Statistical Methods for Image Reconstruction

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Image Reconstruction Methods

(Simplified View)

Analytical (FBP)	**Iterative** (OSEM?)

Image Reconstruction Methods / Algorithms

- **ANALYTICAL**
  - FBP
  - BPF
  - Gridding
  - ...
- **ITERATIVE**
  - **Algebraic** (y = Ax)
    - ART
    - MART
    - SMART
    - ...
  - **Statistical**
    - **(Weighted) Least Squares**
      - CG
      - CD
      - ISRA
      - ...
    - **Likelihood (e.g., Poisson)**
      - EM (etc.)
      - OSEM
      - SAGE
      - CG
      - Int. Point
      - GCA
      - PSCD
      - FSCD ...

Outline

Part 0: Introduction / Overview

Part 1: From Physics to Statistics (Emission tomography)

- Assumptions underlying Poisson statistical model
- Emission reconstruction problem statement

Part 2: Four of Five Choices for Statistical Image Reconstruction

- Object parameterization
- System physical modeling
- Statistical modeling of measurements
- Cost functions and regularization

Part 3: Fifth Choice: Iterative algorithms

- Classical optimization methods
- Considerations: nonnegativity, convergence rate, ...
- Optimization transfer: EM etc.
- Ordered subsets / block iterative / incremental gradient methods

Part 4: Performance Analysis

- Spatial resolution properties
- Noise properties
- Detection performance

Part 5: Miscellaneous topics (?)

- ...

History

- Iterative method for X-ray CT (Hounsfield, 1968)
- ART for tomography (Gordon, Bender, Herman, JTB, 1970)
- Richardson/Lucy iteration for image restoration (1972, 1974)
- Weighted least squares for 3D SPECT (Goitein, NIM, 1972)
- Proposals to use Poisson likelihood for emission and transmission tomography
  Emission: (Rockmore and Macovski, TNS, 1976)
  Transmission: (Rockmore and Macovski, TNS, 1977)
- First expectation-maximization (EM) algorithms for Poisson model
  Emission: (Shepp and Vardi, TMI, 1982)
  Transmission: (Lange and Carson, JCAT, 1984)
- First regularized (aka Bayesian) Poisson emission reconstruction
  Geman and McClure, ASA, 1985
- Ordered-subsets EM algorithm
  Hudson and Larkin, TMI, 1994
- Commercial introduction of OSEM for PET scanners
  circa 1997

Why Statistical Methods?

- Object constraints (*e.g.*, nonnegativity, object support)
- Accurate physical models (less bias $\Rightarrow$ improved quantitative accuracy)
  improved spatial resolution?
  (*e.g.*, nonuniform attenuation in SPECT)
- Appropriate statistical models (less variance $\Rightarrow$ lower image noise)
  (FBP treats all rays equally)
- Side information (*e.g.*, MRI or CT boundaries)
- Nonstandard geometries (“missing” data)

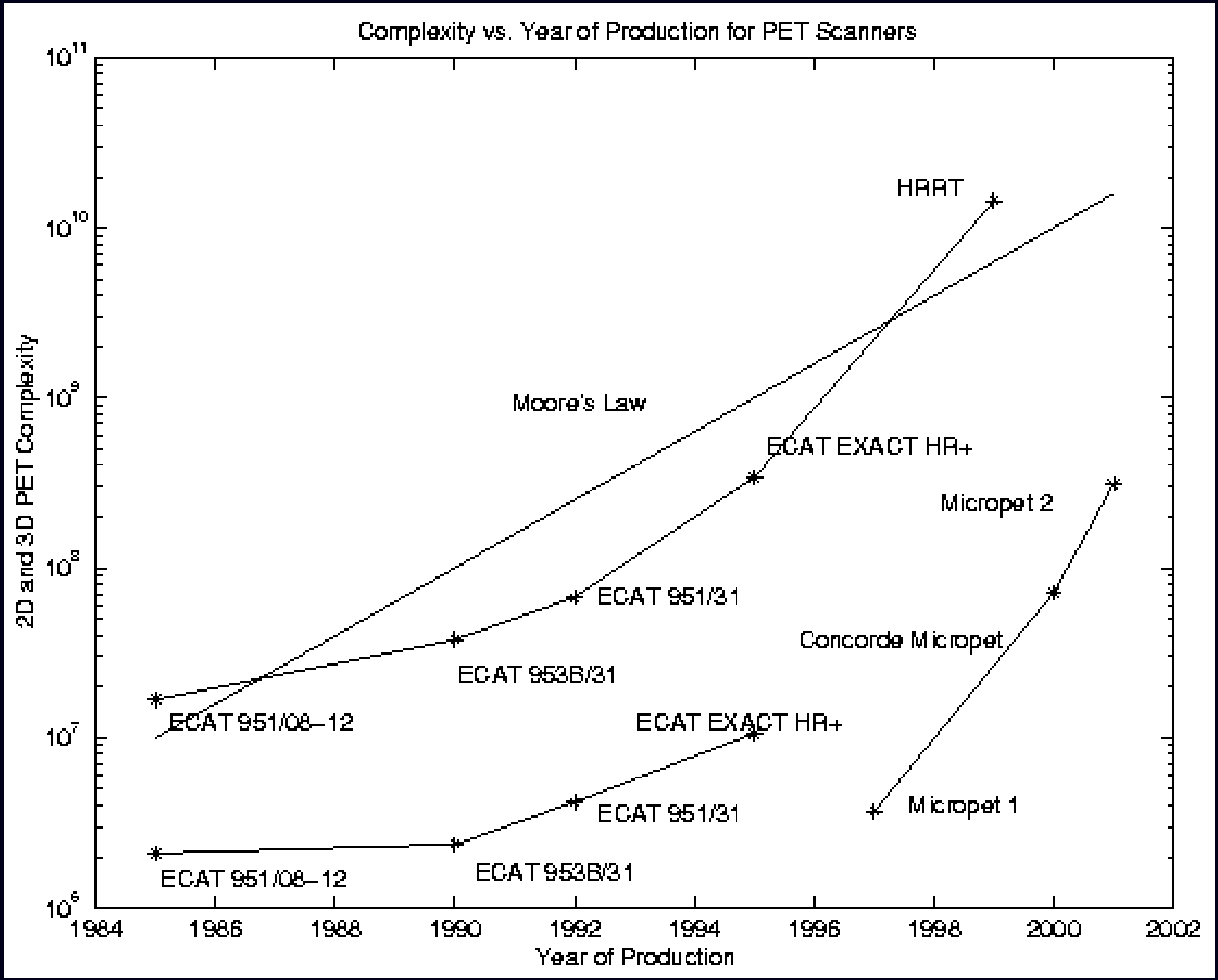
Disadvantages?

- Computation time
- Model complexity
- Software complexity

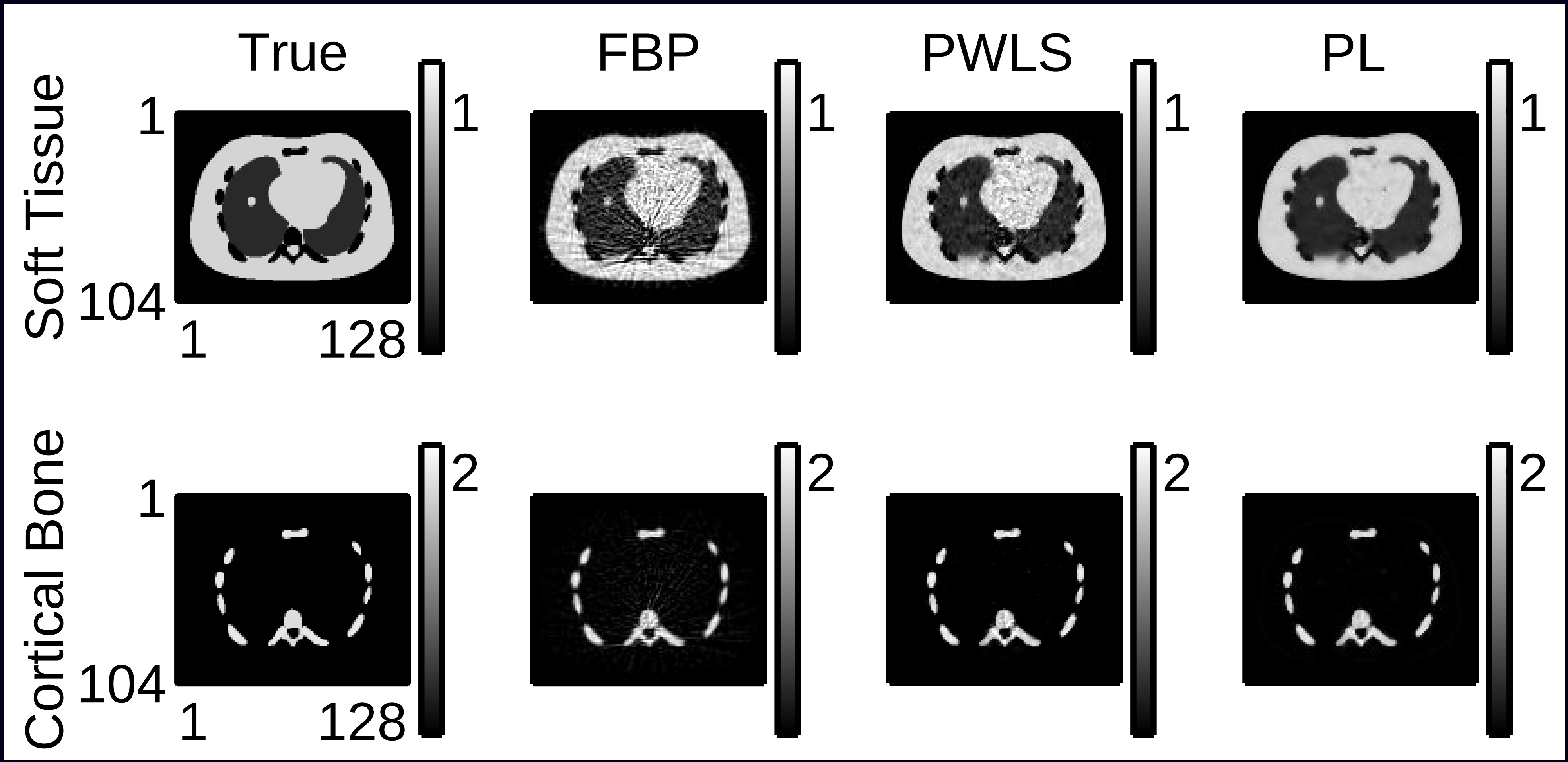
Analytical methods (a different short course!)

- Idealized mathematical model
  - Usually geometry only, greatly over-simplified physics
  - Continuum measurements
- No statistical model
- Easier analysis of properties (due to linearity)
  e.g., Huesman (1984) FBP ROI variance for kinetic fitting

What about Moore’s Law?

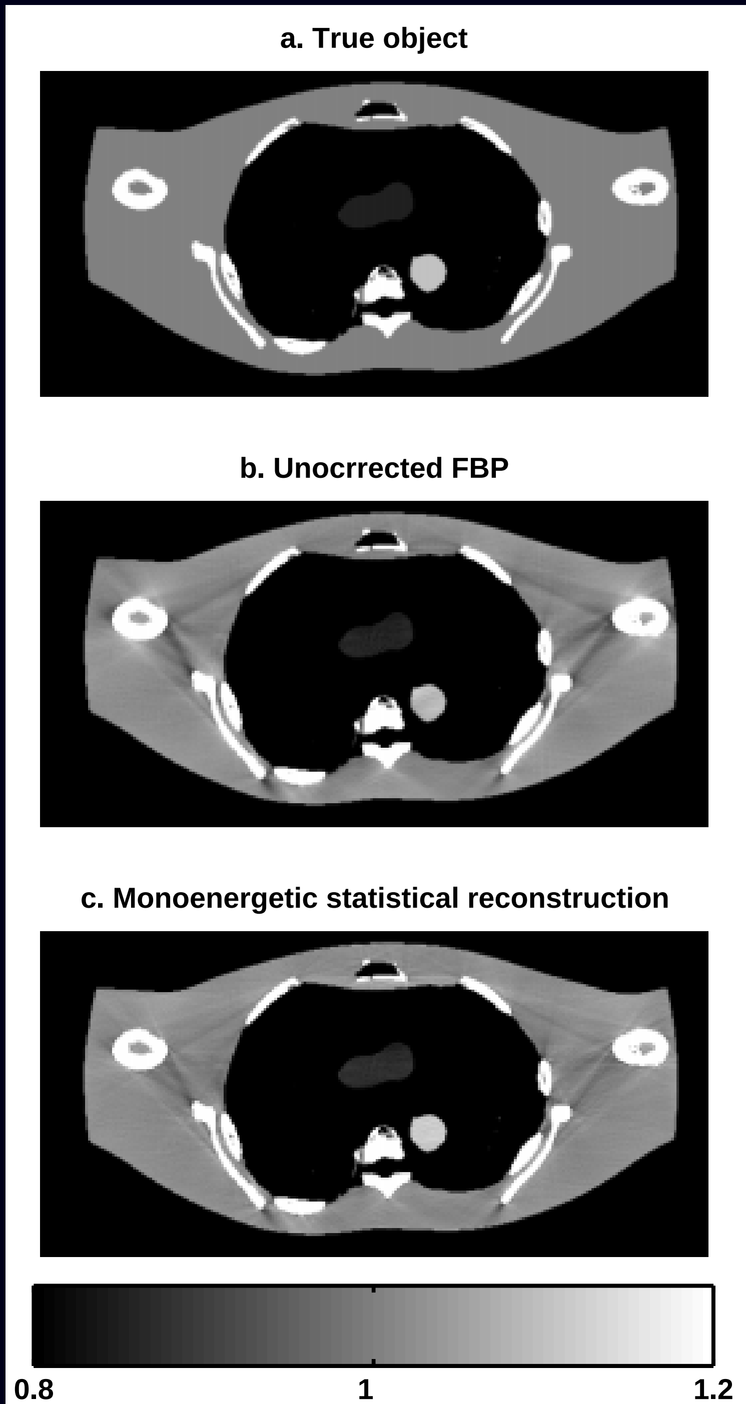


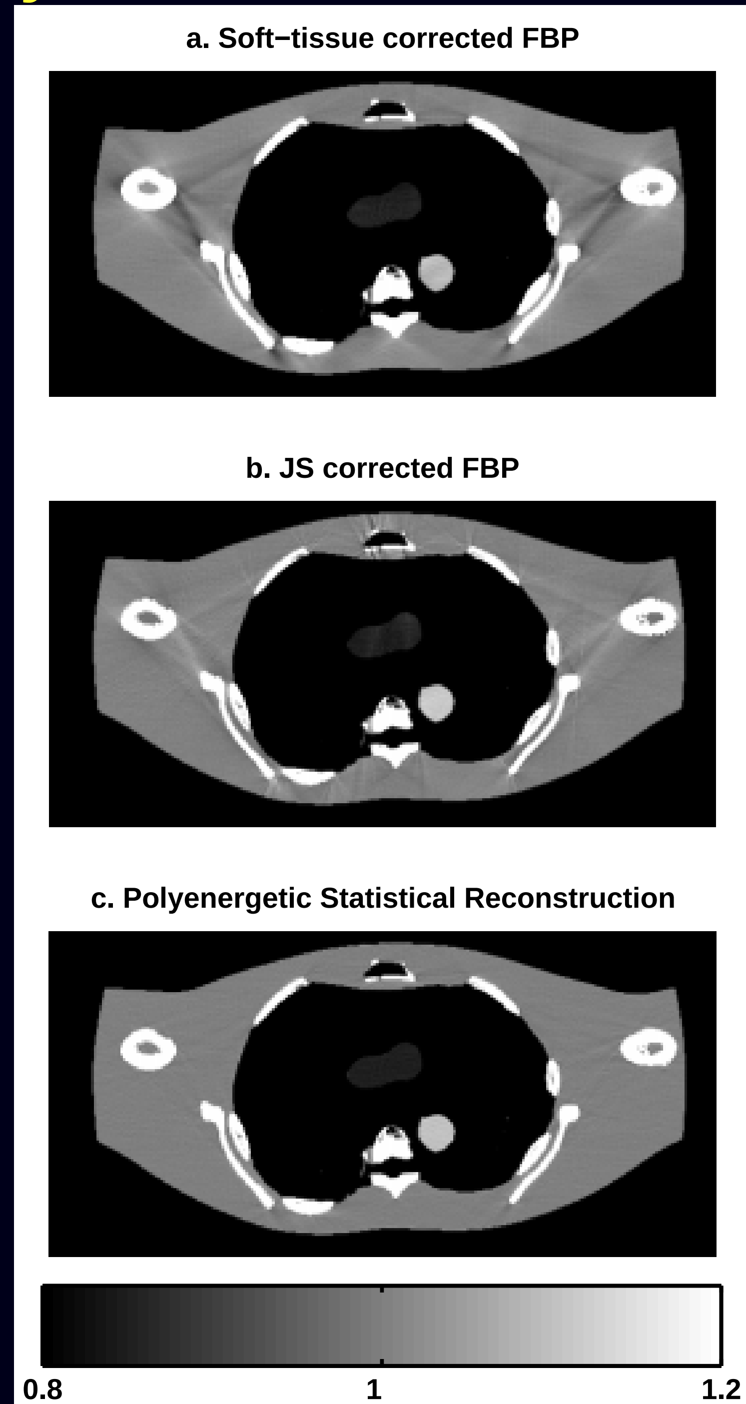
Benefit Example: Statistical Models



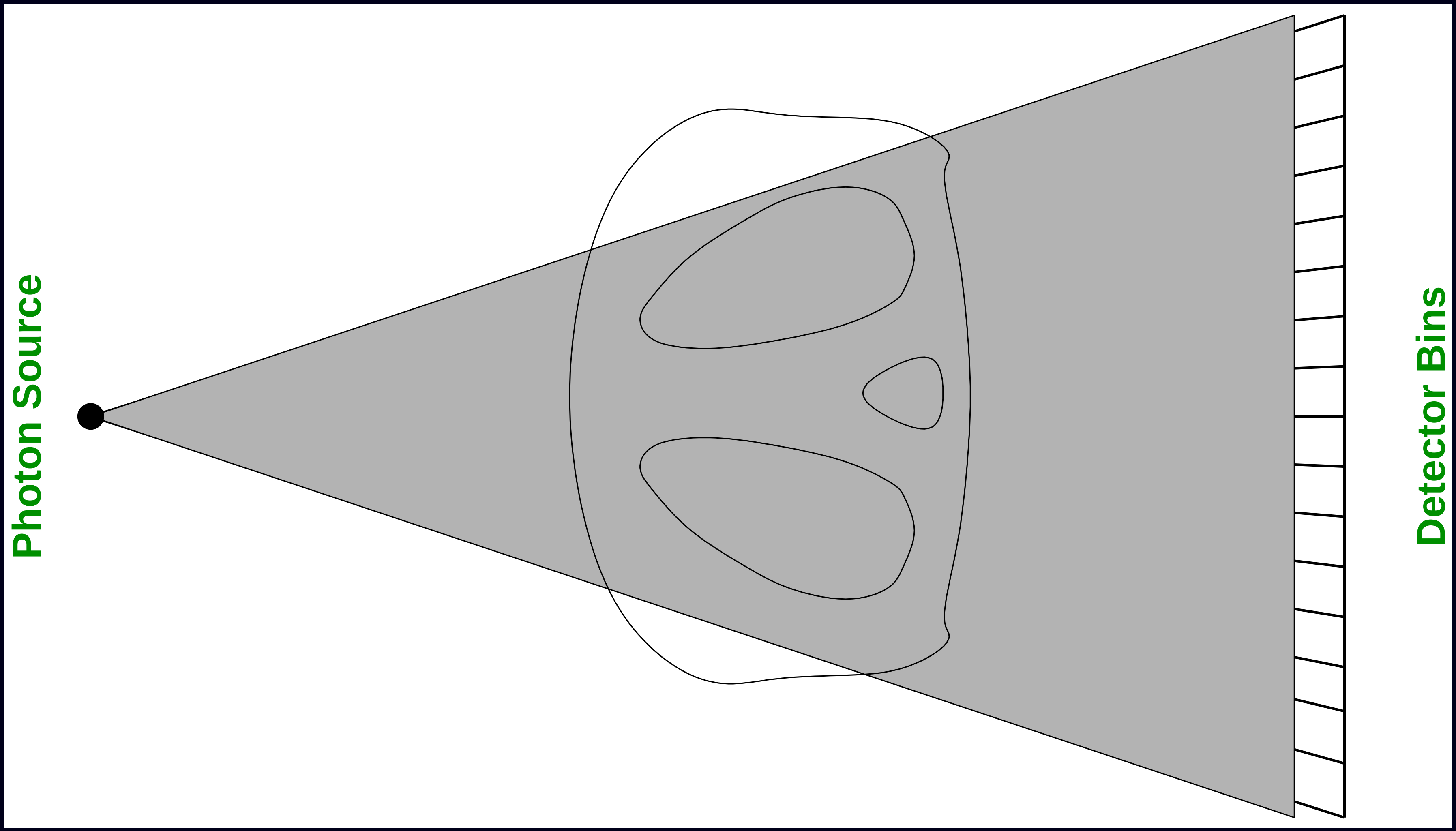
	NRMS Error	
Method	Soft Tissue	Cortical Bone
FBP	22.7%	29.6%
PWLS	13.6%	16.2%
PL	11.8%	15.8%

Benefit Example: Physical Models

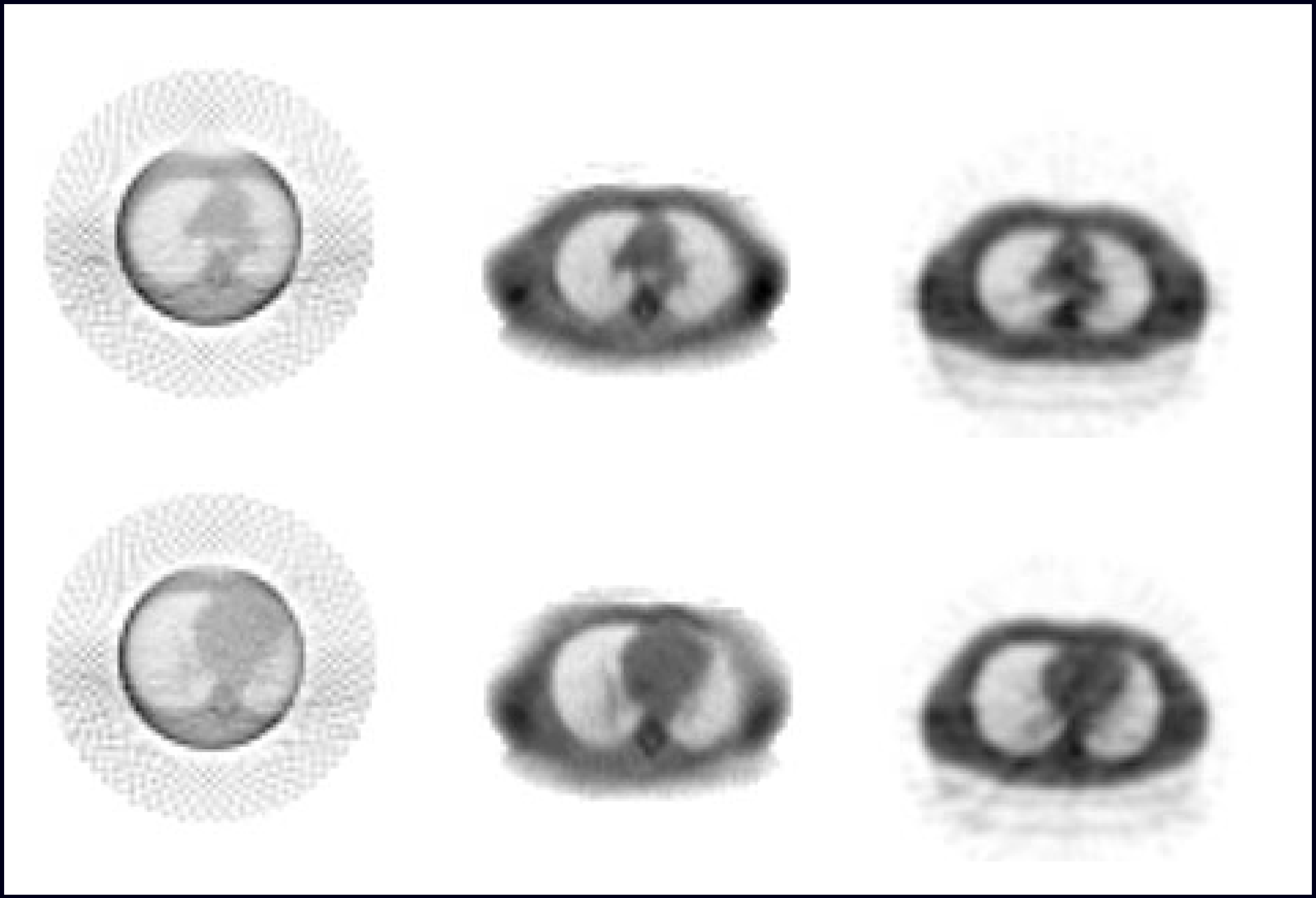




Benefit Example: Nonstandard Geometries



Truncated Fan-Beam SPECT Transmission Scan



Truncated FBP | Truncated PWLS | Untruncated FBP

One Final Advertisement: Iterative MR Reconstruction