A Simplified Approach to Two-Port Analysis in Feedback

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Abstract—In this paper, a new pedagogical approach for analyzing the negative feedback circuits is proposed. The presented approach is in fact the completed form of the well-known twoport network analysis which is the most intuitive method for teaching the negative feedback concept. The two-port network analysis is rewritten in a more general and conceptual format. In analyzing the output series feedback, the presented analysis resolves prior shortcomings. The presented approach helps the students analyze and design all types of negative feedback circuits more intuitively.

Keywords—Feedback, Signal Flow Graph (SFG), Analysis of Feedback Circuits, Feedback Configurations, Loading Effect, Input/Output impedance.

I. INTRODUCTION

THE concept of negative feedback is well recognized in analysis and design of electronic circuits [1]. The basic modules of an operational amplifier which produce greater than unity gain are electronically constructed using active elements. These elements are subject to severe variations versus temperature, supply voltage, process and aging. Providing a constant gain versus the above mentioned variations of active element parameters is one of the most important reasons for employing feedback in practical circuits [2], [3], [4]. Distortion reduction is another outcome of feedback circuits [2], [5]. All these advantages come at the price of gain reduction.

Educating a successful electronic engineer requires providing him or her with the comprehensive understanding of the role of each part of the circuit on the overall system performance features. The very basic and most accurate method for analyzing the feedback circuits is through using the wellknown KVL-KCL relations. This method will definitely yield accurate results. However, writing long and complicated KVL-KCL relations gives rise to the probability of computational mistakes and these equations will hardly enhance the students' intuition. Signal flow graph (SFG) is another well-known technique for feedback analysis [6]. It helps the student perceive the mechanism of feedback by engaging a graphical method, but, due to numerous complicated mathematical equations, it fails to give them any sense of where the equations head to. Furthermore, this method does not give any insight about the effect of feedback on decreasing or increasing different circuit parameters. The other commonly used method for handling feedback circuits is Return-Ratio [1]. This method tries to compute the loop gain using the conventional approach for

analyzing electronic circuits, and then the other parameters will be computed using the calculated loop gain. Still, the computed return ratio and defined loop gain have some differences while they are supposed to be equal. This technique, despite being probably the best method for advanced designers, is a bit too complex for beginning students. Perhaps the more intuitive method of feedback analysis presented in many circuit analysis textbooks, is based on the concept of Two-Port Networks [7]. This analysis scheme is basically based on derivation of the closed-loop parameters from the open-loop gain (a) and the feedback factor (f) [8] and is originated from twoport networks theory [2]. In comparison to other mentioned feedback analysis techniques, this method helps the students perceive the effect of feedback on many circuit parameters such as gain, input and output impedance. Reference [9] comes up with a generalized feedback model based on two-port theory which resolves the issue of analyzing local feedback (source/emitter degeneration). Local feedback, however, can be addressed with some simple derivation not worth using feedback model.

The two-port network analysis approach has its foundations in the circuit theory, but it has been developed to be specifically used in feedback circuits. It employs the concept of network matrices, such as impedance and admittance, while avoiding tedious mathematical calculations. One of the most important problems in analyzing feedback circuits through this technique is calculation of the output impedance which is an important parameter of an operational amplifier. As an example, the main referenced books in teaching circuit analysis are unable to provide a thorough analysis of output series feedback impedance using this recent approach (the case of sensing output current) [2], [10]. There is just a vague explanation of the output series feedback impedance in one special case which is neither complete nor entirely accurate in Gray and Meyer's book [2] and [11]. Another attempt for analyzing the case of output series feedback is reported in [12]. Nevertheless, this work has failed to bring an intuitive method while only considering a special case. A similar work has been reported in [13] which again suffers from the same drawbacks, i.e., neither providing any intuitive understanding nor considering all cases.

In this paper, a complete conceptual approach based on twoport network analysis is provided for analyzing different types of feedback circuits. The output impedance is then calculated in the special case of series feedback at the output (sensing the output current). The paper is organized as follows. In section II, practical configurations and the effect of loading of four possible feedback network configurations are investigated and analytical derivations of the output impedance of series-series

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Fig. 1. Universal categorization of all different input feedback connections (i: input signal, f: feedback signal).

feedback are then developed. We verify the presented method using KVL-KCL, as well as SFG analysis in section III. Section IV concludes the paper. This technique is appropriate for feedback circuits with any kind of transistors. Without loss of generality, we concentrate on bipolar case since it is more general and the results could be easily extended to other circuits with MOS transistors.

II. THE PROPOSED FEEDBACK CIRCUIT ANALYSIS

A. Practical Configurations and the Effect of Loading

In this section, a complete conceptual approach toward the analysis of feedback circuits based on the two-port network solution is proposed. In the two-port solution, the students must compare the circuit against one of the four possible network configurations: shunt-shunt, series-series, series-shunt and shunt-series, where loading issues can be modeled using Y, Z, h and g parameters, respectively. This process is exhaustive and needs remembering all four two-port network configurations and their relations. To make this procedure faster, easier, and more intuitive, we suggest recognizing the nature of the feedback based on the circuit configurations and feedback connections. Fig. 1 and Fig. 2 show all possible configurations

of feedback circuits when connected to the circuit input and output.

The topology with feedback signal applied to the collector/drain of the output transistor is irrelevant and never used [a(3), b(6)]. This is because neither the difference between the output and input voltages nor the difference between the input and output currents are measured and fed to the circuit in this case. Furthermore, the case of a differential pair will be reduced to the case of c(7) in Fig. 1, where the feedback signal is assumed to reach Q1 through its emitter or reduced to the case c(8) in Fig. 1, where the feedback signal is assumed to reach Q1 through its base.

Once the type of the feedback is recognized by matching to the cases presented in Fig. 1 and Fig. 2, which are obviously more intuitive, it remains for the student to take into account the loading effect of the feedback circuit. Referring to the four parameterized boxes (Y, Z, h and g), loading effect calculation would be cumbersome. Sedra and Smith [14] use a linguistic connection relating series with open-circuit and shunt with short-circuit to help the students calculate the loading effect more easily. Instead, the loading could be calculated based on the diagrams presented here. As an example for shunt-series feedback, consider the circuit shown in case (d) of Fig. 3. It is obvious that, $i_{if} = -(\frac{R_2}{R_1+R_2})i_{of}$ and the equivalent



Fig. 2. Universal categorization of all different output feedback connections (i: input signal, f: feedback signal).



Fig. 3. Simple loading analysis after feedback type is recognized by using intuitively clear open/shorts i_{of}/v_{of} .

impedances seen through the input port, R_{if} , and output port, R_{of} , are

$$R_{if} = R_1 + R_2, (1a)$$

$$R_{of} = R_1 || R_2. \tag{1b}$$

Note that, for the circuits connected as above, the output loading computation would require i_{if} to be shorted to ground. For the input loading, we note that the output is already open-circuit due to the nature of the circuit so students not need change anything.

Looking at cases (a) to (d) in Fig. 3, we can see a simplified pattern. In all cases, the loading of the feedback circuit on the forward path can be easily calculated from the bottom pictures without memorizing anything. For the output loading a short-circuit is already there in the input shunt circuits [see Figs. 3(a) and 3(d)], and an open is visible for series circuits [see Figs. 3(b) and 3(c)]. For the input loading voltage sources visible in the output shunt circuits need to be shorted, while the current sources in the output series circuits naturally need to be opened.

The feedback circuit analysis using four-port models is simplified as follows. First the students must recognize the feedback types based on the rules outlined in Fig. 1 and Fig. 2. Then, the feedback circuit must be isolated and driven by voltage or current sources as required. Finally, the loading effects can be extracted by almost simple inspection without memorizing anything.

After determining the type of feedback circuit and computing its loading effect, it remains to calculate the input and output impedances, which is expected to be straightforward. In fact, a missing link in teaching feedback circuit analysis to engineering students is calculating the output impedance in the case of sensing current at output port (series feedback at the output), which is of a great consequence and is not covered in any textbook. For example, in Gray and Meyer's book [2] the only vaguely studied case is limited to the seriesseries feedback with the output assigned to the collector and the output feedback connected to the emitter, and no comment has been made on the other case, when the output is sensed at the emitter and the feedback branch is connected to the collector. We will go through this calculation during the next section.

B. Derivation of the Output Impedance of Series-Series Feedback

While the two-port approach is the most popular method of analyzing feedback loops, not all kinds of feedback circuits have been analyzed with this technique in different main references for teaching circuit analysis [2], [14]. Referring to these textbooks, the output impedance of the output series feedback has remained unclear based on the two-port approach. We find only a vague explanation that the output impedance will increase as a result of the output series feedback for instance (section 8.5.2, in [2] page 575). In its explanation, it has been mentioned that the output impedance is increased by a factor of 1 + T, where T is the loop gain, however what is meant by the output impedance? To enlighten this



Fig. 4. An amplifier circuit with the output series feedback (output assigned to the collector and the current sensed through the emitter): a) Schematic, b) Thevenin equivalent circuit.



Fig. 5. An amplifier circuit with output series feedback (output assigned to the emitter and the current is sensed through collector): a) Schematic b) Thevenin equivalent circuit.

matter, let us go more thoroughly through the concept of output series feedback and its effect. What is controlled by this feedback circuit is the variation in output current which can be interpreted as an enlarged output resistance in the branch determining the changes in the output current, which we call the output branch. We represent all prior stages with a Thevenin equivalent of $v_{th} = K.v_{in}$ and $R_{th} = r_{out}$, where K is the gain of the prior stages and r_{out} is the output resistance of all previous stages. Now, the output current for the typical equivalent circuit shown in Fig. 4(b) (neglecting the base series resistance) could be written as

$$i_o = \frac{K . v_{in}}{R_1 + \frac{r_{out} + r_{\pi}}{\beta + 1}}.$$
 (2)

The branch containing R_1 , $r_e = r_{\pi}/(\beta + 1)$, and r_{out} , is hereafter called the output branch. Therefore, by adding the output series feedback to the circuit [see Fig. 4(b)] the output branch resistance with feedback, R_{bf} , will be equivalent to

$$R_{bf} = (R_1 + \frac{r_{out} + r_{\pi}}{\beta + 1})(1 + af).$$
(3)

As is shown in Fig. 4(a) the series feedback is connected to the emitter of the output transistor and thus the output resistance, is measured in the output transistor collector. For the open-loop circuit, the output impedance could be calculated based on the equation presented in reference [2] for the common emitter output stage with the emitter degeneration resistance, R_E and base resistance, R_S

$$R_X \simeq r_o \frac{1 + g_m R_E + g_m \frac{R_S}{\beta}}{1 + g_m \frac{R_E}{\beta} + g_m \frac{R_S}{\beta}}.$$
 (4)

Considering the closed-loop feedback circuit, the emitter resistance, R_E , is a part of the output branch and could be approximately replaced by R_{bf} [from equation (3)]

$$R_X \simeq r_o \frac{1 + g_m (1 + af) (R_1 + \frac{r_{out} + r_m}{\beta + 1}) + g_m \frac{r_{out}}{\beta}}{1 + g_m \frac{(1 + af)(R_1 + \frac{r_{out} + r_m}{\beta + 1})}{\beta} + g_m \frac{r_{out}}{\beta}}.$$
 (5)

1 + af is approximately equal to $\frac{R_1(K+1) + \frac{r_{out} + r_{\pi}}{\beta+1}}{R_1 + \frac{r_{out} + r_{\pi}}{\beta+1}}$, where K and r_{out} denote the gain and the output resistance of the opamp, respectively, therefore we can rewrite (5) as

$$R_X \simeq r_o \frac{1 + g_m(\frac{R_1(K+1)(\beta+1) + r_{out} + r_\pi}{\beta+1}) + g_m \frac{r_{out}}{\beta}}{1 + g_m(\frac{R_1(K+1)(\beta+1) + r_{out} + r_\pi}{(\beta+1)\beta}) + g_m \frac{r_{out}}{\beta}}.$$
 (6)

So we can rewrite (6) as

$$R_X \simeq r_o \frac{R_1(K+1)(\beta+1) + 2r_{out} + 2r_{\pi}}{\frac{R_1(K+1)(\beta+1) + r_{out}(\beta+1) + r_{\pi}(\beta+1)}{\beta}}.$$
 (7)

If $R_1(\beta + 1)(K + 1)$ is much greater than $r_{out}(\beta + 1) + r_{\pi}$, we can rewrite (6) as

$$R_X \simeq r_o \frac{1 + R_1 g_m(K+1)}{1 + R_1 \frac{g_m(K+1)}{\beta}}.$$
(8)

Thus, the overall output resistance is the parallel equivalent of the value obtained from (7) and the collector resistance, meaning: $R_{out} = R_2 || R_X$. Generally, $R_2 \ll R_X$ and thus the total output resistance will be close to R_2 . This approximate value has always been used in the previous references due to the fact that usually $R_2 \ll R_X$ and there seemed no need to derive R_X . However in today's CMOS design, R_2 is normally replaced by a current source, while on the other hand the output resistance, r_o , of nanometer MOS devices is increasingly becoming smaller, calling for a more rigorous analysis like the above.

The second model for the connection of the output series feedback which could be seen in different circuits is where the feedback network is connected to collector of the output transistors and thus the output voltage and therefore the output resistance is measured at the emitter of this transistor [Fig. 5(a)]. As it can be inferred from this figure, $R_{out} = R_X ||R_2$ where R_X is the resistance seen upward through the emitter. Thus, to complete the relation, we should derive the value of R_X . As it was mentioned in the previous section, the output branch comprises $R_2, r_e = r_\pi/\beta$, and r_{out} , therefore, by adding the output series feedback to the circuit [see Fig. 5(b)] the output branch resistance with feedback, R_{bf} , will be equivalent to

$$R_{bf} = (R_2 + \frac{r_{out} + r_{\pi}}{\beta})(1 + af).$$
(9)

Furthermore, the output branch impedance could be rewritten as

$$R_{bf} = R_2 + R_X, \tag{10}$$

and therefore

$$R_X = R_{bf} - R_2. (11)$$

1 + af is equal to $1 + \frac{KR_1}{R_2 + \frac{r_{out} + r_{\pi}}{\beta}}$, where K and r_{out} denote the gain and the output resistance of the op-amp, respectively, so we can rewrite (11) as

$$R_X = \frac{R_1 K \beta + r_{out} + r_{\pi}}{\beta}.$$
 (12)

As a result, the overall output impedance will be

$$R_{out} = R_X || R_2, \tag{13}$$

while replacing the value of R_X from (13) yields

$$R_{out} = \frac{R_1 K \beta + r_{out} + r_{\pi}}{\beta} ||R_2.$$
(14)

Given that the output series feedback always occurs in one of the two forms considered here, the presented analysis could be used in any kind of output series feedback circuit to derive the output impedance easily. In order to confirm our proposed approach, in the next section, the derived relations for output resistance has been confirmed through the well-known SFG and KVL-KCL methods for a typical amplifier circuit.

III. THE PROPOSED ANALYSIS JUSTIFICATION THROUGH SFG AND KVL-KCL ANALYSIS

In this section, we derive the output resistance of the two discussed types of output series feedback with two well-known methods of SFG and KVL-KCL, for the sample amplifier circuits shown in Fig. 4 and Fig. 5.

The brief derivation of the output impedance for the first case (see Fig. 4) with SFG and KVL-KCL will be explained in the following.

A. First Output Series Feedback Analysis Through SFG

In this section, first type of output series feedback (see Fig. 4) is analyzed through SFG method. The output impedance is derived by substituting transistor's hybrid- π model where $v_{in} = 0$ and a test voltage (v_X) is used to drive the amplifier output, and the resulting i_X is then calculated [see Fig. 6(a)].



Fig. 6. Equivalent circuit of the first case (see Fig. 4), with the circuit replaced with its a) hybrid- π model, b) Flow graph model.



Fig. 7. The hybrid- π model of the first case (see Fig. 4), for output impedance calculation.

This model simplifies to

$$i_o = \frac{v_\pi}{r_\pi} + g_m v_\pi + \frac{v_X - v_C}{r_o},$$
 (15)

in which v_C and i_X is given by

$$v_C = i_o R_1, \tag{16}$$

and

$$i_X = \frac{v_X - v_C}{r_o} + g_m v_\pi,$$
 (17)

respectively, where

$$v_{\pi} = \frac{(Kv_{diff} - v_C)r_{\pi}}{r_{out} + r_{\pi}},\tag{18}$$

and

$$v_{diff} = -R_1 i_o \frac{R_{in}}{R_{in} + R_1}.$$
 (19)

Hence, $R_{in} \gg R_1$, then $R_{in}/(R_{in} + R_1) \simeq 1$.

So we can rewrite (19) as

$$v_{diff} = -R_1 i_o. \tag{20}$$

These equations can be represented graphically as shown in Fig. 6(b). According to Mason's formula [15], the transmission gain, $1/R_X$, from node v_X to node i_X is

$$\frac{i_X}{v_X} = \frac{1}{R_X} = \frac{1}{\Delta} \sum_k P_k \Delta_k,$$
(21)

where P_k is the gain of the k^{th} forward path, Δ is the determinant of the graph, and Δ_k is the determinant with the k^{th} forward path eliminated.

The determinant is then calculated by

$$\Delta = 1 - \left[\frac{Kr_{\pi}}{r_{out} + r_{\pi}}(g_m + \frac{1}{r_{\pi}})(-R_1) + R_1 \frac{-1}{r_o} + R_1 \frac{-r_{\pi}}{r_{out} + r_{\pi}}(g_m + \frac{1}{r_{\pi}})\right].$$
(22)

 P_k and Δ_k for k = 1, 2, 3, 4 is as

$$P_1 = \frac{1}{r_o},\tag{23a}$$



Fig. 8. Equivalent circuit of the second case (see Fig. 5), with the circuit replaced with its a) hybrid- π model, b) Flow graph model.

$$\Delta_1 = \Delta, \tag{23b}$$

$$P_2 = \frac{1}{r_o} R_1 \frac{-1}{r_o},$$
 (23c)

$$\Delta_2 = 1, \tag{23d}$$

$$P_3 = \frac{1}{r_o} R_1 \frac{-r_\pi}{r_{out} + r_\pi} g_m,$$
 (23e)

$$\Delta_3 = 1, \tag{23f}$$

$$P_4 = \frac{1}{r_o} (-R_1) \frac{K r_{\pi}}{r_{out} + r_{\pi}} g_m,$$
 (23g)

$$\Delta_4 = 1. \tag{23h}$$

Substituting (22) through (23h) into (21) and rearranging gives

$$R_X = \frac{r_o(1 + R_1 \frac{(\beta+1)(K+1)}{r_{out} + r_{\pi}}) + R_1}{1 + \frac{R_1(K+1)}{r_{out} + r_{\pi}}}.$$
 (24)

Rearranging (24) obtains

$$R_X \simeq \frac{r_o(R_1(K+1)(\beta+1) + r_{out} + r_{\pi}) + R_1(r_{out} + r_{\pi})}{R_1(K+1) + r_{out} + r_{\pi}}.$$
(25)

If
$$r_{\pi} \gg r_{out}$$
 and $\frac{r_o(\beta+1)(K+1)}{r_{out}+r_{\pi}} \gg 1$, we can rewrite (24) as

$$R_X = r_o \frac{1 + R_1 g_m(K+1)}{1 + R_1 \frac{g_m(K+1)}{\beta}},$$
(26)

which confirms the obtained equation in (8).

In order to find total output resistance, R_X should be considered in shunt with R_2 .

B. First Output Series Feedback Analysis Through KVL-KCL Analysis

In this part, we are going to reconfirm the derived equation for output impedance in the first type of output series feedback through KVL-KCL analysis. Therefore, the circuit can be analyzed by writing the KVL-KCL nodal relations of its hybrid- π model where $v_{in} = 0$ and a test voltage (v_X) is used to drive the amplifier output and the resulting i_X is then calculated (see Fig. 7).

KCL at node A gives

$$\frac{v_A}{R_{in}} + \frac{v_A}{R_1} = \frac{v_\pi}{r_\pi} + g_m v_\pi + \frac{v_X - v_A}{r_o}.$$
 (27)

KCL at node B gives

$$\frac{v_A + v_\pi - K v_{diff}}{r_{out}} + \frac{v_\pi}{r_\pi} = 0.$$
 (28)

KVL around loop I gives

$$v_{diff} = -v_A. (29)$$

KCL at node C gives

$$i_X = \frac{v_X - v_A}{r_o} + g_m v_\pi.$$
 (30)

Combination of (27) to (30) obtains

$$\frac{v_X}{i_X} = R_X = \frac{r_o(1 + R_1 \frac{(\beta+1)(K+1)}{r_{out} + r_{\pi}}) + R_1}{1 + \frac{R_1(K+1)}{r_{out} + r_{\pi}}}.$$
 (31)



Fig. 9. The hybrid- π model of the second case (see Fig. 5), for output impedance calculation.

Rearranging (31) obtains

$$R_X \simeq \frac{r_o(R_1(K+1)(\beta+1) + r_{out} + r_{\pi}) + R_1(r_{out} + r_{\pi})}{R_1(K+1) + r_{out} + r_{\pi}}.$$
(32)

If $r_{\pi} \gg r_{out}$ and $\frac{r_o(\beta+1)(K+1)}{r_{out}+r_{\pi}} \gg 1$, we can rewrite (31) as

$$R_X \simeq r_o \frac{1 + R_1 g_m(K+1)}{1 + R_1 \frac{g_m(K+1)}{\beta}},$$
(33)

which again confirms (8).

To find total output resistance, R_X should be considered in shunt with R_2 .

For typical values of the circuit parameters, *i.e.*, $\beta = 100$, K = 1000, $r_{out} = 500k\Omega$, $R_1 = 1k\Omega$, $r_{\pi} = 2.5k\Omega$, $r_o = 100K\Omega$, equation (7) provides an output impedance, $R_X = 6.724M\Omega$ while equations (25) and (32) give $R_X = 6.758M\Omega$, that results in an error of 0.5%.

In a similar manner, the output impedance for the second type of output series feedback (see Fig. 5) is also verified with both SFG and KVL-KCL analysis while the corresponding equations are briefly described in the next two subsections.

C. Second Output Series Feedback Analysis Through SFG

In this subsection, the second type of output series feedback circuit is analyzed by substituting the transistor's hybrid- π model where $v_{in} = 0$ and a test voltage (v_X) is used to drive the amplifier output, and the resulting i_X is then calculated [see Fig. 8(a)]. The corresponding circuit equations are briefly presented here.

This model simplifies to

$$i_X = \frac{-v_\pi}{r_\pi} - g_m v_\pi - \frac{v_C - v_X}{r_o}.$$
 (34)

As $g_m + \frac{1}{r_\pi} \simeq g_m$, rearranging (34) gives

$$v_{\pi} = \frac{-i_X}{g_m} + \frac{-v_C}{g_m r_o} + \frac{v_X}{g_m r_o}.$$
 (35)

 v_C in (35) can be derived as

$$v_C = i_o R_1. \tag{36}$$

The test voltage (v_X) is given by

v

$$v_X = i_o r_o + v_\pi g_m r_o + v_C, \tag{37}$$

where

$$_{diff} = \frac{v_{\pi}(r_{out} + r_{\pi})}{Kr_{\pi}} + \frac{v_X}{K},$$
 (38)

and

$$R_1 i_o \frac{R_{in}}{R_{in} + R_1} = v_{diff}.$$
 (39)

Since $R_{in} \gg R_1$, we can rewrite (39)

$$i_o = \frac{v_{diff}}{R_1}.$$
(40)

These equations can be represented graphically as shown in Fig. 8(b). According to Mason's formula, the transmission gain, R_X , from node i_X to node v_X is

$$\frac{v_X}{i_X} = R_X = \frac{1}{\Delta} \sum_k P_k \Delta_k, \tag{41}$$

where P_k is the gain of the k^{th} forward path, Δ is the determinant of the graph, and Δ_k is the determinant with the k^{th} forward path eliminated.

The determinant can be calculated by

$$\Delta = -\frac{r_o\beta + r_{out} + r_\pi}{R_1 K \beta}.$$
(42)

 P_k and Δ_k for k = 1, 2, 3 is as

$$P_1 = \frac{-1}{g_m} \frac{r_{out} + r_\pi}{K r_\pi} \frac{1}{R_1} r_o,$$
 (43a)

$$\Delta_1 = 1, \tag{43b}$$

$$P_2 = \frac{-1}{g_m} \frac{r_{out} + r_\pi}{K r_\pi} \frac{1}{R_1} R_1 1,$$
(43c)

$$\Delta_2 = 1, \tag{43d}$$

$$P_3 = \frac{-1}{g_m} g_m r_o, \tag{43e}$$

$$\Delta_3 = 1. \tag{43f}$$

Substituting (42) through (43f) into (41) and rearranging gives

$$R_X = \frac{r_o R_1 K \beta + r_o (r_{out} + r_\pi) + R_1 (r_{out} + r_\pi)}{r_o \beta + r_{out} + r_\pi}.$$
 (44)

Since $r_o\beta \gg r_{out} + r_{\pi}$ and $r_o \gg R_1$,

we can rewrite (44) as

$$R_X = \frac{R_1 K \beta + r_\pi + r_{out}}{\beta},\tag{45}$$

which is exactly similar to the derived equation for R_X in (12).

In order to find total output resistance, R_X should be considered in shunt with R_2 .

D. Second Output Series Feedback Analysis Through KVL-KCL Analysis

Finally, the derived output resistance for the second type of output series feedback circuit is approved here through KVL-KCL equations. For this calculation, consider the equivalent circuit shown in Fig. 9, where $v_{in} = 0$ and a test voltage (v_X) is used to drive the amplifier output and the resulting i_X is then calculated.

KCL at node A gives

$$i_X + g_m v_\pi + \frac{v_B - v_X}{r_o} + \frac{v_\pi}{r_\pi} = 0.$$
(46)

KCL at node B gives

$$\frac{v_B}{R_1||R_{in}} + \frac{v_B - v_X}{r_o} + g_m v_\pi = 0.$$
 (47)

KVL around loop I gives

$$v_{diff} = v_B. \tag{48}$$

KCL at node C gives

$$\frac{v_{\pi}}{r_{\pi}} = \frac{K v_{diff} - (v_X + v_{\pi})}{r_{out}}.$$
(49)

Combination of (46) to (49) obtains

$$R_X = \frac{r_o R_1 K \beta + r_o (r_{out} + r_\pi) + R_1 (r_{out} + r_\pi)}{r_o \beta + r_{out} + r_\pi}.$$
 (50)

Since $r_o\beta \gg r_{out} + r_{\pi}$ and $r_o \gg R_1$,

we can rewrite (50) as

$$\frac{v_X}{i_X} = R_X = \frac{R_1 K \beta + r_\pi + r_{out}}{\beta}.$$
(51)

Again this equation is in good agreement with (12).

To find total output resistance, R_X should be considered in shunt with R_2 .

For typical values of the circuit parameters, *i.e.*, $\beta = 100$, K = 1000, $r_{out} = 500k\Omega$, $R_1 = 1k\Omega$, $r_{\pi} = 2.5k\Omega$, $r_o = 100k\Omega$, equation (12) provides an output impedance, $R_X = 1.005M\Omega$ and equations (44) and (50) give $R_X = 0.956M\Omega$ that results in an error of 5.01%.

The main contributors to this error are the op-amp's non idealities. For instance, the output resistance is assumed to be $500k\Omega$, whereas in a well designed op-amp in which a proper output stage is utilized, this value will be as small as 10Ω , resulting in a negligible error.

IV. DISCUSSION AND CONCLUSIONS

This paper demonstrated a simple method of deciding the feedback types followed by a more intuitive two-port analysis. This analysis covers all types of possible feedback circuits. Furthermore, in the second part of this paper, a simplified approach to the analysis of the ambiguous series feedback at the output was presented and rigorously proven via the well-known SFG and KVL-KCL methods.

The new method was used in class for more than six years. The methodology was shown on 20 circuit examples on the author's class website. ¹ The same circuits were also analyzed using Blac kmans's method [16] and our experience shows that the proposed feedback type recognition method removed many ambiguities that students confront when identifying the feedback box in the two-port analysis. The issue of determining the output impedance in series feedback at the output also removes ambiguities.

¹http://ee.sharif.edu/~elecprinc-SatMon.

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