Regularized Estimation of Magnitude and Phase of Multiple-Coil B1 Field via Bloch-Siegert B1 Mapping

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Introduction: Parallel excitation pulse design usually requires accurate magnitude and phase maps of the B_1 field produced by each coil. Bloch-Siegert (BS) B_1 mapping [1] has been shown to be fast and accurate; however, the B_1 map produced by this phase-based method may suffer from low SNR in low magnitude regions having insufficient excitation or low spin density. This problem has been mitigated in [2] by using combinations of multiple coils for imaging excitation. However, it does not help for low spin density regions and insufficient excitation is still possible as accurate B_1 maps are unknown; furthermore, estimation of B_1 phase needs another set of scans, which is time-consuming and information redundant. In this work, we propose a regularized method to jointly estimate the magnitude and (relative) phase of multi-coil B_1 maps from BS B_1 mapping data without using additional scans for phase estimation. By utilizing the prior knowledge that B_1 maps are smooth [3], the regularization terms can help improve quality of the B_1 maps in low magnitude regions. The method was demonstrated by phantom experiments.

Theory: We propose to acquire the standard BS B_1 mapping data [1] that needs 2*R scans (R = the number of coils). In each scan, the same coil combination is used for the BS pulse and its corresponding slice excitation pulse. The composite B_1^+ field $E_r(\boldsymbol{x})$ produced at each time r is described in (1), where $\alpha_{r,j}$ is a complex weight that indicates how the coils are combined at each time, $B_j(\boldsymbol{x})$ is the magnitude of the B_1 map produced by the *jth* coil with a unit input current, and $\phi_j(\boldsymbol{x})$ is the corresponding B_1 phase map. A convenient choice of $\alpha_{r,j}$ is the "all-but-one" strategy, where $\alpha_{r,r}=0$ and $\alpha_{r,j}=1$ when $j \neq r$. The signal models for the BS data (reconstructed images) of the *rth* pair of scans are described in (2), where r = 1, 2, …, R; the superscripts +/– denote the scan with $+\omega_{RF}$ or $-\omega_{RF}$ BS pulse, $I_r^{\pm}(\boldsymbol{x})$ is the image of each scan, μ is the ratio between the actual flip and $|E_r(\boldsymbol{x})|$, $m_r^{\pm}(\boldsymbol{x})$ is the magnitude related to spin density, T_1, T_2, T_R, T_E, flip angle, receive sensitivity, magnetization transfer (MT) effect, etc., $\phi_b(\boldsymbol{x})$ is the corresponding background phase, and $K_{BS}^{\pm}(\boldsymbol{x})$ is the BS pulse constant that incorporates the B_0 field map $\omega_0(\boldsymbol{x})$ [1]. We simplify (2) into (3) by changing variables: $z_r(\boldsymbol{x}) \triangleq E_r(\boldsymbol{x})e^{i\phi_b(\boldsymbol{x})}$, $\tilde{\phi}_r(\boldsymbol{x}) \triangleq \angle z_r(\boldsymbol{x})$, $\tilde{B}_r(\boldsymbol{x}) \triangleq |z_r(\boldsymbol{x})|$, and $M_r^{\pm}(\boldsymbol{x}) \triangleq sin(\mu|z_r(\boldsymbol{x})|)\, m_r^{\pm}(\boldsymbol{x})$. Thus we can obtain the magnitude and relative phase of the B_1 maps by only estimating $\tilde{B}_r(\boldsymbol{x})$ and $\tilde{\phi}_r(\boldsymbol{x})$. $M_r^{\pm}(\boldsymbol{x})$ is a set of nuisance parameters that needs to be jointly estimated, but they are fortunately linear terms that can be easily estimated.

$$E_r(\boldsymbol{x}) = \sum_{j=1}^{R} \alpha_{r,j} B_j(\boldsymbol{x}) e^{i\phi_j(\boldsymbol{x})} \quad (1)$$

$$\begin{cases} I_r^+(\boldsymbol{x}) = \sin(\mu|E_r(\boldsymbol{x})|)\, e^{i\angle E_r(\boldsymbol{x})} m_r^+(\boldsymbol{x}) e^{i\phi_b(\boldsymbol{x})} e^{iK_{BS}^+(\boldsymbol{x})|E_r(\boldsymbol{x})|^2} \\ I_r^-(\boldsymbol{x}) = \sin(\mu|E_r(\boldsymbol{x})|)\, e^{i\angle E_r(\boldsymbol{x})} m_r^-(\boldsymbol{x}) e^{i\phi_b(\boldsymbol{x})} e^{iK_{BS}^-(\boldsymbol{x})|E_r(\boldsymbol{x})|^2} \end{cases} \quad (2)$$

$$\begin{cases} I_r^+(\boldsymbol{x}) = M_r^+(\boldsymbol{x}) e^{i[K_{BS}^+(\boldsymbol{x})\tilde{B}_r(\boldsymbol{x})^2 + \tilde{\phi}_r(\boldsymbol{x})]} \\ I_r^-(\boldsymbol{x}) = M_r^-(\boldsymbol{x}) e^{i[K_{BS}^-(\boldsymbol{x})\tilde{B}_r(\boldsymbol{x})^2 + \tilde{\phi}_r(\boldsymbol{x})]} \end{cases} \quad (3)$$

$$\Psi\left(\tilde{B}(\boldsymbol{x}), \tilde{\phi}(\boldsymbol{x}), M(\boldsymbol{x})\right) = \sum_{r=1}^{R} \sum_{\varsigma=+,-} \left\| I_r^{\varsigma}(\boldsymbol{x}) - M_r^{\varsigma}(\boldsymbol{x}) e^{i[K_{BS}^{\varsigma}(\boldsymbol{x})\tilde{B}_r(\boldsymbol{x})^2 + \tilde{\phi}_r(\boldsymbol{x})]} \right\|^2 + \beta_1 \sum_{r=1}^{R} \left\| C\tilde{B}_r(\boldsymbol{x}) \right\|^2 + \beta_2 \sum_{r=2}^{R} \left\| C e^{i[\tilde{\phi}_r(\boldsymbol{x}) - \tilde{\phi}_1(\boldsymbol{x})]} \right\|^2 \quad (4)$$

$$\begin{bmatrix} B_1(\boldsymbol{x}) e^{i\phi'_1(\boldsymbol{x})} \\ \vdots \\ B_R(\boldsymbol{x}) e^{i\phi'_R(\boldsymbol{x})} \end{bmatrix} = \begin{bmatrix} \alpha_{1,1} & \cdots & \alpha_{1,R} \\ \vdots & \ddots & \vdots \\ \alpha_{R,1} & \cdots & \alpha_{R,R} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{B}_1(\boldsymbol{x}) e^{i\tilde{\phi}_1(\boldsymbol{x})} \\ \vdots \\ \tilde{B}_R(\boldsymbol{x}) e^{i\tilde{\phi}_1(\boldsymbol{x})} \end{bmatrix} \quad (5)$$

Regularization enforces prior knowledge to improve estimation. It is reasonable to assume that the magnitudes of the composite B_1 maps, $\tilde{B}_r(\boldsymbol{x})$, are spatially smooth. Although the absolute phase $\tilde{\phi}_r(\boldsymbol{x})$ is not necessarily smooth, the difference of it relative to a reference coil, e.g., $\tilde{\phi}_r(\boldsymbol{x}) - \tilde{\phi}_1(\boldsymbol{x})$, should be smooth. Therefore, a finite differencing matrix C can be applied in regularization terms to penalize roughness. Since $\tilde{\phi}_r(\boldsymbol{x}) - \tilde{\phi}_1(\boldsymbol{x})$ is likely to have phase wrap, we use the regularizer proposed in [4] that instead regularizes the roughness of $e^{i[\tilde{\phi}_r(\boldsymbol{x}) - \tilde{\phi}_1(\boldsymbol{x})]}$. Our final cost function for estimating B1 is in (4), where $\tilde{B}(\boldsymbol{x}) = [\tilde{B}_1(\boldsymbol{x}), \ldots, \tilde{B}_R(\boldsymbol{x})]$, $\tilde{\phi}(\boldsymbol{x}) = [\tilde{\phi}_1(\boldsymbol{x}), \ldots, \tilde{\phi}_R(\boldsymbol{x})]$, $M(x) = [M_1(x), \ldots, M_R(x)]$, β_1 and β_2 are scalar regularization parameters. We estimate all the unknowns by minimizing $\Psi\left(\tilde{B}(\boldsymbol{x}), \tilde{\phi}(\boldsymbol{x}), M(\boldsymbol{x})\right)$, during which $\tilde{B}(\boldsymbol{x})$, $\tilde{\phi}(\boldsymbol{x})$ and $M(\boldsymbol{x})$ are cyclically updated. We update $M(\boldsymbol{x})$ by simply taking the real least square solution of (4) in each iteration. We use conjugate gradients with line search algorithm [5] to update $\tilde{B}(\boldsymbol{x})$ and $\tilde{\phi}(\boldsymbol{x})$, where backtracking line search [6] and monotonic line search [5] are used for $\tilde{B}(\boldsymbol{x})$ and $\tilde{\phi}(\boldsymbol{x})$ respectively. The standard approach [1] produces good initial guess for $\tilde{B}(\boldsymbol{x})$, and initial guess of $\tilde{\phi}(\boldsymbol{x})$ can then be solved analytically from equation (3) once $\tilde{B}(\boldsymbol{x})$ is initialized, without knowing $M(x)$. Once $\tilde{B}(\boldsymbol{x})$ and $\tilde{\phi}(\boldsymbol{x})$ are estimated, the magnitude and relative phase of the original coils can be derived easily by (5), where $\phi'_r(\boldsymbol{x}) = \phi_r(\boldsymbol{x}) + \phi_b(\boldsymbol{x})$ which does not change the relative phase of the *r*th coil.

Methods and Results: The proposed method was tested by a phantom experiment on a 3T GE scanner equipped with an 8-coil custom parallel transmit/receive system [7]. Other than a routine B_0 mapping, we did a total of 16 scans with "all-but-one" coil combinations for the excitation and the BS pulse, where 8 ms Fermi BS pulses with ± 4000 Hz off-resonance were followed by 2D spin-warp readout. Sequence parameters: 5 mm slice thickness, matrix 64*64, 26 cm FOV. Fig. 1 shows the results by the proposed method and the conventional method (B_1 magnitude is computed as in [1] and B_1 phase is computed by solving (3)).

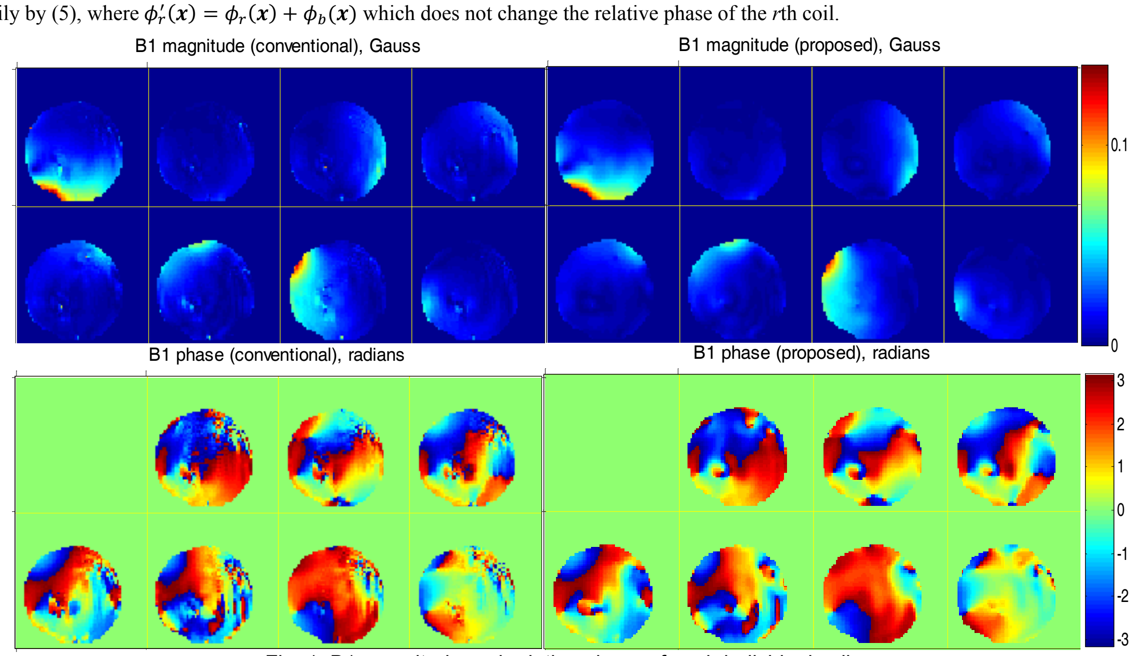

Fig. 1: B1 magnitude and relative phase of each individual coil

Conclusions: The proposed method uses the same coil combinations for the slice excitation and BS pulses to jointly estimate B_1 phase and magnitude, which saves the set of R scans for phase estimation. This iterative regularized estimation produces improved B1 magnitude and phase maps for low SNR regions. Future work will be to optimize the coil combinations, $i.e., \alpha_{r,j}$ in (1), to better reduce low excitation regions.

References: [1] Sacolick et al., MRM 63:1315-1322, 2010. [2] Sacolick et al., Proc. ISMRM 19:2926 (2011). [3] Funai et al., IEEE ISBI, 2007. [4] Zhao et al., Proc. ISMRM 19:2841 (2011). [5] Fessler et al., IEEE TIP, 8: 688-99, 1999. [6] K. Lange. *Numerical analysis for statisticians*, 1999. [7] System provided by group from Texas A&M University.

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