Image Reconstruction: Algorithms and Analysis

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Preface

This book describes the theory and practice of iterative methods for tomographic image reconstruction and related inverse problems such as image restoration. I emphasize methods that are rooted in *statistical* models for the measurement noise; previous texts have emphasized primarily *analytical* image reconstruction methods or iterative methods that are based on algebraic principles rather than statistical models.

Some of the earliest tomographic images created by Hounsfield [1, 2] for X-ray computed tomography were formed using iterative methods. And even earlier than that, Muehllehner and Wetzel described an iterative method for (4 view) SPECT reconstruction [3]. Around the same time, Kuhl et al. also developed iterative methods for SPECT [4, p. 183] [5]. However, by the mid 1970's the analytical filter-backproject (FBP) method quickly became the method used exclusively in routine clinical practice for tomographic reconstruction for many years [6, p. 695]. In the late 1990's, **statistical image reconstruction** (**SIR**) methods (of the type described in Chapter 16) were first introduced commercially for reconstructing SPECT and PET images, following many years of academic research demonstrating their benefits. SIR methods soon supplanted the venerable FBP algorithm clinically for PET and SPECT. Full SIR methods for X-ray CT (of the type described in this book) became commercially available in about 2011, motivated primarily by concerns about patient X-ray dose. The enormous computation time of SIR methods for X-ray CT has slowed its adoption and spurred considerable research into acceleration methods.

Image reconstruction methods for magnetic resonance imaging (MRI) are poised to undergo a revolution similar to that of PET, SPECT and X-ray CT in the near future. Historically, the exclusive reconstruction method used for clinical MRI has been the inverse fast Fourier transform (FFT). Many factors have increased interest in iterative reconstruction methods for MRI, including the introduction of parallel imaging with multiple coils, problems such as field inhomogeneity and non-Cartesian k-space trajectories, and incomplete sampling, often called **compressed sensing** [7, 8], particularly in dynamic imaging problems. Therefore, this book also describes MR image reconstruction problems even though the primary considerations in MRI are sampling, not noise statistics.

The increasing use of and interest in statistical image reconstruction methods motivated this text. The concepts needed for research and implementation of statistically based iterative reconstruction methods are spread far and wide over the literature, in journals with homes in engineering, mathematics, physics, radiology, and statistics. This book is an attempt to bring together in one place many of the key ingredients.

Although focused on PET, SPECT, X-ray CT, and MRI, tomographic reconstruction problems arise in numerous applications, some surprising, such as ecological inference from aggregate data [9, Section 6.2.4], and ionospheric measurements [10].

I had in mind two distinct audiences when writing this book. One audience is the researchers and students involved in developing new methods for image reconstruction. The other audience is the practitioners of medical (and nonmedical) imaging (medical physicists, etc.) who need to use image reconstruction for their work, and would benefit from making informed choices among the many reconstruction methods available. For the first audience, I have included the many mathematical details that one must master before one reaches a point where one can contribute useful new algorithms. The theorems and proofs throughout the text are aimed primarily at this audience. For the benefit of the more applied readers, I try to explain the practical implications of the theory, rather than adopting the traditional mathematical style where the theorems and proofs stand on their own with relatively little explanatory text. In other words, practical readers will find the text too theoretical, and mathematical readers will find the text too verbose. *C'est la vie...*

For the benefit of all readers, I have tried to include explicit algorithms in forms that are very close to their actual implementations. In addition, I provide free software (suitable for use with MATLAB or OCTAVE) that illustrates many of the algorithms described in this book an accompanying web site `http://web.eecs.umich.edu/~fessler`. Many sections of the book refer to this software.

The web site also lists any errata found for this book.

The book is designed to be reasonably self contained for readers who have familiarity with the basic principles of tomographic imaging systems. Some background in basic probability is essential for working with statistical methods. The appendices summarize some "well known" useful mathematical and statistical tools that are used in the text.

This book is organized into several parts.

- Part I contains chapters that describe some of the image reconstruction *problems*. The first chapter describes statistical methods for image restoration and the analysis of the properties of those methods. This chapter encapsulates many of the principal ideas of the book in a relatively simple setting. The next chapter considers regularization methods; these correspond to prior information about the object and are used throughout the applications. The next few chapters describe tomography problems: idealized analytical tomography, reconstruction from Fourier samples (*e.g.*, MRI), transmission tomography (X-ray CT), and emission tomography (PET, SPECT).

- Part II describes optimization *algorithms*. The first chapter in this part reviews the types of general purpose optimization methods that are often applied for image reconstruction. The next chapter emphasizes **optimization transfer** methods which are special-purpose algorithms that can be tailored to the specific problems in image reconstruction. The next chapter discusses convergence of such algorithms. Then there are chapters describing algorithms for specific problems: least squares problems, emission tomography, and transmission tomography. Regularized methods are treated in detail.
- Part III addresses the analysis of the *properties* of statistical image reconstruction methods, including spatial resolution and noise characteristics. The analysis of these properties aids in the design of regularization methods and in prediction of algorithm performance for specific imaging tasks such as signal detection.
- Part IV addresses more advanced topics such as dynamic image reconstruction and motion-compensated image reconstruction.
- The appendices provide mathematical background (probability, matrix analysis, etc.) needed for the main text.

Synopsis of image reconstruction

This book treats image reconstruction as an inverse problem of the following form. We are given a finite-length measurement vector $\boldsymbol{y}$, from which we want to recover a function f that describes some property of an object. For example, in emission tomography, f represents the 3D spatial distribution of a radiotracer. A very general block diagram for image reconstruction problems is the following.

$$\begin{array}{c}\text{unknown object}\\ f\end{array} \to \boxed{\begin{array}{c}\text{Imaging}\\ \text{system}\end{array}} \to \begin{array}{c}\text{data}\\ \boldsymbol{y}\end{array} \to \boxed{\begin{array}{c}\text{Reconstruction}\\ \text{algorithm}\end{array}} \to \begin{array}{c}\text{object estimate}\\ \hat{f}\end{array}$$

To design image reconstruction algorithms usually we begin by modeling the deterministic aspects of the imaging system, *i.e.*, modeling the measurements $\bar{\boldsymbol{y}}_{\mathrm{c}}(f)$ that would be recorded ideally in the absence of noise. The function $\bar{\boldsymbol{y}}_{\mathrm{c}}(\cdot)$ is a "**continuous-to-discrete**" (**C-D**) mapping from the set of continuous-space functions f to the space of n_{d}-dimensional vectors $\bar{\boldsymbol{y}}$. In practice the measurements are always contaminated by noise; in some cases the measurement noise is well modeled as being additive, as illustrated by the following diagram.

e,synopsis,model,2

$$\begin{array}{c}\text{unknown object}\\ f\end{array} \to \boxed{\begin{array}{c}\text{Ideal C-D}\\ \text{System}\end{array}} \to \begin{array}{c}\text{noiseless data}\\ \bar{\boldsymbol{y}}_{\mathrm{c}}(f)\end{array} \to \underset{\substack{\uparrow\\ \text{noise}}}{\bigoplus} \to \begin{array}{c}\text{noisy data}\\ \boldsymbol{y}\end{array} \qquad (0.0.1)$$

We can never determine f exactly for noisy data, but we can try to find an estimate $\hat{f}$ that hopefully is a useful approximation of f. In choosing $\hat{f}$, often we have two conflicting goals. We would like $\hat{f}$ to "fit the data," *i.e.*, we would like some measure of data mismatch $d(\bar{\boldsymbol{y}}_{\mathrm{c}}\left(\hat{f}\right), \boldsymbol{y})$ to be small. But we do not want to fit the *noise* in the data, *i.e.*, we want $\hat{f}$ to be compatible with any prior expectations about the characteristics of f. For example, often we assume that f is smooth or piecewise smooth.

Because f is a continuous-space function whereas $\boldsymbol{y}$ is a finite-length vector, often we choose a finite-dimensional parametric model for f, where the finite-length vector $\boldsymbol{x}$ denotes the model parameters (*e.g.*, pixel values for a pixelized model). Usually the model involves a linear combination of basis functions, *i.e.*,

e,synposis,f=Bx

$$f = \boldsymbol{\mathcal{B}}_{\square}\boldsymbol{x}, \qquad (0.0.2)$$

for some linear operator $\boldsymbol{\mathcal{B}}_{\square}$ that maps the n_{p}-dimensional vector $\boldsymbol{x}$ into a continuous-space function f. In other words, each "column" of $\boldsymbol{\mathcal{B}}_{\square}$ is a basis function. With this parameterization, the block diagram for the measurement model becomes the following.

$$\begin{array}{c}\text{unknown}\\ \text{parameters}\\ \boldsymbol{x}\end{array} \to \boxed{\begin{array}{c}\text{Basis}\\ \text{functions}\\ \boldsymbol{\mathcal{B}}_{\square}\end{array}} \to \begin{array}{c}\text{unknown}\\ \text{object}\\ f = \boldsymbol{\mathcal{B}}_{\square}\boldsymbol{x}\end{array} \to \boxed{\begin{array}{c}\text{Ideal C-D}\\ \text{System}\end{array}} \to \begin{array}{c}\text{noiseless}\\ \text{data}\\ \bar{\boldsymbol{y}}_{\mathrm{c}}(f)\end{array} \to \underset{\substack{\uparrow\\ \text{noise}}}{\bigoplus} \to \begin{array}{c}\text{noisy}\\ \text{data}\\ \boldsymbol{y}\end{array}$$

By using a parametric model for the object f, we can express the noiseless measurements in terms of the parameter vector $\boldsymbol{x}$ using a "**discrete-discrete**" (**D-D**) mapping

e,synopsis,yb,x

$$\bar{\boldsymbol{y}}(\boldsymbol{x}) \triangleq \bar{\boldsymbol{y}}_{\mathrm{c}}(\boldsymbol{\mathcal{B}}_{\square}\boldsymbol{x}) \qquad (0.0.3)$$

that maps an n_p-dimensional image parameter vector $\boldsymbol{x}$ into a n_d-dimensional ideal measurement vector $\bar{\boldsymbol{y}}$. With this definition we can simplify the block diagram by combining the first two blocks as follows.

$$\begin{array}{ccccccccc} \text{unknown} & & & & \text{noiseless} & & & & \text{noisy} \\ \text{parameters} & \rightarrow & \boxed{\begin{array}{c}\text{Ideal D-D}\\ \text{System}\end{array}} & \rightarrow & \text{data} & \rightarrow & \bigoplus & \rightarrow & \text{data} \\ \boldsymbol{x} & & & & \bar{\boldsymbol{y}}(\boldsymbol{x}) & & \uparrow & & \boldsymbol{y} \\ & & & & & & \text{noise} & & \end{array} \qquad (0.0.4)$$ e,synopsis,x-y

With this D-D formulation, we first compute an estimate $\hat{\boldsymbol{x}}$ of the parameter vector $\boldsymbol{x}$ and then use that to synthesize $\hat{f}$ if needed using (0.0.2), as illustrated below.

$$\begin{array}{ccccccccc} \text{noisy} & & & & \text{parameter} & & & & \text{object} \\ \text{data} & \rightarrow & \boxed{\begin{array}{c}\text{Reconstruction}\\ \text{algorithm}\end{array}} & \rightarrow & \text{estimate} & \rightarrow & \boxed{\begin{array}{c}\text{Synthesis}\\ \text{(D/A)}\end{array}} & \rightarrow & \text{estimate} \\ \boldsymbol{y} & & & & \hat{\boldsymbol{x}} & & & & \hat{f} = \boldsymbol{\mathcal{B}}_\square\, \hat{\boldsymbol{x}} \end{array}$$

The last synthesis step is a kind of "**digital to analog**" (**D/A**) conversion that is usually done implicitly by using a digital display, rather than done computationally.

This book (and the image reconstruction literature) focuses primarily on the D-D model (0.0.4) and algorithms for computing $\hat{\boldsymbol{x}}$, but it is important to be aware that the model (0.0.1) is closer to reality. Too many papers generate simulated data using the same model (0.0.4) that is used for the reconstruction algorithm, a process called the **inverse crime** [11]. This "crime" can be avoided by using simulators based on (0.0.1), *e.g.*, [12].

In statistical methods for inverse problems, typically we determine an estimate $\hat{\boldsymbol{x}}$ of the parameter vector $\boldsymbol{x}$ by minimizing a cost function of the following form:

e,synopsis,xh
$$\hat{\boldsymbol{x}} = \operatorname*{arg\,min}_{\boldsymbol{x} \in \mathcal{X}} d(\bar{\boldsymbol{y}}(\boldsymbol{x}), \boldsymbol{y}) + \mathsf{R}(\boldsymbol{x}), \qquad (0.0.5)$$

where $\mathsf{R}(\boldsymbol{x})$ is a **regularizer** (*e.g.*, a roughness penalty) that controls the trade-off between spatial resolution and noise.

The principal topics of this book are all present in the expression (0.0.5). To perform image reconstruction, one must consider the following components.

- System model $\bar{\boldsymbol{y}}_\mathrm{c}(f)$. For any given f, what would noiseless measurements be? This depends on the sensor physics.
- Log-likelihood / data fit term $d(\bar{\boldsymbol{y}}, \boldsymbol{y})$. This term depends on the statistical model for the measurements.
- Regularization method / prior $\mathsf{R}(\boldsymbol{x})$. How do we control noise without degrading desired signal features?
- Object model parameterization $f = \boldsymbol{\mathcal{B}}_\square\, \boldsymbol{x}$. One must compromise between accuracy and computation time.
- Constraint set $\mathcal{X}$. For example, in tomography usually f is nonnegative. In MRI, although f is often complex, sometimes its phase is (assumed) known.
- Minimization algorithm (argmin). How do we ensure convergence to the minimizer, and how to provide rapid convergence?
  Ideally the choice of the cost function and the iterative algorithm should be kept distinct, *i.e.*, the cost function should be chosen based on models and statistical principles, and then the algorithm should be chosen based on how fast it minimizes the chosen cost function.
- Properties of $\hat{\boldsymbol{x}}$ or $\hat{f} = \boldsymbol{\mathcal{B}}_\square\, \hat{\boldsymbol{x}}$, such as spatial resolution and noise. Here the analysis is complicated because $\hat{\boldsymbol{x}}$ is often a nonlinear function of $\boldsymbol{y}$.
- Consideration of departures from modeling assumptions. For example, what is the effect of object motion during imaging, and how should the reconstruction method be designed to compensate for such effects?

As an example, Fig. 0.0.1 shows a simulation of a true object $\boldsymbol{x}_\mathrm{true}$, and noisy and blurry data $\boldsymbol{y}$ for an image restoration problem with shift-invariant blur, as discussed in Chapter 1. Also shown are two restorations $\hat{\boldsymbol{x}}$; one computed by a non-iterative, non-statistical method, and the other by an iterative, statistical method described in Chapter 1.

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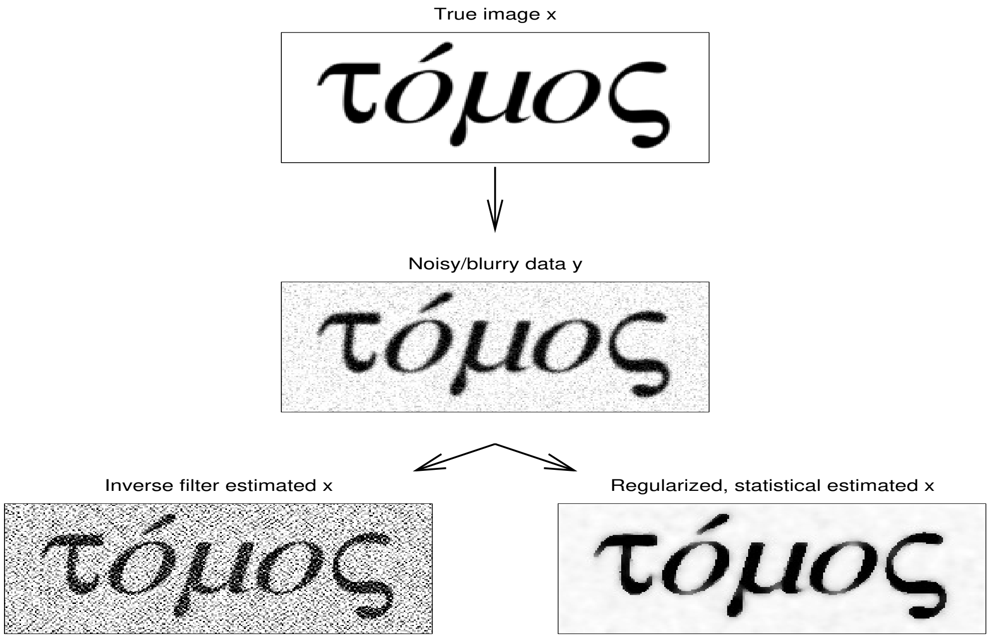

fig,front,ex

Figure 0.0.1: Illustration of image restoration by non-statistical (left) and statistical (right) methods.

Related references

There are several existing books on tomography, most of which were published several years ago and emphasize analytical image reconstruction methods. An exception is the more recent book by Natterer [13] that includes a chapter about maximum-likelihood methods.

Many books describe mathematical aspects of Radon transform [13–18]. Books that describe general issues in tomographic reconstruction and treat algebraic reconstruction methods include [19–22]. Regularization and inverse problem treatments include [23–32]. Webb gives a fascinating history of medical tomography [4]. Mathematical image processing texts include [28, 29, 33–36]. Many books cover the general principles and physics of tomographic imaging systems, including [37–43]. For MRI, I have found [41] to be particularly helpful.

There have been numerous review papers and book chapters that summarize statistical image reconstruction methods, including [44–56].

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