Homework #5, EECS 516, F09. Due **Due Fri. Oct. 30**, by 1:30PM

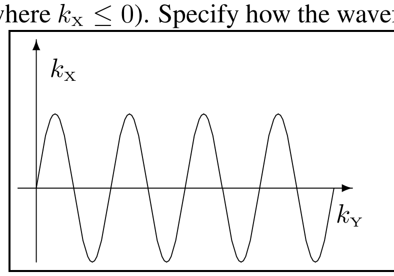
MR physics

1. [10] Let the total number of spins per unit volume be $N = N_+ + N_-$, for $I_z = \pm 1/2$.
   (a) [5] Find an expression for $\mathsf{E}[N_+ - N_-]$ in terms of N and the physical parameters.
   (b) [5] When kT is large relative to the energy difference between the two spin states, simplify your answer to part (a).

2. [15] Consider two materials A and B having the same equilibrium magnetization M_{Z0}, but with relaxation parameters (T_{1A}, T_{2A}) and (T_{1B}, T_{2B}) respectively. For a 90° excitation, let $\Delta S_{xy}(t) = M_{xyA}(t) - M_{xyB}(t)$ be the difference in transverse magnetization and $\Delta S_z(t) = M_{zA}(t) - M_{zB}(t)$ be the difference in longitudinal magnetization.
   (a) [5] Determine the time t the maximizes $|\Delta S_{xy}(t)|$.
   (b) [5] Determine the time t the maximizes $|\Delta S_z(t)|$.
   (c) [5] Evaluate your expressions for the case of 1T brain imaging of white matter ($T_1 = 680\mathrm{ms}, T_2 = 92\mathrm{ms}$), and gray matter ($T_1 = 820\mathrm{ms}, T_2 = 100\mathrm{ms}$).
   Assume the spins of interest in the two materials have the same resonant frequency and remain in phase.

MR imaging

3. [10] (a) [5] For the sinusoidally shaped k-space trajectory below, that starts at the origin, draw the timing diagram for the pulse sequence. Note that two excitations will be used to cover k-space; one for the top-half of k-space (where $k_{\mathrm{Y}} \geq 0$) and the other for the bottom half (where $k_{\mathrm{X}} \leq 0$). Specify how the waveforms change between measurements.



   (b) [5] Determine the parameters (amplitudes and timing) of the gradient waveforms from part (a) that will approximately achieve $\mathrm{FOV}_{\mathrm{Y}} = 19.2$ cm and $\Delta_{\mathrm{X}} = \Delta_{\mathrm{Y}} = 0.3$ cm. (Do not worry about $\mathrm{FOV}_{\mathrm{X}}$.) Assume that Δ_{Y} and $\mathrm{FOV}_{\mathrm{Y}}$ are based on the k-space coverage along the k_{Y} axis. Also assume that the maximum gradient amplitude is 1 G/cm.

4. [10] A 2D FT imaging sequence was designed using rectangular waveforms $G_{\mathrm{Y}}(t)$ of duration τ (and various amplitudes) for phase encoding. The design was based on the usual assumption that the induced field is linear, *i.e.*, $\vec{B}(\boldsymbol{r}, t) = (B_{\mathrm{Z0}} + \boldsymbol{r} \cdot G_{\mathrm{Y}}(t))\,\vec{k}$. However, the actual field produced by the phase-encode gradient coil is nonlinear: $(y + \alpha y^3)\, G_{\mathrm{Y}}(t)$.
   (a) [5] Rewrite the signal equation for $s(t)$ in light of the nonlinearity in the y gradient.
   (b) [5] If the object is an impulse at (x_0, y_0), *i.e.*, $m(x, y) = \delta_2(x - x_0, y - y_0)$, then determine at what position the impulse will be reconstructed by a usual reconstruction algorithm that is unware of the gradient nonlinearity.
   (Such spatial distortions can be an important consideration when using MR for applications like surgical planning.)