Simultaneous Estimation of Image and Inhomogeneity Field Map

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1 Abstract

A new image reconstruction technique is described that allows the simultaneous estimation of a running field map and field inhomogeneity corrected images while acquiring only part of k-space at each time point. A new method of estimating field maps is derived and applied to the case of using only one interleave of a two-interleaved spiral sequence at each of two different echo times.

2 Introduction

To perform a reconstruction that corrects for field inhomogeneities, it is standard practice to get an estimate of the field map from an initial reconstruction of data collected at two different echo times. Let $y(t_i)$ and $z(t_i)$ be the data collected at time point t_i in the two data sets with $\boldsymbol{z}$ collected with an echo time that is τ seconds longer than that used for $\boldsymbol{y}$. Typically τ is 2 ms or less to avoid 2π wrap-arounds of phase while estimating the fieldmap. Using the discretized signal equation, the expected data for the two acquisitions at time point i is given by,

$$\begin{aligned} u_i &= \sum_j m_j e^{-i\left(2\pi k(t_i) r_j + \omega_o(r_j) t_i\right)} \\ v_i &= \sum_j m_j e^{-i\left(2\pi k(t_i) r_j + \omega_o(r_j)(t_i+\tau)\right)}, \end{aligned} \tag{1}$$

where m_j is the discretized object's magnetization at position r_j, $k(t_i)$ is k-space trajectory, and $\omega_o(r_j)$ is the field inhomogeneity at position r_j. Letting $a_{ij} = e^{-i\left(2\pi k(t_i) r_j + \omega_o(r_j) t_i\right)}$ and $b_{ij} = e^{-i\left(2\pi k(t_i) r_j + \omega_o(r_j)(t_i+\tau)\right)}$, (1) becomes,

$$\begin{aligned} \mathbf{u} &= A\mathbf{m} \\ \mathbf{v} &= B\mathbf{m}. \end{aligned} \tag{2}$$

The data vectors $\boldsymbol{y}$ and $\boldsymbol{z}$ are given by, $y(t_i)=u_i+\varepsilon_i$, and $z(t_i)=v_i+\xi_i$, where ε_i and ξ_i are the noise at time points i in each acquisition. The goal is to estimate the vectors $\boldsymbol{\omega_o}$ and $\boldsymbol{m}$ from the data $\boldsymbol{y}$ and $\boldsymbol{z}$.

2.1 Conventional Method

The conventional method proceeds as described in [1], it assumes that the action of the inhomogeneity phase accrual occurs at the echo time, TE, so that reconstruction of the data vector $\boldsymbol{y}$ gives an image, $\hat{m}_y(r_j)$, such that,

$$\hat{m}_y(r_j) \approx m_j e^{-i\omega_o(r_j)TE}.$$

A similar assumption results in the reconstruction of $\boldsymbol{z}$ as

$$\hat{m}_z(r_j) \approx m_j e^{-i\omega_o(r_j)TE} e^{-i\omega_o(r_j)\tau}.$$

The standard approach is to then divide $\hat{m}_y(r_j)$ by $\hat{m}_z(r_j)$ resulting in $e^{i\omega_o(r_j)\tau}$. By taking the phase of this quantity and dividing by τ, an estimate of the inhomogeneity map is formed.

An alternative approach that we investigate here is to form a cost function and use an optimization method of gradient descent to find the field map that best fits the data.

3 Proposed Method

The proposed method alternates between two phases, first we use our best estimate of the field map, starting with zeros or the result of the standard method, and estimate the image that best corresponds to the object fitting all of the data from ***y*** and ***z***. Then we use this estimate of the object to find the field map that minimizes a cost function involving ***y*** and ***z***. Our cost function *J*, which has data fit terms and a roughness penalty, is given by

$$J(\mathbf{m},\omega_o,\mathbf{t}) = \sum_i \frac{1}{2}(y_i - u_i)^*(y_i - u_i) + \sum_i \frac{1}{2}(z_i - v_i)^*(z_i - v_i) + \frac{\beta}{2}\sum_{j_1,j_2:neighbors}\left(\omega_o(r_{j1}) - \omega_o(r_{j2})\right)^2, \tag{3}$$

where ' * ' denotes the complex conjugate transpose. The roughness penalty part of the cost function can be represented in matrix notation as, $\beta\omega_o{}'C'C\omega_o$, where C is constructed to take the differences between pairs of neighboring pixels. To minimize this cost function, we find the gradient of J with respect to $\omega_o(r_j)$, as

$$\begin{aligned}\frac{\partial}{\partial\omega_o(r_j)}J = {} & \sum_i \frac{1}{2}\left(-it_i m_j^* e^{i(2\pi k(t_i)r_j + \omega_o(r_j)t_i)}(y_i - u_i) + it_i m_j e^{-i(2\pi k(t_i)r_j + \omega_o(r_j)t_i)}(y_i - u_i)^*\right) + \\ & \sum_i \frac{1}{2}\left(-i(t_i+\tau)m_j^* e^{i(2\pi k(t_i)r_j + \omega_o(r_j)(t_i+\tau))}(z_i - v_i) + i(t_i+\tau)m_j e^{-i(2\pi k(t_i)r_j + \omega_o(r_j)(t_i+\tau))}(z_i - v_i)^*\right) + \\ & \beta\sum_{j',j:neighbors}\left(\omega_o(r_j) - \omega_o(r_{j'})\right).\end{aligned} \tag{4}$$

Letting

$$g_j = \sum_i - it_i m_j^* e^{i(2\pi k(t_i)r_j + \omega_o(r_j)t_i)}(y_i - u_i),$$

and

$$h_j = \sum_i - i(t_i+\tau)m_j^* e^{i(2\pi k(t_i)r_j + \omega_o(r_j)(t_i+\tau))}(z_i - v_i),$$

and using our expressions for A and B as in (2), we see that,

$$\mathbf{g} = \mathbf{m}^* \cdot * A'(-i\mathbf{t} \cdot *(\mathbf{y} - A\mathbf{m}))$$

$$\mathbf{h} = \mathbf{m}^* \cdot * B'(-i(\mathbf{t}+\tau) \cdot *(\mathbf{z} - B\mathbf{m})),$$

where '.*' denotes element-wise multiplication. We can then express the gradient of J from (4) as a vector for all values of r_j as

$$\begin{aligned}\frac{\partial}{\partial\omega_o}J &= \frac{1}{2}(\mathbf{g} + \mathbf{g}^*) + \frac{1}{2}(\mathbf{h} + \mathbf{h}^*) + \beta C'C\omega_o \\ &= \mathrm{Re}\{\mathbf{g}\} + \mathrm{Re}\{\mathbf{h}\} + \beta C'C\omega_o.\end{aligned} \tag{5}$$

Therefore, our reconstruction is performed by using the conjugate gradient method to reconstruct our image based on the current guess of the field map. This reconstructed image is then put into the cost function (3) and we use gradient descent on J using the gradient from (5). The field map is updated as follows,

$$\omega_o^{new} = \omega_o^{old} - \alpha\frac{\partial}{\partial\omega_o}J,$$

with α chosen small enough to ensure convergence.

4 Results

The gradient-descent field map estimation technique was applied to a simulation study acquiring only one interleave of a two-interleave spiral sequence at each of two different echo times. The number of k-space points for both spirals was 4746 for an image matrix size of 64. A large mask was used to eliminate some aliasing artifact. Reconstruction was performed using Matlab (The MathWorks, Inc.) on a 700 MHz Pentium III workstation running Red Hat Linux, (Red Hat), taking approximately 80 seconds per iteration, which included three steps of the conjugate gradient method on updating the image, followed by three steps of gradient descent on the field map. The reconstruction was started with an initial guess of zeros for the image and the field map. The results after 1500 iterations and the normalized root mean squared error (NRMSE) over the iterations are given in Figure 1. The simulated object was a uniform rectangle with a small rectangular region of inhomogeneity of 200 rad/s (~32 Hz). The uncorrected image in Figure 1 shows the blurring that occurs near the edges of the off-resonance region. As can be seen in the reconstructed image after 1500 iterations, most of this blurring has been corrected by the proposed reconstruction method. The estimated field map closely matches the simulated field map and includes the jump discontinuity despite the roughness penalty. We hope to be able to reduce the number of iterations required for convergence by tuning α and β properly and by improving our initial guess. Further reductions in computation time may be realized by using the NUFFT as in [3] and time segmentation as in [2].

The proposed method was also applied to a phantom study acquiring only one interleave of a two-interleaved spiral sequence at each of two different echo times on a GE Signa 3.0T scanner with a TE/TR of 20/2000 ms, FOV of 20 cm, and a matrix size of 64, yielding 4744 k-space points for both spirals. An initial guess of zeros was used for the image and field map, and 1000 iterations were run of the same reconstruction scheme as the simulations. Figure 2 shows the field map from the standard method using both interleaves, the field map from the proposed method using both interleaves at each time point (initial estimate was the standard method) and the field map from the proposed method using only 1 interleave per time point (initial estimate of zeros). The iterative method with the full data set, the second image in Figure 2, should give the best estimate of the true field map as it exists in undistorted image space and takes into account the time evolution of the inhomogeneities during the readout. The iterative method with half the data set, the third image in the figure, gives a field map close to that of the full data set, although further tuning of the parameters α and β should give better results.

5 Conclusion

A new field map estimation technique was introduced that allows the estimation of a running field map in cases where k-space has been alternatively subsampled with alternating echo times between time points. Unlike the conventional estimation technique, this method accounts for the time course of the effect of inhomogeneities and is not limited by 2π wrap-around effects.

References

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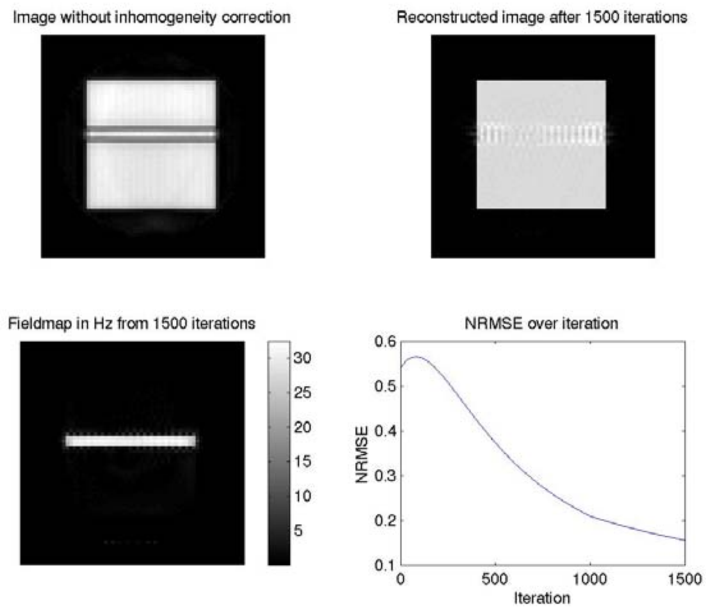

Figure 1: Results from Simulation. Estimated undistorted image and field map are shown from the proposed iterative method for iteration 1500. NRMSE is shown over iterations. The distorted image is shown for reference.

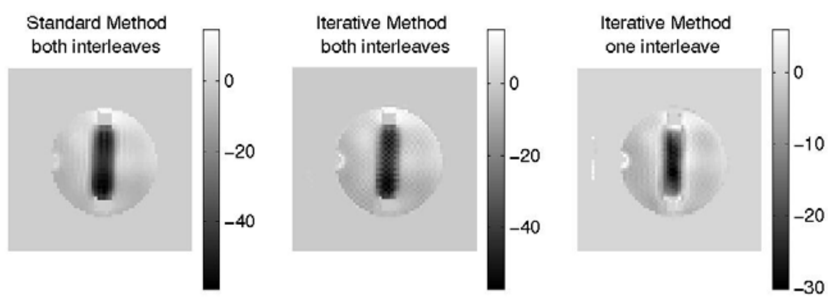

Figure 2: Preliminary results from phantom study, field maps for standard and proposed iterative methods. Iterative method is shown using full and reduced data sets while the standard method is shown with a full data set.