Homework #3, EECS 516, F09. Due **Due Fri. Oct. 2**, by 1:30PM

Ultrasound basics

1. [10] Suppose that a certain region of the body can be modeled as a uniform array of point scatterers and a fluid-filled, non-reflecting cyst, with the following reflectivity:

$$R(x,y,z) = \begin{cases} \frac{1}{A^3}\operatorname{comb}\left(\frac{x}{A}\right)\operatorname{comb}\left(\frac{y}{A}\right)\operatorname{comb}\left(\frac{z}{A}\right)\left(1-\operatorname{rect}\left(\frac{\sqrt{x^2+y^2+(z-z_0)^2}}{B}\right)\right), & z>0 \\ 0, & \text{otherwise,} \end{cases}$$

where $z_0 > B/2 + A$. Supposed this region is imaged in A-mode with a $L \times L$ transducer at $z = 0$, where $L < A \ll B$.

(a) [5] Sketch the estimated reflectivity vs depth, *i.e.*, $\hat{R}(0,0,z)$, assuming a near-field model (ignoring diffraction). Assume that appropriate gain compensation has been applied and that the effective pulse envelope is $\operatorname{rect}\left(\frac{t-\tau/2}{\tau}\right)$, *i.e.*, rectangular with width τ, where $c\tau/2 < A$. Hint: first sketch $R(0,0,z)$.

(b) [5] Repeat for the case where the envelope is broader: $3A < c\tau/2 < 4A$.

2. [10] Our analysis has largely assumed weak reflections, ignoring multiple reflections. Consider two reflecting surfaces modeled by:

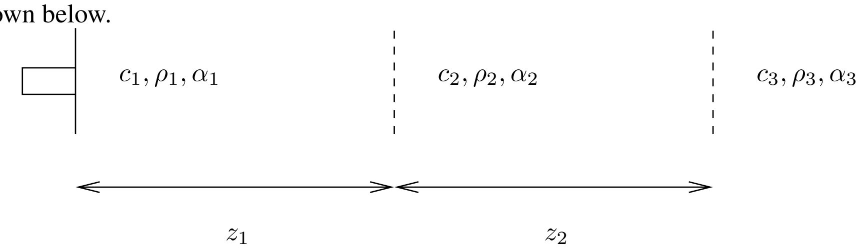
$$R(x,y,z) = R_1\,\delta(z-z_1) + R_2\,\delta(z-z_2),$$

where $z_2 > z_1$.

(a) [5] Suppose this object is imaged in A-mode, using an ideal Dirac impulse as the pulse envelope. Find the estimated reflectivity $\hat{R}(0,0,z)$ accounting for multiple reflections and for the pressure losses in transmission across the surfaces. You need consider only the first received of the multiple reflections, and you can ignore attenuation.

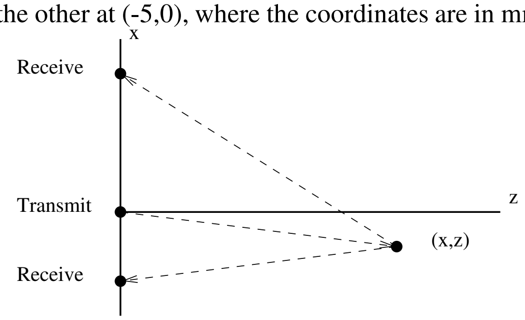
(b) [5] Compare your expression $\hat{R}(0,0,z)$ in the case where $R_1 = R_2 = 0.1$ to the simpler model for $\hat{R}(0,0,z)$ based on the weakly reflecting assumption.

3. [10] An ultrasonic pulse with envelope $a(t) = \operatorname{rect}(t/T)$ is transmitted through the volume containing the material properties and interfaces shown below.



Sketch the envelope of the received signal, labeling important features. Neglect multiple reverberations and use the weakly reflecting assumption. Also neglect $1/r$ spreading losses. Assume that $cT/2 \ll \min(z_1, z_2)$.

4. [10] An ultrasonic tracking system consists of a single "point" transmitter located at $(x,z) = (0,0)$ and two "point" receivers, one located at (10,0) and the other at (-5,0), where the coordinates are in mm, as shown below.



Assume that the object consists of a single reflector at unknown position (x,z). If the receiver at (10,0) detects an echo at $t = 53.74838498865699\,\mu$sec, and the receiver at (-5,0) detects an echo at $t = 54.36156333177956\,\mu$sec, determine the location (x,z) of the reflector. Assume $c = 1.5$mm/μsec. This process is called **triangulation**.

Hint: if an analytical solution is elusive, try using the Matlab `fsolve` function, initialized with an on-axis guess of about the correct depth.

5. [20] The purpose of this problem is to use MATLAB to visualize the effects of dispersion in ultrasonic pulses with frequency-dependent attenuation. Use the following parameters:
   - $f_0 = 1.5$ MHz
   - c = 1500 m/s
   - attenuation coefficient is 1dB/cm/MHz (and linear in f near f_0).

   Use the following two choices of amplitude modulation envelopes:
   - quadratic: $\left(1 - 4(tf_0/3)^2\right) \operatorname{rect}(tf_0/3)$
   - gaussian: $\exp\left(-(tf_0/2)^2\right)$

   Compute and plot the envelope of the (attenuation corrected) received signals for ideal plane reflectors at depths $z = 0$, 4, 8, and 12 cm.
   A skeleton Matlab program will be available on the web site.