Splitting-Based Statistical X-Ray CT Image Reconstruction with Blind Gain Correction

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ABSTRACT

Variational methods are useful for solving ill-posed inverse imaging problems by minimizing a cost function with a data fidelity term and a regularization term. For statistical X-ray computed tomography (CT) image reconstruction, penalized weighted least-squares (PWLS) criteria with edge-preserving regularization can improve quality of the reconstructed image compared to traditional filtered back-projection (FBP) reconstruction. Nevertheless, the huge dynamic range of the statistical weights used in PWLS image reconstruction leads to a highly shift-variant local impulse response, making effective preconditioning difficult. To overcome this problem, iterative algorithms based on variable splitting were proposed recently.[1,2] However, existing splitting-based iterative algorithms do not consider the (unknown) gain fluctuations that can occur between views.[3] This paper proposes a new variational formulation for splitting-based iterative algorithms where the unknown gain parameter vector and the image are estimated jointly with just simple changes to the original algorithms. Simulations show that the proposed algorithm greatly reduces the shading artifacts caused by gain fluctuations yet with almost unchanged computational complexity per iteration.

1. INTRODUCTION

The effective X-ray source intensity in CT scan can fluctuate from view to view due to the attenuation of thin items like sheets that partially block reference channels. Thibault *et al.* proposed to modify the cost function so that it depends on both the unknown image $\mathbf{x}$ and an unknown gain parameter vector $\mathbf{g}$, where $[\mathbf{g}]_k$ denotes the gain fluctuation of the kth view, and to minimize jointly over both $\mathbf{x}$ and $\mathbf{g}$ by solving the following convex optimization problem:[3]

$$(\hat{\mathbf{x}}, \hat{\mathbf{g}}) \in \operatorname*{argmin}_{\mathbf{x},\mathbf{g}} \left\{ \Psi(\mathbf{x},\mathbf{g}) \triangleq \tfrac{1}{2} \left\| \mathbf{y} - \mathbf{A}\mathbf{x} - \mathbf{g} \otimes \mathbf{1} \right\|_{\mathbf{W}}^2 + \mathsf{R}(\mathbf{x}) \right\}, \tag{1}$$

where $\mathbf{y}$ denotes the noisy post-logarithm sinogram that may suffer from gain fluctuations, $\mathbf{A}$ denotes the system matrix, $\mathbf{W}$ denotes the diagonal weighting matrix that accounts for measurement variance, $\otimes$ denotes the Kronecker product operator, $\mathbf{1}$ denotes the vector with all entries equal to unity and of length equal to the number of beams, and R is an edge-preserving regularizer. Compared with Eq. (1), existing splitting-based iterative algorithms reconstruct image without considering the effect of $\mathbf{g}$, or equivalently, setting $\mathbf{g}$ to be $\mathbf{0}$. This introduces visible shading artifacts as shown in Figure 2c. We propose splitting-based iterative algorithms based on a simplification of the joint cost function in Eq. (1) that improves image quality compared to conventional splitting-based iterative algorithms[1,2] that assume $\mathbf{g} = \mathbf{0}$.

2. METHOD

2.1 Joint gain-image estimation for X-ray CT image reconstruction

Let $\mathbf{y}_k$, $\mathbf{A}_k$, and $\mathbf{W}_k$ for $k = 1, \ldots, K$ denote data, system matrix, and diagonal weighting matrix associated with the kth view in a CT scan, respectively. The optimization problem in Eq. (1) is equivalent to

$$\begin{aligned}(\hat{\mathbf{x}}, \hat{\mathbf{g}}) &\in \operatorname*{argmin}_{\mathbf{x},\mathbf{g}} \left\{ \textstyle\sum_{k=1}^{K} \tfrac{1}{2} \left\| \mathbf{y}_k - \mathbf{A}_k\mathbf{x} - g_k \mathbf{1} \right\|_{\mathbf{W}_k}^2 + \mathsf{R}(\mathbf{x}) \right\} \\ &= \operatorname*{argmin}_{\mathbf{x}} \left\{ \textstyle\sum_{k=1}^{K} \tfrac{1}{2} \min_{g_k} \left\{ \left\| \mathbf{y}_k - \mathbf{A}_k\mathbf{x} - g_k \mathbf{1} \right\|_{\mathbf{W}_k}^2 \right\} + \mathsf{R}(\mathbf{x}) \right\}. \end{aligned} \tag{2}$$

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The inner 1D minimization problem in Eq. (2) has minimizer

$$\hat{g}_k(\mathbf{x}) = \frac{\mathbf{1}^\top \mathbf{W}_k(\mathbf{y}_k - \mathbf{A}_k\mathbf{x})}{\mathbf{1}^\top \mathbf{W}_k \mathbf{1}} \tag{3}$$

with minimum

$$\|\mathbf{y}_k - \mathbf{A}_k\mathbf{x} - \hat{g}_k(\mathbf{x})\,\mathbf{1}\|^2_{\mathbf{W}_k} = \left\| \left(\mathbf{I} - \frac{\mathbf{1}\mathbf{1}^\top \mathbf{W}_k}{\mathbf{1}^\top \mathbf{W}_k \mathbf{1}} \right) (\mathbf{y}_k - \mathbf{A}_k\mathbf{x}) \right\|^2_{\mathbf{W}_k} = \|\mathbf{y}_k - \mathbf{A}_k\mathbf{x}\|^2_{\tilde{\mathbf{W}}_k} \,, \tag{4}$$

where we define the following positive semi-definite symmetric "diagonal + rank-1" weighting matrix:

$$\tilde{\mathbf{W}}_k = \mathbf{W}_k - \frac{\mathbf{W}_k \mathbf{1}\mathbf{1}^\top \mathbf{W}_k}{\mathbf{1}^\top \mathbf{W}_k \mathbf{1}} \,. \tag{5}$$

Plugging Eq. (4) into Eq. (2) yields the following problem formulation that is equivalent to Eq. (1) yet also equivalent to the kind of cost function used in "conventional" splitting-based iterative algorithms except that it uses a non-diagonal weighting matrix:

$$\hat{\mathbf{x}} \in \operatorname*{argmin}_{\mathbf{x}} \left\{ \Psi(\mathbf{x}) \triangleq \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|^2_{\tilde{\mathbf{W}}} + \mathsf{R}(\mathbf{x}) \right\} \,, \tag{6}$$

where $\tilde{\mathbf{W}}$ is a block diagonal matrix with block described in Eq. (5). Since $\tilde{\mathbf{W}}$ is positive semi-definite, Eq. (6) is a convex optimization problem, and any existing optimization methods such as nonlinear conjugate gradient[4] (**NCG**), ordered subsets algorithm[5] (**OS**), and fast iterative shrinkage-thresholding algorithm[6] (**FISTA**) are still applicable. Here, we propose to solve Eq. (6) by using splitting-based iterative algorithms[2,7] with variable splitting involving the sinogram $\mathbf{u} = \mathbf{A}\mathbf{x}$. The subproblem of $\mathbf{u}$ has a closed form solution:

$$\mathbf{u}^{(j+1)} = \mathbf{D}_\rho^{-1} \left(\tilde{\mathbf{W}}\mathbf{y} + \rho \left(\mathbf{A}\mathbf{x}^{(j+1)} + \mathbf{d}^{(j)} \right) \right) \,, \tag{7}$$

where $\mathbf{D}_\rho \triangleq \tilde{\mathbf{W}} + \rho\mathbf{I}$, and $\mathbf{d}$ is the scaled dual variable of $\mathbf{u}$ in the alternating direction method of multipliers[8] (**ADMM**). When there is no gain correction, i.e., $\mathbf{g} = \mathbf{0}$, $\mathbf{D}_\rho$ is a diagonal matrix, and Eq. (7) can be computed efficiently in $O(S)$, where S is the size of the sinogram. When there is gain correction, i.e., $\mathbf{g} \neq \mathbf{0}$, $\mathbf{D}_\rho$ is block diagonal matrix with block:

$$\mathbf{D}_{\rho,k} = (\mathbf{W}_k + \rho\mathbf{I}) + \left(\frac{-\mathbf{w}_k}{\mathbf{1}^\top \mathbf{w}_k} \right) \mathbf{w}_k^\top \,, \tag{8}$$

where $\mathbf{w}_k \triangleq \mathbf{W}_k \mathbf{1}$ is the diagonal entries of $\mathbf{W}_k$. The inverse of $\mathbf{D}_\rho$ will also be a block diagonal matrix with block:

$$(\mathbf{D}_{\rho,k})^{-1} = (\mathbf{W}_k + \rho\mathbf{I})^{-1} + \frac{(\mathbf{W}_k + \rho\mathbf{I})^{-1} \mathbf{w}_k \mathbf{w}_k^\top (\mathbf{W}_k + \rho\mathbf{I})^{-1}}{\mathbf{1}^\top \mathbf{w}_k - \mathbf{w}_k^\top (\mathbf{W}_k + \rho\mathbf{I})^{-1} \mathbf{w}_k} \tag{9}$$

by the Sherman-Morrison formula. Note that the matrix-vector multiplication of $(\mathbf{D}_{\rho,k})^{-1}$ and a vector of proper size involves only componentwise division, vector inner product, and vector outer product. Therefore, the computational complexity of Eq. (7) is still $O(S)$ in the presence of gain correction. That is, we can estimate the unknown gain parameter vector and the image jointly with almost unchanged computational complexity per iteration.

2.2 Applying prior knowledge of gain parameter to the joint gain-image estimation

The optimization problem described in Eq. (1) can be thought of as an X-ray CT image reconstruction with *blind* gain correction since we have no prior knowledge of the gain parameter vector, and we apply gain correction to every view. However, sometimes we do have some prior knowledge of the gain parameter vector. For example, when the sinogram is truncated between some view angles, we know that the object is outside the field of view, and the reference channels might be blocked by the object in these views with high probability. Hence, it is better to apply gain correction to these views. Similarly, when the object is well bounded in the field of view in some view angles, i.e., the projection is not truncated, the reference channel is less likely to be blocked, and we can turn off gain corrections in these views. To incorporate such prior knowledge about the support of the gain parameter vector, we propose a constrained optimization problem for *non-blind* gain correction:

$$(\hat{\mathbf{x}}, \hat{\mathbf{g}}) \in \operatorname*{argmin}_{\mathbf{x},\mathbf{g}} \left\{ \Psi(\mathbf{x}, \mathbf{g}) \triangleq \tfrac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x} - \mathbf{g} \otimes \mathbf{1}\|_{\mathbf{W}}^2 + \mathsf{R}(\mathbf{x}) \right\} \text{ s.t. } [\mathbf{g}]_{k \notin \mathcal{K}} = 0 \,, \tag{10}$$

where $\mathcal{K} \subseteq \{1, \dots, K\}$ is the index set of the *candidate* views which may suffer from gain fluctuations. Note that since the gain fluctuations are assumed to be only view-dependent, following the same procedure in the blind case, we can get exactly the same equivalent problem formulation as in Eq. (6), where $\tilde{\mathbf{W}}$ is a block diagonal matrix with block:

$$\tilde{\mathbf{W}}_k = \begin{cases} \mathbf{W}_k & \text{, if } k \notin \mathcal{K} \\ \mathbf{W}_k - \frac{\mathbf{W}_k \mathbf{1}\mathbf{1}^\top \mathbf{W}_k}{\mathbf{1}^\top \mathbf{W}_k \mathbf{1}} & \text{, otherwise} . \end{cases} \tag{11}$$

When we solve the X-ray CT image reconstruction with non-blind gain correction using splitting-based methods with variable splitting involving the sinogram, we will solve the subproblem of $\mathbf{u}$ using Eq. (7), where $(\mathbf{D}_\rho)^{-1}$ is a block diagonal matrix with block:

$$(\mathbf{D}_{\rho,k})^{-1} = (\mathbf{W}_k + \rho\mathbf{I})^{-1} + \frac{(\mathbf{W}_k + \rho\mathbf{I})^{-1} \mathbf{w}_k \mathbf{w}_k^\top (\mathbf{W}_k + \rho\mathbf{I})^{-1}}{\mathbf{1}^\top \mathbf{w}_k - \mathbf{w}_k^\top (\mathbf{W}_k + \rho\mathbf{I})^{-1} \mathbf{w}_k} \mathbb{1}_{k \in \mathcal{K}} \,, \tag{12}$$

where $\mathbb{1}_{k \in \mathcal{K}}$ is the indicator function of $\mathcal{K}$. Clearly, when $\mathcal{K} = \emptyset$, it reduces to the X-ray CT image reconstruction without gain correction; when $\mathcal{K} = \{1, \dots, K\}$, it is the X-ray CT image reconstruction with blind gain correction. Furthermore, if desired, we can shrink the set $\mathcal{K}$ as the iterative algorithm proceeds. For example, we can reset the very small estimated gain fluctuations to be zero after several iterations.

2.3 Joint gain-image estimation using other optimization methods

Although we focus on solving Eq. (6) using a splitting-based method in this paper, it can be solved by any other optimization method. For example, the cost function in Eq. (6) has gradient

$$\nabla\Psi(\mathbf{x}) = \mathbf{A}^\top \tilde{\mathbf{W}} (\mathbf{A}\mathbf{x} - \mathbf{y}) + \nabla\mathsf{R}(\mathbf{x}) \tag{13}$$

assuming that the regularization term $\mathsf{R}(\mathbf{x})$ is differentiable. This gradient can be used for any first-order method such as **NCG**. One popular iterative method in X-ray CT image reconstruction is called **OS**[5] (or in particular, **OS-SQS**), which is an *accelerated* version of a convergent separable quadratic surrogate (**SQS**) algorithm. This ordered subsets method is successful because of the high complexity of computing the forward and back-projection in X-ray CT image reconstruction problems. The basic idea of **SQS** algorithm is to find a separable quadratic surrogate function that majorizes the original cost function and to minimize it. Since the only difference between the conventional variational formulation and our proposed formulation is the weighted least-squares (WLS) term, we just focus on the majorizer of that part. A quadratic majorizer of a function f has the general form

$$f\big(\mathbf{x}^{(j)}\big) + \big(\mathbf{x} - \mathbf{x}^{(j)}\big)^\top \nabla f\big(\mathbf{x}^{(j)}\big) + \tfrac{1}{2} \big\|\mathbf{x} - \mathbf{x}^{(j)}\big\|_{\mathbf{D}}^2 \tag{14}$$

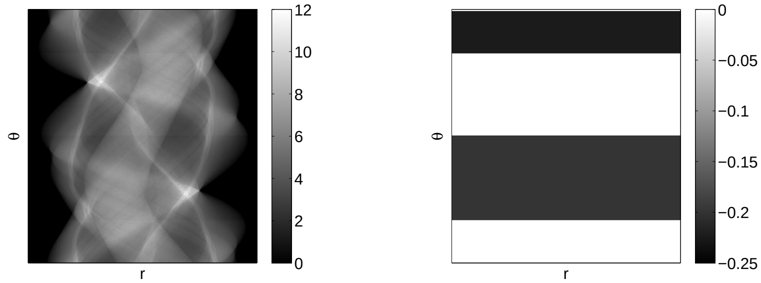
with $\mathbf{D} \succeq \nabla^2 f$. When f is the conventional WLS cost, one particular choice of $\mathbf{D}$ is[5]

$$\mathbf{D}_{\text{WLS}} \triangleq \operatorname{diag}\Big\{|\mathbf{A}|^\top \mathbf{W} \, |\mathbf{A}| \, \mathbf{1}\Big\} \succeq \mathbf{A}^\top \mathbf{W} \mathbf{A} \,. \tag{15}$$

Since $\mathbf{W} \succeq \tilde{\mathbf{W}}$, we have $\mathbf{A}^\top \mathbf{W} \mathbf{A} \succeq \mathbf{A}^\top \tilde{\mathbf{W}} \mathbf{A}$. When f is the proposed WLS cost, it is clear that

$$\mathbf{D}_{\text{WLS}} \succeq \mathbf{A}^\top \tilde{\mathbf{W}} \mathbf{A} \,, \tag{16}$$

and $\mathbf{D}_{\text{WLS}}$ is also a valid **SQS** diagonal matrix for the proposed WLS cost. Therefore, it is very easy to modify the existing **OS** algorithm to enable gain correction.



(a) (b)

Figure 1: 2D simulation: (a) The noisy sinogram with gain fluctuations, and (b) the corresponding gain fluctuations, where the horizontal and vertical axes are the radial axis (r) and the projection view angle (θ), respectively.

3. RESULTS

To evaluate our proposed method, we consider both 2D and 3D X-ray CT image reconstruction problems. In each case, the statistical weight w_i is set to be e^{-y_i}, where y_i denotes the line-integral projection *with* gain fluctuations. We are interested in edge-preserving regularizer R in the form:

$$\mathsf{R}(\mathbf{x}) \triangleq \beta \sum_{n=1}^{N} \sum_{m=1}^{M} \kappa_n \kappa_{n+s(m)} \Phi([\mathbf{D}_m \mathbf{x}]_n) \ , \tag{17}$$

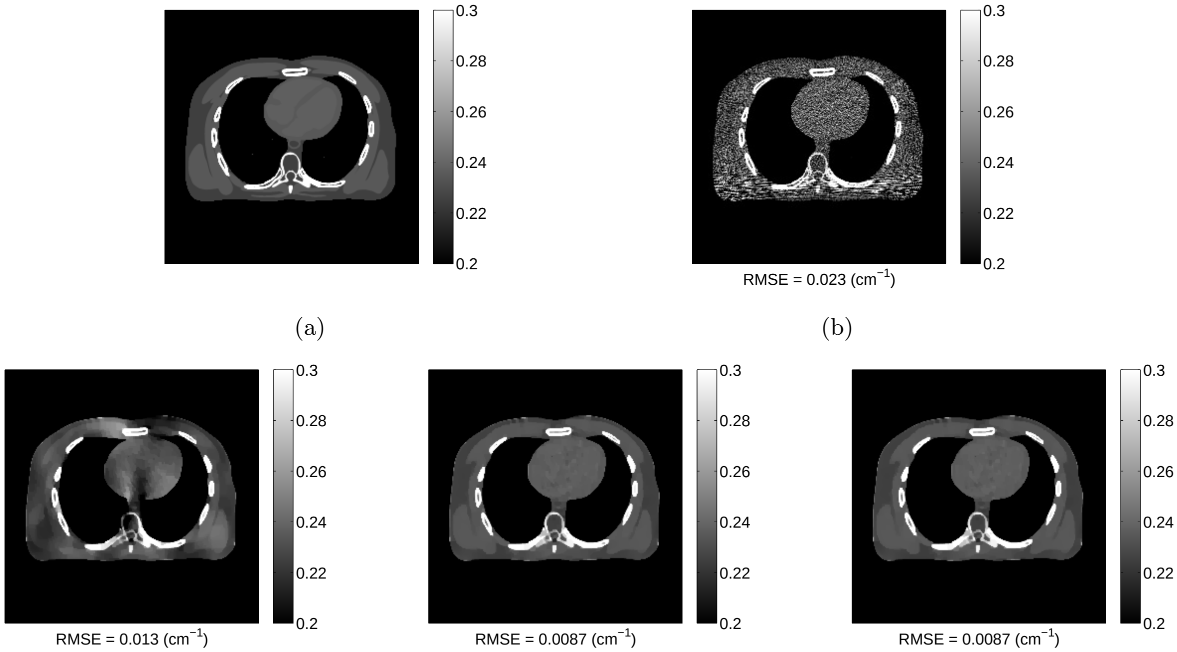
where β is the regularization parameter, N is the number of voxels, M is the number of offsets, κ_n is the voxel-dependent weight for $n = 1, \ldots, N$, $\mathbf{D}_m$ is the first-order finite-difference matrix in the mth direction with offset $s(m)$ for $m = 1, \ldots, M$, and Φ is an edge-preserving potential function. We choose Φ to be the Fair potential function $\Phi_{\mathrm{FP}}(x) \triangleq |x|/\delta - \ln(1 + |x|/\delta)$ with parameter δ. Following the voxel-dependent weight proposed by Fessler *et al.*,[9] κ_n is set to be $\sqrt{[\mathbf{A}^\top \mathbf{W} \mathbf{1}]_n / [\mathbf{A}^\top \mathbf{1}]_n}$. For 2D case, $M = 2$ for the horizontal and vertical neighbors; for 3D case, $M = 13$ for the thirteen nearest neighbors. The minimization problem in Eq. (6) is solved by using **ADMM** for 500 iterations.[2] The FBP/FDK reconstruction from the gain-fluctuated noisy sinogram is used as the initial guess $\mathbf{x}^{(0)}$ for the iterative algorithm.

3.1 2D fan beam X-ray CT image reconstruction

We first consider a 2D X-ray CT image reconstruction from simulated NCAT phantom data with gain fluctuations. We use a 256×256 2D slice of NCAT phantom to numerically generate a 444×492 gain-fluctuated noisy sinogram with GE LightSpeed fan-beam geometry downsampled by two corresponding to a monoenergetic source with 10^5 incident photons per ray without background events. Two sections of angular samples suffer from gain fluctuations due to partially blocked reference channels with 20% and 18% attenuation, respectively, as shown in Figure 1. We set $\delta = 10^{-5}$ and $\beta = 3 \times 10^{-6}$ for edge-preserving regularization in this case. Figure 2 shows the true image, the initial guess, the conventional reconstruction without gain correction, the proposed reconstruction with blind gain correction, and the reference reconstruction from a noisy sinogram without gain fluctuations as a comparison, from left to right, top to bottom. As can be seen from Figure 2, our proposed method greatly reduced the shading artifacts resulting from gain fluctuations. Figure 3 shows the estimated gain parameter vector and its RMS error. As can be seen from Figure 3, our proposed method estimates the gain parameter vector accurately, and therefore, we have a comparable reconstruction with the reconstruction from the gain-fluctuation-free noisy sinogram as shown in Figure 2d and Figure 2e. The RMS difference between them is about $3.12 \times 10^{-5}\ \mathrm{cm}^{-1}$, which means that they are very close to each other.

3.2 3D axial X-ray CT image reconstruction

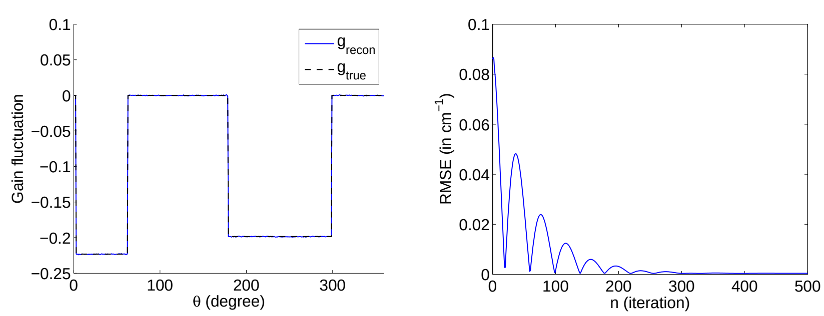
We now consider a 3D X-ray CT image reconstruction from simulated phantom data with gain fluctuations. Assuming that the gain fluctuations are changing linearly in the angular direction and are constant in the transaxial direction, we use a $128 \times 120 \times 100$ 3D phantom (cylinder bone-like inserts) to analytically generate a



(a) (b)

(c) (d) (e)

Figure 2: 2D simulation: (a) The true phantom (in cm^{-1}), (b) the initial guess using the FBP reconstruction, (c) the conventional reconstruction without gain correction, (d) the proposed reconstruction with blind gain correction, and (e) the reference reconstruction from a noisy sinogram without gain fluctuations.



(a) (b)

Figure 3: 2D simulation: (a) The estimated gain parameter vector as a function of projection view angle, and (b) the RMS error of the estimated gain parameter vector versus iteration.

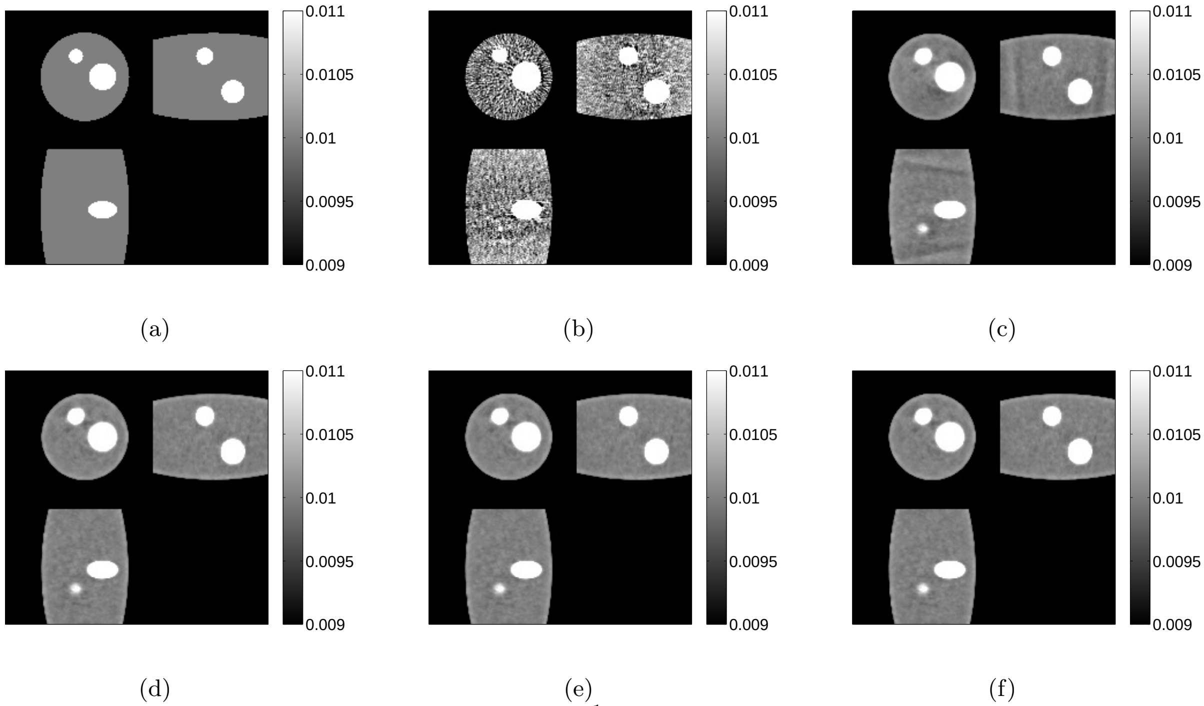
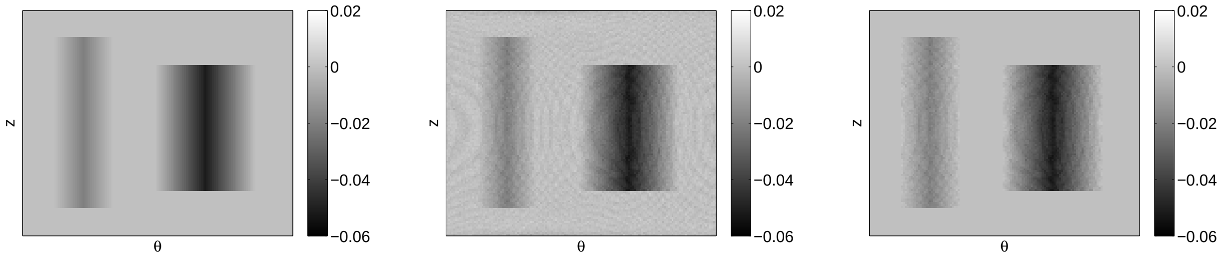


Figure 4: 3D simulation: (a) The true phantom (in cm^{-1}), (b) the initial guess using the FDK reconstruction, (c) the conventional reconstruction without gain correction, (d) the proposed reconstruction with blind gain correction, (e) the proposed reconstruction with non-blind gain correction, and (f) the reference reconstruction from a noisy sinogram without gain fluctuations. Each subfigure shows the middle transaxial, coronal, and sagittal planes of the volume.

$128 \times 120 \times 144$ noisy sinogram with axial geometry corresponding to a monoenergetic source with 10^4 incident photons per ray without background events. Then, we numerically add 2% and 5% (peak) attenuation to two separate sections of views, respectively. In this case, we set $\delta = 10^{-3}$ and $\beta = 2 \times 10^{-3}$ for edge-preserving regularization. Figure 4 shows the middle transaxial, coronal, and sagittal planes of the true image, the initial guess, the conventional reconstruction without gain correction, the proposed reconstruction with blind gain correction, the proposed reconstruction with non-blind gain correction, and the reference reconstruction from a noisy sinogram without gain fluctuations as a comparison, from left to right, top to bottom. As can be seen from Figure 4, our proposed method effectively reduced the shading artifacts under such small attenuations. Figure 5 shows the true gain fluctuations and the estimated gain fluctuations for both blind and non-blind cases. The estimated gain fluctuations are a little bit noisier due to the small peak attenuation and show a ringing pattern no matter the reconstruction is blind or not. Figure 6 shows the RMS difference between the image at the nth iteration and the *converged* reference reconstruction for each method. Note that all methods show almost the same convergence rates in the early iterations before they start deviating from the solution. That is, gain correction does not change the convergence rate of the algorithm very much but rather improves the overall accuracy of the method.

4. CONCLUSION

In this paper, a new variational formulation for splitting-based X-ray CT image reconstruction algorithm for jointly estimating the true gain parameter vector and the image is proposed. We evaluate our proposed method in both 2D and 3D cases. The shading artifacts due to gain fluctuations are greatly reduced, while the computational complexity per iteration is almost unchanged in our proposed method. Similar concepts can be applied to any splitting-based iterative algorithm with variable splitting on the sinogram, and we are going to extend our proposed method to other 3D X-ray CT geometries.



(a) (b) (c)

Figure 5: 3D simulation: (a) the true gain fluctuations, (b) the estimated gain fluctuations for the blind case, and (c) the estimated gain fluctuations for the non-blind case, where the horizontal and vertical axes are the projection view angle (θ) and the transaxial axis (z), respectively.

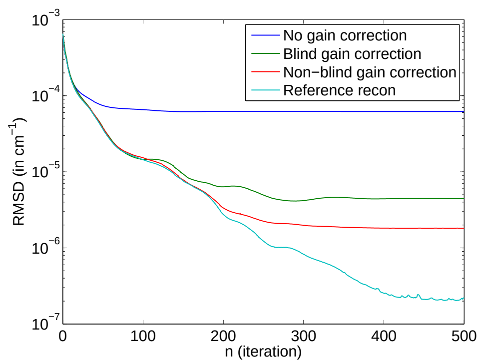


Figure 6: 3D simulation: for each method, RMS difference between the image at the nth iteration and the converged reference reconstruction.

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