Improved gradient waveforms for small-tip 3D spatially tailored excitation using Iterated Local Search

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Synopsis

We propose a strategy for the joint design of gradient and radiofrequency waveforms for small-tip 3D spatially tailored excitation, that may lead to more globally optimal excitation k-space trajectories. Currently, gradients are either pre-defined or restricted to certain classes such as echo-planar or concentric shells. Our method makes use of a recently proposed optimization method that expresses k-space with a 2nd-order B-spline basis permitting arbitrary k-space trajectories. We employ this method in an Iterated Local Search strategy, and show that this approach reduces the sensitivity of the excited pattern to the choice of initial k-space trajectory that "seeds" the optimization.

Introduction

The joint design of gradient and radiofrequency (RF) waveforms for small-tip 3D spatially tailored excitation poses a non-linear, non-convex, and constrained optimization problem that remains unsolved. Most existing approaches to tailored RF design either pre-define the gradients and only solve the least-squares RF design sub-problem, or restrict the gradients to certain classes such as echo-planar[1] or concentric shells[2] that permit low-order parametrization. Recently, a more general method for joint gradient and RF design was proposed[3], in which the excitation k-space trajectory is expressed using a 2nd-order B-spline basis leading to the following constrained optimization problem:

$$\arg\min_{\mathbf{c},\mathbf{b}} ||\mathbf{A}(\mathbf{c}) - \mathbf{b}||_2^2 + R(\mathbf{b}), \text{ subject to gradient hardware constraints,} \quad (1)$$

where $\mathbf{c}$ is the B-spline coefficients, $\mathbf{A}$ is the small-tip system matrix[4], and $\mathbf{b}$ is the RF waveform. The method in [3] alternates between updating $\mathbf{c}$ and $\mathbf{b}$, resulting in locally optimal gradients that are not restricted to the heuristic trajectories of previous approaches. However, the gradients obtained depend on the initial k-space trajectory that "seeds" the algorithm. For example, in [3] it was observed that a stack-of-spirals initial trajectory produced a relatively poor excitation (after local optimization) compared to the other initial trajectories tested. Here we propose an extension of [3] that reduces the dependence of the final excitation patterns on the initial (seed) k-space trajectory, possibly leading to designs that are closer to being globally optimal.

Methods

Our approach is based on the Iterated Local Search (ILS) method[6], an iterative optimization strategy that employs a local optimization routine at each iteration. Our ILS search begins by finding a local optimum k_0^* starting from some initial k-space trajectory (e.g., stack-of-spirals or concentric shells). We then attempt to sample "nearby" local minima by "perturbing" k_0^* multiple times to produce a set of N_{test} candidate seed trajectories $k_{1,j}, j = 1...N_{test}$, for the next iteration (Fig. 1). For each $k_{1,j}$, Eq. (1) is descended (in parallel on a 32-core desktop computer) using the method in [3] to yield a local optimum $k_{1,j}^*$, and among all $k_{1,j}^*, j = 1...N_{test}$, the trajectory producing the miminum cost function value is chosen as the value k_1^* for the next iteration. This procedure is then repeated until convergence. We generate the candidate trajectories $k_{i+1,j}, j = 1...N_{test}$, at iteration i by expressing the locally optimal trajectory k_i^* using a reduced number $N_{basis,j}$ ($< N_{basis}$) of B-spline basis functions, i.e., by projecting k_i^* onto a set of N_{test} lower-dimensional B-spline bases. For a given j we do this projection by first obtaining an unconstrained least-squares fit $\mathbf{c}_j^u$ to k_i^* using $N_{basis,j}$ 2nd-order B-spline basis functions, and then minimizing the following constrained cost function using Matlab's quadprog function:

$$\arg\min_{\mathbf{c}_j} ||\mathbf{c}_j - \mathbf{c}_\mathbf{j}^\mathbf{u}||_2^2, \text{ subject to gradient hardware constraints.}$$

Here we use $N_{basis} = 50$ and $N_{basis,j} = \{10, 12, 14, \ldots, 50\}$ (i.e., $N_{test} = 20$, with the last value of 50 corresponding to the unperturbed trajectory). In essence, the $k_{i+1,j}, j = 1...N_{test}$, correspond to smoothed versions of the current iterate k_i^* with varying degree of smoothness. We evaluated three different k-space initializations:

stack-of-spirals, SPINS[5], and the extended-KT trajectory from [3]. The target pattern was a 3D cube. RF pulse duration was approximately equal (4ms) for all three k-space initializations. Simulations and experiments were done for single-coil transmission, however the principles introduced here would also apply to parallel transmission. We ran a total of five ILS iterations.

Results

Figure 2 shows the design cost function value (Eq. (1)) versus computation time for both local and the proposed global (ILS) optimization, for the three different k-space initializations. Each curve shows the cost-vs-time for the best local minimum k_i^* obtained at each iteration. A sharp spike along a curve indicates that the ILS algorithm identified a nearby local minimum with lower cost than the unperturbed trajectory. Stack-of-spirals produces significantly higher cost (poorer excitation accuracy) after local optimization compared to the other two initializations, however after the ILS search all three k-space intializations produce more similar final costs.

Figure 3 shows Bloch simulated excitation patterns obtained with both local and ILS optimization. All three k-space initializations produce similar excitation accuracy only after the proposed ILS optimization, as expected from Fig. 2. Figure 4 validates these excitation results experimentally, for the case of stack-of-spirals initialization [GE 3T scanner; uniform gel phantom; body coil RF transmission; 8-channel headcoil reception; matrix 240x240x48; FOV 24x24x24 cm^3; TR=100ms; flip angle 7.5º].

Conclusion

We have proposed an Iterated Local Search strategy for the joint design of gradients and RF waveforms for small-tip 3D tailored excitation, that reduces the influence of the choice of initial (seed) k-space trajectory on excitation accuracy.

Acknowledgements

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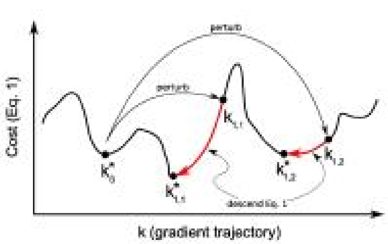
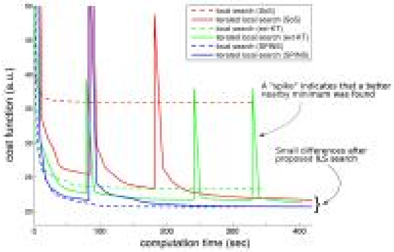
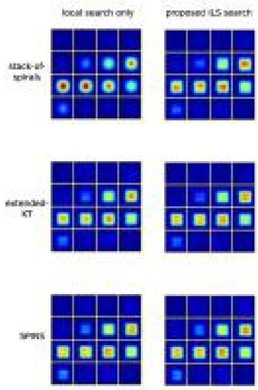
Figures



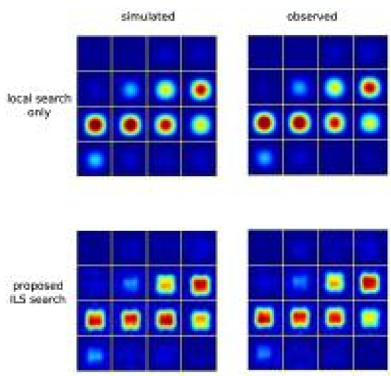
Iterated Local Search strategy. The local optimization (red paths) is done by descending Eq. (1) using the method in [3].



Cost function (Eq. (1)) vs. computation time for three different k-space initializations, for both local-only search (dashed curves) and the proposed ILS search (solid curves). When using the proposed ILS approach, all three k-space initializations converge to nearly the same final cost function value.



Simulated excitation patterns for the three k-space initializations (seeds) tested in Fig. 2, after either local search (left) or the proposed ILS search (right).



Experimental validation of the Bloch simulation results in Fig. 3, for stack-of-spirals k-space initialization. Simulated and observed excitation patterns are in excellent agreement.