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Examples of EECS 401 Prerequisite Material (Not Necessarily Completely Inclusive)

Differentiation from First Principles

$$\frac{d}{dx} f(x) = \lim_{\delta \searrow 0^+} \frac{f(x+\delta) - f(x)}{\delta} = \lim_{\delta \searrow 0^+} \frac{f(x) - f(x-\delta)}{\delta}, \quad \text{if } f(\cdot) \text{ is continuous at } x \text{ and the limits are equal,}$$

where $\delta \searrow 0^+$ means δ approaches 0 from the right (positive δ)

Riemann Integration
If f is a continuous function on $[a, b]$, then

$$\int_a^b f(x)\, dx = \lim_{n\to\infty} \frac{a-b}{n} \sum_{i=1}^n f\left(a + \frac{i}{n}(b-a)\right).$$

Similarly, if δ is small, then

$$\int_a^{a+\delta} f(x)\, dx \approx \delta \cdot f(a).$$

Multivariate Integration (Especially Double Integration Limits)
If $A = \{(x, y) : \ x^2 + y^2 \leq 2, \ x \geq 0, \ y \geq 1\}$, then

$$\iint_A xy \ dxdy = \int_1^{\sqrt{2}} \int_0^{\sqrt{2-y^2}} xy \ dxdy = \int_1^{\sqrt{2}} \frac{1}{2}(2-y^2)y \ dy = \frac{1}{2}(y^2 - \frac{1}{4}y^4)\Big|_1^{\sqrt{2}} = \frac{1}{2}(2-1) - \frac{1}{2}(1-\frac{1}{4}) = 1/8$$

Simple Matrix Inversion

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \text{if} \quad ad \neq bc$$

Leibniz's Rule

$$\text{If } G(x) = \int_{a(x)}^{b(x)} h(x,y)\ dy \quad \text{then} \quad \frac{d}{dx} G(x) = h(x, b(x)) \frac{d}{dx} b(x) - h(x, a(x)) \frac{d}{dx} a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} h(x,y)\ dy$$

Geometric Series

$$\sum_{i=0}^n a^n = \frac{1-a^{n+1}}{1-a}, \quad \text{if } a \neq 1; \qquad \sum_{i=0}^\infty a^n = \frac{1}{1-a}, \quad \text{if } |a| < 1$$

Proof: $S = a^0 + a^1 + \cdots + a^n$ so $aS = a^1 + a^2 + \cdots + a^{n+1}$ so $(1-a)S = S - aS = a^0 - a^{n+1}$.