Joint design of trajectory and RF pulses for parallel excitation

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Introduction : In current parallel excitation pulse design methods (e.g., [1]), one pre-determines the gradient waveforms (hence the excitation k-space trajectory [2]), and subsequently computes the RF pulses for a desired excitation pattern. Inspired by [3], we propose an alternating optimization framework for *joint* design of trajectory and RF pulses for parallel excitation. The method alternates between optimizing the pulses using conjugate gradient (CG), and modifying trajectory parameters in the gradient descent (GD) direction. The cost function gradient, with respect to the parameters, can be computed using an analytical formula. Compared to pulses designed with pre-determined trajectory, joint designs excite with significantly higher accuracy and lower RF power. These benefits come at a modest cost in computation time.

Theory

The joint design framework is based on the spatial-domain pulse design method [1]. Let $\mathbf{b}_r = (b_r(t_0),\ldots,b_r(t_{N-1}))$ be RF pulse samples for the rth coil, and $\mathbf{k}_x(\boldsymbol{\phi}_x) = (k_x(t_0;\phi_x),\ldots,k_x(t_{N-1};\phi_x))$, $\mathbf{k}_y(\boldsymbol{\phi}_y) = (k_y(t_0;\phi_y),\ldots,k_y(t_{N-1};\phi_y))$ be the k-space trajectory parameterized by $\boldsymbol{\phi}_x = (\phi_{x1},\ldots,\phi_{xL})$, $\boldsymbol{\phi}_y = (\phi_{y1},\ldots,\phi_{yL})$. Let $\mathbf{A}(\mathbf{k}_x(\boldsymbol{\phi}_x),\mathbf{k}_y(\boldsymbol{\phi}_y))$ be the pulse design system matrix [1], $\mathbf{S}_r = diag(s_r(\mathbf{x}_0),\ldots, s_r(\mathbf{x}_{M-1}))$ be the sensitivity pattern samples of the rth coil, $\mathbf{d} = (d(\mathbf{x}_0),\ldots,d(\mathbf{x}_{M-1}))$ be the desired excitation pattern, and $\mathbf{W}$ be a diagonal ROI matrix [1]. We can define a cost function comprising excitation error and integrated pulse power terms, balanced by regularization parameter β:

$$\Psi(\mathbf{b}_1,\ldots,\mathbf{b}_R,\boldsymbol{\phi}_x,\boldsymbol{\phi}_y) = \left\| \sum_{r=1}^{R} \mathbf{S}_r \mathbf{A}(\mathbf{k}_x(\boldsymbol{\phi}_x),\mathbf{k}_y(\boldsymbol{\phi}_y))\mathbf{b}_r - \mathbf{d} \right\|_{\mathbf{W}}^2 + \beta \sum_{r=1}^{R} \|\mathbf{b}_r\|^2 . \qquad (1)$$

We design *locally optimal* RF pulses and trajectory parameters by the following alternating optimization process:

$$for\ i = 0 : N_{alt} - 1$$

$$(\hat{\boldsymbol{\phi}}_x^{(i+1)}, \hat{\boldsymbol{\phi}}_y^{(i+1)}) = \arg\min_{\boldsymbol{\phi}_x,\boldsymbol{\phi}_y} \Psi(\hat{\mathbf{b}}_1^{(i)},\ldots,\hat{\mathbf{b}}_R^{(i)},\boldsymbol{\phi}_x,\boldsymbol{\phi}_y) \qquad (2)$$

$$(\hat{\mathbf{b}}_1^{(i+1)},\ldots,\hat{\mathbf{b}}_R^{(i+1)}) = \arg\min_{\mathbf{b}_1,\ldots,\mathbf{b}_R} \Psi(\mathbf{b}_1,\ldots,\mathbf{b}_R,\hat{\boldsymbol{\phi}}_x^{(i+1)},\hat{\boldsymbol{\phi}}_y^{(i+1)}) \qquad (3)$$

$$end$$

This process is initialized by $(\hat{\boldsymbol{\phi}}_x^{(0)},\hat{\boldsymbol{\phi}}_y^{(0)}),(\hat{\mathbf{b}}_1^{(0)},\ldots,\hat{\mathbf{b}}_R^{(0)})$, and halts when a convergence criterion is satisfied. The pulse design problem (Eq. (3)) can be solved with CG [1]. The trajectory optimization (Eq. (2)) is indirectly constrained by maximum gradient amplitude and slew rate limits, and can be tackled with the GD method (possibly, with only one step). If we parameterize the trajectory by weights of temporal basis functions ($\mathbf{k}_x = \mathbf{H}_x \boldsymbol{\phi}_x$ and/or $\mathbf{k}_y = \mathbf{H}_y \boldsymbol{\phi}_y$, where $\mathbf{H}_x$, $\mathbf{H}_y$ denote basis function matrices), the cost function gradients can be computed analytically via

$$\nabla_{\boldsymbol{\phi}_x}\Psi = \mathbf{H}_x{}' \cdot \nabla_{k_x}\Psi, \quad where \quad \nabla_{k_x}\Psi = 2\,\mathrm{Re}\{-i \cdot \sum_{r=1}^{R} diag(\mathbf{b}'_r)\mathbf{A}'\mathbf{X}\mathbf{S}_r{}'\mathbf{W}\mathbf{e}\}, \qquad (4)$$

$$\nabla_{\boldsymbol{\phi}_y}\Psi = \mathbf{H}_y{}' \cdot \nabla_{k_y}\Psi, \quad where \quad \nabla_{k_y}\Psi = 2\,\mathrm{Re}\{-i \cdot \sum_{r=1}^{R} diag(\mathbf{b}'_r)\mathbf{A}'\mathbf{Y}\mathbf{S}_r{}'\mathbf{W}\mathbf{e}\}, \qquad (5)$$

with $\mathbf{X} = diag(x_0,\ldots,x_{M-1})$, $\mathbf{Y} = diag(y_0,\ldots, y_{M-1})$, and $\mathbf{e} = \sum_{r=1}^{R} \mathbf{S}_r \mathbf{A}(\mathbf{k}_x(\boldsymbol{\phi}_x),\mathbf{k}_y(\boldsymbol{\phi}_y))\mathbf{b}_r - \mathbf{d}$. Essentially, the trajectory parameters are modified in the directions of $-\nabla_{\boldsymbol{\phi}_x}\Psi$ and $-\nabla_{\boldsymbol{\phi}_y}\Psi$ in between pulse optimization iterations.

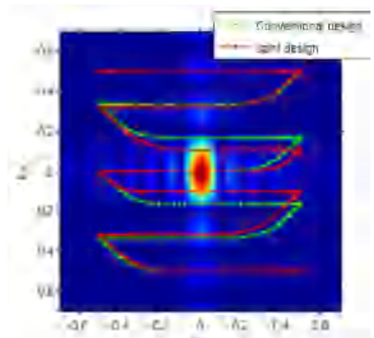

Fig. 1: Original (XFOV = 6 cm, green) and jointly designed (red) echo-planar trajectories. Joint design automatically adjusts strategy of sampling the desired pattern spectrum (underlying).

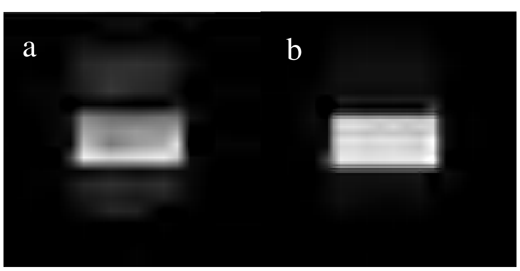

Fig. 2: (a) Pattern excited by conventional design; (b) pattern excited by joint design, which effectively adjusts phase encoding (Fig. 1) to reduce aliasing.

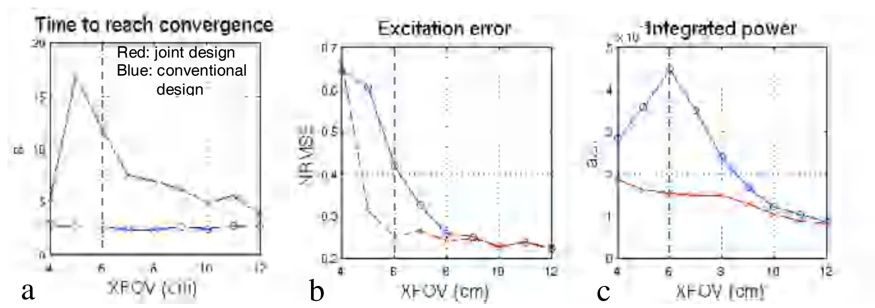

Fig. 3: In a range of acceleration factors, the joint design is beneficial in terms of excitation accuracy and pulse power, at modest computation cost. (a) Computation time for 99.9% convergence in the design methods; (b) mean square excitation error in ROI; (c) total integrated power of the eight pulses. (dash line: measures corresponding to Fig. 1 and 2)

Experiment

By 2D Bloch simulation of eight-coil parallel excitation, we evaluated the joint design method in terms of excitation accuracy and pulse power. The joint design was initialized with RF pulses designed using an echo-planar (EP) trajectory [4] with uniform phase encoding locations (Fig. 1). The jointly optimal trajectory and pulses were compared to pulses conventionally designed [1], with the same EP trajectory as for joint design initialization. In the joint design, we parameterized the EP trajectory by phase encoding locations ($\boldsymbol{\phi}_y$) via expressing its y component as $\mathbf{k}_y = \mathbf{H}_y \boldsymbol{\phi}_y$, where $\mathbf{H}_y$ was a matrix whose columns were time-shifted rectangular functions. The x component was not parameterized. This parameterization strategy turned the trajectory optimization process virtually constraint-free. We alternately ran 5 CG iterations and 1 GD step (with very small step size) until 99.9% convergence in cost function.

Fig. 1 shows the initial EP trajectory (green, excitation FOV = 6 cm), and the optimal EP trajectory sought by the joint design (red). They are overlaid on the spectrum of the desired pattern, a uniform block with flip angle = 30 degrees. Over the GD steps in the joint design, the phase-encoding locations were automatically adjusted to attain a local minimum in cost. The resulting improvement in excitation accuracy is prominent (Fig. 2), as aliasing in the phase-encoded dimension (y) is significantly reduced. Integrated power of the joint design was one third of the original value (1.5E-3 compared to 4.5E-3, in a.u.). These gains come at the cost of a modest increase in computation time (12 s compared to 2.5 s, Matlab implementation). Such cost-benefit tradeoff was also observed at other acceleration factors with the same simulation and design settings (Fig. 3) – the joint design is superior to the conventional design, in terms of accuracy and power, over a range of excitation FOV.

References: [1] W Grissom et al, MRM(56), 2006 [2] JM Pauly et al, JMR(81), 1989 [3] CJ Hardy et al, JMR(77), 1988 [4] Y Zhu, MRM(51), 2004

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