Homework #7, EECS 516, F09. Due **Due Fri. Dec. 4**, by 1:30PM

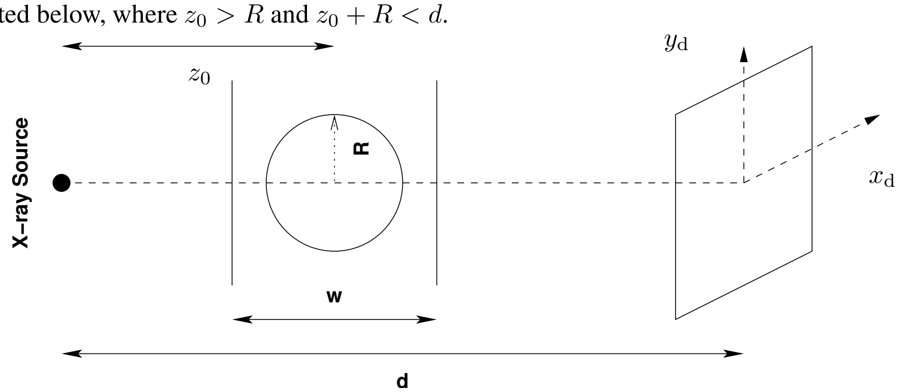
X-ray radiography: physics

1. [15] One region of the body has 10 cm thickness of muscle. Another region has 10 cm of muscle and 2 cm of bone. The densities of the muscle and bone are 1.0 and 1.75 g/cm^3 respectively.
To solve this problem, use the tables of mass attenuation coefficients for cortical bone and skeletal muscle from NIST.
`http://physics.nist.gov/PhysRefData/XrayMassCoef/tab4.html`
(a) [10] Calculate the X-ray transmission through each of the regions at energies of 30 and 100 keV.
(b) [5] Which energy is preferable for bone-muscle contrast?
(c) [0] Which energy is preferable for visualizing variations in muscle thickness in the presence of bone?

2. [20] Consider a cube of depth l have a total of N incident photons on one face.
(a) [10] Determine the total (mean) number of scattered photons if the total linear attenuation coefficient is μ and the Compton coefficient is μ_c. Hint: use the general relationship for the number of interactions in a thin section: $\Delta n = n_{\rm in}\mu\Delta x$. Neglect multiple scattering.
(b) [5] As a concrete example, determine the fraction of the photons that are Compton scattered when transmitting 50 keV photons through 20 cm of water, for which $\mu = 0.2269$ cm^{-1} and $\mu_c = 0.2$ cm^{-1}.
(c) [5] Returning to the general expression in part (a), what fraction of the photons are scattered for a very thick object?
(d) [0] In the previous part, what does "very thick" mean?

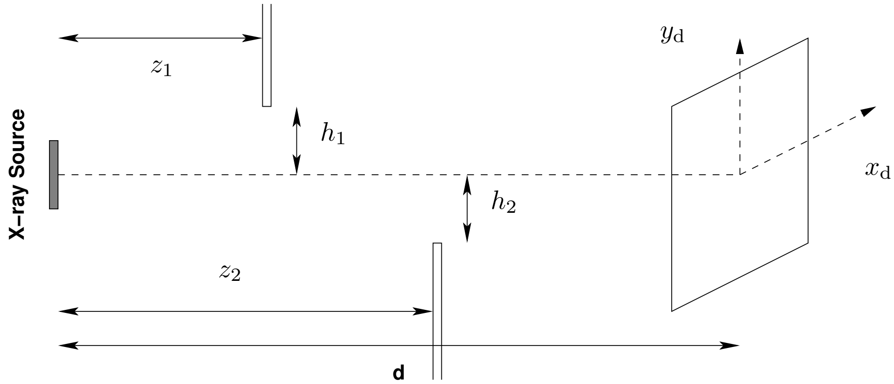
X-ray radiography: source considerations

3. [15] A monoenergetic X-ray point-source illuminates a cylindrical object having uniform attenuation coefficient μ_0 positioned as illustrated below, where $z_0 > R$ and $z_0 + R < d$.



(a) [10] Determine $I_{\rm d}(x, y)$, neglecting the falloff of the source intensity over the detector plane due to obliquity.
(b) [5] Suppose now that the cylinder is embedded a layer of soft tissue of thickness w as illustrated above, and of infinite extent in x and y, with attenuation coefficient μ_t. Repeat part (a) in this case.

4. [10] A uniform rectangular X-ray source $s(x, y) = \mathrm{rect}(x/X)\,\mathrm{rect}(y/Y)$ illuminates two opaque, semi-infinite thin planes as shown below. Ignoring all obliquity factors, sketch the intensity $I_{\rm d}(0, y_{\rm d})$ vs $y_{\rm d}$ on the detector plane, labeling all important features. Assume $h_1 > Y/2$ and $h_2 > Y/2$.



5. [10] A $L \times L$ square X-ray source, having unity intensity, parallel to and a distance d from the recorder, is used to image a planar transparency (of infinite extent) a distance z from the source and having a transmission $t = \frac{1}{2} + \frac{1}{2}\cos(2\pi a y)$. Ignoring obliquity, determine the intensity at the recorder plane. (Do not leave in convolution form.)

X-ray radiography: recorder considerations

6. [15] Film emulsion is placed on both sides of a phosphor screen of thickness d. The resultant transparencies are combined such as to provide an overall transparency having a small-signal normalized frequency response $\bar{H}_0 = \frac{1}{2}\left(\bar{H}_f + \bar{H}_b\right)$, where $\bar{H}_f$ and $\bar{H}_b$ are the individual normalized responses.
   (a) [10] Find the high-frequency cutoff $\rho_{1/2}$ of the overall transparency using appropriate approximations.
   (b) [5] By what factor does this cutoff frequency differ from the case of a single emulsion on the front side where the X-rays impinge?

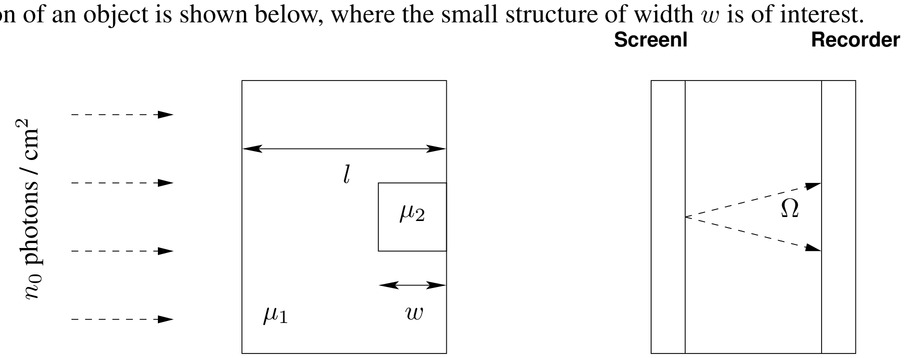
7. [15] An extended-source X-ray system is used with the source parallel to the recorder plane and with distribution $s(r) = \mathrm{e}^{-\pi(r/u)^2}$. The recorder, a distance d from the source, has impulse response $h(r) = \mathrm{e}^{-\pi(r/v)^2}$. Interpret u and v as the "width" of the source and the detector response, respectively.
   (a) [10] Neglecting all obliquity factors, determine the "optimal" distance z_0 from the source to place a thin transparency to be imaged so as to maximize the overall frequency response at a spatial frequency ρ_0 (relative to to the DC response).
   (b) [5] Discuss the optimal z_0 in cases where $v \gg u$ or $u \gg v$.

X-ray radiography: noise and SNR

8. [15] An object has attenuation $\mu(x,y,z) = \mu_0 \left(\mathrm{rect}\left(\frac{z-l/2}{l}\right) + \mathrm{rect}\left(\frac{z+t/2}{t}\right)\mathrm{rect}(x)\,\mathrm{rect}(y)\right)$ where $\mu t \ll 1$. In words, the object is an infinite slab of thickness l with a small "lesion" of thickness t protruding from it.
   (a) [10] Neglecting scatter and assuming ideal detector efficiency ($\eta = 1$), find the SNR for imaging the thicker region containing the lesion. Assume N_0 incident photons.
   (b) [5] Suppose that the material has energy dependence $\mu(\mathcal{E}) = A\,\mathrm{e}^{-B\mathcal{E}}$. Determine the optimal energy by maximizing the SNR.

9. [10] An X-ray transparency $t(x,y) = a + b\cos(2\pi f_0 x)$ at a depth z is imaged using a $L \times L$ square source having an intensity of n photons per unit area at a distance d from the recorder. Find the SNR of the resulting image, where here we define SNR as the ratio of the peak amplitude of the sinusoid over the standard deviation of the average value. Assume that the area of each recorder pixel has neglible effect on the ability to resolve the sinusoid. Interpret "peak amplitude" to mean the difference between the peak value and the average value.

10. [10] The cross section of an object is shown below, where the small structure of width w is of interest.



   (a) [5] Determine the SNR of the X-ray photons emerging from the object, where the size of a resolution element is A cm^2. Assume $|\mu_2 - \mu_1|\,w \ll 1$.
   (b) [5] Determine the SNR of the recording where the phosphor screen has a capture efficiency η and produces L light photons per X-ray photon, and R events per light photon are recorded. Due to critical angle considerations, only emitted light photons over a solid angle Ω are received by the recorder. Assume L and R are Poisson random variables.