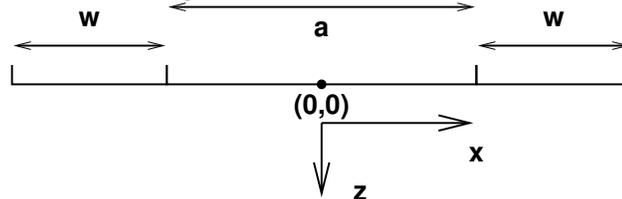


Homework #4, EECS 516, F09. Due **Due Fri. Oct. 9**, by 1:30PM

1. [20] Consider a 1D ultrasound transducer consisting of 3 elements as shown below:



The electronics for this array are configured such that the *outer* two elements are used to transmit simultaneously, and the *inner* element is used to receive.

- (a) [10] Determine (analytically) the overall far-field beam pattern (product of transmit and receive patterns) for this system. (Use polar coordinates.)
- (b) [10] For the case  $a = 3w$  and  $w = 5\lambda$ , use plots to compare the overall beam pattern of this system with that of a system where all 3 elements are used together for both transmit and receive.
- (c) [0] Describe the advantage and disadvantage of each design.
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2. [20] An ultrasound system has a circular transducer of diameter  $D$  that includes an acoustic lens that focuses the beam on the point  $(0, 0, z')$ , where  $z' \gg D$ .
- (a) [10] Determine analytically the (approximate) 2D beam pattern in the focal plane  $z = z'$ . Think about the symmetries of the problem and express your answer in an appropriate coordinate system.
- (b) [10] Define the beam width at the focal plane to be the distance between the innermost zeros of the beam pattern. Determine the beam width for this system when  $D = 10$  mm,  $\lambda = 0.5$  mm, and  $z' = 40$  mm.
- (c) [0] What is the diffraction-limited beam width at depth  $z' = 40$  mm?
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3. [20] In class we focused on using different phases for each element in a transducer array. It can also be useful to use different *gains* on each element in a transducer array.
- (a) [15] Determine the (approximate) angular beam pattern (in polar coordinates) for a linear array having triangular apodization, *i.e.*, unity gain at the array center, and the gain decreases towards zero at some point at the edges (or beyond the edges) of the array. Hint: for simplicity you can ignore sampling and treat the array as a 1D continuum.
- (b) [5] Discuss advantage(s) and disadvantage(s) relative to the non-apodized array.
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4. [10] In practice, each element of an ultrasound transducer has a gain applied to it to correct for individual variations in the piezoelectric elements, electronics, etc. Suppose that for a 128 element array one of those gains is set incorrectly to be  $1 + \alpha$  times as large as it should be, where  $\alpha > 0$ .
- (a) [10] Determine the far-field beam pattern (in polar coordinates) for the system by decomposing the aperture function  $s_x(x)$  into the sum of two components, one of which represents the aperture function with the gains set correctly.
- (b) [0] Describe qualitatively the effect of the incorrect gain.
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5. [10] A power supply fault causes complete signal loss for every other element of a 129 element ultrasound array, *e.g.*, only the signals from elements  $-64, -62, -60, \dots, 0, 2, 4, \dots, 62, 64$  are recorded. Determine the far-field receive beam pattern when this phased array is electronically steered in the direction  $\beta = \sin \theta_0$ . Assume that if all elements were working, there would be no gaps between the elements.
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6. [10] A 1D ultrasound array having equally spaced elements and with total aperture width  $D$  is focused by electronic processing, but is *mechanically translated* to form the image. How should the aperture be translated (*i.e.*, what is the maximum acceptable step size) between beam lines assuming that the system will always operate at an *f*-number (*i.e.*,  $z/D$ ) of at least 3?
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7. [5] You have available four ultrasound linear arrays: two wide (W) arrays with  $w = d = \lambda/2$ , and two narrow (N) arrays with  $w = d = \lambda/4$ . You must design a transducer from these four arrays, by choosing from the six possible orderings: WWNN, NNWW, NWWN, WNNW, WNWN, NWNW. Choose your design and explain why it is the best.