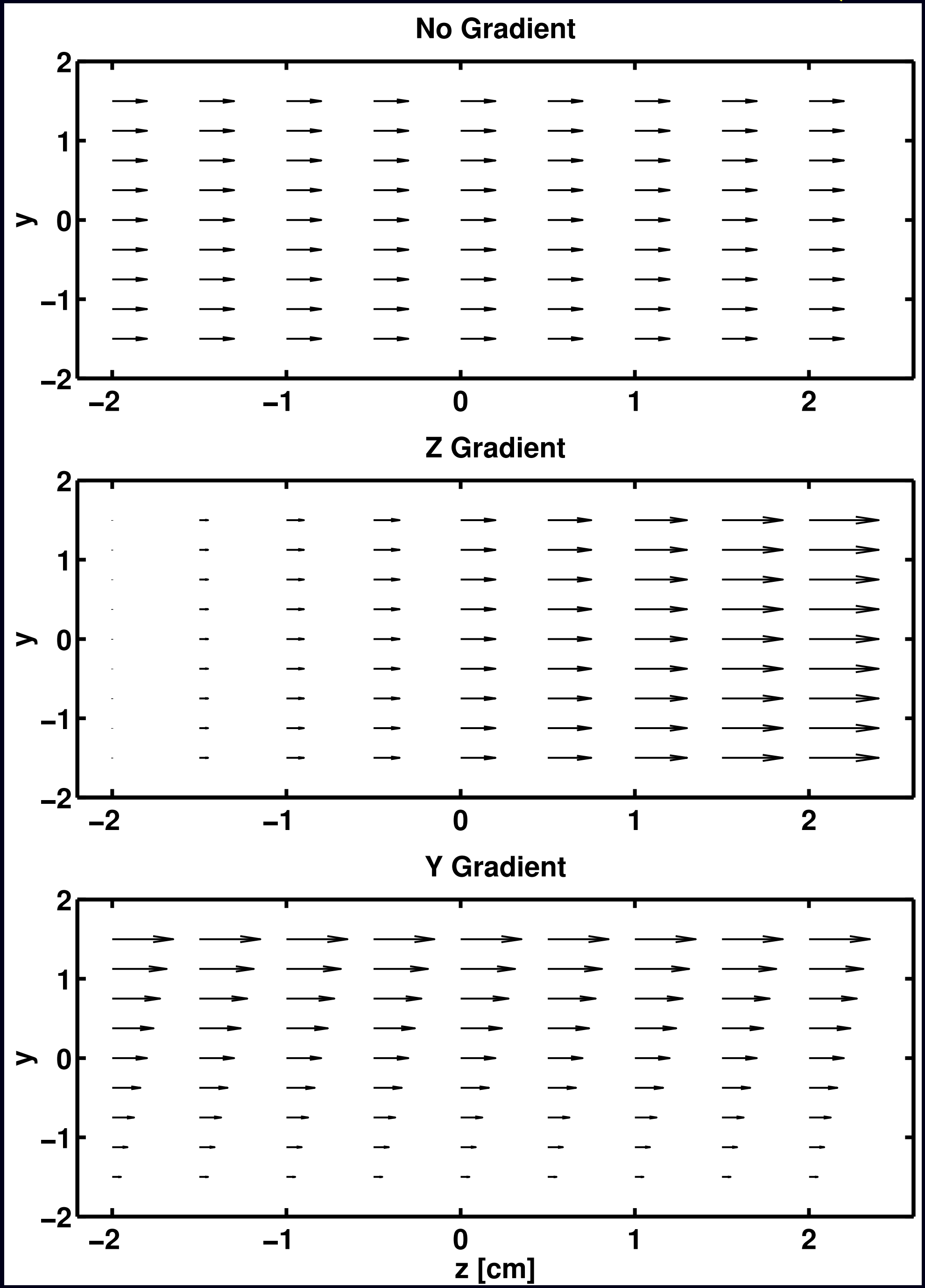
$$\vec{B}(x,y,z) = \begin{bmatrix} 0 \\ 0 \\ B_0 + xG_\mathrm{X} \end{bmatrix} = (B_0 + xG_\mathrm{X})\vec{k}.$$

Here, x location becomes “encoded” through the Larmor relationship:

$$\omega(x,y,z) = \gamma|\vec{B}(x,y,z)| = \gamma(B_0 + xG_\mathrm{X}) = 2\pi\left(f_0 + x\frac{\partial f}{\partial x}\right), \quad \frac{\partial f}{\partial x} = \frac{\gamma}{2\pi}G_\mathrm{X}.$$

- $B_0 \approx 10^4$ Gauss $\Rightarrow f_0 \approx 60$ MHz
- $G_\mathrm{X} \approx 1$ Gauss/cm $\Rightarrow \frac{\partial f}{\partial x} \approx$ 6 KHz / cm

Effect of gradient fields: $\vec{B}(0, y, z)$



Bloch Equation

Phenomenological description of time evolution of local magnetization $\vec{M}(\vec{r},t)$:

$$\frac{d\vec{M}}{dt} = \vec{M}\times\gamma\vec{B} - \frac{M_{\mathrm{X}}\vec{i}+M_{\mathrm{Y}}\vec{j}}{T_2} - \frac{(M_{\mathrm{Z}}-M_0)\vec{k}}{T_1}$$

Precession ↑

Relaxation ↑ ↑

Equilibrium ↑

Driving term $\vec{B}(\vec{r},t)$ includes

- Main field $\vec{B}_0$
- RF field $\vec{B}_1(t)$
- Field gradients $\vec{r}\cdot\vec{G}(t) = xG_{\mathrm{X}}(t)+yG_{\mathrm{Y}}(t)+zG_{\mathrm{Z}}(t)$

$$\vec{B}(\vec{r},t) = \vec{B}_0+\vec{B}_1(t)+\vec{r}\cdot\vec{G}(t)\vec{k}$$

Excitation Considerations

Excitation

- Selective (for slice selection)
- Non-selective (affects entire volume)

Design parameters (functions!):

- RF amplitude signal $B_1(t)$
- Gradient strengths $\vec{G}(t)$

Non-Selective Excitation

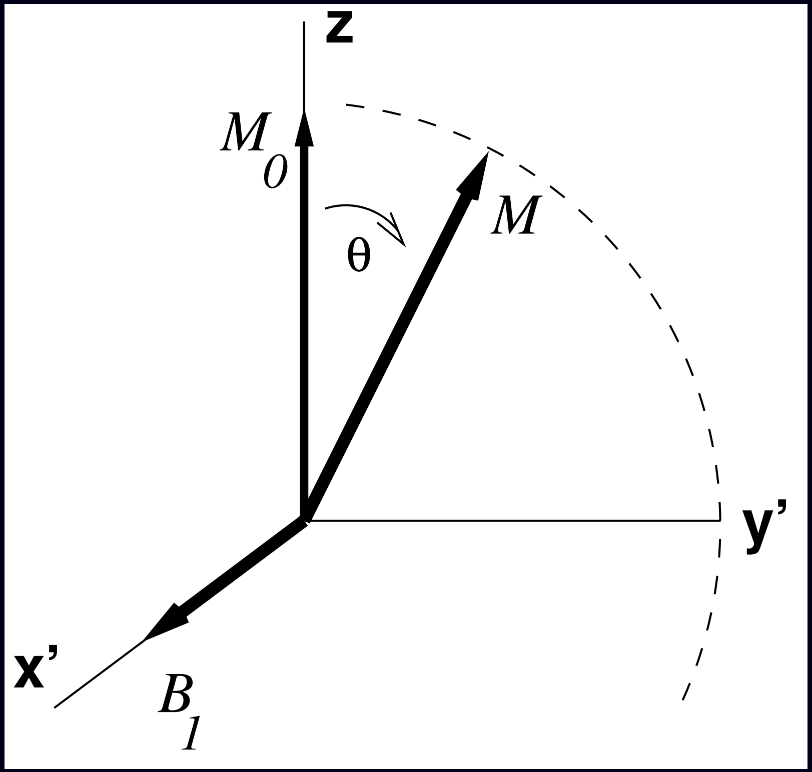
- No gradients: excite entire volume
- $\vec{B}_1(t) = B_1(t)(\vec{i}\cos\omega_0 t - \vec{j}\sin\omega_0 t)$ (circularly polarized)
- Ignore relaxation

Solution to Bloch equation:

$$\vec{M}_{\text{rot}}(t) = R_{\text{x}}\left(\int_0^t \omega_1(s)\,\mathrm{d}s\right)\vec{M}_{\text{rot}}(0)$$

$$\omega_1(t) = \gamma B_1(t)$$

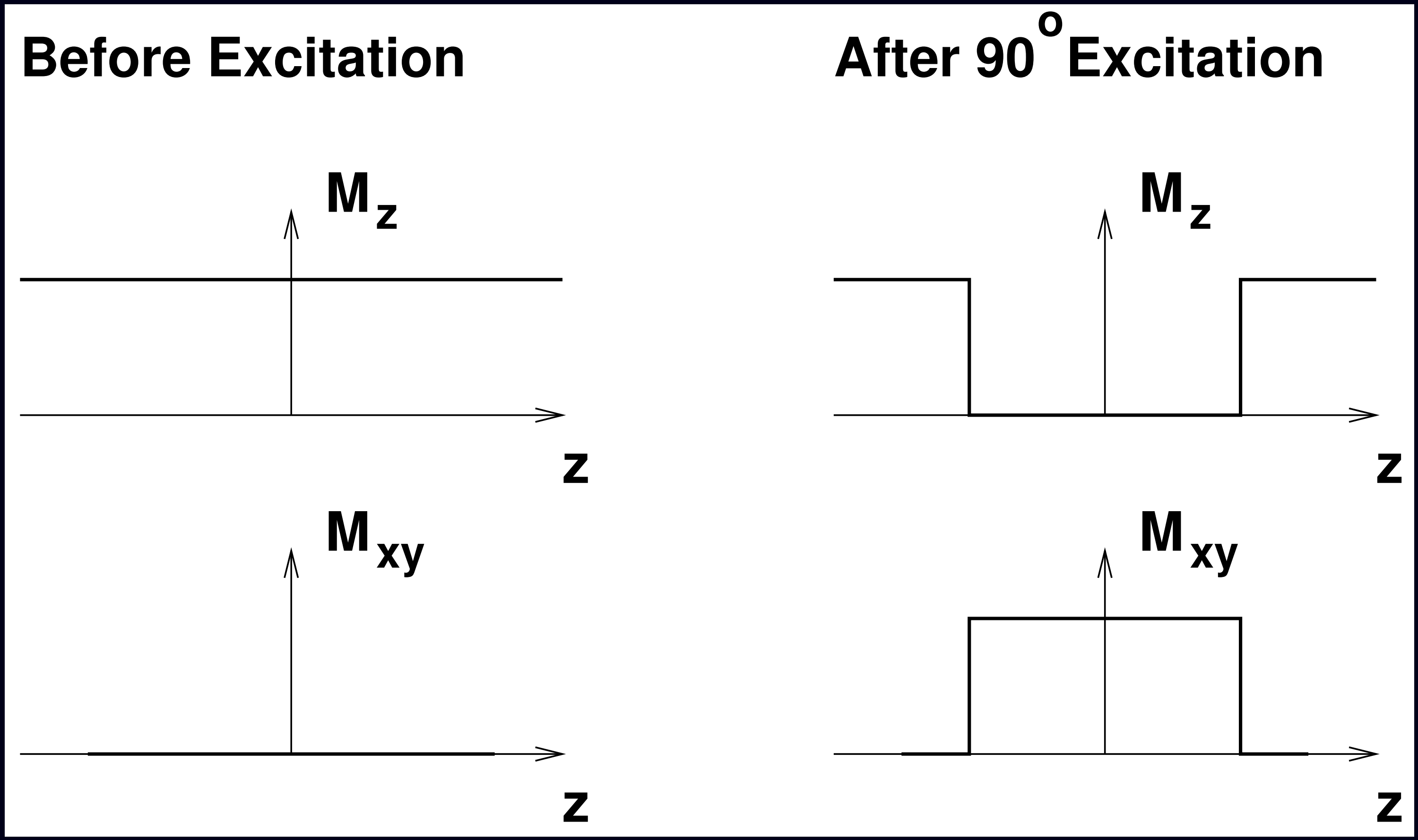
$$R_{\text{x}}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$



Typically $\theta = 90°$ or $\theta = 180°$.

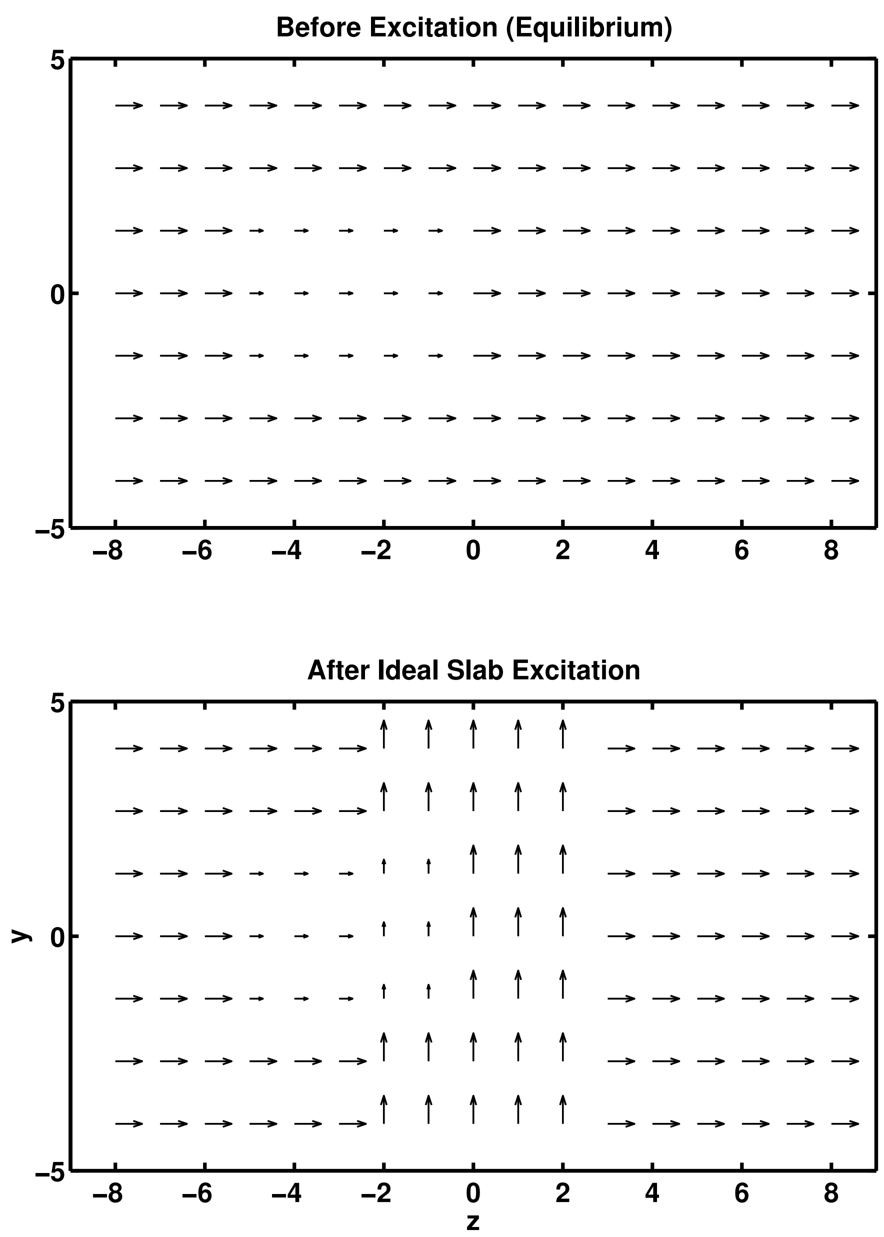
Selective Excitation

Ideal slice profile:



Requires non-zero gradient so the resonant frequencies vary with z.

90° Selective Excitation



RF Pulse Design: Small Tip-Angle Approximation

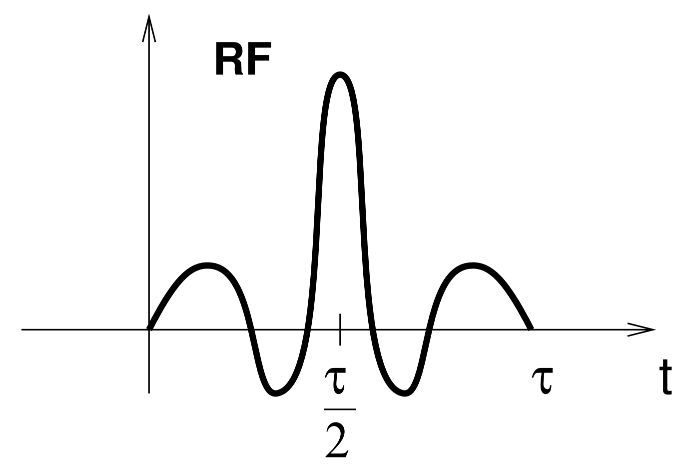
Assumptions:

- $\theta < 30^\circ$ (weak RF pulse)
- $M_Z(t) \approx M_0$
- $d/dt M_Z(t) \approx 0$
- Constant gradient in z direction

(Approximate) solution to Bloch equation:

$$M(z) \propto \mathcal{F}_1\{B_1(t)\}\Big|_{f=z\frac{\gamma}{2\pi}G_Z}$$

For a rectangular slice profile
we a need sinc-like RF pulse:



Receive Considerations

Fundamental Equation of MR

After excitation, magnetization continues to evolve via Bloch equation.

Solve Bloch equation for RF=0 to find evolution of transverse magnetization: $M(\vec{r},t) = M_{\mathrm{X}}(\vec{r},t) + \imath M_{\mathrm{Y}}(\vec{r},t)$.

Fundamental Equation of MR Imaging

$$M(\vec{r},t) \;=\; m_0(\vec{r}) \;\; \mathrm{e}^{-\imath\omega_0 t} \;\; \mathrm{e}^{-t/T_2(\vec{r})} \;\; \mathrm{e}^{-\imath\phi(\vec{r},t)}$$

Post-excitation ________ ↑

Precession ________________ ↑

Relaxation ________________________ ↑

User-controlled phase term ________________ ↑

$$\phi(\vec{r},t) = \gamma \int_0^t \vec{r} \cdot \vec{G}(\tau)\, \mathrm{d}\tau.$$

- Key design parameters are gradients: $\vec{G}(t)$.
- How to manipulate the gradients to make an image of $m_0(\vec{r})$?

Received Signal

From Faraday's law (assuming uniform receive coil sensitivity):

$$s_r(t) = \int_{\text{vol}} \frac{\partial}{\partial t} M(\vec{r}, t) \, \mathrm{d}\vec{r}.$$

Using the Fundamental Equation of MR and converting to baseband:

$$s(t) = s_r(t) \, \mathrm{e}^{-\imath \omega_0 t} = \int_{\text{vol}} m_0(\vec{r}) \, \mathrm{e}^{-t/T_2(\vec{r})} \, \mathrm{e}^{-\imath \phi(\vec{r},t)} \, \mathrm{d}\vec{r}.$$

Disregarding T_2 decay:

$$\boxed{s(t) = \int_{\text{vol}} m_0(\vec{r}) \, \mathrm{e}^{-\imath \phi(\vec{r},t)} \, \mathrm{d}\vec{r},}$$

where

$$\phi(\vec{r}, t) = \gamma \int_0^t \vec{r} \cdot \vec{G}(\tau) \, \mathrm{d}\tau.$$

$\therefore$ The baseband signal is a phase-weighted volume integral of $m_0(\vec{r})$.

Signal Equation

Rewriting baseband signal:

$$\boxed{s(t) = \iiint m(x,y,z)\, \mathrm{e}^{-\imath 2\pi[x k_{\mathrm{X}}(t) + y k_{\mathrm{Y}}(t) + z k_{\mathrm{Z}}(t)]} \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z}$$

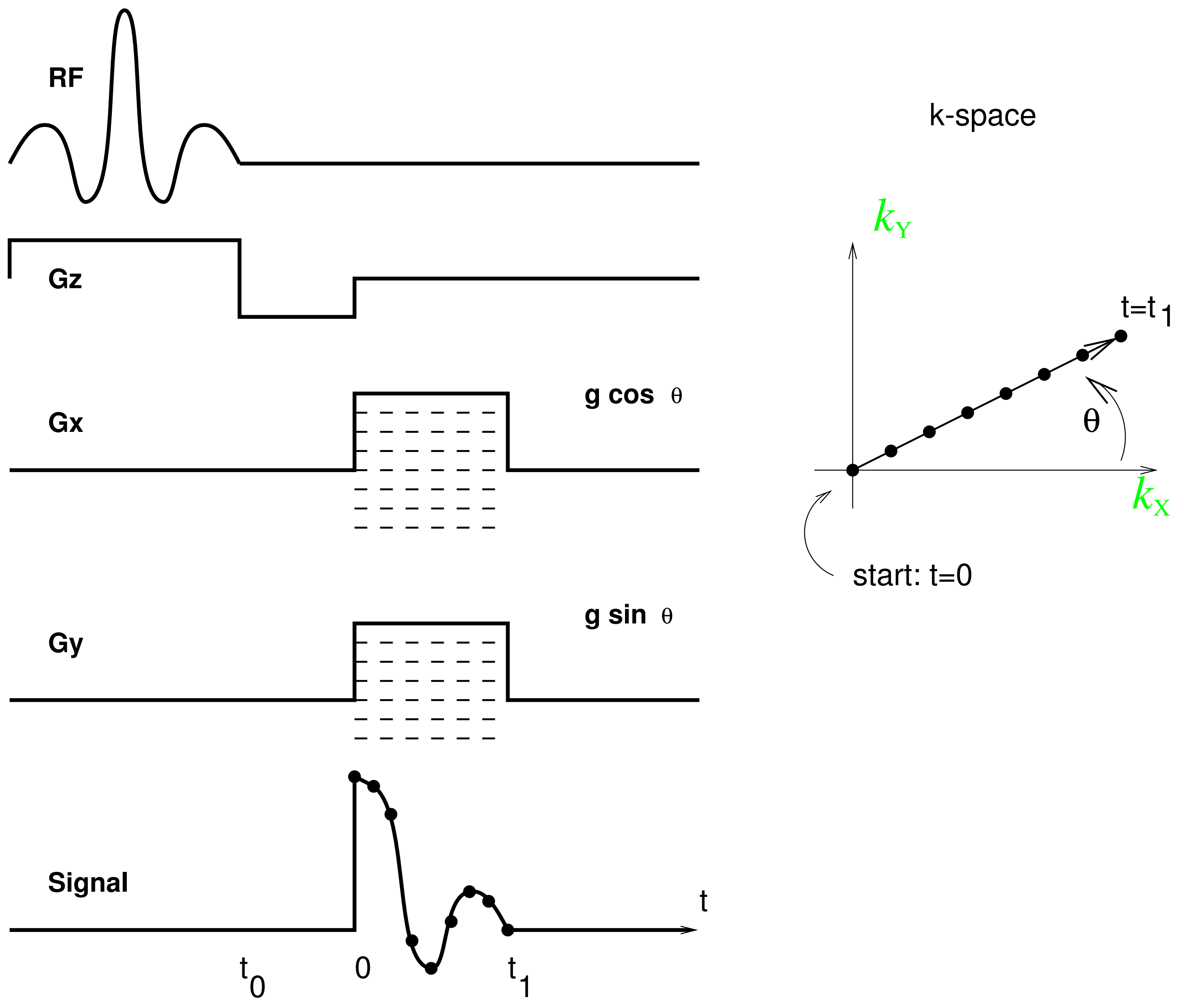
where the “k-space trajectory” is defined in terms of the gradients:

$$\begin{aligned} k_{\mathrm{X}}(t) &= \frac{\gamma}{2\pi} \int_0^t G_{\mathrm{X}}(\tau) \, \mathrm{d}\tau \\ k_{\mathrm{Y}}(t) &= \frac{\gamma}{2\pi} \int_0^t G_{\mathrm{Y}}(\tau) \, \mathrm{d}\tau \\ k_{\mathrm{Z}}(t) &= \frac{\gamma}{2\pi} \int_0^t G_{\mathrm{Z}}(\tau) \, \mathrm{d}\tau. \end{aligned}$$

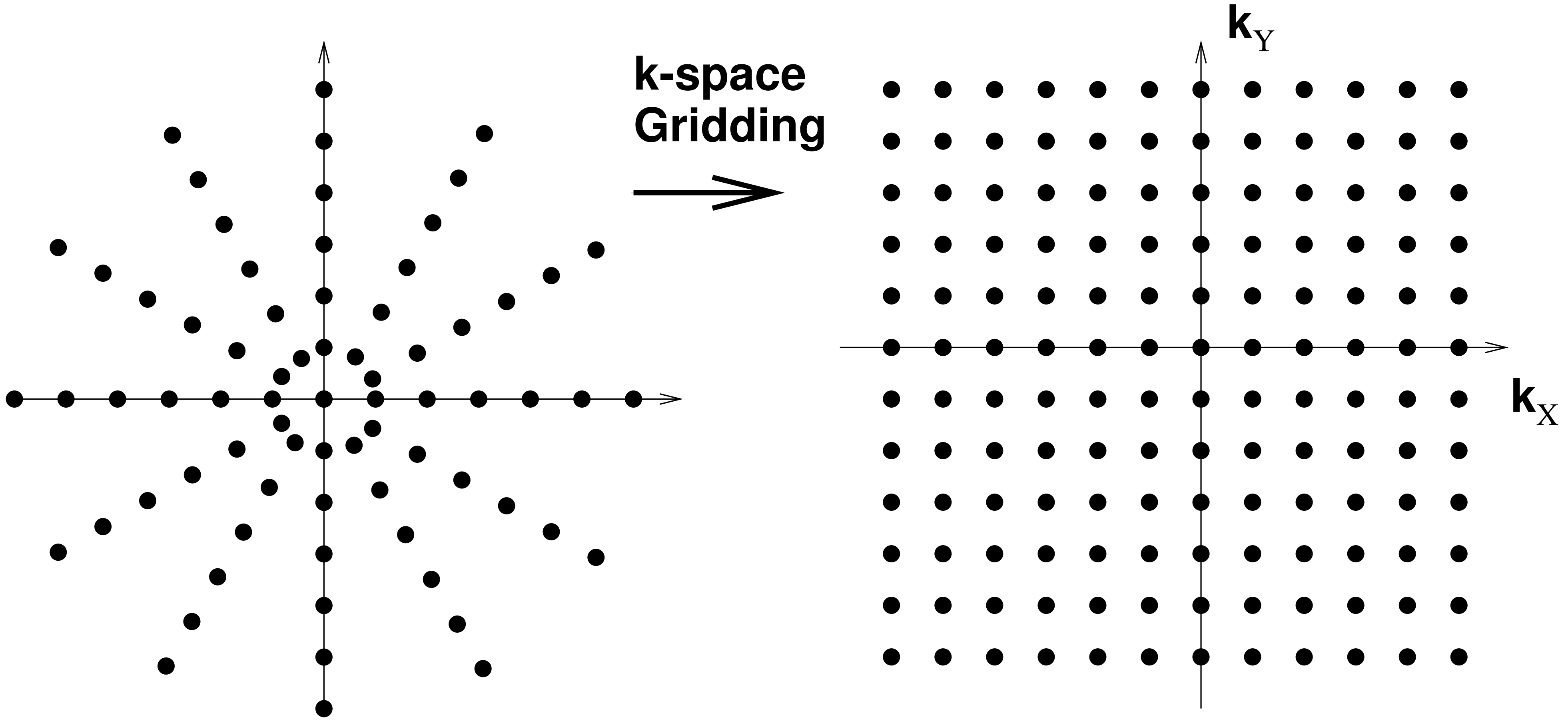
⇒ MRI measures samples of the FT of the object’s magnetization.

(Ignoring relaxation and off-resonance effects.)

2D Projection-Reconstruction Pulse Sequence



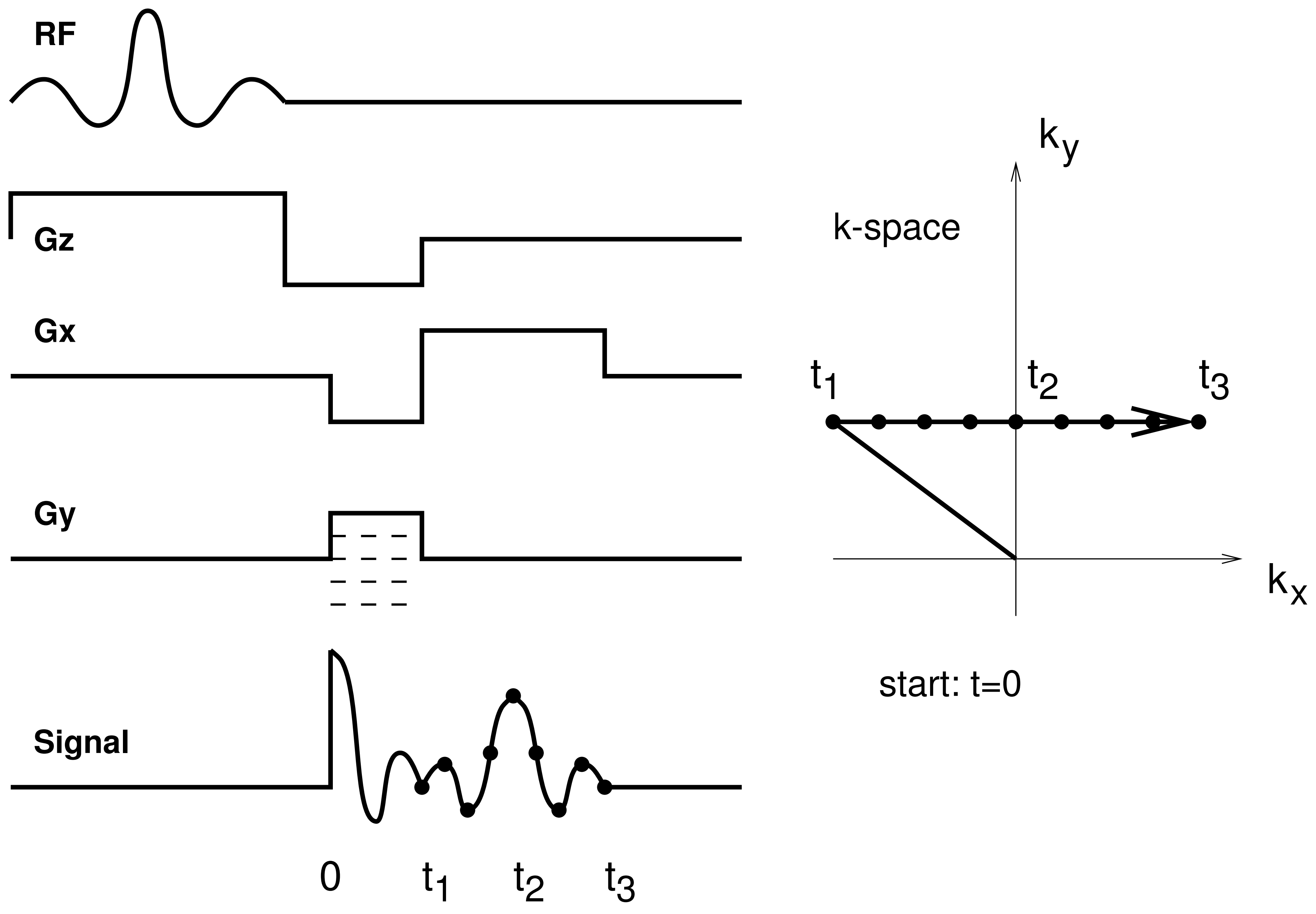
Gridding



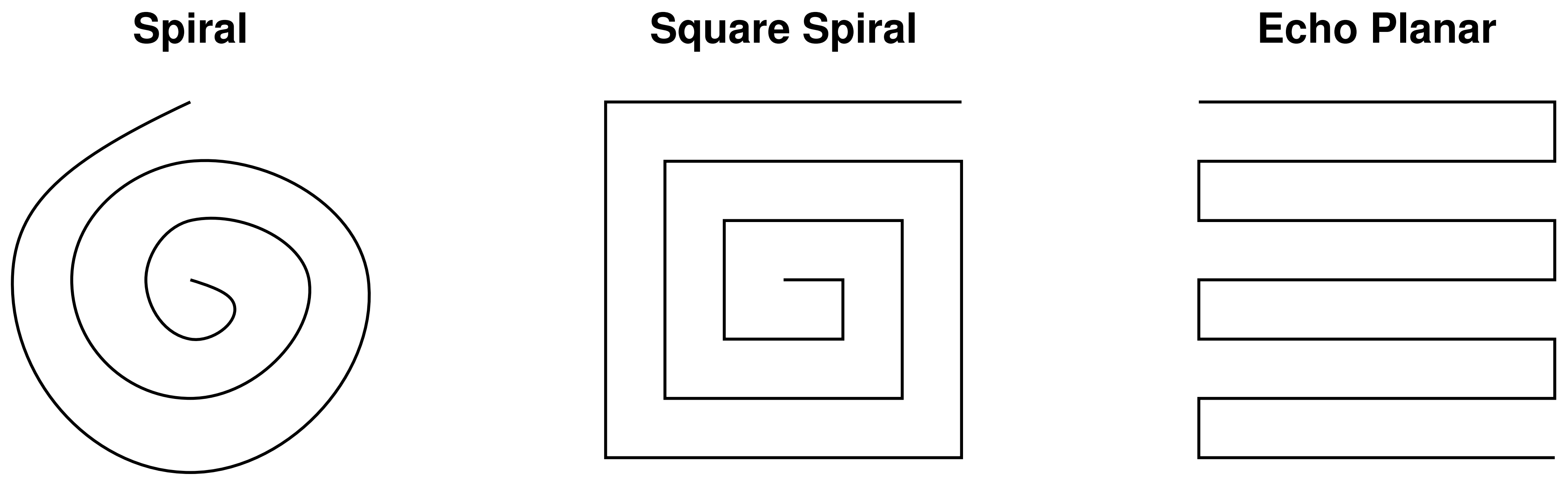
Followed by inverse 2D FFT to reconstruct image of magnetization.

Alternatively, one can apply the filtered backprojection algorithm.

Spin-Warp Imaging (Cartesian k-space)

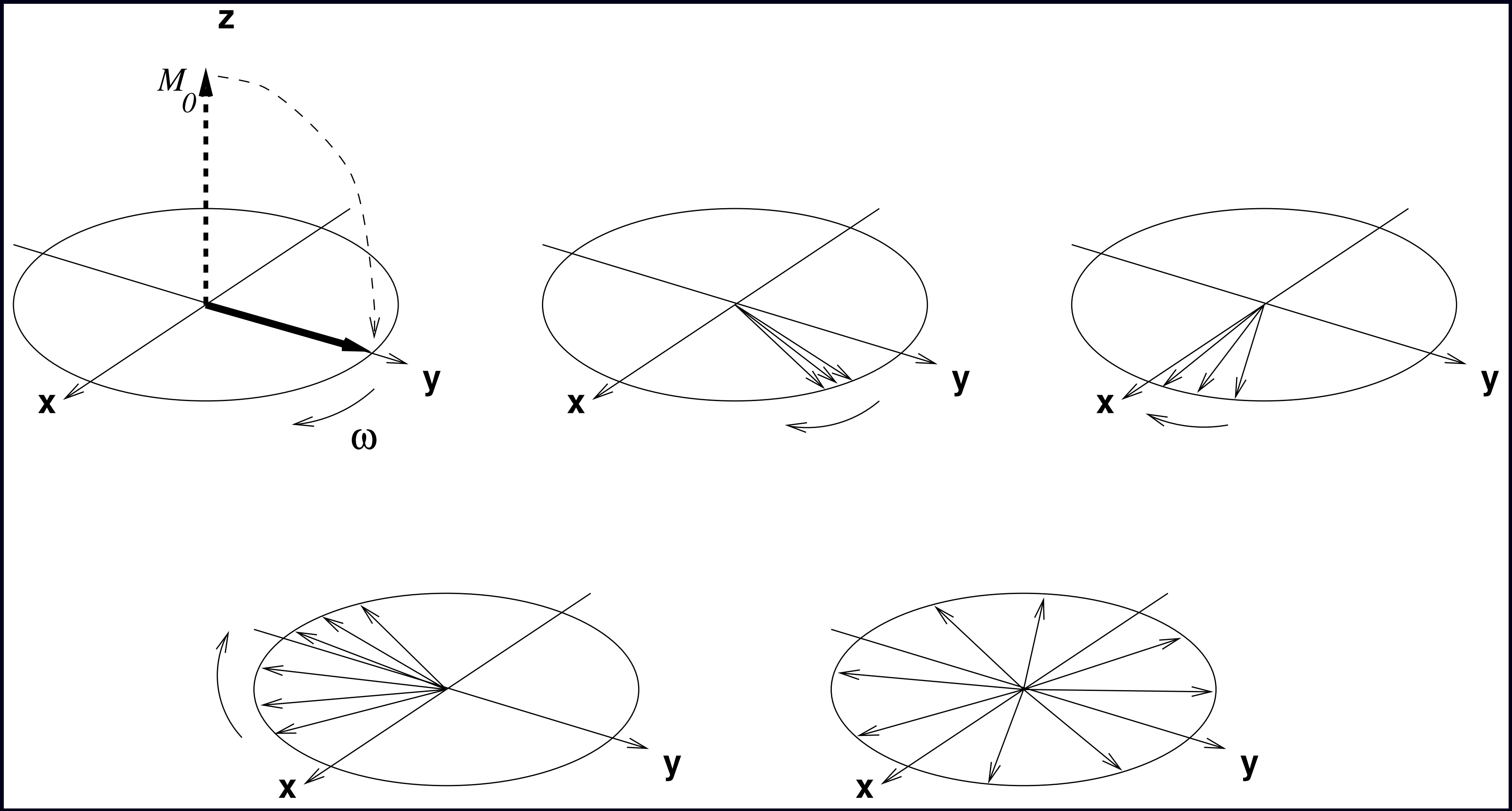


Fast Imaging Methods



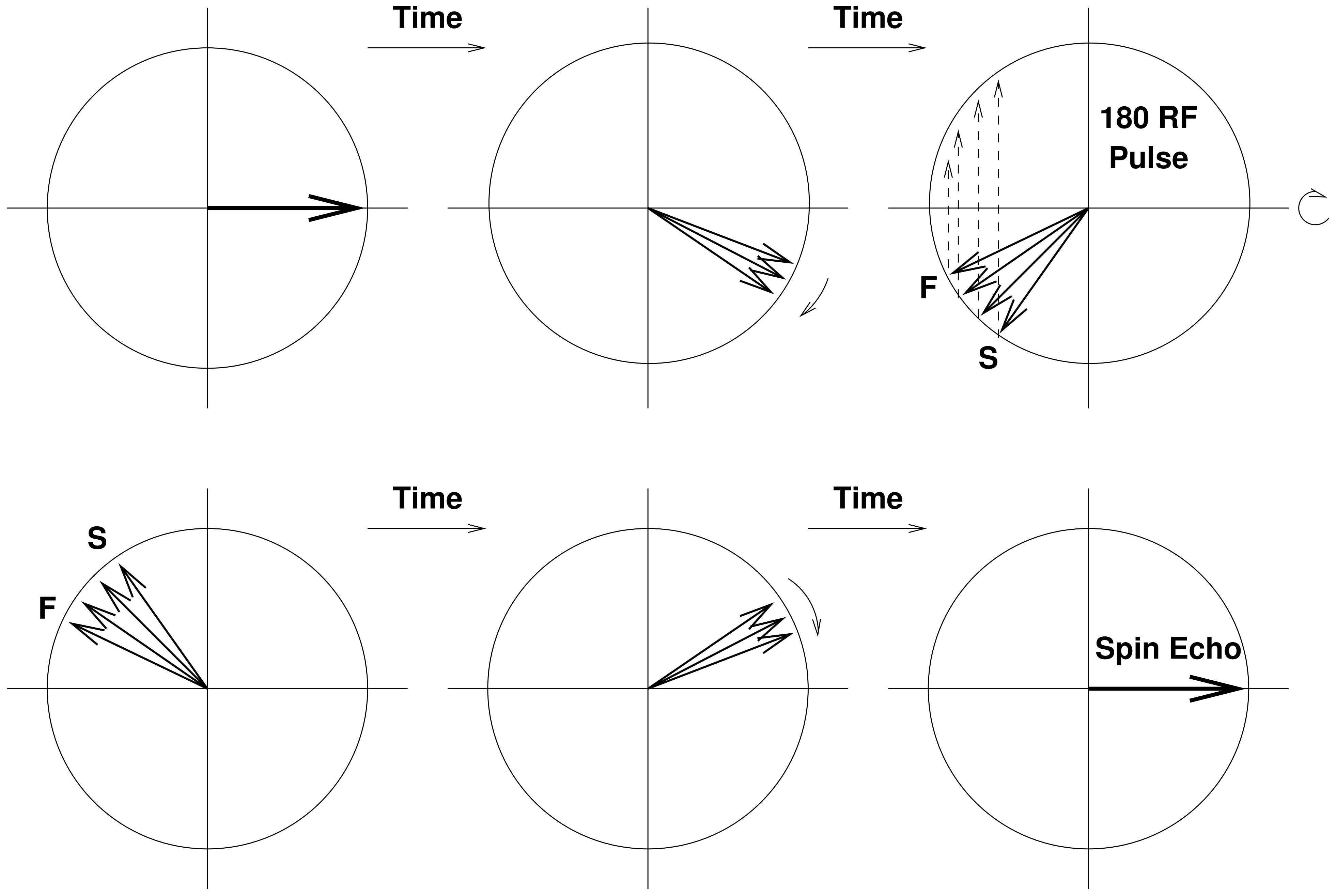
Off-Resonance Effects

Main field inhomogeneity, susceptibility variations, chemical shift.



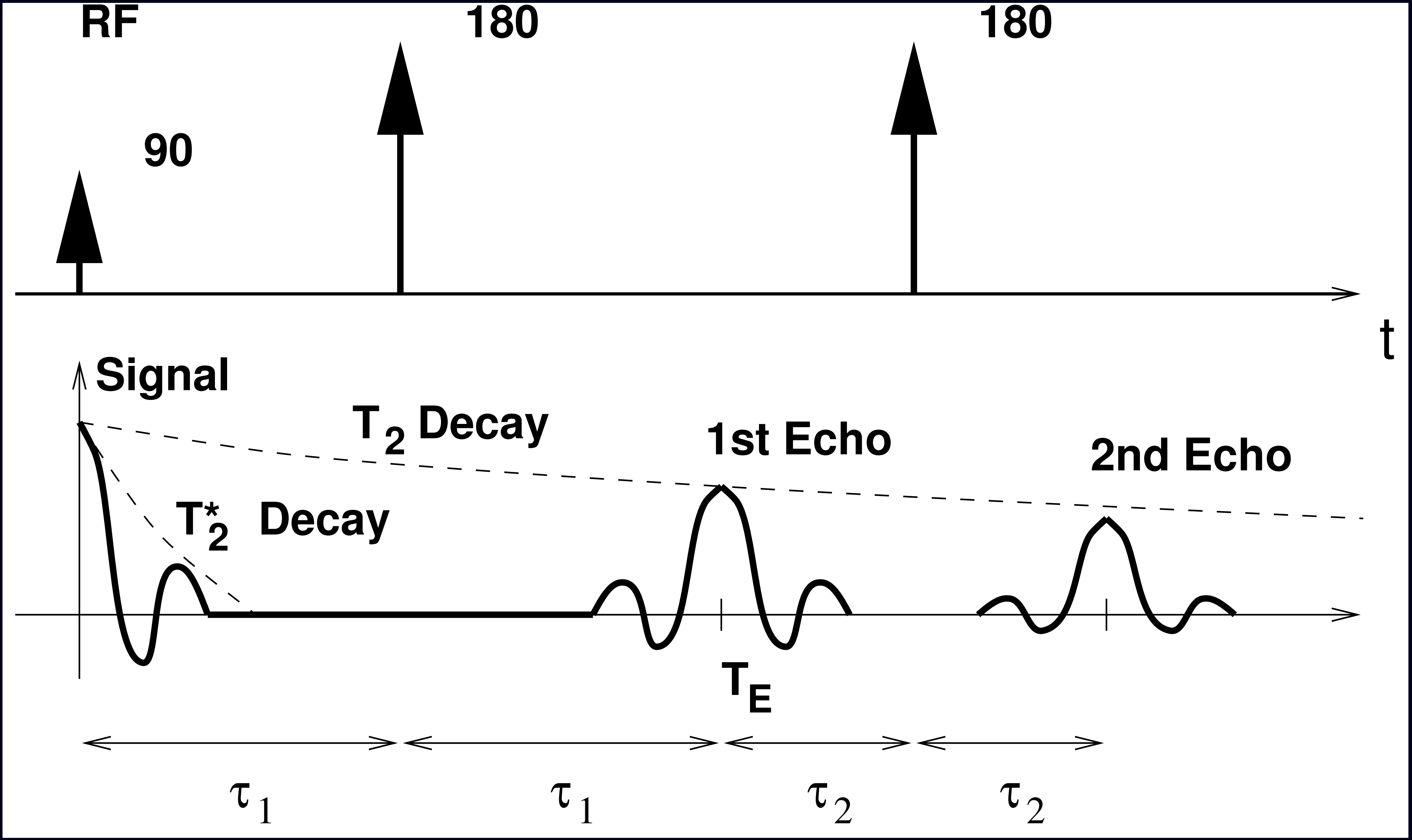
Dephasing causes destructive interference and signal loss.

Spin Echo



Fessler, Univ. of Michigan

Spin Echo and Signal



2DFT Spin-Echo Spin-Warp Pulse Sequence

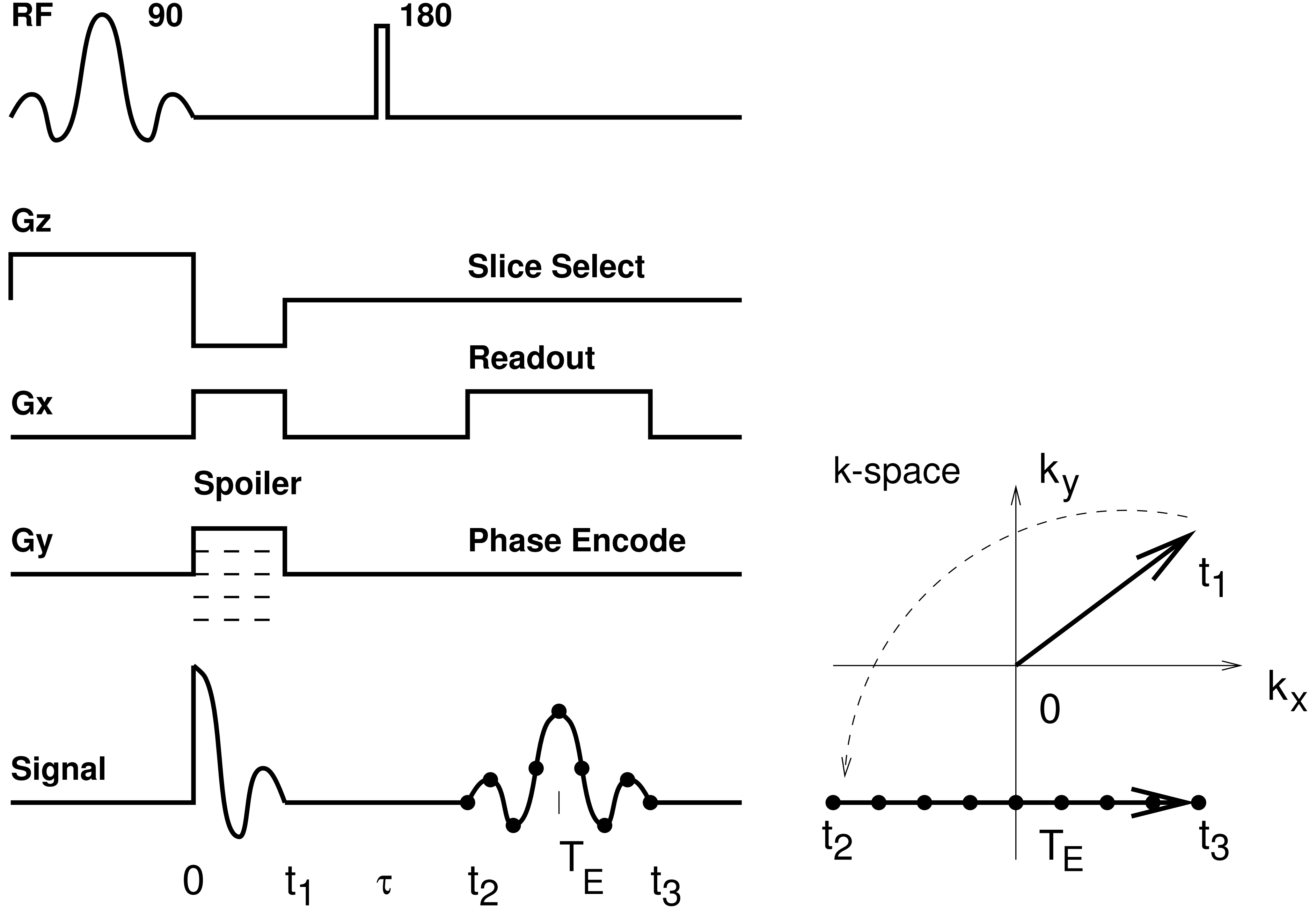


Image Reconstruction Options

- Cartesian sampling patterns: inverse FFT
- Non-Cartesian sampling patterns:
  - gridding then inverse FFT
  - iterative reconstruction (regularized least squares)
- Iterative reconstruction with field inhomogeneity compensation:

$$s(t) = \int m_0(\vec{r})\, \mathrm{e}^{-\imath \omega(\vec{r}) t}\, \mathrm{e}^{-\imath 2\pi \vec{k}(t) \cdot \vec{r}}\, d\vec{r}$$

Not a Fourier transform!

Summary

NMR images represent the local magnetic environment of ^{1}H protons

- Large main field establishes resonance
- RF pulses and gradient fields perturb magnetization and phases
- Measured signal related to Fourier transform of object
- Reconstruction can be simple: inverse FFT
- Time-varying gradients control k-space trajectory
- Timing parameters control T_1 and T_2 contrast

Slides and lecture notes available from:

`http://www.eecs.umich.edu/~fessler`

Some of Many Omitted Topics

- Spectroscopic imaging
- Dynamic imaging
- Functional imaging
- Blood flow imaging
- Multi-slice imaging
- Main-field hardware (shimming, open magnets, ...)
- RF coil design
- Gradient nonlinearities
- Signal to noise considerations
- MR contrast agents
- Interventional MR
- Gating (cardiac and respiratory)
- ...