Comparing Estimator Covariances at Matched Spatial Resolutions for Imaging System Design

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Abstract — **The design of imaging systems by optimizing parameters such as aperture sizes is complicated by the dependence of the overall system properties on the regularization method used in the image reconstruction algorithm. This paper describes a simple method for holding fixed the overall point spread function (PSF) by varying the regularization matrix simultaneously with the system design parameters, so that the estimator covariance can be minimized at some fixed desired spatial resolution. The expression derived for estimator covariance is a simple function of the pseudo-inverse of the measurement Fisher information matrix and of the desired PSF matrix, and illuminates the inherent resolution/noise tradeoffs in imaging system design.**

I. Introduction

The design of imaging systems has long been a topic of considerable research interest, and activity in this area is particularly timely in light of ongoing advances in detectors that enable a greater variety of system design choices. Numerous criteria for "optimal" system design have been proposed, and it is unlikely that any single criterion will ever be universally accepted as the unique best method. The method described in this paper is a potentially useful addition to the family of methods for comparing imaging system designs and choosing system design parameters.

Our past efforts have focused on statistical criteria for system design, particularly those based on the Cramer-Rao bound. The conventional CR bound is poorly suited to imaging problems since practical estimators in ill-conditioned problems must be regularized, which induces bias. The uniform Cramer-Rao bound of Hero *et al.* [1] allows for biased estimators of the type used in imaging problems. An appealing property of the uniform CR bound is that it is estimator independent. However, the tradeoff between variance and bias is parameterized by *bias gradient length*, a quantity that can be difficult to translate into more familiar imaging metrics such as spatial resolution. In particular, a given bias gradient length can correspond to different spatial resolutions for different imaging systems.

In this paper, we forgo the estimator-independence of the uniform CR bound, and instead focus on the class of penalized weighted least-squares estimators. This class has been shown to achieve the uniform CR bound under certain conditions [1], so it is a reasonable class of methods for consideration. Furthermore, fast converging iterative algorithms are available for this class of estimators, *e.g.* [2–4], so system designs that are optimized with respect to this class of estimators can be implemented in practice. Nevertheless, the dependence on the particular estimator is a limitation that one must bear in mind when applying this technique.

As described in [5], the spatial resolution properties of penalized-likelihood estimators are nontrivial functions of the imaging system response and the penalty function. If one considers different system designs while holding the penalty function fixed, the spatial resolution can change. Even if one adjusts the regularization parameter simultaneously with the system design parameters to hold fixed some scalar aspect of the PSF of the reconstructed images, such as the full-width at half-maximum (FWHM), the *shape* of the PSF can change as one changes system design parameters. This greatly complicates the task of making "fair" comparisons between alternative imaging system designs.

We would like to have a method for comparing the noise properties (namely the estimator covariance) of images reconstructed for different system designs, but subject to the constraint that the overall PSF of the imaging system and reconstruction method is held fixed, so that the spatial resolution properties are consistent. In this paper, we show that by suitably varying the *entire penalty function* simultaneously with the system design parameters, this goal is achievable, at least analytically. Remarkably, the final form for the estimator covariance depends quite simply on the Fisher information matrix of the imaging system and on the target PSF matrix, and lends insight in the tradeoffs between spatial resolution and noise.

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