Statistical Reconstruction in CT

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...
Why we are here

A picture is worth 1000 words
(and perhaps several 1000 seconds of computation?)

Thin-slice FBP
Seconds

ASIR
A bit longer

Statistical
Much longer*

(* ask the vendors)
Why statistical methods for CT?

• Accurate physical models
  ○ X-ray spectrum, beam-hardening, scatter, ... reduced artifacts? quantitative CT?
  ○ X-ray detector spatial response, focal spot size, ... improved spatial resolution?
  ○ detector spectral response (e.g., photon-counting detectors)

• Nonstandard geometries
  ○ transaxial truncation (big patients)
  ○ long-object problem in helical CT
  ○ irregular sampling in “next-generation” geometries
  ○ coarse angular sampling in image-guidance applications
  ○ limited angular range (tomosynthesis)
  ○ “missing” data, e.g., bad pixels in flat-panel systems

• Appropriate statistical models
  ○ weighting reduces influence of photon-starved rays (FBP treats all rays equally)
  ○ reducing image noise or dose
and more...

- Object constraints
  - nonnegativity
  - object support
  - piecewise smoothness
  - object sparsity (e.g., angiography)
  - sparsity in some basis
  - motion models
  - dynamic models
  - ...

Disadvantages?
- Computation time (Dr. Thrall says super computer)
- Must reconstruct entire FOV
- Model complexity
- Software complexity
- Algorithm nonlinearities
  - Difficult to analyze resolution/noise properties (cf. FBP)
  - Tuning parameters
  - Challenging to characterize performance
“Iterative” vs “Statistical”

- **Traditional** successive substitutions iterations
  - e.g., Joseph and Spital (JCAT, 1978) bone correction
  - usually only one or two “iterations”
  - not statistical

- **Algebraic** reconstruction methods
  - Given sinogram data $y$ and system model $A$, reconstruct object $x$ by “solving” $y = Ax$
  - ART, SIRT, SART, ...
  - iterative, but typically not statistical
  - Iterative filtered back-projection (FBP):
    \[
    x^{(n+1)} = x^{(n)} + \alpha \text{FBP}(y - Ax^{(n)})
    \]

- **Statistical** reconstruction methods
  - Image domain
  - Sinogram domain
  - Fully statistical (both)
  - Hybrids (e.g., AIR, 7961-18, Bruder et al.)
“Statistical” methods: Image domain

- Denoising methods

\[
\text{sinogram } y \rightarrow \text{FBP} \rightarrow \text{noisy reconstruction } \tilde{x} \rightarrow \text{iterative denoiser} \rightarrow \text{final image } \hat{x}
\]

- Remarkable advances in denoising methods in last decade
  
  Zhu & Milanfar, T-IP, Dec. 2010, using “steering kernel regression” (SKR) method
  
  - Typically assume white noise
  - Streaks in low-dose FBP appear like edges (highly correlated noise)
Denoising methods “guided by data statistics”

- Image-domain methods are fast (thus practical)
- The technical details are a mystery...
- ASIR? IRIS? ...

Diagram:

sinogram $y \rightarrow \text{FBP} \rightarrow$ noisy reconstruction $\tilde{x} \rightarrow$ magical iterative denoiser $\uparrow$ sinogram statistics? $\rightarrow$ final image $\hat{x}$
“Statistical” methods: Sinogram domain

- Sinogram restoration methods

  \[ \text{noisy sinogram} \xrightarrow{\text{adaptive or iterative denoiser}} \text{cleaned sinogram} \xrightarrow{\text{FBP}} \text{final image} \]

  - fast, but limited denoising without resolution loss (local, no edges)

FBP, 10 mA

Wang et al., T-MI, Oct. 2006, using PWLS-GS on sinogram

FBP from denoised sinogram
(True? Fully? Slow?) Statistical reconstruction

- Object model
- Physics/system model
- Statistical model
- Cost function (log-likelihood + regularization)
- Iterative algorithm for minimization

“Find the image \( \hat{x} \) that best fits the sinogram data \( y \) according to the physics model, the statistical model and prior information about the object”

- Repeatedly revisiting the sinogram data can use statistics fully
- Repeatedly updating the image can exploit object properties
- But repetition is expensive...
History: Statistical reconstruction for PET

- Iterative method for emission tomography (Kuhl, 1963)
- Weighted least squares for 3D SPECT (Goitein, NIM, 1972)
- Richardson/Lucy iteration for image restoration (1972, 1974)
- Poisson likelihood (emission) (Rockmore and Macovski, TNS, 1976)
- Expectation-maximization (EM) algorithm (Shepp and Vardi, TMI, 1982)
- Regularized (aka Bayesian) Poisson emission reconstruction (Geman and McClure, ASA, 1985)
- Ordered-subsets EM algorithm (Hudson and Larkin, TMI, 1994)
- Commercial introduction of OSEM for PET scanners circa 1997

Today, most commercial PET systems include unregularized OSEM.

15 years between key EM paper (1982) and commercial adoption (1997) (25 years if you count the R/L paper in 1972 which is the same as EM)
History: Statistical reconstruction for CT

- Iterative method for X-ray CT (Hounsfield, 1968)
- ART for tomography (Gordon, Bender, Herman, JTB, 1970)
- ...
- Roughness regularized LS for tomography (Kashyap & Mittal, 1975)
- Poisson likelihood (transmission) (Rockmore and Macovski, TNS, 1977)
- EM algorithm for Poisson transmission (Lange and Carson, JCAT, 1984)
- Iterative coordinate descent (ICD) (Sauer and Bouman, T-SP, 1993)
- Ordered-subsets algorithms (Manglos et al., PMB 1995)
  (Kamphuis & Beekman, T-MI, 1998)
  (Erdoğan & Fessler, PMB, 1999)
- ...
- Commercial introduction for CT scanners circa 2010

(* numerous omissions)
Zhou Yu, Jean-Baptiste Thibault, Charles Bouman, Jiang Hsieh, Ken Sauer

https://engineering.purdue.edu/BME/AboutUs/News/HomepageFeatures/ResultsOfPurdueResearchUnveiledAtRSNA
Five Choices for Statistical Reconstruction

1. Object model
2. System physical model
3. Measurement statistical model
4. Cost function: data-mismatch and regularization
5. Algorithm / initialization

No perfect choices - one can critique all approaches!

Historically these choices are often left implicit in publications, but being explicit facilitates reproducibility
Choice 1. Object Parameterization

Finite measurements: \( \{y_i\}_{i=1}^M \). Continuous object: \( f(\vec{r}) = \mu(\vec{r}) \).

“All models are wrong but some models are useful.”

Linear *series expansion* approach. Represent \( f(\vec{r}) \) by \( x = (x_1, \ldots, x_N) \) where

\[
f(\vec{r}) \approx \tilde{f}(\vec{r}) = \sum_{j=1}^{N} x_j b_j(\vec{r}) \leftarrow \text{“basis functions”}
\]

Reconstruction problem becomes “discrete-discrete:” estimate \( x \) from \( y \).

Numerous basis functions in literature. Two primary contenders:
- voxels
- blobs (Kaiser-Bessel functions)
  - Blobs are approximately band-limited (reduced aliasing?)
  - Blobs have larger footprints, increasing computation.

Open question: how small should the voxels be?

Choice 2. System model / Physics model

- scan geometry
- source intensity $I_0$
  - spatial variations (air scan)
  - intensity fluctuations
- resolution effects
  - finite detector size / detector spatial response
  - finite X-ray spot size / anode angulation Inhomogeneous
  - detector afterglow
- spectral effects
  - X-ray source spectrum
  - bowtie filters
  - detector spectra response
- scatter
- ...

Trade-off
- computation time
- accuracy/artifacts/resolution/contrast
Exponential edge-gradient effect

Fundamental difference between emission tomography and CT:

Recorded intensity for $i$th ray:

$$I_i = \int_{\text{source}} \int_{\text{detector}} I_0(\vec{p}_s, \vec{p}_d) \exp \left( - \int_{\mathcal{L}(\vec{p}_s, \vec{p}_d)} \mu(\vec{r}) \, d\ell \right) \, d\vec{p}_d \, d\vec{p}_s$$

$$\neq I_0 \exp \left( - \int_{\text{source}} \int_{\text{detector}} \int_{\mathcal{L}(\vec{p}_s, \vec{p}_d)} \mu(\vec{r}) \, d\ell \, d\vec{p}_d \, d\vec{p}_s \right).$$

Usual “linear” approximation:

$$I_i \approx I_0 \exp \left( - \sum_{j=1}^{N} a_{ij} x_j \right), \quad a_{ij} \triangleq \int_{\text{source}} \int_{\text{detector}} \int_{\mathcal{L}(\vec{p}_s, \vec{p}_d)} b_{ij}(\vec{r}) \, d\ell \, d\vec{p}_d \, d\vec{p}_s$$

elements of system matrix $\mathbf{A}$
“Line Length” System Model

Assumes (implicitly?) that source is a point and detector is a point.

\[ x_1 \quad x_2 \]

\[ a_{ij} \triangleq \text{length of intersection} \]
“Strip Area” System Model

Account for finite detector width. Ignores nonlinear partial-volume averaging.

Practical (?) implementations in 3D include

- Distance-driven method (De Man and Basu, PMB, Jun. 2004)
- Separable-footprint method (Long et al., T-MI, Nov. 2010)
- Further comparisons needed...
Lines versus strips

From (De Man and Basu, PMB, Jun. 2004)

MLTR of rabbit heart

Ray-driven

Distance-driven
Forward- / Back-projector “Pairs”

Typically iterative algorithms require two key steps.

- **forward projection** (image domain to projection domain):
  \[
  \bar{y} = Ax, \quad \bar{y}_i = \sum_{j=1}^{N} a_{ij} x_j = [Ax]_i
  \]

- **backprojection** (projection domain to image domain):
  \[
  z = A'y, \quad z_j = \sum_{i=1}^{M} a_{ij} y_i
  \]

The term “forward/backprojection pair” often refers to some implicit choices for the object basis and the system model.

Sometimes \( A'y \) is implemented as \( By \) for some “backprojector” \( B \neq A' \). Especially in SPECT and sometimes in PET.

Least-squares solutions (for example):

\[
\hat{x} = \arg \min_x \|y - Ax\|^2 = [A'A]^{-1} A'y \neq [BA]^{-1} By
\]
Mismatched Backprojector $B \neq A'$

$\hat{x}$ \hspace{1cm} $\hat{x}(PWLS - CG)$ \hspace{1cm} $\hat{x}(PWLS - CG)$

Matched \hspace{5cm} Mismatched

*cf.* SPECT/PET reconstruction – usually unregularized
Acceleration

- Projector/backprojector algorithm
  - Approximations (e.g., transaxial/axial separability)
  - Symmetry

- Hardware / software
  - GPU, CUDA, OpenCL, FPGA, SIMD, pthread, OpenMP, MPI, ...

- ...
Choice 3. Statistical Model

The physical model describes measurement mean, e.g., for a monoenergetic X-ray source and ignoring scatter etc.:

\[ \bar{I}_i([Ax]_i) = I_0 e^{-\sum_{j=1}^{N} a_{ij}x_j}. \]

The raw noisy measurements \( \{I_i\} \) are distributed around those means. Statistical reconstruction methods require a model for that distribution.

Trade offs: using more accurate statistical models

- may lead to less noisy images
- may incur additional computation
- may involve higher algorithm complexity.

CT measurement statistics are very complicated (cf. PET)

- incident photon flux variations (Poisson)
- X-ray photon absorption/scattering (Bernoulli)
- energy-dependent light production in scintillator (?)
- shot noise in photodiodes (Poisson?)
- electronic noise in readout electronics (Gaussian?)

Whiting, SPIE 4682, 2002; Lasio et al., PMB, Apr. 2007
To log() or not to log() – That is the question

Models for “raw” data $I_i$ (before logarithm)

- **compound Poisson** (complicated)  
  Whiting, SPIE 4682, 2002; Elbakri & Fessler, SPIE 5032, 2003; Lasio *et al.*, PMB, Apr. 2007

- **Poisson + Gaussian** (photon variability and electronic readout noise):  
  $$I_i \sim \text{Poisson}\{\bar{I}_i\} + \mathcal{N}(0, \sigma^2)$$  

- **Shifted Poisson** approximation (matches first two moments):  
  $$\bar{I}_i \triangleq [I_i + \sigma^2]_+ \sim \text{Poisson}\{\bar{I}_i + \sigma^2\}$$  
  Yavuz & Fessler, MIA, Dec. 1998

- **Ordinary Poisson** (ignore electronic noise):  
  $$I_i \sim \text{Poisson}\{\bar{I}_i\}$$  
  Rockmore and Macovski, TNS, Jun. 1977; Lange and Carson, JCAT, Apr. 1984

All are somewhat complicated by the nonlinearity of the physics:  
$$\bar{I}_i = e^{-[Ax]_i}$$
After taking the log()

Taking the log leads to a linear model (ignoring beam hardening):

\[ y_i \triangleq -\log \left( \frac{I_i}{I_0} \right) \approx [Ax]_i + \varepsilon_i \]

Drawbacks:
- Undefined if \( I_i \leq 0 \) (e.g., due to electronic noise)
- It is biased (by Jensen’s inequality):
  \[ \mathbb{E}[y_i] \geq -\log(\bar{I}_i/I_0) = [Ax]_i \]
- Exact distribution of noise \( \varepsilon_i \) intractable

Practical approach: assume Gaussian noise model: \( \varepsilon_i \sim \mathcal{N}(0, \sigma_i^2) \)

Options for modeling noise variance \( \sigma_i^2 = \text{Var}\{\varepsilon_i\} \)
- consider both Poisson and Gaussian noise effects: \( \sigma_i^2 = \frac{I_i+\sigma_0^2}{I_i} \) (Thibault et al., SPIE 6065, 2006)
- consider just Poisson effect: \( \sigma_i^2 = \frac{1}{I_i} \) (Sauer & Bouman, T-SP, Feb. 1993)
- pretend it is white noise: \( \sigma_i^2 = \sigma_0^2 \)
- ignore noise altogether and “solve” \( y = Ax \)

Whether using pre-log data is better than post-log data is an open question.
Choice 4. Cost Functions

Components:
- \textit{Data-mismatch} term
- \textit{Regularization} term (and regularization parameter $\beta$)
- Constraints (\textit{e.g.}, nonnegativity)

Reconstruct image $\hat{x}$ by minimizing a cost function:

$$\hat{x} \triangleq \arg \min_{x \geq 0} \Psi(x)$$

$$\Psi(x) = \text{DataMismatch}(y, Ax) + \beta \text{Regularizer}(x)$$

Forcing too much “data fit” alone would give noisy images.

Equivalent to a Bayesian MAP (maximum \textit{a posteriori}) estimator.

Distinguishes “statistical methods” from “algebraic methods” for “$y = Ax$.”
Choice 4.1: Data-Mismatch Term

Standard choice is the negative log-likelihood of statistical model:

$$\text{DataMismatch} = -L(x; y) = -\log p(y|x) = \sum_{i=1}^{M} -\log p(y_i|x).$$

- For pre-log data $I$ with shifted Poisson model:

$$-L(x; I) = \sum_{i=1}^{M} (\bar{I}_i + \sigma^2) - [I_i + \sigma^2] + \log(\bar{I}_i + \sigma^2), \quad \bar{I}_i = I_0 e^{-[Ax]_i}$$

  This can be non-convex if $\sigma^2 > 0$; it is convex if we ignore electronic noise $\sigma^2 = 0$. Trade-off ...

- For post-log data $y$ with Gaussian model:

$$-L(x; y) = \sum_{i=1}^{M} w_i \frac{1}{2} (y_i - [Ax]_i)^2 = \frac{1}{2} (y - Ax)'W(y - Ax), \quad w_i = 1/\sigma_i^2$$

  This is a kind of (data-based) weighted least squares (WLS). It is always convex in $x$. Quadratic functions are “easy” to minimize.

- ...
Choice 4.2: Regularization

How to control noise due to ill-conditioning?

**Noise-control methods in clinical use in PET reconstruction today:**
- Stop an unregularized algorithm before convergence
- Over-iterate an unregularized algorithm then post-filter

**Other possible “simple” solutions:**
- Modify the raw data (pre-filter / denoise)
- Filter between iterations
- ...

**Appeal:**
- simple / familiar
- filter parameters have intuitive units (e.g., FWHM), unlike a regularization parameter $\beta$
- Changing a post-filter does not require re-iterating, unlike changing a regularization parameter $\beta$

Dozens of papers on regularized methods for PET, but little clinical impact. (USC MAP method is available in mouse scanners.)
Edge-Preserving Reconstruction: PET Example

Phantom
Quadratic Penalty
Huber Penalty

Quantification vs qualitative vs tasks...
More “Edge Preserving” PET Regularization

Chlewicki et al., PMB, Oct. 2004; “Noise reduction and convergence of Bayesian algorithms with blobs based on the Huber function and median root prior”
Regularization in PET

Nuyts et al., T-MI, Jan. 2009: MAP method outperformed post-filtered ML for lesion detection in simulation

Noiseless images:

<table>
<thead>
<tr>
<th>Phantom</th>
<th>ML-EM filtered</th>
<th>Regularized</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Regularization options

Options for $R(x)$

In increasing complexity:

- quadratic roughness
- convex, non-quadratic roughness
- non-convex roughness
- total variation
- convex sparsity
- non-convex sparsity

Goal: reduce noise without degrading spatial resolution

Many open questions...
Roughness Penalty Functions

\[ R(x) = \sum_{j=1}^{N} \frac{1}{2} \sum_{k \in \mathcal{N}_j} \psi(x_j - x_k) \]

\( \mathcal{N}_j \triangleq \text{neighborhood} \) of \( j \)th pixel (e.g., left, right, up, down)
\( \psi \) called the potential function

### Quadratic vs Non-quadratic Potential Functions

- **Parabola (quadratic):** \( \psi(t) = t^2 \)
- **Hyperbola:** \( \psi(t) = \sqrt{1 + (t/\delta)^2} \)
  (edge preservation)
**Regularization parameters: Dramatic effects**


“*q generalized gaussian*” potential function with tuning parameters: $\beta, \delta, p, q$:

$$\beta \psi(t) = \beta \frac{\frac{1}{2} |t|^p}{1 + \frac{|t/\delta|^{p-q}}{p-q}}$$

<table>
<thead>
<tr>
<th>$p = q$</th>
<th>$p = 2, q = 1.2, \delta = 10$ HU</th>
<th>$p = q = 1.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>11.1 10.9 10.8</td>
<td>10.8 8.2</td>
</tr>
<tr>
<td>4.2</td>
<td>10.9 7.2</td>
<td></td>
</tr>
</tbody>
</table>

Noise: 11.1 10.9 10.8

(\#lp/cm): 4.2 7.2
Summary thus far

1. Object parameterization
2. System physical model
3. Measurement statistical model
4. Cost function: data-mismatch / regularization / constraints

Reconstruction Method \( \triangleq \) Models + Cost Function + Algorithm

5. Minimization algorithms:

\[
\hat{x} = \arg\min_x \Psi(x)
\]
Choice 5: Minimization algorithms

- **Conjugate gradients**
  - Converges slowly for CT
  - Difficult to precondition due to weighting and regularization
  - Difficult to enforce nonnegativity constraint
  - Very easily parallelized

- **Ordered subsets**
  - Initially converges faster than CG if many subsets used
  - Does not converge without relaxation etc., but those slow it down
  - Computes regularizer gradient $\nabla R(x)$ for every subset - expensive?
  - Easily enforces nonnegativity constraint
  - Easily parallelized

- **Coordinate descent** (Sauer and Bouman, T-SP, 1993)
  - Converges high spatial frequencies rapidly, but low frequencies slowly
  - Easily enforces nonnegativity constraint
  - Challenging to parallelize

- **Block coordinate descent** (Benson et al., NSS/MIC, 2010)
  - Spatial frequency convergence properties depend...
  - Easily enforces nonnegativity constraint
  - More opportunity to parallelize than CD
Convergence rates

(De Man et al., NSS/MIC 2005)

In terms of iterations: CD < OS < CG < Convergent OS
In terms of compute time? (it depends...)
Ordered subsets convergence

Theoretically OS does not converge, but it may get “close enough,” even with regularization.

CD  
200 iter

OS  
41 subsets  
200 iter

difference  
0 ± 10HU

display: 930 HU ± 58 HU

(De Man et al., NSS/MIC 2005)

Ongoing saga...

(SPIE, ISBI, Fully 3D, ... )
Example

(movie)

82-subset OS with two different (but similar) edge-preserving regularizers. One frame per every 10th iteration.
Resolution characterization challenge

Shape of edge response depends on contrast for edge-preserving regularizers.
Assessing image quality

- Several talks in Session 4
- Poster 7961-115, Rongping Zeng, Kyle J. Myers
  Task-based comparative study on iterative image reconstruction methods for limited-angle x-ray tomography
- Poster 7961-113, Pascal Theriault Lauzier, Jie Tang, Guang-Hong Chen
  Quantitative evaluation method of noise texture for iteratively reconstructed x-ray CT images
- ...

Very important to be specific about which statistical reconstruction method is being evaluated because results may vary significantly for different choices of models, parameters, stopping rules, ...

- What does “MTF” mean for nonlinear, shift-variant systems?
- “Less dose reduction for larger patients”
- “Resolution degrades as dose decreases”
Some open problems

- **Modeling**
  - Statistical modeling for very low-dose CT
  - Resolution effects
  - Spectral CT
  - Object motion

- **Parameter selection / performance characterization**
  - Performance prediction for nonquadratic regularization
  - Effect of nonquadratic regularization on detection tasks
  - Choice of regularization parameters for nonquadratic regularization

- **Algorithms**
  - Optimization algorithm design
  - Software/hardware implementation
  - Moore's law alone will not suffice
    (dual energy, dual source, motion, dynamic, smaller voxels ...)

- **Clinical evaluation**

- ...