Homework #6, EECS 516, F09. Due **Due Fri. Nov. 6**, by 1:30PM

1. [15] An object consists of two impulses having the same x position but separated in the y direction by distance L. The object is images using a 2D FT sequence with phase encode lines at $\left\{\pm\frac{1}{2}\Delta_{k_Y}, \pm\frac{3}{2}\Delta_{k_Y}, \pm\frac{5}{2}\Delta_{k_Y}, \ldots\right\}$.
   (a) [5] At what $\mathrm{FOV_Y}$ value(s) will the image equal zero?
   (b) [5] At what $\mathrm{FOV_Y}$ value(s) will the image be a single impulse (within one FOV)?
   (c) [5] For the previous parts, does is matter whether the to impulses are centered about the x axis?

2. [20] Suppose that an object's magnetization is a single spin at (0,α): $m(x, y) = \delta_2(x - \alpha, y)$, where $\alpha \in (0, 1)$. Consider a spin-warp imaging sequence corresponding to a $N \times M$ image with FOV = $N\Delta_X \times M\Delta_Y$, but for simplicity assume that $\Delta_X = \Delta_Y = 1$.
   (a) [10] Determine analytically an expression for the sampled signal and for the inverse DFT reconstructed object for an image grid that is $\{-N/2, \ldots, N/2 - 1\}$. Note that when $\alpha \in (0, 1)$ the spin does not lie on the grid!
   (b) [10] For $N = 2^8$, plot your solutions $x[n, 0]$ for $\alpha = 0$ and $\alpha = 0.25$. Zoom in to display the range $-20 \leq n \leq 20$.
   (c) [0] Comment on whether MRI image formation / reconstruction is shift invariant.

MR excitation

3. [10] A uniform object is excited by a nonselective 90° pulse applied along X′. Following this excitation is a z-gradient pulse of amplitude G and duration T. Then we apply a nonselective $-90°$ pulse along X′.
   Plot the resulting $M_Z(z)$ after this sequence. Ignore T_2 effects and assume $\omega = \omega_0$.

MR imaging

4. [0] Modify the single spin-echo 2DFT pulse sequence into a dual spin-echo 2D FT sequence (*i.e.*, using 90-180-180). Design the sequence so that the early echo image corresponds to $\mathrm{TE}_1 = 40$ ms and the late-echo image corresponds to $\mathrm{TE}_1 = 100$ ms. Use a constant phase-encoding interval of 4 ms, and for each echo acquisition use a readout gradient interval of 8 ms centered about the echo. Draw a timing diagram and plot the k-space trajectory for this sequence.

5. [5] Dr. T claims that in a 2D FT spin-echo sequence, the magnitude of the baseband signal peaks at time T_E, the center of the readout, for each phase-encode level. Dr. C argues that the maximum signal magnitude is at the center of each readout when the phase encoding lobe is zero, but is not necessarily in the center for other phase-encode levels. Discuss the validity of these claims using theoretical arguments, physical reasoning, or examples.