Digital Breast Tomosynthesis Reconstruction with Detector Blur and Correlated Noise

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1. Background

2. Reconstruction method
   - Formulating the reconstruction problem
   - Solving the reconstruction problem

3. Phantom and patient results
   - DBT system
   - Experimental phantom
   - Patient case

4. Conclusion and future work
DBT has been developed to deal with overlapping tissue in mammogram.

Goal: improve DBT reconstruction by modeling detector blur and correlated noise.

A first step towards systematic MBIR for DBT.

Hope to improve image quality for both subtle microcalcifications and mass spiculations.
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Measurement model combines detector blur and Lambert-Beer law:

\[ \bar{Y}_i = I_0 B_i e^{-A_i f} \]

- \( \bar{Y}_i \): expected projection view for the \( i \)th view angle.
- \( f \): unknown 3D attenuation image.
- \( A_i \): forward projector for \( i \)th view angle.
- \( B_i \): the blurring operation:
  - Allowed to be projection view angle dependent.
  - Assumed linear shift-invariant within each projection.
  - Determined from published system MTF [1]
- \( I_0 \): expected projection value in absence of imaged object
  (Can be a constant or a diagonal matrix for nonuniform flux.)
- monoenergetic approximation
- \( e^{-x} \) for vector \( x \) denotes element-wise exponentiation
We prefer the reconstruction problem to have a quadratic data-fit term.
The non-diagonal blur matrix $B_i$ before the exponential is complicating.
In DBT, we assume that the image $f$ is composed of two parts:

$$f = f_{\text{background}} + f_{\text{signal}}$$

- low-frequency background $f_{\text{background}}$ is approximately uniform within the support of the blurring kernel:
  $$B_i A_i f_{\text{background}} \approx A_i f_{\text{background}}$$
- small structures $f_{\text{signal}}$ (such as MC) contribute very little to the projection value:
  $$A_i f_{\text{signal}} \ll 1$$

Combining yields the simpler approximation (for DBT, not CT):

$$\bar{Y}_i = I_0 B_i e^{-A_i f} \approx I_0 e^{-B_i A f}.$$  

(cf. exponential edge-gradient effect [2])

Thus the expected log-transformed projection is approximately linear:

$$y_i \triangleq \log(I_0/Y_i) \implies \bar{y}_i \approx B_i A_i f.$$
Correlated noise: The Covariance Matrix

Cost function needs the measurement covariance matrix.

Physics of CsI phosphor / a:Si Active Matrix Flat Panel Detector:

\[ \text{X-ray photons} \Rightarrow \text{Visible light photons} \Rightarrow \text{Electronic signal (measured)} \]

Quantum noise depends on detector blur but electronic noise does not.

Covariance matrix for the \( i \)th projection view is non-diagonal due to blur:

\[
K_i = B_i K^q_i B_i' + K^r_i
\]

- \( K^q_i \): diagonal covariance of quantum noise
- \( K^r_i \): diagonal covariance of readout noise

(See related CT work of Tilley, Siewerdsen, Stayman [3], [4], [5], [6], [7].)
Assuming $y_i$ has approximately a Gaussian distribution: $y_i \sim \mathcal{N}(\bar{y}_i, K_i)$ leads to a regularized reconstruction problem with non-diagonal weighting:

$$
\hat{f} = \arg \min_f \frac{1}{2} \sum_{i=1}^{m} \|y_i - B_i A_i f\|_2^2 K_i^{-1} + R(f)
$$

$$
= \arg \min_f \frac{1}{2} \sum_{i=1}^{m} \|S_i (y_i - B_i A_i f)\|_2^2 + R(f)
$$

- Regularizer: $R(f) = \beta \sum_k \psi([Cf]_k)$
  - $Cf$ computes 2D (in-plane) finite differences
  - edge-preserving hyperbola potential: $\psi(z) = \delta^2 (\sqrt{1 + (z/\delta)^2} - 1)$

- Inverse matrix square root of noise covariance:
  $$
  S_i \triangleq K_i^{-1/2} = (B_i K_i^q B_i' + K_i^r)^{-1/2}.
  $$

This non-diagonal term is the computational challenge.
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Implementing $S_i$ efficiently in DBT

- Noise covariance $K_i = B_i K_i^q B_i' + K_i^r$ is non-diagonal.
- Implementing $S_i = K_i^{-1/2} S_i$ is challenging in general, particularly in body CT where bones etc. cause very nonuniform noise.
- In DBT, compressed breasts have fairly uniform thickness, mainly composed of soft tissue.
- Key idea: we approximate quantum noise as a constant for all detector elements for each projection view:

$$K_i^q = \sigma_{i,q}^2 I.$$ 

- We also assume all detector elements have similar readout noise variance for each projection view:

$$K_i^r = \sigma_{i,r}^2 I.$$ 

- Thus the non-diagonal noise covariance matrix simplifies:

$$K_i \approx \sigma_{i,q}^2 B_i B_i' + \sigma_{i,r}^2 I.$$
Implementing $S_i$ efficiently for DBT (cont.)

- Now we can simplify inverting the noise covariance matrix:

$$K_i \approx \sigma_{i,q}^2 B_i B_i' + \sigma_{i,r}^2 I.$$

- For periodic boundary conditions, the blur matrix is circulant and diagonalizable by a DFT:

$$B_i = Q^{-1} H_i Q.$$  

  - $Q$ is the 2D discrete Fourier Transform (DFT) matrix.
  - $H_i$ is the blur frequency response for the $i$th view.

- The square-root inverse of the noise covariance simplifies:

$$S_i = K_i^{-1/2} = Q^{-1} (\sigma_{i,q}^2 H_i H_i' + \sigma_{i,r}^2 I)^{-1/2} Q.$$  

- Multiplying $S_i$ by a vector is a simple (high-pass) filter using FFTs.

- No iterative method for matrix inversion required for DBT!

  - $\sigma_{i,q}^2$ estimated using a Lucite slab of appropriate thickness.
  - $\sigma_{i,r}^2$ estimated from the dark current image of the detector.
Overall cost function

- Overall cost function for DBT image reconstruction:

\[
\hat{f} = \arg \min_f \Psi(f), \quad \Psi(f) \triangleq \frac{1}{2} \sum_{i=1}^{m} \| S_i (y_i - B_i A_i f) \|^2 + R(f) \\
= \frac{1}{2} \| \tilde{y} - \tilde{A} f \|^2 + R(f)
\]

- Prewhitened projection data via FFT-based filtering: \( \tilde{y} \triangleq \begin{bmatrix} S_1 y_1 \\ \vdots \\ S_m y_m \end{bmatrix} \).

- Prewhitened system matrix (for analysis): \( \tilde{A} \triangleq \begin{bmatrix} S_1 B_1 A_1 \\ \vdots \\ S_m B_m A_m \end{bmatrix} \).

- Hessian matrix for data-fit term: \( \tilde{A}' \tilde{A} = \sum_{i=1}^{m} A_i' B_i' S_i' S_i B_i A_i \).
Algorithm: Separable Quadratic Surrogates (SQS)

- Cost function: \( \Psi(f) = \frac{1}{2}\|\tilde{y} - \tilde{A}f\|^2_2 + R(f) \)
- Both the quadratic data-fit term and the regularizer are convex.
- To apply SQS we need upper bounds on their Hessians [8].
- As usual for SQS: \( \nabla^2 R(f) \preceq \beta C'C \preceq\beta \text{diag}\{|C|'|C| \cdot 1\} = 8\beta I \).
- Need to find diagonal majorizing matrix \( D \) such that \( \tilde{A}'\tilde{A} \preceq D \).
- Then modified SQS algorithm for minimizing DBT cost function is:
  \[
  f^{(n+1)} = f^{(n)} - [D + 8\beta I]^{-1}\nabla\Psi(f^{(n)}).
  \]
- We use ordered subsets (OS), with one view at a time (ala SART), to accelerate early convergence.
- We call this the SQS-DBCN method, where DBCN stands for Detector Blur and Correlated Noise.
The usual choice would be $D = \text{diag}\left\{|\tilde{A}'|\tilde{A}| \right\}$.

Implementing this would be difficult due to negative values in $S_i$.

Instead, note that because $H_i H_i' \preceq I$:

\[
B_i' S_i' S_i B_i = Q^{-1} H_i' (\sigma_{i,q}^2 H_i H_i' + \sigma_{i,r}^2 I)^{-1} H_i Q \preceq Q^{-1} ((\sigma_{i,q}^2 + \sigma_{i,r}^2)^{-1} I) Q = (\sigma_{i,q}^2 + \sigma_{i,r}^2)^{-1} I.
\]

That inequality leads to the following diagonal majorizer:

\[
\tilde{A}' \tilde{A} = \sum_{i=1}^{m} A_i' B_i' S_i' S_i B_i A_i \preceq \sum_{i=1}^{m} (\sigma_{i,q}^2 + \sigma_{i,r}^2)^{-1} A_i' A_i \preceq D
\]

\[
D \triangleq \sum_{i=1}^{m} (\sigma_{i,q}^2 + \sigma_{i,r}^2)^{-1} \text{diag}\left\{ A_i' A_i 1 \right\}.
\]

This diagonal majorizer is as easy to implement as usual SQS case.
Geometry of the DBT System

- GE GEN2 prototype DBT system
- 21 projections within $\pm 30^\circ$ sequentially with $3^\circ$ increment.
- Used central 9 views over $\pm 12^\circ$
- Detector resolution = $1920 \times 2304$
- Detector pixel size = 0.1mm
- Voxels: $dx = dy = 0.1mm$, $dz = 1mm$.
- Initialized with uniform image: $f^{(0)} = 0.05/mm$
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Experimental phantom

- Stack of five 1-cm-thick 50% adipose/50% glandular heterogeneous slabs that mimic the composition and parenchymal pattern of the breast.
- Clusters of calcium carbonate specks of three nominal size ranges (0.25-0.30mm, 0.18-0.25mm, and 0.15-0.18mm), sandwiched at random locations between the slabs to simulate MCs of different conspicuities.
- SQS-DBCN reconstructions compared with SART (3 iterations) [9], [10].
Reconstructed microcalcification (MC) comparison

<table>
<thead>
<tr>
<th>Reconstructed microcalcification (MC) Comparison</th>
<th>0.15-0.18mm MC</th>
<th>0.18-0.25mm MC</th>
<th>0.25-0.30mm MC</th>
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</thead>
<tbody>
<tr>
<td>SART (3 iterations)</td>
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<tr>
<td>(a) CNR = 3.9</td>
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<tr>
<td>FWHM = 0.21 mm</td>
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<td>(b) CNR = 5.6</td>
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<td>FWHM = 0.24 mm</td>
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<tr>
<td>(c) CNR = 9.1</td>
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<tr>
<td>FWHM = 0.29 mm</td>
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<tr>
<td>SQS-DBCN</td>
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<tr>
<td>$\beta = 80$</td>
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<tr>
<td>$\delta = 0.002$ mm</td>
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<tr>
<td>(d) CNR = 6.9</td>
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<tr>
<td>FWHM = 0.17 mm</td>
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<tr>
<td>(e) CNR = 8.4</td>
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<tr>
<td>FWHM = 0.22 mm</td>
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<tr>
<td>(f) CNR = 20.6</td>
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<tr>
<td>FWHM = 0.25 mm</td>
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</table>
Comparison of CNR and FWHM

Results averaged over 5 clusters of each size in the phantom:
- 49 of 0.15-0.18mm MCs
- 66 of 0.18-0.25mm MCs
- 64 of 0.25-0.30mm MCs

SQS-DBCN generally enhanced CNR and decreased FWHM, *i.e.*, the MCs appear sharper.
Regularization parameter selection

- Average CNR vs. $\beta$, $\delta$
- Small and medium MC sizes
- Red lines indicate SART
- For different MC sizes, CNR-optimal $\beta$, $\delta$ varies
- Reducing $\delta$ improves max CNR
- Proposed SQS-DBCN outperforms SART over a large range of parameters.
MC CNR increases from (a) to (c). However, spiculations and tissue textures become more patchy and artificial in (c). Need better figures of merit.
Conclusion

- Proposed DBT reconstruction method incorporates detector blur and a correlated noise model.
  A step towards developing model-based iterative reconstruction for DBT.
- Computationally efficient algorithm adds just one 2D FFT pair per view per iteration.
  - Update per view $\approx 2.7$ seconds with 8 threads and modified SF [11]
  - One 2D FFT pair $\approx 0.03$ seconds $\Rightarrow 1\%$ overhead
  - SQS-DBCN for 9 views and 10 iterations $\approx 5 \text{ min}$
- Both quantitatively and visually the new SQS-DBCN method can better enhance MCs compared with the SART while preserving the image quality of spiculations and tissue texture, if parameters are chosen well.
- SQS-DBCN method relies on good parameter selection and accurate estimation of noise variance.
- Future work: develop an adaptive parameter selection method, improve estimation of noise variances, generalize model to relax the assumptions.
Bibliography


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Backup: derivation

1. \( f = f_{\text{background}} + f_{\text{signal}} \)

Assume:

2. smooth background: \( BA f_{\text{background}} \approx A f_{\text{background}} \)

3. low-contrast details: \( A f_{\text{signal}} \ll 1 \)

4. unity DC response of blur: \( B1 = 1 \)

\[
B e^{-Af} = B e^{-A(f_{\text{signal}} + f_{\text{background}})}
\]

\[
= B e^{-Af_{\text{signal}}} e^{-Af_{\text{background}}}
\]

\[
\approx B (1 - Af_{\text{signal}}) e^{-BAf_{\text{background}}}
\]

\[
= (1 - BAf_{\text{signal}}) e^{-BAf_{\text{background}}}
\]

\[
\approx e^{-BAf_{\text{signal}}} e^{-BAf_{\text{background}}} = e^{-BA(f_{\text{signal}} + f_{\text{background}})} = e^{-BAf}
\]

For a 0.2 mm MC with \( \mu = 1.5 / \text{mm} \), \( A f_{\text{signal}} = 0.3 \ll 1 \)
Noise standard deviation from lucite slab

![Graph showing standard deviation vs. angle](chart.png)

- Standard deviation [AU]: \( \sigma_q \)
- Standard deviation [AU]: \( \sigma_r \)

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