RELAXED ORDERED−SUBSETS ALGORITHM FOR IMAGE RESTORATION OF CONFOCAL MICROSCOPY

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Overview

The expectation−maximization (EM) algorithm for maximum likelihood image recovery converges very slowly. Thus, the ordered subsets EM (OS−EM) algorithm has been widely used in image reconstruction for tomography due to an order−of−magnitude acceleration over the EM algorithm [1]. However, OS−EM is not guaranteed to converge. The recently proposed ordered subsets, separable paraboloidal surrogates (OS−SPS) algorithm with relaxation has been shown to converge to the optimal point while providing fast convergence [2]. In this paper, we develop a relaxed OS−SPS algorithm for image restoration [3]. Because data acquisition is different in image restoration than in tomography, we adapt a different strategy for choosing subsets in image restoration which uses pixel location rather than projection angles. Simulation results show that the order−of−magnitude acceleration of the relaxed OS−SPS algorithm can be achieved in image restoration.

Algorithms	Convergence		Image Reconstruction	Image Restoration
	Converge	Rate		
EM	Yes	Slow	Yes	Yes
OS−EM	No	Fast	Yes	No
OS−SPS	No	Fast	Yes	No
Relaxed OS−SPS	Yes	Fast	Yes	No

Measurement Model

Measurement Model for Confocal Microscopy:

$$Y_i \sim \text{Poisson}\{[Ax]_i + b_i\} \qquad i = 1,\ldots,N$$

A = System matrix which is assumed to be known
x = Unknown image to be estimated
b_i= Background noise and dark current

Objective Function: $\Phi(x) = L(x) - \beta R(x)$

b = Regularization parameter

Log−Likelihood Function: $L(x) = \sum_{i=1}^{N} h_i(l_i)$

where $h_i(l) = y_i \log(l + b_i) - (l + b_i)$

and $l_i = [Ax]_i = \sum_{j=1}^{p} a_{ij} x_j$

Penalty Function: $R(x) = \sum_{k=1}^{r} \psi([Cx]_k)$

y = Potential function
C = Penalty matrix (first−order neighborhood: horizontal & vertical cliques)

Goal: $\hat{x} = \underset{x \geq 0}{\text{argmax}}\ \Phi(x)$

The Algorithms

OS Technique −− Decompose the objective function in sub−objective functions f_m

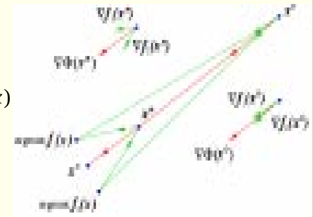
$$\Phi(x) = \sum_{m=1}^{M} \underbrace{\sum_{i \in S_m} h_i(l_i) - \frac{\beta}{M} R(x)}_{f_m}$$

M = Number of subsets

"Subset−Balance"−like Conditions:

$$\nabla f_1(x) \cong \nabla f_2(x) \cong \ldots \cong \nabla f_M(x)$$

Thus $\nabla \Phi(x) \cong M \nabla f_m(x)$



OS−SPS Algorithm

$$x_j^{(n,m)} = \left[x_j^{(n,m-1)} + M \frac{\nabla_j f_m}{d_j + \beta p_j} \right]_+$$

d_j = Precomputed curvature of the likelihood function
p_j = Precomputed curvature of the penalty function

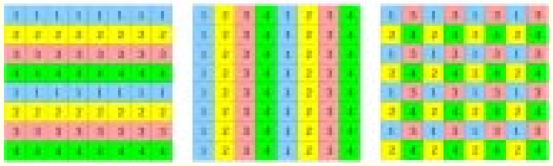
Relaxed OS−SPS Algorithm

$$x_j^{(n,m)} = \left[x_j^{(n,m-1)} + \alpha_n M \frac{\nabla_j f_m}{d_j + \beta p_j} \right]_+$$

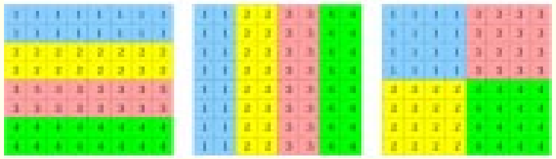
a_n = Positive relaxation parameter, $\sum_n \alpha_n = \infty$ and $\sum_n \alpha_n^2 < \infty$

Subset Design

"Good" Choices for 4 subsets (satisfy "subset−balance"−like conditions)



"Bad" Choices for 4 subsets (violate "subset−balance"−like conditions)

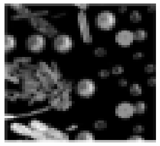


Simulation Results

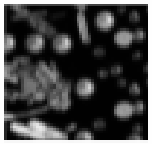
Simulated Data: A 256x256 cell image was degraded by a 15x15 *xz* confocal PSF and Poisson Noise

Restoration: • Perform relaxed OS−SPS (8 subsets) for 50 iterations

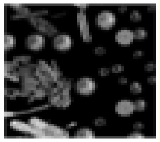
• Relaxation parameter $\alpha_n = 11/(10+n)$
• Nonquadratic penalty $\psi(t) = \delta^2 \left[\left|\frac{t}{\delta}\right| - \log\left(1 + \left|\frac{t}{\delta}\right|\right) \right]$



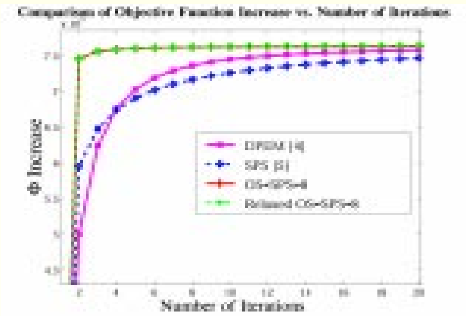
Original Image



Degraded Image



Restored Image



Algorithms	Time/iter (s)	Time Comparison	Number of FLOPS	FLOPS Comparison
DPEM	1.03	0.92	84,937,142	0.92
SPS	1.12	1	92,406,026	1
OS−SPS−2	1.23	1.10	92,522,010	1.00
OS−SPS−4	1.86	1.66	95,944,812	1.04
OS−SPS−8	3.65	3.26	102,919,258	1.11

Conclusion

We demonstrated that the relaxed OS−SPS algorithm, conventionally used for tomography, can be adapted to use in image restoration by choosing appropriate subsets. Essentially, we based this choice on the pixel location. Similarly to tomography, we are able to achieve the order−of−magnitude acceleration over the nonrelaxed version algorithm.

References

[1] H. M. Hudson and R. S. Larkin, "Accelerated Image Reconstruction Using Ordered Subsets of Projection Data," *IEEE Trans. Medical Imaging*, vol. 13, no. 4, pp. 601−−609, December 1994.
[2] S. Ahn and J. A. Fessler, "Globally Convergent Ordered Subsets Algorithms: Application to Tomography," in *Proc. IEEE Nuc. Sci. Symp. Med. Im. Conf.*, 2001.
[3] S. Sotthivirat and J. A. Fessler, "Relaxed ordered−subsets algorithm for penalized−likelihood image restoration," *J. Opt. Soc. Am. A*, Submitted, Available: http://www.eecs.umich.edu/~fessler
[4] A. R. De Pierro, "A Modified Expectation Maximization Algorithm for Penalized Likelihood Estimation in Emission Tomography," *IEEE Trans. Med. Imaging*, vol. 14, no. 1, pp. 132−−137, March 1995.
[5] H. Erdo?an and J. A. Fessler, "Monotonic Algorithms for Transmission Tomography," *IEEE Trans. Med. Imaging*, vol. 18, no. 9, pp. 801−−814, September 1999.