Eng. 100: Music Signal Processing
DSP Lecture 2: Lab 2 overview

Curiosity: http://www.youtube.com/watch?v=qybUFnY7Y8w

Announcements:

- DSP lecture notes on Google Drive
- Lab 1
  - Finish RQ on Canvas by Thu. at 10:30AM (usually due *before* lab)
  - Finish Lab 1 this week; due Friday 5PM on Gradescope (grace)
- Read Lab 2 before next week's lab!
  Finish RQ on Canvas by next Thu. at 10:30AM.
- Midterm (on schedule): Wed. Mar. 20, in class
- See syllabus for office hours. Come say hi!

Outline

- Previous class summary (TC, A/D, frequency, `Julia`)
- Part 0. Lab 1 questions? (cut-and-paste vs understanding / HW0)
- Part 1: Terminology

Lab 2: Computing and visualizing the frequencies of musical tones

- Part 2. Sampling signals, especially sinusoids
- Part 3: Computing the frequency of a sampled sinusoid
  (with a computer, rather than by hand and eye like in previous class)
- Part 4: Visualizing, modeling and interpreting data
  using semi-log and log-log plots
- Part 5: Basic dimension analysis (units)

Part 1: Terminology

Terminology

Help me remember to define each new term!

(First overview class may have been rushed but not now...)

Course title: Music Signal Processing
What is a signal?

Wikipedia (electronics) 2012:
a signal is any time-varying or spatial-varying quantity

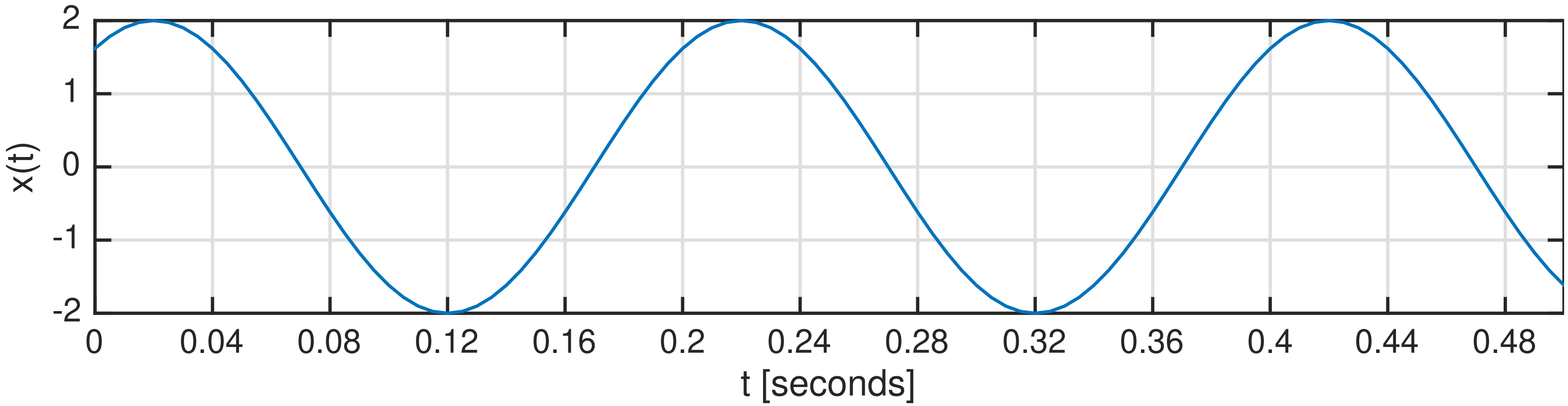
2021: *"In signal processing, a signal is a function that conveys information about a phenomenon."*

Q0.1 Common use of "signal" ? (short answer; introduce neighbors)

??

Part 2:
Sampling analog signals, especially sinusoidal signals

Sinusoidal "pure" tones (simple signal)



From Lab 1:

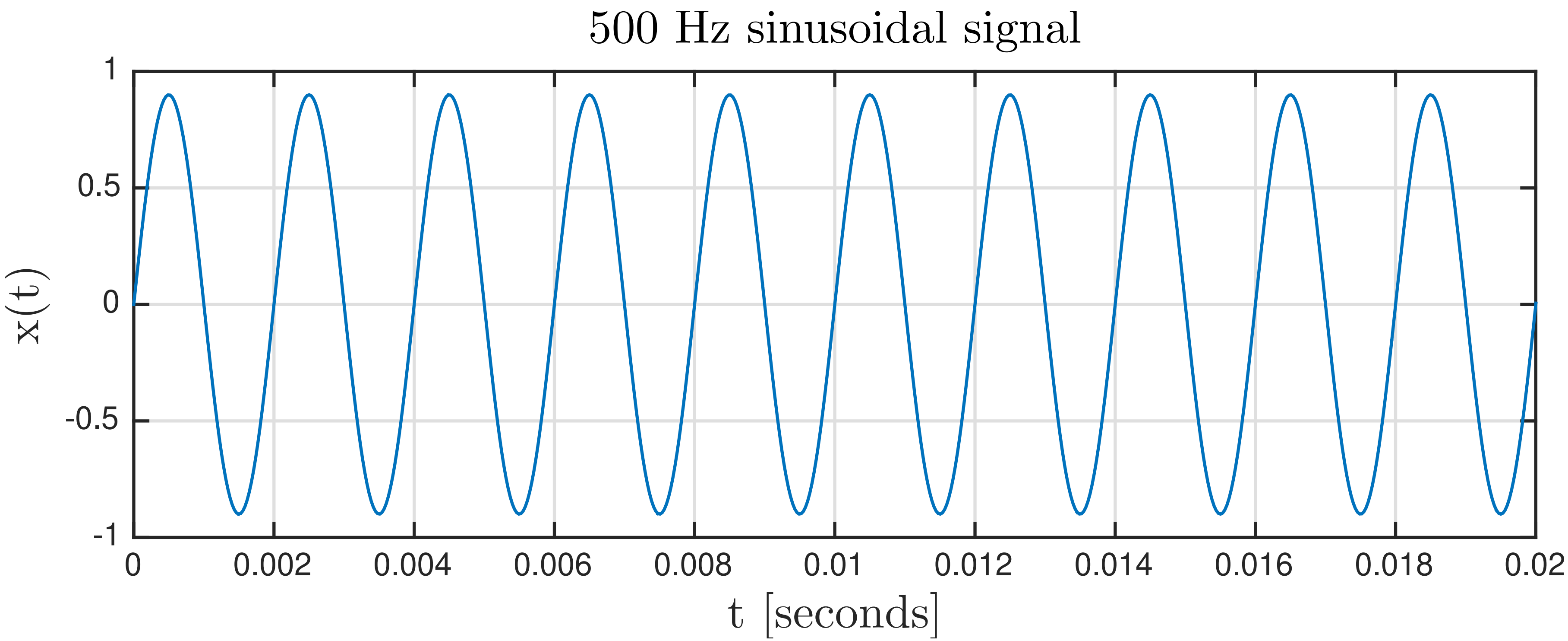
$$x(t) = 2\cos(2\pi 5\,(t - 0.02)) = 2\cos(2\pi 5 t - \pi/5)$$

- Amplitude: $A = 2$
- Frequency: $f = 5$ Hz (cycles per second)
- Period: $T = 1/f = 0.2$ s
- Phase: $\theta = -\pi/5$ radians

"musical?" (*cf.* instruments, *cf.* hearing range)

Sinusoidal signal at 500 Hz

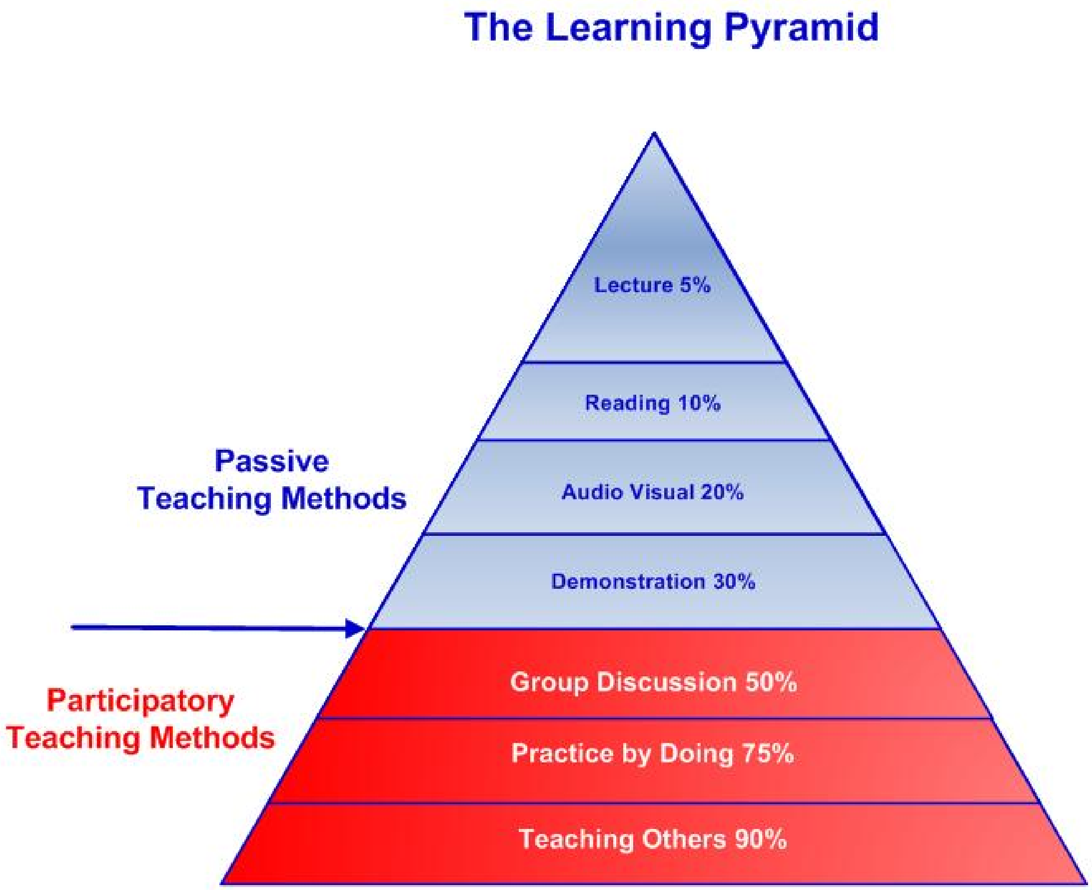
$$x(t) = 0.9\cos(2\pi 500 t - \pi/2)$$ play



Q0.2 What is the period of this signal (in seconds)?

A: 2 B: 0.2 C: 0.02 D: 0.002 E: 500 ??

Learning pyramid



https://thepeakperformancecenter.com/educational-learning/learning/principles-of-learning/learning-pyramid (citation !!)

Sampling an analog signal

Analog signal (continuous-time signal): $x(t)$,
where t can be any real number. Units of "t" are seconds.

$$\underbrace{x(t)}_{\substack{\text{analog: } t \in \mathbb{R} \\ \text{(e.g., vinyl)}}} \to \boxed{\text{A/D converter}} \to \underbrace{x[n]}_{\substack{\text{digital: } n \in \mathbb{Z} \\ \text{(e.g., CD, MP3)}}}$$

Crucial "system" quantities for an A/D converter (*i.e.*, for sampling):

- Sampling rate: S. Units: $\frac{\text{Sample}}{\text{Second}}$ or Hz
- Sampling interval: $\boxed{\Delta = 1/S.}$ Units: seconds

On paper, or in software, sampling means: substitute $\boxed{t = n/S.}$

Digital signal (discrete-time signal):

$$\boxed{x[n] = x(n/S) = x(n\Delta)}$$

where n can be any *integer*.

Q0.3 What are the units of "n" above? (Choose best answer.)

A: unitless B: seconds C: Hz D: $\frac{\text{Sample}}{\text{Second}}$ E: none of these ??

Q0.4 What are the units of "n/S" ?

A: unitless B: seconds C: Hz D: $\frac{\text{Sample}}{\text{Second}}$ E: none of these ??

Analog signal $x(t)$ and digital signal $x[n]$ are related but quite different!

Terminology:

Q0.5 Common use of term *sample* or *sampling*?

??

Example: Sampled sinusoidal signal

Analog signal

$$x(t) = 2\cos(2\pi(5\text{Hz})t - \pi/5)$$

Choose sampling rate: $S = 50$ Hz.

Q0.6 What is the sampling interval Δ?

A: 20 B: 2 C: 0.2 D: 0.02 E: None of these ??

Mars Climate Orbiter loss 1999-08-23 ($\approx \$190$ M)

https://science.nasa.gov/mission/mars-climate-orbiter

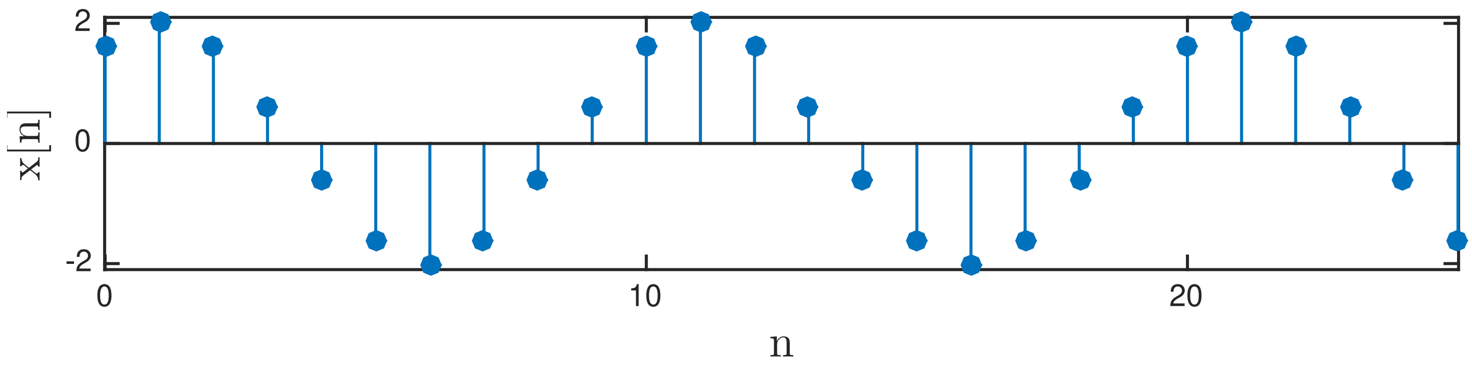
Analog signal:

$$x(t) = 2\cos(2\pi(5\text{Hz})t - \pi/5)$$

For $S = 50$ Hz, we substitute $t = n/S = n/(50\text{Hz})$ to emulate A/D.

Digital signal:

$$x[n] = x(0.02n) = 2\cos(2\pi(5\text{Hz})(n/50\text{Hz}) - \pi/5)$$
$$= 2\cos(0.2\pi n - \pi/5) \text{ after simplifying}$$



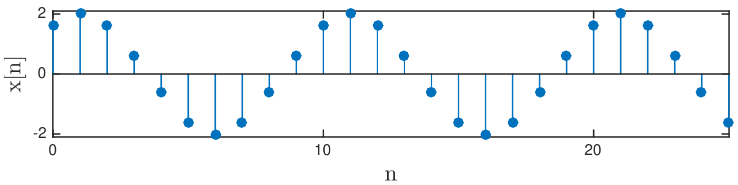
Q0.7 What are the units of "$0.2\pi n - \pi/5$" above? (Choose best.)

A: unitless B: radians C: Hz D: $\frac{\text{Sample}}{\text{Second}}$ E: degrees ??

Example: Sampled sinusoidal signal (continued)

Digital signal as a formula: $x[n] = 2\cos(0.2\pi n - \pi/5)$

Digital signal as a plot:



But in a computer (or DSP chip in phone) it is an array (list) of numbers:

n	$x[n]$ (value)	$x[n]$ (formula)
0	1.62	$2\cos(0.2\pi 0 - \pi/5)$
1	2.00	$2\cos(0.2\pi 1 - \pi/5)$
2	1.62	$2\cos(0.2\pi 2 - \pi/5)$
3	0.62	$\vdots$
4	-0.62	
5	-1.62	
$\vdots$	$\vdots$	

(Actually stored in binary (base 2) not in decimal.)

Sampling a sinusoid in general

Given a pure sinusoidal (analog) signal: $x(t) = A\cos(2\pi f t + \theta)$.

Q0.8 Units of the product ft? (not feet)

A: unitless B: seconds C: Hz D: $\frac{\text{Sample}}{\text{Second}}$ E: none of these ??

If we sample it at S $\frac{\text{Sample}}{\text{Second}}$ (by substituting $t = n/S$),
we get a digital sinusoidal signal (formula):

$$x[n] = A\cos(2\pi f n/S + \theta) = A\cos\left(2\pi \frac{f}{S} n + \theta\right)$$

Q0.9 Units of f/S?

A: unitless B: seconds C: Hz D: $\frac{\text{Sample}}{\text{Second}}$ E: none of these ??

The quantity $\boxed{\Omega = 2\pi f/S}$ is called the *digital frequency* in DSP.

Sampling a sinusoid - summary

Analog
sinusoidal signal
$x(t) = A\cos(2\pi f t + \theta)$
(frequency f)
$\rightarrow$ [Sampling A/D rate S] $\rightarrow$
Digital
sinusoidal signal
$x[n] = A\cos\left(2\pi \frac{f}{S} n + \theta\right)$
(list of numbers)

Real world:

- A computer sound card samples a microphone input signal.
- Applications like Shazam/Zoom/TikTok use digital audio signals.

Basic music transcription requires that we *reverse* this process!

Digital sinusoidal signal
x[n]
$\rightarrow$ [DSP magic] $\rightarrow$
frequency (pitch)
f

Part 3:
Computing the frequency of a sampled sinusoidal signal

Reconstructing a sinusoid from its samples

Given samples of a digital sinusoid in a computer:

$$x[n] = A\cos(\Omega n + \theta), \quad n = 1, 2, \ldots, N.$$

(Stored as a list of N numbers, not as a formula, for known rate S.)

How can we find the frequency f of the original analog sinusoidal signal?

- Step 1. Determine the "digital frequency" Ω
- Step 2. Relate Ω to the original (analog) frequency. (*cf.* "$F = ma$")

  This step is easy because $\Omega = 2\pi \frac{f}{S}$, so rearranging: $\boxed{f = \frac{\Omega}{2\pi} S.}$

Example. $x[n] = 3\cos(0.0632n + 5)$ with $S = 8192$ Hz.
Original frequency is $f = \frac{0.0632}{2\pi} 8192\text{Hz} = 82.4\text{Hz}$ (low E)

But what if we are given an array of signal values instead of a formula?

Finding a digital frequency

(The computer or DSP chip perspective)

Given:

- Signal values:

n	0	1	2	3	4	5	...
$x[n]$	1.62	2.00	1.62	0.62	-0.62	-1.62	...

- Sinusoidal assumption (model): $x[n] = A\cos(\Omega n + \theta)$

Goal: Determine the digital frequency Ω, a model parameter.

This problem arises in many applications, including music DSP.
EE types have proposed many solutions.
Lab 2 uses an elegantly simple method based on trigonometry.

Everything in signal processing / data science / statistics starts with arrays of numbers (data) and uses models to extract (hopefully useful) information.

Key trigonometric identities

Angle sum and difference formulas [wiki]

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

Product-to-sum identities:

$$2\cos a \cos b = \cos(a-b) + \cos(a+b)$$
$$2\sin a \sin b = \cos(a-b) - \cos(a+b)$$

Wikipedia says these identities date from 10th century Persia. [wiki]

We will use these (ancient) identities repeatedly!

Practical example: Tuning a piano

Rewriting product-to-sum identity:

$$\cos(a+b)+\cos(a-b) = 2\cos a \cos b.$$

Substitute $a = 2\pi 442 t$ and $b = 2\pi 2 t$:

$$\cos(2\pi 444 t)+\cos(2\pi 440 t) = 2\cos(2\pi 442 t)\cos(2\pi 2 t).$$

The sum of two sinusoids having close frequencies is a sinusoid at the average of the frequencies with a (sinusoidally) time-varying amplitude!

440:	play	444:	play	440&444:	play
		441:	play	440&441:	play

The combined (sum) signal gets louder and softer, "beating" with period = 0.25 sec. Why do we consider the sum? ??
The slower the period, the closer the 2 frequencies.

Why is this relevant to tuning a piano (or guitar or ...)? ??

Back to finding a digital frequency

Repeating product-to-sum identity:

$$\cos(a+b)+\cos(a-b)=2\cos a\cos b.$$

As another practical application of this identity, some clever DSP expert suggested substituting $a=\Omega n+\theta$ and $b=\Omega$:

$$\cos(\Omega(n+1)+\theta)+\cos(\Omega(n-1)+\theta)=2\cos(\Omega)\cos(\Omega n+\theta).$$

Now use sinusoidal model assumption: $x[n]=A\cos(\Omega n+\theta)$, yielding:

$$x[n+1]+x[n-1]=2\cos(\Omega)\,x[n].$$

Rearranging yields $\cos(\Omega)=\dfrac{x[n+1]+x[n-1]}{2x[n]}$, or equivalently:

$$\boxed{\Omega=\arccos\left(\frac{x[n+1]+x[n-1]}{2x[n]}\right).}$$

Example: find a sinusoid's digital frequency

Q0.10 How many signal samples do we need to find Ω?

A: 1 B: 2 C: 3 D: 4 E: 5 ??

Earlier example:

n	0	1	2	3	4	5	...
$x[n]$	1.62	2.00	1.62	0.62	−0.62	−1.62	...

Apply the arccos formula to this data using $n = 2$:

$$\Omega = \arccos\left(\frac{x[n+1]+x[n-1]}{2x[n]}\right) = \arccos\left(\frac{x[3]+x[1]}{2x[2]}\right)$$
$$= \arccos\left(\frac{0.62+2}{2 \cdot 1.62}\right) = \arccos(0.8086) = 0.629 \approx 0.2\pi.$$

cf. sampled sinusoidal signal on p. 13.

Is this useful for a guitar tuner app yet? ??

Example of finding a sinusoid's frequency

Recall from "Step 2" on p. 17 that $\boxed{f = \frac{\Omega}{2\pi} S}$

Combining Step 1 and Step 2:

$$\boxed{f = \frac{S}{2\pi} \arccos\left(\frac{x[n+1]+x[n-1]}{2x[n]}\right).}$$

Use this formula in Lab 2 to compute the frequency from a digital signal corresponding to samples of a sinusoid.

Example: $S = 1500$ Hz and $x[n] = (\ldots, ?, ?, ?, 3, 7, 4, ?, ?, ?, \ldots)$
The signal values denoted "?" are, say, lost or garbled.
Solution: $f = \frac{1500}{2\pi} \arccos\left(\frac{3+4}{2\cdot 7}\right) = \frac{1500}{2\pi} \arccos\left(\frac{1}{2}\right) = \frac{1500}{2\pi}\frac{\pi}{3} = 250\text{Hz}.$

Historical note: this approach is a simplification of Prony's method from (!) 1795.

Exercise

Suppose $S = 8000$ Hz and the signal samples are
$x[n] = (\ldots, 0, 5, 0, -5, 0, 5, 0, -5, 0, \ldots)$

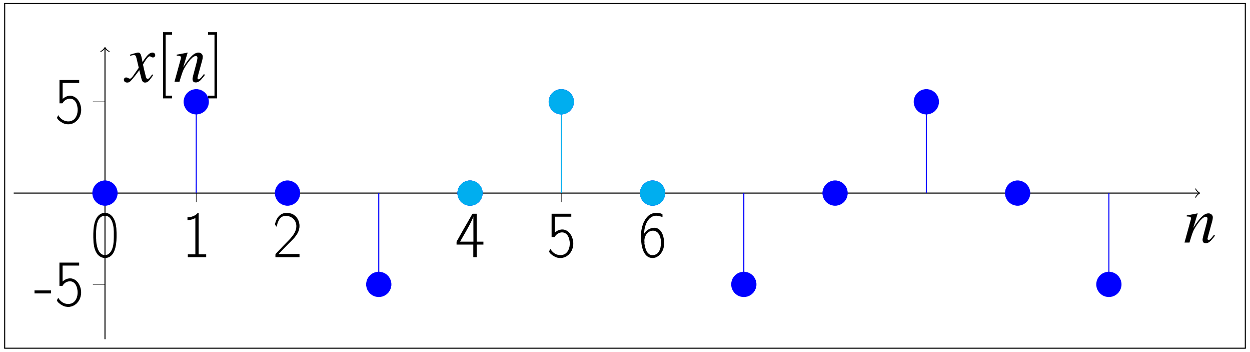
Q0.11 What is the frequency f of the sinusoid (in Hz)?
A: 1000 B: 2000 C: 4000 D: 8000 E: None of these ??

Hint. Here is the arccos frequency formula repeated for convenience:

$$f = \frac{S}{2\pi} \arccos\left(\frac{x[n+1] + x[n-1]}{2x[n]}\right).$$

Illustration

Stem plot of the signa $x[n]$ from previous page.



(Dis)Advantages of this method

Many frequency estimation methods have been proposed.
Apparently not everyone just uses this one; why not?

Advantages of this method:

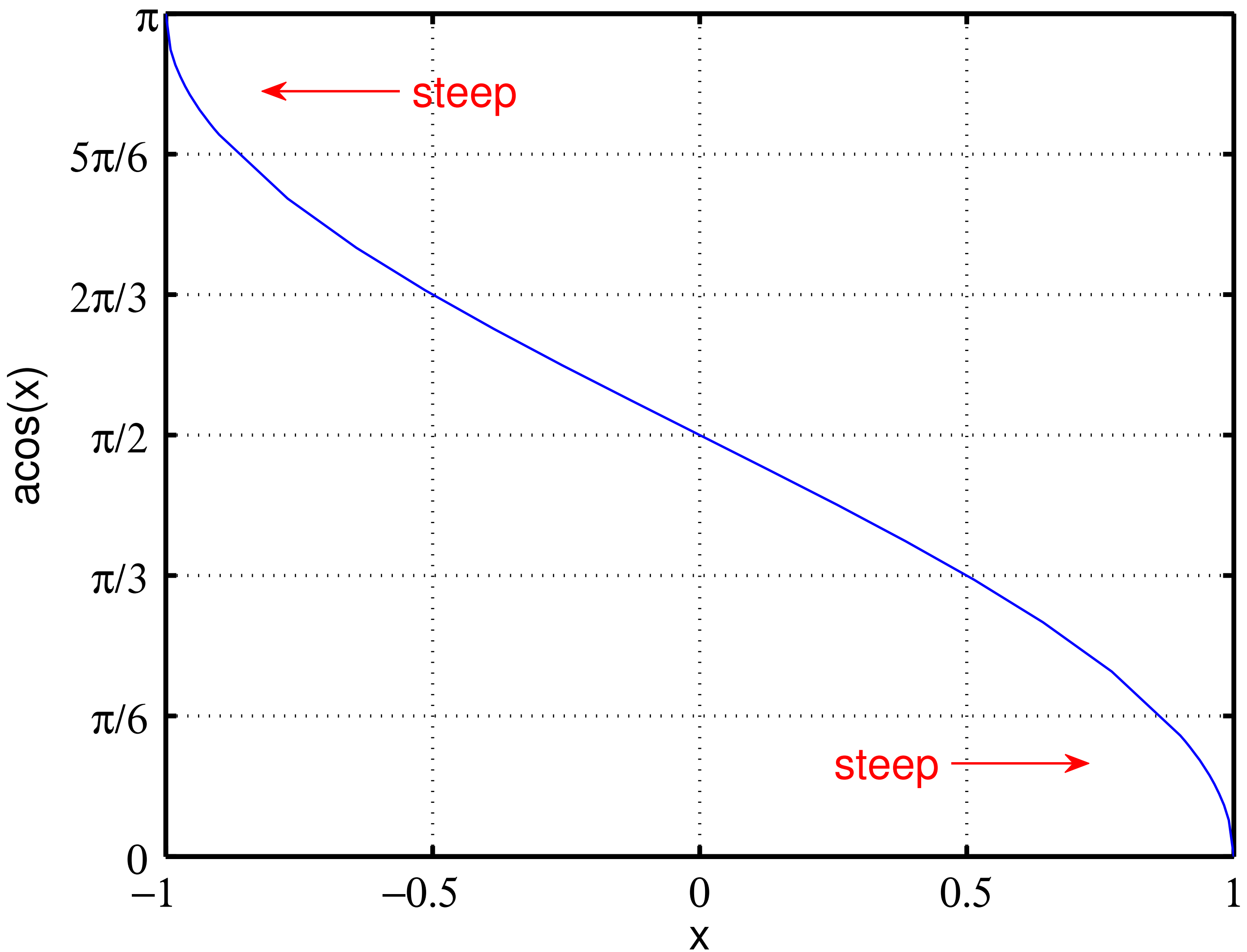
- Very simple to implement, can use simple DSP chip.
- Fast tracking of sudden frequency changes.
- Can use outliers (weird values) to segment long signals.
- Requires knowledge of trigonometry only.

Disadvantages of this method

- Very sensitive to additive noise in the data $x[n]$.
- What if $x[n] = 0$ for some n? Divide by 0!
- Arc-cosine function is sensitive to small changes.

A useful starting point for music DSP...

Arc-cosine



Part 4:
Visualizing, modeling, and interpreting data using semi-log and log-log plots

Data visualization and modeling

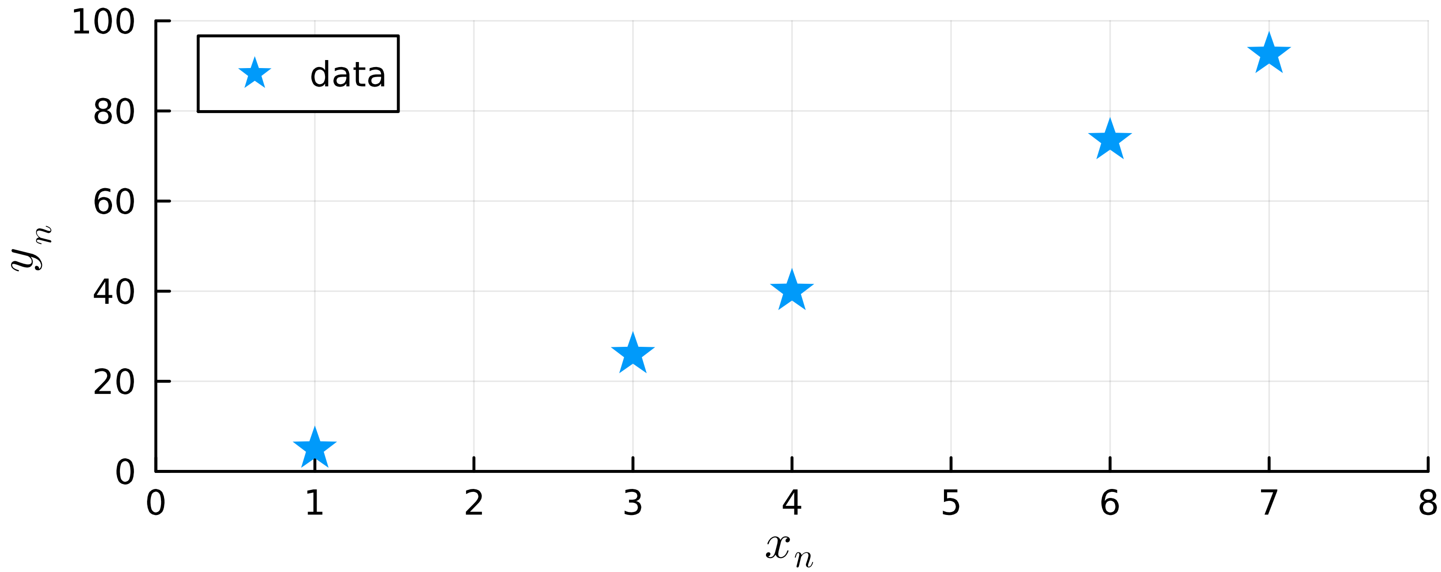
Given: N pairs of data values: $(x_1, y_1), (x_2, y_2), \ldots (x_N, y_N)$

Goal: Find a relationship between the values (a model):
- $y = g(x)$
- $y_n = g(x_n)$, $n = 1, \ldots, N$

- Example (Physics 140):
  x = height above ground a ball is released
  y = velocity on impact with ground.
- Example (Physics 240):
  x = electrical current through a light bulb
  y = energy released in the form of heat by the bulb
- Example (Engin 100-430):
  x = piano key "number" (1 to 88)
  y = frequency of note played by a key

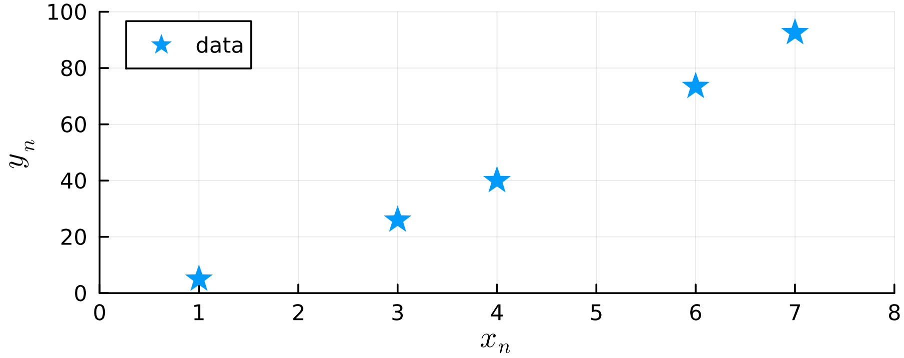
Visualizing data using scatter plots

Example (scatter plot of 5 pairs of data values):



Do your eyes try to "connect the dots?"
Your brain is trying to build a model!

Making a scatter plot in Julia




```
using Plots; default(markerstrokecolor=:auto, markersize=8)
using LaTeXStrings # optional: for plot labels with equations
x = [1, 3, 4, 6, 7]
y = [5.00, 25.98, 40.00, 73.48, 92.60]
scatter(x, y; marker=:star, label="data", size=(500,200),
 xaxis=(L"x_n", (0,8), 0:8), yaxis=(L"y_n", (0,100)))
# savefig("scatter1.pdf")
```

Julia notes:

- The arguments `x, y` here are called "positional arguments" and their order matters.
- The arguments `xlabel=L"x_n"` etc. are called named keyword arguments or named parameters or name-value pairs in Julia and Python. (Matlab has some such capability in recent versions.) They are optional and they can appear in any order (after the semicolon).
- The `:star` value here is a **Symbol** based on string interning.

Common mathematical models

Linear model: $\boxed{y = ax}$

one parameter: slope a

Affine model: $\boxed{y = ax + b}$

two parameters: slope a and intercept b

Quadratic (parabola) model: $\boxed{y = ax^2 + bx + c}$

Simple "power" model: $\boxed{y = bx^p}$

power parameter p, scale factor b

Simple "exponential" model: $\boxed{y = ba^x}$

Note that the independent variable x is in the exponent here.

Q0.12 How many *parameters* does the quadratic model have?

A: 0 B: 1 C: 2 D: 3 E: 4 ??

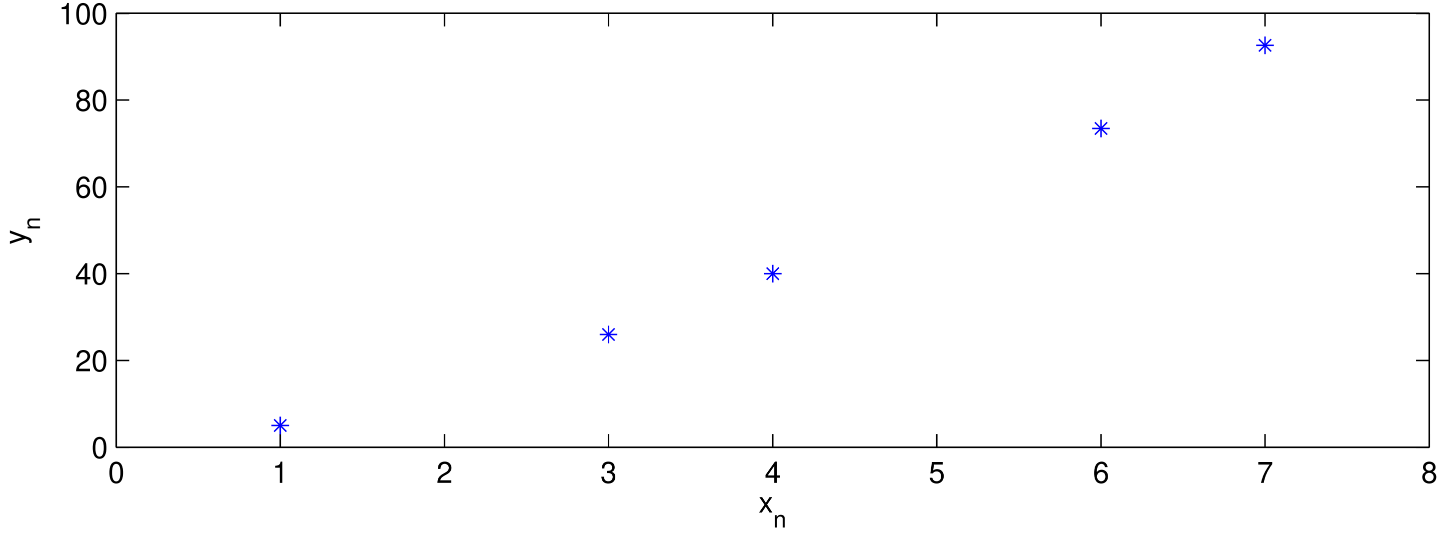
Q0.13 Which model is appropriate for height/velocity example?

(skip: Phys 140)

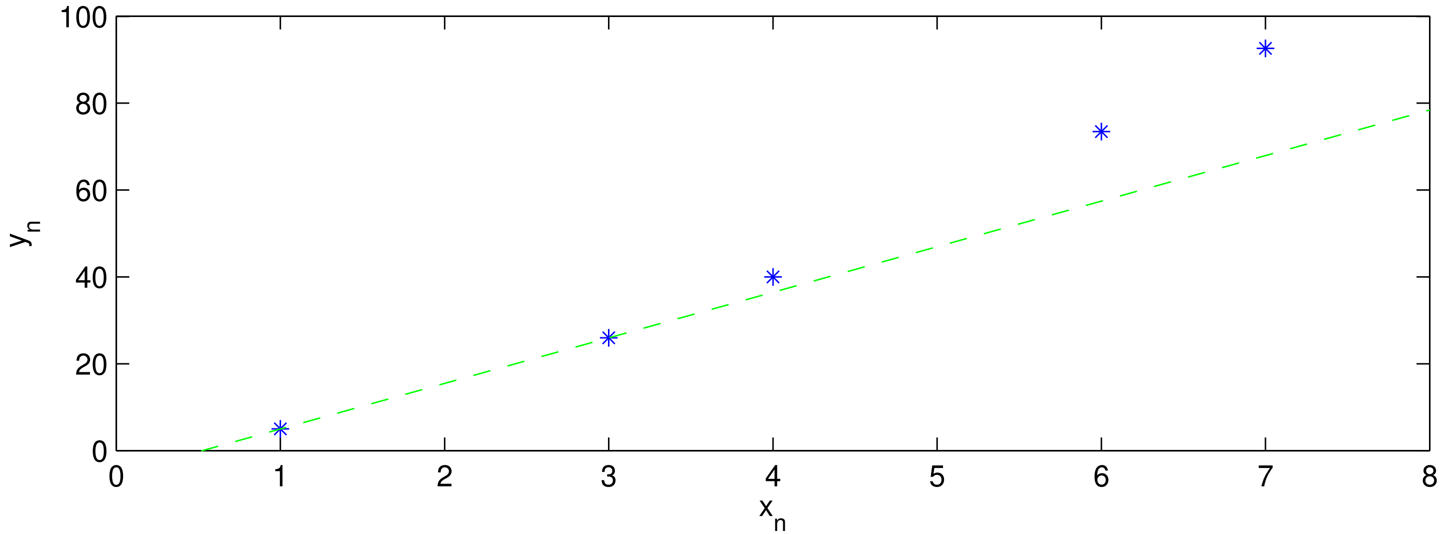
A: linear B: affine C: quadratic D: power E: exponential ??

How do we choose among these models *given data*?

Example: Linear or affine model?



Example: Linear or affine model?



Any two points determine the equation for a line.
Does a line fit this data? ??

So we rule out both linear and affine models by visualization.

For power and exponential model, we need logarithms.

Review of logarithms

- In `Julia`, C, C++, python, `Matlab` and in this class, `log` means natural log, (base e), which might be `ln` on your calculator.
- For base-10 logarithm use `log10` in `Julia`, C, C++, `Matlab`, python, ..., and write $\log_{10}$ on paper.
- Properties of logarithms (for any base $b > 0$): (Math 105)
  - $b^{\log_b(x)} = x$ if $x > 0$ (the defining property)
  - $\log_b(xy) = \log_b(x) + \log_b(y)$ if $x > 0$ and $y > 0$ log of product
  - $\log_b(x^p) = p \log_b(x)$ if $x > 0$ log of power
- Other related properties:
  - $e^{\log(x)} = x$ and $10^{\log_{10}(x)} = x$ if $x > 0$
  - $\log(e) = 1$ and $\log_{10}(10) = 1$ and $\log_b(1) = 0$
  - $e^{a+b} = e^a e^b$

Q0.14 Exercise: Simplify $e^{c \log(z)}$ for $z > 0$.

A: z^c B: c^z C: $z e^c$ D: $c z$ E: None of these

Simple "exponential" model

$$\boxed{y = b\,a^x}$$

Take the logarithm (any base) of both sides:

$$\log(y) = \log(b\,a^x) = \log(b) + \log(a^x) = \log(b) + x\log(a)$$

$$\boxed{\log(y) = \log(a)\,x + \log(b)}$$

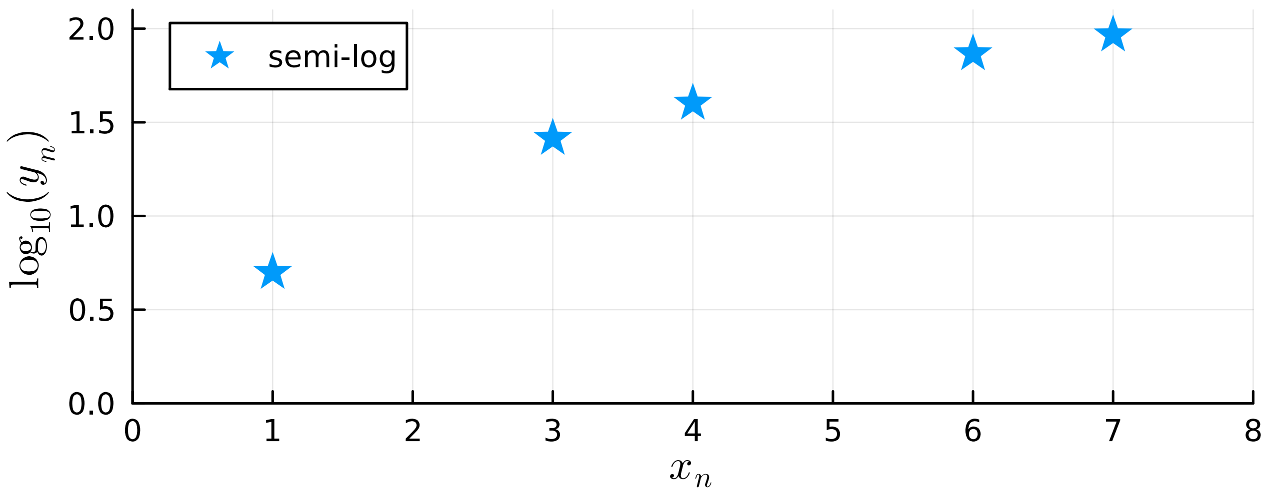
This is the equation of a line on a log scale:

$$\underbrace{\log(y)}_{\tilde{y}} = \underbrace{\log(a)}_{\text{slope}}\; x + \underbrace{\log(b)}_{\text{intercept}}$$

To see if the exponential model fits some data, make a scatter plot of $\boxed{\log(y_n) \text{ versus } x_n}$ and see if it looks like a straight line.

This is called a semi-log plot.

Making a semi-log plot in Julia




```julia
using Plots; default(markerstrokecolor=:auto, markersize=8)
using LaTeXStrings
x = [1, 3, 4, 6, 7]
y = [5.00, 25.98, 40.00, 73.48, 92.60]
scatter(x, log10.(y); marker=:star, size=(500,200), label="semi-log",
 xaxis=(L"x_n", (0,8), 0:8), yaxis=(L"\log_{10}(y_n)", (0,2.1)))
# savefig("scatter2.pdf")
```

Q0.15 The exponential model provides a good fit for this data.

A: True B: False ??

Simple "power" model

$$\boxed{y = b x^p}$$

Take the logarithm (any base) of both sides:

$$\log(y) = \log(b x^p) = \log(b) + \log(x^p) = \log(b) + p \log(x)$$

$$\boxed{\log(y) = p \log(x) + \log(b)}$$

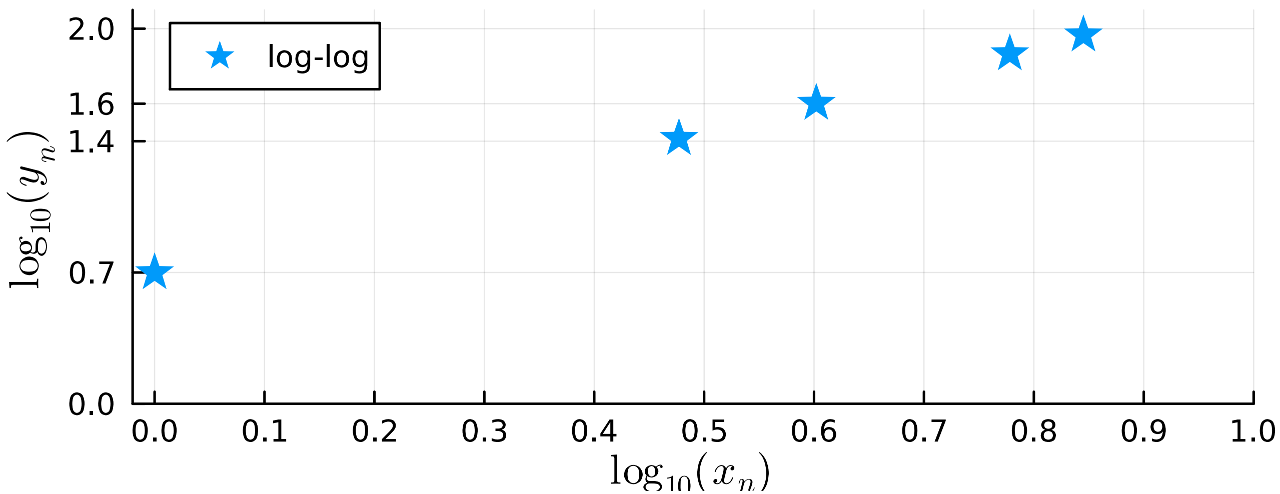
This is the equation of a line on a log-log scale:

$$\underbrace{\log(y)}_{\tilde{y}} = \underbrace{p}_{\text{slope}} \underbrace{\log(x)}_{\tilde{x}} + \underbrace{\log(b)}_{\text{intercept}}$$

To see if the power model fits some data, make a scatter plot of $\boxed{\log(y_n) \text{ versus } \log(x_n)}$ and see if it looks like a straight line.

What do you suppose this type of plot is called? ??

Making a log-log plot in Julia

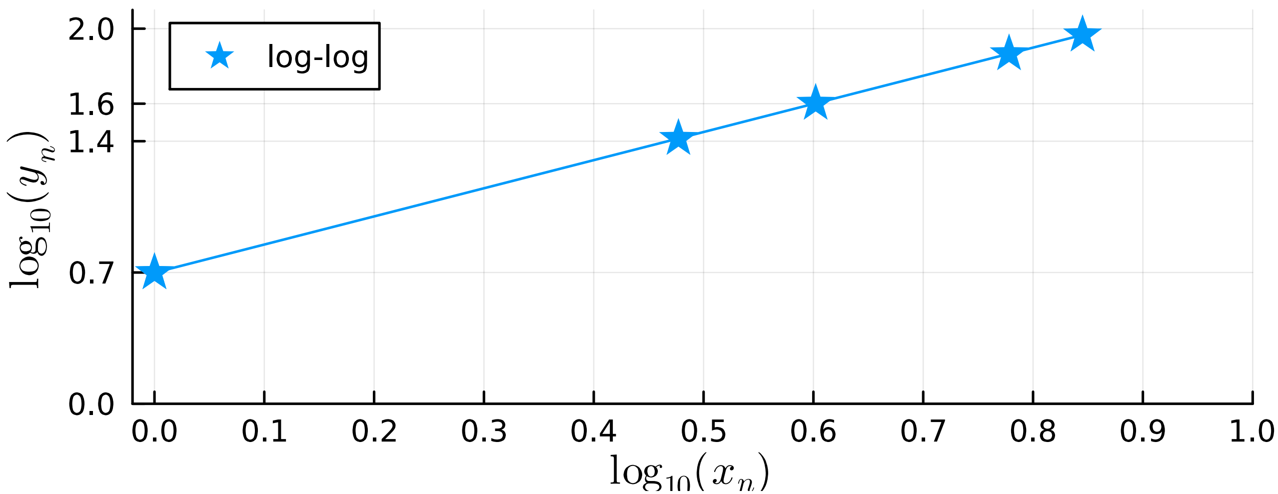



```
using Plots; default(markerstrokecolor=:auto, markersize=8)
using LaTeXStrings
x = [1, 3, 4, 6, 7]
y = [5.00, 25.98, 40.00, 73.48, 92.60]
scatter(log10.(x), log10.(y); marker=:star, size=(500,200), label="log-log",
 xaxis=(L"\log_{10}(x_n)", (-0.02,1), 0:0.1:1),
 yaxis=(L"\log_{10}(y_n)", (0.,2.1), [0, 0.7, 1.4, 1.6, 2]), )
# savefig("scatter3.pdf")
```

Q0.16 The power model provides a good fit for this data.

A: True B: False ??

Checking a log-log scatter plot in Julia




```
scatter!(log10.(x), log10.(y); smooth=true) # along with other style args...
```

`smooth=true` : adds "least squares best-fit line" to scatter plot

Yes! log-log plot lies along a line $\Longrightarrow$ power model is a good fit.
Now we just need the parameters to write our equation model.

- intercept $\approx 0.7 = \log_{10}(b) \Longrightarrow b = 10^{0.7} \approx 5.0$
- slope $= \frac{\Delta \tilde{y}}{\Delta \tilde{x}} \approx \frac{1.6-0.7}{0.6-0.0} = 1.5 \Longrightarrow p = 1.5$

Model: $y = b\,x^p = 5\,x^{1.5}$

Checking a model

Recall given data:

```
x = [1, 3, 4, 6, 7]
y = [5.00, 25.98, 40.00, 73.48, 92.60]
```

Model found on previous slide: $y = 5x^{1.5}$

To check model in `Julia` (using broadcast):

```
5 * (x .^ 1.5)
```

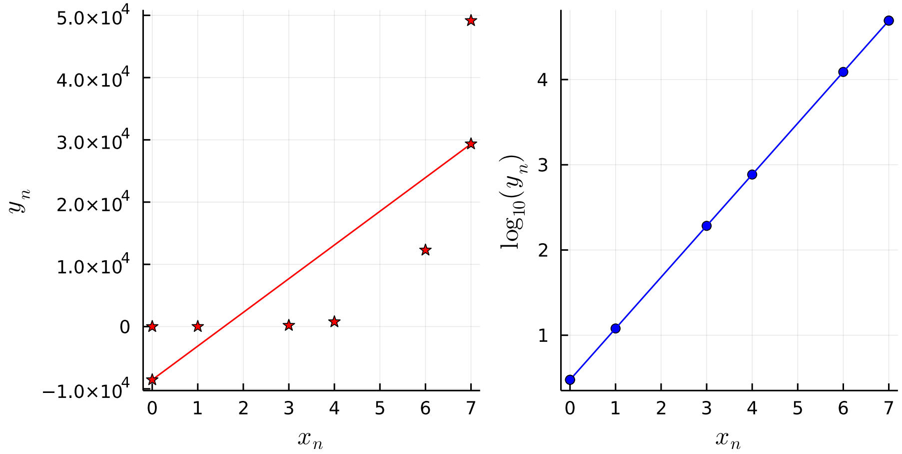
Output is:

```
5-element Vector{Float64}:
5.0 25.981 40.0 73.485 92.601
```

So our power model fits the given data very well.

A semi-log example

```
using Plots; default(label="")
x = [0, 1, 3, 4, 6, 7]
y = [3, 12, 192, 768, 12288, 49152]
p1 = scatter(x, y; color=:red, marker=:star, smooth=true)
p2 = scatter(x, log10.(y); color=:blue, marker=:circle, smooth=true)
plot(p1, p2)
```



Q0.17 Exercise: determine a model for this data. (after Lab 2 overview)

??

Summary of two important models

- Simple exponential model: $\boxed{y = b\,a^x}$

  Use semi-log plot:

$$\underbrace{\log(y)}_{\tilde{y}} = \underbrace{\log(a)}_{\text{slope}}\,x + \underbrace{\log(b)}_{\text{intercept}}$$

- Simple power model: $\boxed{y = b\,x^p}$

  Use log-log plot:

$$\underbrace{\log(y)}_{\tilde{y}} = \underbrace{p}_{\text{slope}}\,\underbrace{\log(x)}_{\tilde{x}} + \underbrace{\log(b)}_{\text{intercept}}$$

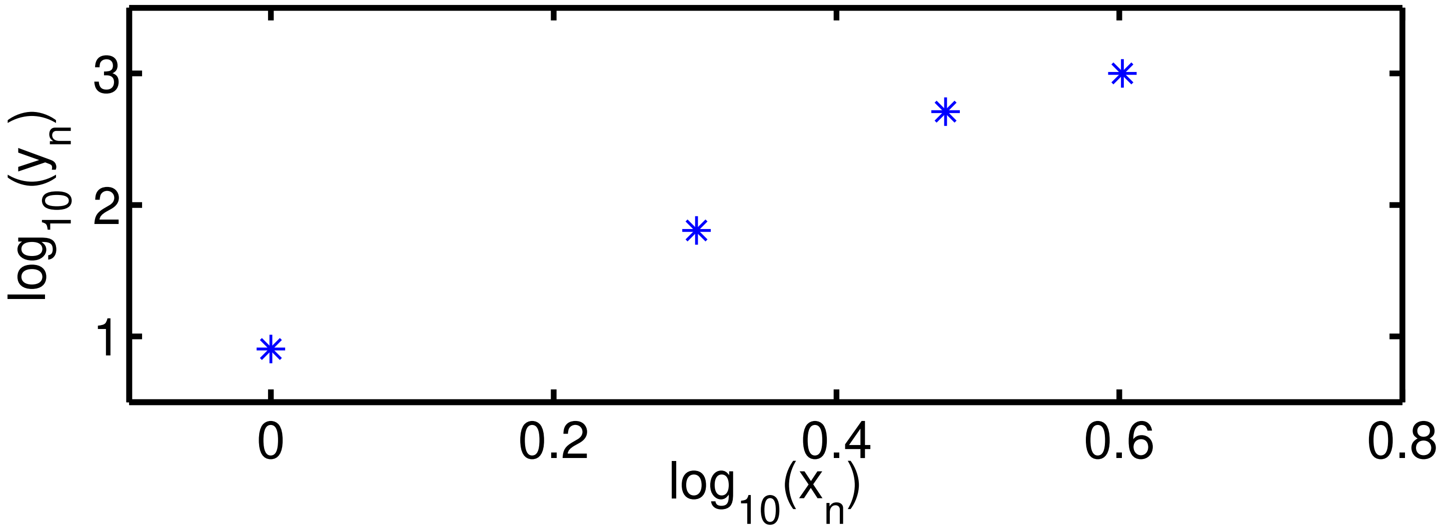
Even though Lab 2 uses these models for musical notes, they are ubiquitous in science and engineering and more.

Models with missing data (read yourself)

Musical context: song without all 88 notes

Given: `y = [8, 64, 512, 1000]`
Given: `x` is *four values* (in order) from the set $\{1, 2, 3, 4, 5\}$
i.e.: [1, 2, 3, 4] or [1, 2, 3, 5] or [1, 2, 4, 5] or [1, 3, 4, 5] or [2, 3, 4, 5]

First try: `x = 1:4; scatter(log10.(x), log10.(y))`

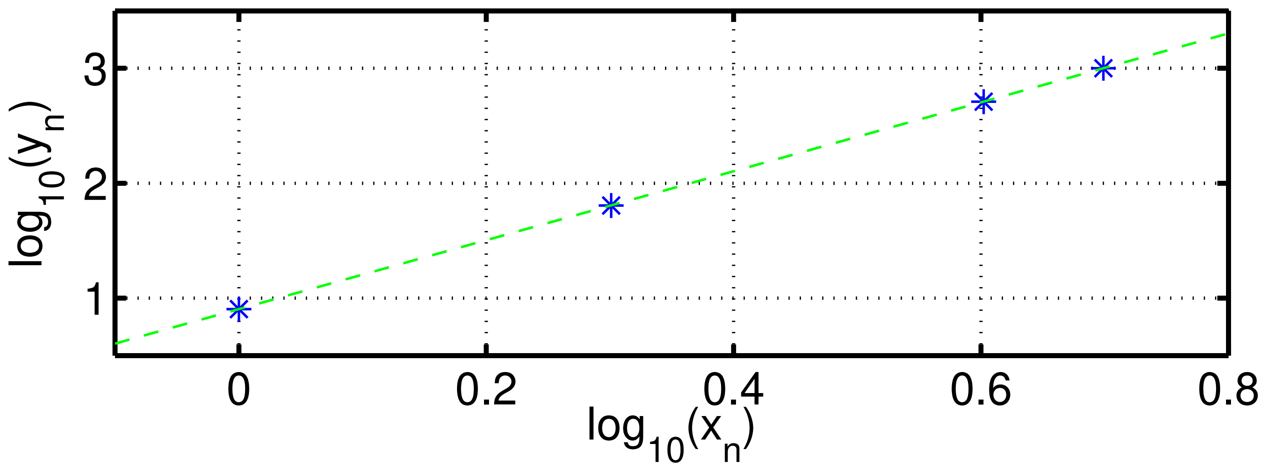


Looks like a "gap" or "jump" at 3rd value

Example with missing data continued

Second try:

```
using Plots
y = [8, 64, 512, 1000]
x = [1, 2, 4, 5]
scatter(log10.(x), log10.(y); smooth=true)
```

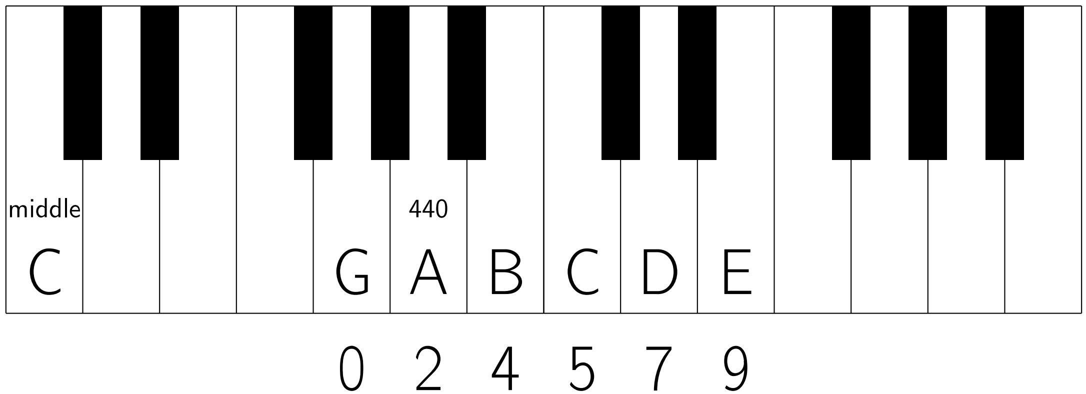


slope $= (3 - 0.9)/(0.7 - 0) = 3 = p$

intercept $= 0.9 = \log_{10}(b) \implies b = 10^{0.9} = 8$

Model: $y = 8x^3$

Missing frequencies in "The Victors"



Missing: 1 3 6 8 10 11 12 13 ... -1 -2 -3 ...

Lab 2 has lots of missing data!

"The Victors" only uses a few of the 88 keys on a piano.

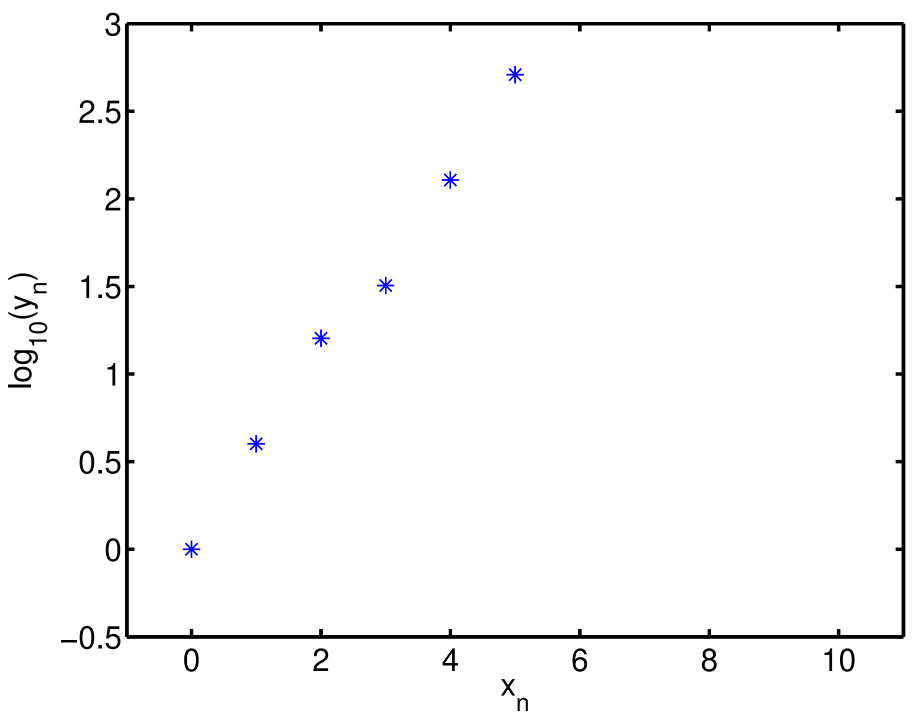
Example: `y = [1, 4, 16, 32, 128, 512]`
where each x_n is one of the numbers in the set $\{0, 2, \ldots, 87\}$

First try:

```
x = 0:5
```

```
scatter(x, log10.(y))
```

Lots of jumps!



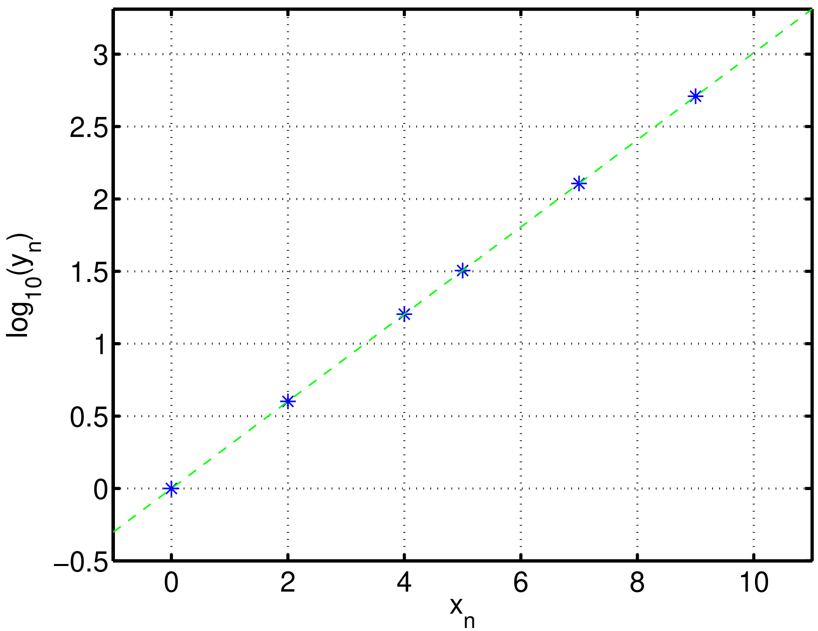
Lots of missing data continued

Second try:

```
y = [1, 4, 16, 32, 128, 512]
```

```
x = [0, 2, 4, 5, 7, 9]
```

```
scatter(x, log10.(y))
```



slope $= (2.7 - 0)/(9 - 0) = 0.3 = \log_{10}(a) \implies a = 10^{0.3} = 2$
intercept $= 0 = \log_{10}(b) \implies b = 10^0 = 1$

Model: $y = 2^x$

You will see a similar situation in Lab 2 (with different data).

What you will do in Lab 2

- Download a sampled signal from Canvas site:
  A tonal version of the chorus of "The Victors."
- Load into `Julia`; segment (chop up) into notes using `reshape`.
- Apply arccos formula to compute frequency of each note.
- Make log-log and semi-log plots of frequencies.
- Determine the formula relating frequencies of notes.
- Note: "Accidentals" are all missing; but you can infer their existence & frequencies from your plot!

Part 5:
Basic dimension analysis
by example

(read yourself if time runs out in class)

Dimensional Analysis Example 1

- Goal: Determine formula for the period of a swinging pendulum, without any physics!
- Find ingredients: mass, length, gravity
- Model: Period = $(\text{mass})^a \, (\text{length})^b \, g^c$
  where g = acceleration of gravity $9.80665\text{m}/\text{s}^2$
  a, b, c are unknown constants to be found
- Approach: Find exponents using dimensional analysis:

$$\text{time} = (\text{mass})^a \, (\text{length})^b \, (\text{length}/\text{time}^2)^c$$

- ◦ No "mass" on LHS so $a = 0$
- ◦ No "length" on LHS so $0 = b + c$
- ◦ For time: $1 = -2c \Longrightarrow c = -1/2$, so $b = 1/2$

Model: period = $\text{length}^{1/2} g^{-1/2} = (\text{length}\ /g)^{1/2}$
From physics: period = $2\pi \, (\text{length}\ /g)^{1/2}$
(2π is a unitless constant; cannot be found from dimension analysis.)

Dimensional Analysis Example 2

Prof. Yagle asks:

- If 1.5 people can build 1.5 cars in 1.5 days,
  how many cars can 9 people build in 9 days?
- Do problems like this give you a headache?
- Would you like to solve problems like this with minimal thinking?

Dimensional Analysis Example 2

Prof. Yagle asks:

- If 1.5 people can build 1.5 cars in 1.5 days,
  how many cars can 9 people build in 9 days?
- Do problems like this give you a headache?
- Would you like to solve problems like this with minimal thinking?

Given:
(1.5 cars) / (1.5 days) / (1.5 people) = 2/3 cars / day / people

Now match units:
(2/3 cars / day / people) (9 days) (9 people) = 54 cars

Simply matching the units suffices.

Summary

- Sampling: a computer can determine frequency of a pure sinusoid from 3 consecutive samples.
- Semi-log plot of $y = b a^x$ (exponential model) is a straight line.
- Log-log plot of $y = b x^p$ (power model) is a straight line.
- Dimensional analysis often gives you correct answers.

Assignment: Read Lab 2 before (next week's) lab!

References

[1] M-Z. Poh, N. C. Swenson, and R. W. Picard. A wearable sensor for unobtrusive, long-term assessment of electrodermal activity. *IEEE Trans. Biomed. Engin.*, 57(5):1243–52, May 2010.