Homework #8, EECS 516, F09. Due **Due Fri. Dec. 11**, by 1:30PM

Tomography

1. [0] The "volume" under a 2D function is $\iint f(x,y)\,\mathrm{d}x\,\mathrm{d}y$.
   (a) [0] Find an expression for that volume in terms of its projections $p_\phi(r)$.
   (b) [0] Show that $p_\phi(r)$ can have the form $h_1(r)h_2(\phi)$ only if $h_2(\phi)$ is a constant.

2. [0] In a CT system, each projection is measured using a uniform scanning beam of width w instead of an infinitesimal pencil beam.
   (a) [0] The data from this system is reconstructed by a conventional FBP algorithm that ignores the finite beam width, yielding an image $\hat{f}(x,y)$. Relate the resulting image $\hat{f}(x,y)$ to the ideal image $f(x,y)$. (Ignore sampling issues and focus on the blur.) Hint: for simplicity, relate the two images in the frequency domain.
   (b) [0] For a small challenge, find the image-domain relationship between $\hat{f}(x,y)$ and $f(x,y)$.
   (c) [0] Describe how you would modify the filter in the FBP method to (mostly) compensate for the finite beam width.
   (d) [0] Are there any undesirable consequences of your modification?

3. [0] (a) [0] Use the Fourier slice theorem to find the Radon transform $p_\phi(r)$ of the 2D function $f(x,y) = \cos(2\pi f_0 x)$.
   (b) [0] Apply (on paper) the FBP reconstruction method to $p_\phi(r)$, finding the resulting signals at each step of the method. Verify that the resulting backprojected image is correct.

4. [0] (a) [0] Describe a class of objects for which a single projection angle is adequate to determine which object in the class was scanned, assuming that the reconstruction method knows the class under consideration, but not the specific member of that class.
   (b) [0] Now describe a class of objects for which two projection angles would be adequate. Try to be as general as you can.

5. [10] Please do the course evaluation online and submit the receipt/acknowledgment that you receive. (By paper or by email.)