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Multi-stage Strategic Investment in CCGTs and Wind Power Units via Progressive Hedging

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Abstract

Generation expansion decision-making problems within electricity markets are commonly complicated. Their complexity comes from the fact that modern power systems are complex dynamic systems with hardly foreseen short- and long-term financial and operational risks. As a result, appropriate decision-making tools are required to generate informed investment decisions.

This thesis focuses on a multi-stage capacity expansion problem from a strategic power producer point of view. This problem is formulated as a large-scale stochastic complementarity model that allows deriving the optimal timing, sizing, and siting of combined cycle gas turbines and wind power production technologies appropriately anticipating the market functioning and all inherent uncertainties. This way, the strategic producer can identify the optimal investment actions (long-term decisions) along with the offering strategy in the day-ahead and balancing markets (short-term decisions) for all assets in its generation portfolio including existing and candidate generation units.

The proposed model is finally recast as a stochastic mixed-integer linear programming problem. The tractability of the resulting model is limited by a significant number of discrete variables and constraints. In this line, the progressive hedging algorithm is applied to achieve a tractable solution with acceptable quality and simulation time. Particularly, two frameworks are developed to decompose the original problem (*i*) per long-term scenarios only and (*ii*) per both long- and short-term scenarios.

The practical interest of the proposed decision-making tool is demonstrated with a set of numerical experiments. In the first instance, a small-scale power system is considered to estimate the quality of the solution, the advantage of the multi-scale decomposition, and the benefit of the computational implementation concerning simulation time. Two larger case studies are additionally performed to prove the potential scalability of the tool for real-life applications.

Preface

This thesis was prepared in the Department of Electrical Engineering at the Technical University of Denmark (DTU). The thesis is set to be 30 ECTS and submitted in fulfillment of the requirements for the *Master of Science in Engineering (Sustainable Energy)*. The study was carried out from January 2017 to June 2017 under the supervision of Professor Jalal Kazempour and Professor Pierre Pinson from the Electrical Engineering Department of DTU and Professor Luis Baringo from the School of Industrial Engineering of University of Castilla-La Mancha.

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My foreign experience would not have been as rich had I not been a member of my large family. Thank you for giving me love, freedom of thought, and a great sense of purpose. All of these accompany me being abroad. I'm so proud to be one of you.

Contents

Nomenclature

Sets and Indexes

$\Delta^{\text{LL,D}}$ Set of lower-level dual decision variables

$\Delta^{\text{LL,P}}$ Set of lower-level primal decision variables

Δ^{UL} Set of upper-level decision variables

$\mathcal{B}$ Set of demand blocks, indexed by b.

$\mathcal{C}$ Set of candidate generation units, indexed by c.

$\mathcal{C}^{\text{Flex}}$ Subset of $\mathcal{C}$ comprising candidate flexible generation units, indexed by c.

$\mathcal{C}^{\text{WP}}$ Subset of $\mathcal{C}$ comprising candidate wind power generation units, indexed by c.

$\mathcal{D}$ Set of demand units, indexed by d.

$\mathcal{E}$ Set of existing generation units, indexed by c.

$\mathcal{E}^{\text{Flex}}$ Subset of $\mathcal{E}$ comprising existing flexible generation units, indexed by c.

$\mathcal{E}^{\text{WP}}$ Subset of $\mathcal{E}$ comprising existing wind power generation units, indexed by c.

$\mathcal{G}$ Set of long-term scenarios, indexed by either γ or γ'.

$\mathcal{H}$ Set of representative days, indexed by h.

$\mathcal{J}^{\text{Own,Flex}}$ Subset of $\mathcal{J}^{\text{Own}}$ comprising flexible generation units belonging to the strategic producer, indexed by j.

$\mathcal{J}^{\text{Own,WP}}$ Subset of $\mathcal{J}^{\text{Own}}$ comprising wind power generation units belonging to the strategic producer, indexed by j.

$\mathcal{J}^{\text{Own}}$ Set of generation units belonging to the strategic producer, indexed by j.

$\mathcal{J}^{\text{Sys,Flex}}$ Subset of $\mathcal{J}^{\text{Sys}}$ comprising all flexible generation units in the system, indexed by j.

$\mathcal{J}^{\text{Sys,WP}}$ Subset of $\mathcal{J}^{\text{Sys}}$ comprising all wind power generation units in the system, indexed by j.

$\mathcal{J}^{\text{Sys}}$ Set of all generation units in the system, indexed by j.

$\mathcal{K}$ Set of market scenarios, indexed by either k or k'.

$\mathcal{N}$ Set of nodes, indexed by either n or m.

$\mathcal{O}$ Set of available capacity options of candidate generation units, indexed by o.

$\mathcal{R}$ Set of rival generation units, indexed by r.

$\mathcal{R}^{\text{Flex}}$ Subset of $\mathcal{R}$ comprising rival flexible generation units, indexed by r.

$\mathcal{R}^{\text{WP}}$ Subset of $\mathcal{R}$ comprising rival wind power generation units, indexed by r.

$\mathcal{T}$ Set of time stages, indexed by either t and τ.

$\mathcal{U}$ Uncertainty space

$\mathcal{W}$ Set of wind power generation scenarios, indexed by ω.

$\Theta_{t\gamma}$ Set of adjacent nodes to node γ of the long-term decision tree at time period t

C Superscript defining candidate generation units.

E Superscript defining existing generation units.

R Superscript defining rival generation units.

Parameters

$\alpha_c^{\text{C}\uparrow}, \alpha_c^{\text{C}\downarrow}$ Share of installed capacity of candidate unit c capable of providing up- and down-reserve [%].

$\alpha_e^{\text{E}\uparrow}, \alpha_e^{\text{E}\downarrow}$ Share of installed capacity of existing unit e capable of providing up- and down-reserve [%].

χ^{SoS} Sequirity of supply factor [-].

ϵ Convergence criterion [MW].

$\hat{X}_{(\cdot)}^{\text{C}(i)}$ Probabilistic average of investment solutions of the adjacent nodes of decision tree [MW].

$\overline{F}_{nm}$ Capacity of transmission line between node n and m [MW].

$\overline{I}_t$ Investment budget for time stage t [\$].

$\overline{P}_{t\gamma db}^{\text{D}}$ Demand of block b of demand unit d at time stage t in long-term scenario γ.

$\overline{R}_r^{\text{R}\uparrow}, \overline{R}_{t\gamma r}^{\text{R}\downarrow}$ Up- and down-reserve capacity of rival unit r at time stage t in long-term scenario γ.

$\overline{X}_{co}^{\text{C}}$ Size of capacity option o of candidate unit c [MW].

$\pi_{\gamma}^{\mathrm{L}}$ Probability of long-term scenario γ [-].

$\pi_{\omega}^{\mathrm{W}}$ Probability of wind power generation scenario ω [-].

π_{k}^{MS} Probability of market scenario k [-].

DR_t Discount rate at time stage t [-].

a_t Amortization rate at time stage t [-].

b_{tkd}^{D} Utility of demand unit d at time stage t in scenario k [$/MWh]

c^{VoLL} Value of lost load [$/MWh].

$c_{t\gamma c}^{\mathrm{C}}$ Marginal production costs of candidate unit c at time stage t in long-term scenario γ [$/MWh].

$c_{t\gamma c}^{\mathrm{Inv}}$ Capital costs of candidate unit c at time stage t in long-term scenario γ [$/MW].

$c_{t\gamma e}^{\mathrm{E}}$ Marginal production costs of existing unit e at time stage t in long-term scenario γ [$/MWh].

$c_{t\gamma kr}^{\mathrm{R}\uparrow}, c_{t\gamma kr}^{\mathrm{R}\downarrow}$ Up- and down-reserve deployment costs of rival unit r at time stage t in long-term scenario γ in market scenario k [$/MWh].

$c_{t\gamma kr}^{\mathrm{R}}$ Marginal production costs of rival unit r at time stage t in long-term scenario γ in market scenario k [$/MWh].

$g_{(\cdot)}^{(i)}$ Convergence parameter [MW].

K_{hj}^{CF} Capacity factor of wind power generation unit j in representative day j [-].

K_{h}^{DF} Demand factor in representative day j [-].

$K_{j\omega}^{\mathrm{WS}}$ Wind scenario factor of wind power generation unit j in wind power scenario ω [-].

M Auxiliary large enough value [-].

$m_{(\cdot)}^{\mathrm{PH}(i)}$ Progressive hedging multiplier at iteration i [$/MW].

$N_{(\cdot)}$ Number of elements in vector $(\cdot)$.

N_{h}^{RD} Weight of the h^{th} representative day [-].

S_{nm} Susceptance of transmission line between node n and m [S].

X_{e}^{E} Installed capacity of existing unit e.

X_{r}^{R} Installed capacity of rival unit e.

Primal decision variables

$\beta^{(\cdot)\downarrow}_{t\gamma hk(\cdot)}$ Optimal down-reserve price bid of strategic producer's flexible generation units at time stage t in long-term scenario γ in representative day h in market scenario k [\$/MW].

$\beta^{(\cdot)\uparrow}_{t\gamma hk(\cdot)}$ Optimal up-reserve price bid of strategic producer's flexible generation units at time stage t in long-term scenario γ in representative day h in market scenario k [\$/MW].

$\beta^{(\cdot)}_{t\gamma hk(\cdot)}$ Optimal day-ahead price bid of strategic producer's generation units at time stage t in long-term scenario γ in representative day h in market scenario k [\$/MWh].

$\overline{P}^{(\cdot)}_{t\gamma hk(\cdot)}$ Optimal day-ahead energy bid of strategic producer's generation units at time stage t in long-term scenario γ in representative day h in market scenario k [MW].

$\overline{R}^{(\cdot)\downarrow}_{t\gamma hk(\cdot)}$ Optimal down-reserve quantity bid of strategic producer's flexible generation units at time stage t in long-term scenario γ in representative day h in market scenario k [MW].

$\overline{R}^{(\cdot)\uparrow}_{t\gamma hk(\cdot)}$ Optimal up-reserve quantity bid of strategic producer's flexible generation units at time stage t in long-term scenario γ in representative day h in market scenario k [MW].

$\theta^{\mathrm{RT}}_{t\gamma hkn\omega}$ Real-time voltage angle at bus n at time stage t in long-term scenario γ in representative day h in market scenario k in wind power scenario ω [-].

$\theta^{\mathrm{DA}}_{t\gamma hkn}$ Day-ahead voltage angle at bus n at time stage t in long-term scenario γ in representative day h in market scenario k [-].

$l^{\mathrm{sh}}_{t\gamma hkd\omega}$ Load shedding of block of demand unit d at time stage t in long-term scenario γ in representative day h in market scenario k in wind power scenario ω [MWh].

$P^{(\cdot),\mathrm{sp}}_{t\gamma hk(\cdot)\omega}$ Wind spillage of a wind power producer at time stage t in long-term scenario γ in representative day h in market scenario k in wind power scenario ω [MWh].

$P^{(\cdot)}_{t\gamma hk(\cdot)}$ Scheduled quantity of a generation unit at time stage t in long-term scenario γ in representative day h in market scenario k [MWh].

$P^{\mathrm{D}}_{t\gamma hkd}$ Scheduled quantity of demand unit d at time stage t in long-term scenario γ in representative day h in market scenario k [MWh].

$r^{(\cdot)\downarrow}_{t\gamma hk(\cdot)\omega}$ Down-reserve deployment of a flexible unit at time stage t in long-term scenario γ in representative day h in market scenario k in wind power scenario ω [MWh].

$r^{(\cdot)\uparrow}_{t\gamma hk(\cdot)\omega}$ Up-reserve deployment of a flexible unit at time stage t in long-term scenario γ in representative day h in market scenario k in wind power scenario ω [MWh].

$u^{\mathrm{C}}_{t\gamma co}$ Binary variable which decides whether to invest in capacity option o of candidate unit c at time-stage t in long-term scenario γ or not [-].

$X^{\mathrm{C}}_{t\gamma c}$ Installed capacity of candidate unit c to be built at time-stage t in long-term scenario γ [MW].

Dual Variables

$\lambda^{\mathrm{RT}}_{t\gamma hkn\omega}$ Dual variable of the real-time power balance constraint at time stage t in long-term scenario γ in representative day h in market scenario k at node n in wind power scenario ω [$/MWh].

$\lambda^{\mathrm{DA}}_{t\gamma hkn}$ Dual variable of the day-ahead power balance constraint at time stage t in long-term scenario γ in representative day h in market scenario k at node n, representing the day-ahead clearing price [$/MWh].

$\underline{\mu}^{(\cdot)}_{t\gamma hk(\cdot)}, \overline{\mu}^{(\cdot)}_{t\gamma hk(\cdot)}$ Dual variables for lower-level problem constraints at time stage t in long-term scenario γ in representative day h in market scenario k .

Auxiliary Variables

$\nu^{(\cdot)+}_{(\cdot)}, \nu^{(\cdot)-}_{(\cdot)}$ Special ordered set of type 1 (SOS1) variables used to linearize the complementary constraints of lower-level problem.

Acronyms

CCGT Combined cycle gas turbine

CPLEX Optimization software package

CVaR Conditional value at risk

DA Day-ahead market

DG Demand growth

DTU Technical University of Denmark

EPEC Equilibrium programming with equilibrium constraints

FC Fuel costs

GAMS General Algebraic Modeling System

IC Investment costs

IEEE Institute of Electrical and Electronics Engineers

KKT Karush–Kuhn–Tucker optimality conditions

LL Lower level

LMP Locational marginal price

LT Long-term

MILP Mixed-integer linear programming

MO Market operator

MPEC Mathematical programming with equilibrium constraints

MS Market scenario

OC Operating costs

PH Progressive hedging

PHA Progressive hedging algorithm

R Revenue

RAM Random-access memory

RI Rival investments

RT Real-time market

RTS Reliability test system

SOS1 Special ordered set of type one

ST Short-term

SW Social welfare

TSO Transmission system operator

UL Upper level

VaR Value at risk

WP Wind power

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CHAPTER 1

Introduction

1.1 Context and motivation

Investment planning is one of the most difficult tasks faced by decision-makers in power systems. In fact, investment decisions are carried out for years ahead, and without a proper preliminary analysis, they could lead to a plethora of barely predictable and costly implications. In an attempt to reduce the financial and operational damage caused by inappropriate investment decisions, research community proposed a broad set of decision-making models to support optimal planning. Practically, these models account for an extensive range of objectives, technical and economic aspects as well as policies to generate optimal power system expansion solutions.

At the moment, the complexity of the investment planning problems is partially enforced by growing uncertainty of multiple scales. First, a global trend towards emission-free power supply technologies leads to a drastic shift in generation mix to the variable and uncontrollable wind and solar generation. In fact, the hardly foreseen weather becomes one of the most significant drivers of market conjecture engendering increasing price volatility [1, 2]. Moreover, electricity energy as a commodity is traded in the closed auctions such that the true valuation of energy is private knowledge of each market participant. Thus, the informational asymmetry around pricing policies of the participants additionally contributes to price stochasticity. Second, technological advances and growing competition among renewable installations suppliers lead to a perceptible increase in availability of the wind and solar assets. According to [3], the levelized cost of offshore wind power in Europe dropped by 56% from 2014 to 2016. Finally, economic cycles prompting downward and upward movements in the industrial development bring a sufficient degree of electricity demand variability questioning the timing, siting and sizing aspects of investment planning and even the necessity of any investment decisions [4].

The structure of the models and modeling assumptions also define the complexity of the investment planning models. There are two types of models considered in the investment studies: static and dynamic models [5]. In the former ones, there is a single year, and all investment decisions are carried out for that target year. Unlike static, in the dynamic models, investment decisions are endogenously made at several time-stages throughout the planning horizon, significantly increasing the computational burden. Moreover, the centralized investment planning proved to be a useful tool to find an optimal expansion planning to maximize the overall social welfare. However, to account for individual objectives of market participants, the consideration of the decentralized planning, i.e. from a strategic power producer point of view, is required.

Unlike the centralized planning, these models are typically fit the bilevel structure of optimization problems resulting in more complex formulations. Apart from that, the models involve a significant number of discrete variables aiming at fulfilling certain technical and economic conditions as well as the linearization of the non-linear terms in objective functions and constraints.

As a result, if the uncertainty is represented by a finite set of scenarios, complex investment planning problems recast as large-scale mixed-integer stochastic programming problems with high computational complexity and limited tractability making the models useless. Typically, there are several ways to achieve tractability and reduce computation burden: (*i*) introduction of simplifying assumptions to relax certain technical and economic sides of power system operations; (*ii*) lessen the number of uncertainty realization scenarios; (*iii*) decomposition of an original problem into a set of smaller sub-problems governed by a common convergence criterion. The first two items might significantly influence the quality of the solution, while the latter allows achieving an appropriate trade-off between the quality of the solution and models' tractability.

1.2 Thesis goal

This project seeks to develop a decision-making tool which supports generation investment planning, i.e. optimal timing, siting and sizing of available production technologies, for a strategic power generation company, participating either at the pool-based or network-constrained electricity market, for years ahead taking into account multiscale uncertainty sources. The investment horizon comprises several time periods, such that the producer is capable of updating its investment plan at later time stages.

In order to achieve model's tractability, the project aims at applying the progressive hedging algorithm and find out to what extend the original problem could be decomposed to improve simulation time providing a reasonable quality of the solution.

1.3 Overview of the investment planning problems

Investment planning problems had been broadly studied in the technical literature. These problems had been solved from different perspectives, i.e. from the various entities' and their objectives' points of view, considering either deterministic or stochastic formulation, using different optimization techniques and solution approaches, resulting in varying degrees of complexity.

One of the most common types of problems is so-called *centralized* investment planning. The problem is solved from the publicly controlled regulator which aims at finding an optimal investment portfolio to maximize the overall welfare or minimize operating costs. In fact, if all agents in the system are price-takers, i.e. enter the market with their actual marginal costs or utilities and energy quantities, the centralized planning results in satisfaction of all individual preferences. For example, the regulator carries out a transmission expansion planning to reinforce the existing network to accommodate new generation and demand units as well as prevent network congestion causing operational issues as well as price differentiation [6, 7]. Solving centralized generation expansion planning results in the most beneficial production assets allocation, timing and sizing from the society point of view satisfying preferences of all generation companies in the system, as illustrated in [8]. It is worth noticing that the separate consideration of network and production expansion problems result only in sub-optimal solutions because this separation does not account for the inherent investment dynamics. However, the coordinated facility planning leads to the synergy effect resulting in more efficient social welfare maximization. This synergy from the co-planning of investment decisions is broadly studied in [9] and [10].

The disadvantage of the centralized planning is the fact that it assumes perfect competition such that all agents in the system follow price-taking policies. Despite that, there is some evidence that market participants are inclined to deviate from the price-taking strategies to gain larger profits or reduce charging costs [11, 12, 13, 14]. In this line, it is reasonable to consider market participants as oligopolistic players with price-making strategies in the framework of the *decentralized* planning. Unlike the centralized approach, under this framework the private entity decides on the own investment plan, and instead of maximizing the social welfare, it maximizes the expectation of the own profit only. Substantially, in the decentralized problem formulation, the actual private valuation of electricity energy reveals such that the investment solution might significantly differ from the centralized one [15, 16]. Moreover, the overall investment planning of a set of strategic price-makers is generally different from the investment equilibrium in the perfect market [17, 18, 19].

It is worth noticing that the network expansion is only carried out in the centralized fashion. In fact, network companies are considered as natural monopolies, and their operations are strictly regulated. Thus, there is no room for network facilities to participate strategically. In this line, it was proposed to coordinate the centralized network and decentralize generation investment plannings [20, 21]. Apparently, this coordination brings additional benefits in the form of synergy in the market with strategic producers.

The centralized problem is generally formulated as a single-level problem. For example, it might be the co-optimization of the system operating costs and investment costs as given in [22]. Contrary to the centralized approach, the decentralized models are commonly formulated as two- or multi-level problems [20, 21, 23, 24]. The multi-level formulation favorably differs from the single-level one at least because it allows accounting for several objectives. For example, [23] formulates a problem for capacity expansion of a single strategic producer which makes investment decisions anticipating the reaction of other generators. The goal of the strategic producer is given by the upper-level objective function while the objectives of others are provided by KKT conditions of the market clearing problems in the lower level. In the transmission and generation co-optimization planning given by [20, 21], the objectives of the system operator are defined by the top-level objective function, preferences of the strategic producers - by the middle-level objective function, and the remaining players' preferences are considered in the lower-level problem.

The multi-level problems are generally more complex to solve due to several reasons. For example, to achieve a tractable single-level equivalent, the lower-level problems are replaced with their KKT conditions, involving stationarity conditions, primal and dual feasibility, as well as complementarity slackness conditions. The former ones are hard equality constraints which significantly reduce the feasibility space. The latter ones are non-linear conditions, linearization of which implicates discrete variables, for instance, binary or SOS1 variables. Moreover, multi-level problems usually comprise non-linear terms in the upper-level objective function. As an example, it could be a product of the dual and primal variables of the lower-level problems. The linearization of these non-linear terms might include an additional set of auxiliary discrete variables, as illustrated in [25].

Investment planning models might be also distinguished as *static* and *dynamic* [5]. Although both models could treat the same planning problem, their solutions are usually different. In the static formulation, the investment decisions have to be performed by a certain target year. Dynamic models, in contrast, allocates the investment decisions among a certain number of time stages inside the planning horizon, and thus result in more flexible and accurate solution. In fact, the solution of the static model is a special case of the dynamic model. In the static approach, the investment decisions are carried out at one single node of the planning decision tree, while the dynamic model allows dispersing the investments among certain time periods. Moreover, the multistage formulation makes decision-makers able to adjust the decisions from the previous stages concerning the updated information throughout the investment horizon, as depicted in Figure 1.1. In general, compared to the static approach, the dynamic investment planning requires more computational resources due to two primary reasons. First, since the dynamic problem considers several time stages, it involves more decision variables, i.e. the same set of investment and operational decisions per each time stage. Second, in the static models, there is only one non-anticipativity condition associated with a single period when the decision has to be performed. In the dynamic planning, the amount of these constraints is larger and defined by a number of the time stages considered in the problem. Notice, the non-anticipativity constraints are essential in the stochastic problems, and their hardness is primarily responsible for a limited tractability of these models.

As long as investment decisions are carried out for years ahead, they are exposed to many uncertainty sources. In fact, decision-makers face multiscale uncertainty associated with both long- and short-term operational and financial risks. The long-term uncertainty commonly includes demand growth, investment costs, fuel prices, rival investments as well as regulatory changes [27]. Short-term risk is usually associated with hourly variability of time-dependent resources, i.e. the wind and solar power generation, as well as demand patterns. Moreover, the privacy around other participants' strategies, i.e. their price and quantity policies, also contributes to the market participation uncertainty. An appropriate treatment of risks results in accurate solutions, whereas ignoring the stochasticity of relevant indicators results in costly implications. For example, [28] demonstrated that by neglecting uncertainty and solving the problem with a simple deterministic planning, the cost of implications is of the same magnitude as the investment

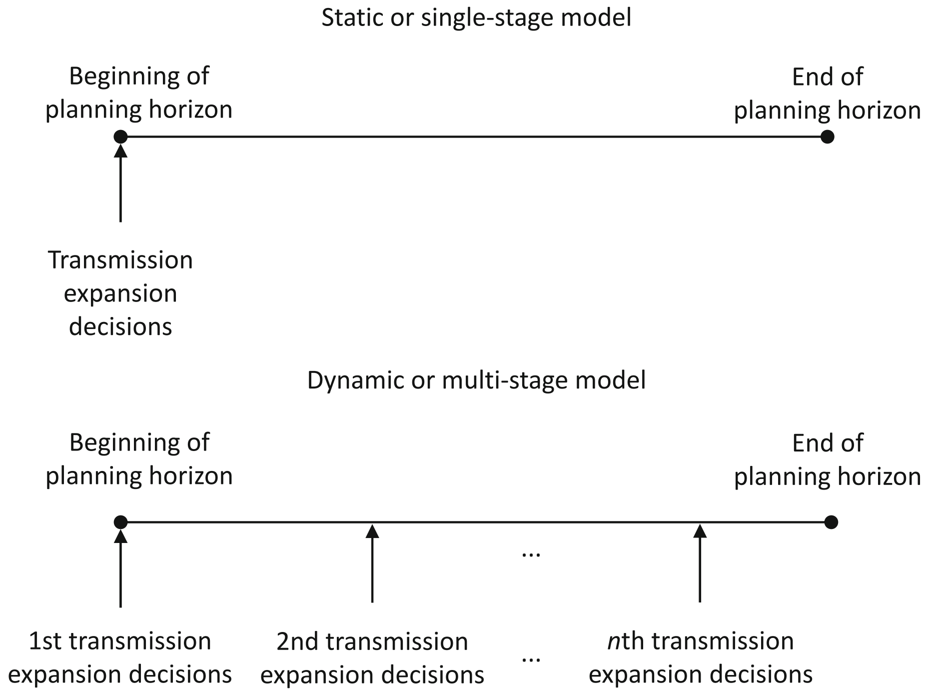


Figure 1.1: Static and dynamic transmission expansion models [26].

costs.

There are two common ways to treat stochastic variables in the optimization problems: stochastic [20, 21, 23, 15] and robust optimization [29, 30]. In the stochastic programming approach, the uncertainty is described through the finite set of scenarios and objective is to maximize or minimize the expected value of the objective function. Robust optimization is an alternative framework to stochastic programming which aims at determining a solution that is feasible under any realization of the uncertain parameters involved in an optimization problem, and optimal in their worst-case realization [31]. Unlike stochastic programming, this approach commonly represents the uncertainty as a polyhedral set, in which the worse case solution is located on one of the vertexes.

As a result, due to a number of existing uncertainty sources and complex structures of the investment planning problems, they are difficult to solve. With a large number of objectives and uncertainty scenarios, these problems might be intractable, i.e. they cannot provide accurate solution being solved directly. In this line, several decomposition techniques were proposed to achieve tractability. One of the most widely used decomposition approaches is Benders decomposition [32], which separates the investment problem from the operations problems [5]. Instances of benders decomposition applications might include [33] for a deterministic case and [34] for a stochastic investment planning problem. According to [35], there is a sufficient number of alternative to benders heuristic decomposition techniques to solve investment problems. For example, greedy randomized methods [36], heuristic solutions to the investment master problem in Benders decomposition [37], search algorithms based on sensitivity analysis [38], genetic algorithms [39], and sequential approximation approaches to account for wind and load variability [40]. Although they potentially could result in good solutions, they do not provide a metric of solution quality, such as bounds on the objective function of the original problem.

Scenario decomposition algorithm based on Progressive Hedging (PHA) proposed by [41] proved to be a useful tool to tackle large-scale investment problems. In [42] PHA was employed to solve stochastic transmission expansion planning, [43] applied the algorithm to solve two-stage capacity investment planning. Later on, [44] introduced a solution framework to obtain a lower bound for the objective function of the original problem. It was then successfully applied to estimate the expectation of the total system costs in the static stochastic mixed-integer transmission planning problem [35]. Moreover, the estimate of the expected total system costs in the multistage investment planning formulation was obtained in [22].

1.4 Thesis objectives

To meet the project's goal in the context above, the following objectives have to be fulfilled:

1. Identify a suitable bilevel structure for the multistage strategic generation investment planning problem

2. Find out a decent approximation for the future market participation of the company. That is, to identify a suitable market clearing problem formulation providing a reasonable trade-off between simulation time and quality of the market approximation

3. Identify potential long-term and short-term operational and economic sources of uncertainty

4. Build long-term and short-term decision-making sequences for the investment planning problem

5. Introduce and justify modeling assumptions

6. Reformulate the original bilevel problem to a sing-level linear problem.

7. Provide theoretical background on the progressive hedging algorithm and formulate the decomposition approach to solve the original problem.

8. Perform a series of case studies to prove the efficiency of the proposed tool.

1.5 Thesis organization

The thesis is organized as follows. Chapter 2 provides a comprehensive description of the multistage strategic capacity expansion problem and gives a mathematical formulation of the problem. Chapter 3 provides some insights into basics of the progressive hedging algorithm, how it could be applied to the considered problem and what relevant modeling and computational issues should be addressed. Chapter 4 introduce a series of case studies. First, the proposed decision-making tool is applied to a small illustrative example to highlight the modeling accuracy and computation performance of the algorithm. Second, the model is tested with a two-area version of IEEE 24-Bus RTS for a two-stage investment planning problem. Third, the model is applied to a pool-based electricity market to solve wind power investment planning for a three-stage investment horizon. Finally, Chapter 5 serves to provide the main findings of this work and emphasize the future research directions.

CHAPTER 2

Description of the multi-stage strategic capacity expansion problem

2.1 Model description and modelling assumptions

2.1.1 General overview

This thesis considers a capacity expansion problem from the perspective of a strategic power producer. This producer competes with other companies in a network-constrained electricity market operating on the hourly basis. Considering the producer as a strategic player implies that it is capable of exerting market power, i.e. offering prices and quantities along with investment decisions of this producer could influence the market outcomes. In fact, if the market share of the generation company is sufficiently large, it has an incentive to offer energy prices and quantities different from the actual marginal costs and production capacities to gain more substantial profit compared to the one under a passive price-taking policy.

The producer faces a dynamic expansion problem. In this case, the investment horizon is divided into a set of time periods indexed by t. At the beginning of each period, it decides on optimal site and size of available power production technologies to be built. Since the investment planning is carried out for years ahead, the company utilizes a series of long-term scenarios, indexed by γ. At the same time, instead of estimating market outcomes for each hour of the planning horizon, it employs plausible aggregated representative days, denoted by h, and wind realizations, denoted by ω, based on the historical data. Apart from that, it assumes a finite set of scenarios on offering strategies of rival producers as well as bidding strategies of consumers, all indexed by k.

The assumptions on a price-making policy of a strategic producer and price-taking policies of rival companies suggest considering the capacity expansion problem through the prism of Stackelberg competition [45]. In this game, the strategic producer is considered as a leader, and other market participants are assumed to be followers. The leader decides its strategy prior to the decisions of the followers and maximizes its profit taking into account their best response. This game is sequential since the leader has an advantage of the first decision. Solving this game as an optimization problem, the producer strategically chooses the offering strategy resulting in generally higher profit than the one in the perfect competition market. Figure 2.1 depicts this sort of completion in the framework of the bilevel optimization.

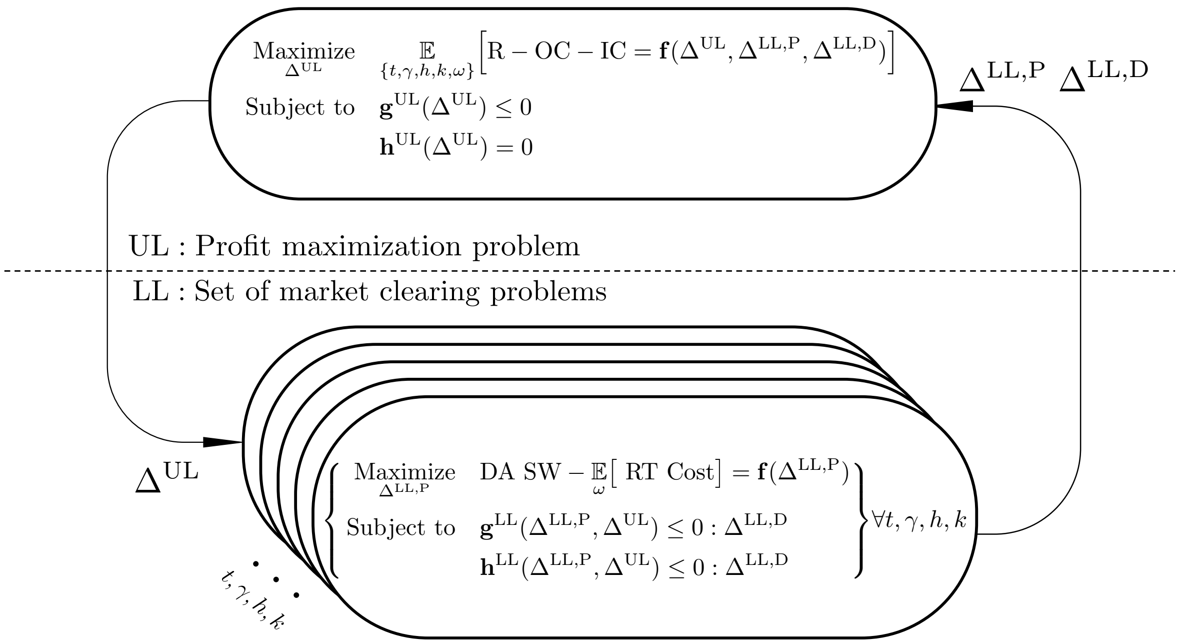


Figure 2.1: Bilevel work flow of the considered problem.

The upper-level problem (UL) aims at maximizing the strategic producer's expected profit over the investment horizon, considering the expected revenue (R), operating costs (OC) and investment costs (IC). The profit is subject to a set of UL decisions collected in Δ^{UL} along with primal and dual decisions of the set of market clearing problems, $\Delta^{LL,P}$ and $\Delta^{LL,D}$ respectively. Δ^{UL} consists of investment decisions and strategic offering decisions. These decisions are considered as variables in the UL problem which enter the lower-level (LL) problem as constants. The objective of the LL problems is to maximize the overall social welfare at the day-ahead stage and minimize the expected balancing costs at the real-time stage. $\Delta^{LL,P}$ primary comprises market operator's decisions on power system scheduling, while $\Delta^{LL,D}$ comprises a set of dual variables, where the market clearing prices are of the main concern. A number of LL problems is defined by the amount of considered uncertainty and operation conditions scenarios. These decisions serve as variables in the LL problem and enter the UL problem as constants. The UL and LL problems are interconnected. In fact, all UL decisions influence the LL decisions which, in turn, have an impact on the value of the UL objective function.

Using the bilevel framework in Figure 2.1, the strategic producer makes long-term decisions on the (*i*) optimal power portfolio to invest at each period of the planning horizon as well as (*ii*) short-term decisions on the optimal offering strategy, such that all these decisions maximize the expected profit over the planning horizon.

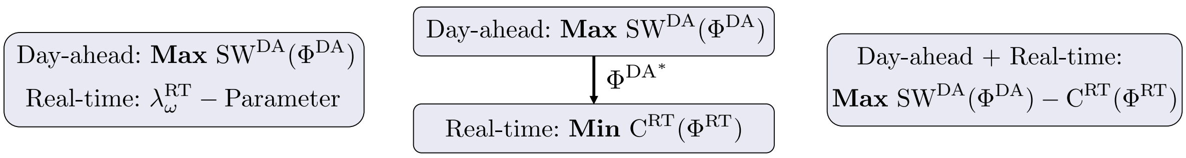
2.1.2 Approximation of the market participation

Strategic producer participates in two trading floors:

- **Day-ahead market.** This market is commonly used to trade energy in the short-run. It is organized as a two-sided auction where power producers, large consumers, and retail companies sell and purchase electricity for each hour of the following day. Power producers submit their offering prices and quantities, whereas consumers submit their bidding prices and quantities. Market operator, who organizes the auction, collects all offers and bids, determines the aggregated supply and demand curves and clears the market, i.g. defines the equilibrium quantities and prices throughout the following day.

- **Real-time market.** This market arises a few minuted before the real-time operations and aims at pricing the actions associated with balancing of power production and consumption which might be different from the day-ahead schedule. It implies that all deviations from the day-ahead quantities are balanced by activation of operating reserve from the flexible units, or deploying last preferable wind spillage and load shedding actions. The balancing actions are scheduled according to the submitted price-quantity pairs as well.

To accurately estimate market outcomes over the planning horizon, a set of lower-level problems embodying day-ahead and real-time markets clearing is employed. Figure 2.2 depicts three possible options to simulate the clearing. The first approach is to consider the day-ahead clearing as an optimization problem and real-time market outcomes as stochastic parameters, described by a finite set of scenarios. This configuration is relatively simple to implement due to a small dimension of the lower-level problem, and thus it provides a relatively faster solution. However, the expectation of the profit is now subject to the accuracy of the real-time price and quantities estimation. An alternative solution would be to consider both day-ahead and real-time clearing as two lower-level optimization problems. In this setup, both day-ahead and real-time outcomes are decision variables. Thus this approximation results in the best accuracy. However, the day-ahead scheduling is considered as a parameter in the real-time clearing problem, which prevents obtaining an exact linear equivalent for the considered investment planning problem. Particularly, the linear equivalent could be achieved, but it would involve a certain number of integer variables related to the day-ahead scheduled quantities desensitization. The accuracy of this discretization significantly affects the quality of solution and simulation time. The third approach relies on the consideration of market auction as a stochastic-integrated day-ahead clearing, which schedules power plants anticipating future wind power fluctuations. In this fashion, the problem involves a single lower-level problem per each uncertainty scenario and operation condition, which considers both day-ahead and real-time prices as decision variables. Although it only approximates the actual market design, it allows obtaining the exact linearization of the upper-level objective function. Thus, the thesis considers this option to approximate the market clearing over the planning horizon.



(a) DA clearing only (b) Sequential DA and RT clearing (c) Stochastic DA clearing

Figure 2.2: Market approximation options.

2.1.3 Uncertainty characterization

Investment planning involves a plethora of risks associated with the uncertainty of different scales. First, investment decisions are carried out taking into account long-term uncertainty which relates to the macroeconomics development. It might be, for instance, barely predictable demand growth, capital costs, fuel costs, generation mix, regulatory conditions et cetera. At the same time, daily participation in the electricity markets grasps the short-term uncertainty which influences the profitability of existing and candidate generation units during each period of the planning horizon. This uncertainty could comprise the stochasticity of market participants strategies, wind power and demand uncertainty, congestion and failure of network facilities. Table 2.3 collects the sources of uncertainty considered in this project.

Table 2.3: Classification of uncertainty sources.

Long-term uncertainties	Market uncertainties	Wind uncertainties
Demand growth, investment cost, fuel prices, rival producers' investments	Rival producers' price offers, demand price bids	WP real-time deviation

Each source of uncertainty is represented by a finite set of plausible scenarios, such that the dimension of uncertainty space $\mathcal{U}$ is defined as follows:

$$\dim_{\mathcal{U}} = N^{\gamma} \times N^{k} \times N^{\omega} \tag{2.1}$$

where $N^{\gamma}, N^{k}, N^{\omega}$ are numbers of long-term, market and wind power production scenarios, respectively.

2.1.4 Decision sequences

As long as the investment planning involves a plethora of uncertainty sources, the long-term and short-term decisions of the producer are shaped by assumed uncertainty sets of scenarios. Figure 2.4 depicted the decision sequence for investment decisions throughout the planning horizon involving N^t time stages. At the beginning of each stage, the strategic producer makes its investment decisions. These are *here-and-now* investment decisions, independent of future long-term scenario realizations. Future investment decisions are carried out at the following stages, i.e. *wait-and-see* decisions, such that they depend on the long-term uncertainty realizations and express the optimal adjustment of the investment plan in future. A number of these decisions is defined by a number of uncertainty scenarios involved at this stage.

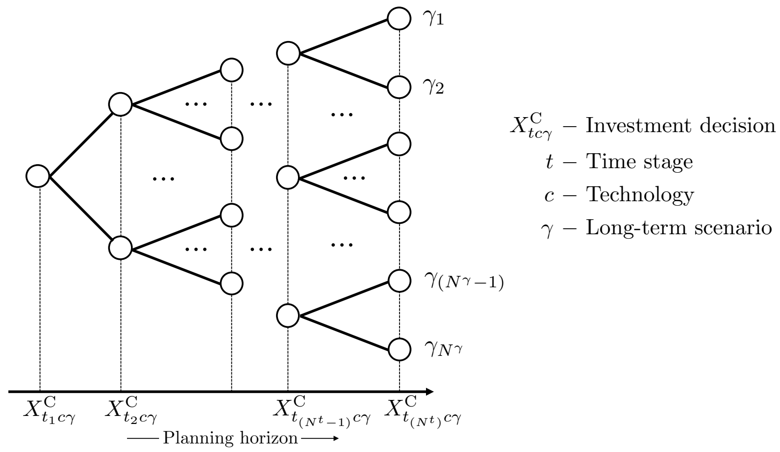


Figure 2.4: Long-term decisions sequence.

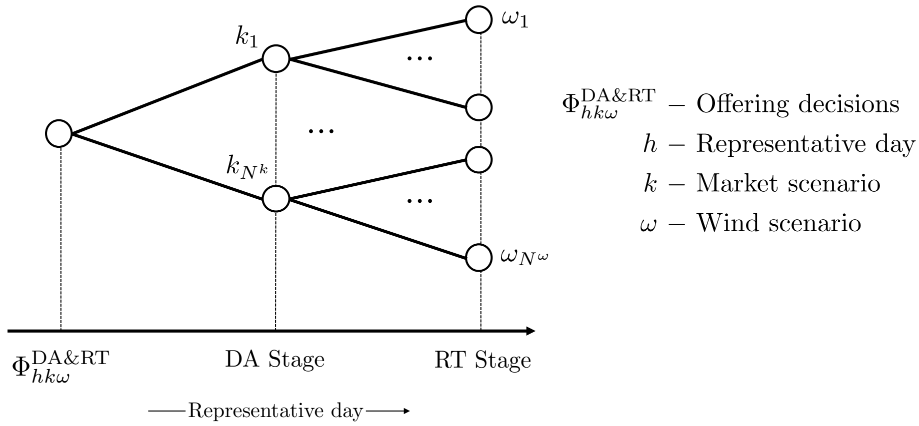


Figure 2.5: Short-term decisions sequence.

Inside each stage of the planning horizon, the producer faces a short-term decision sequence, depicted in Figure 2.5. First, it decides the optimal offering strategy, i.e. the day-ahead energy quantities and prices for a portfolio of existing and candidate units as well as up- and down-reserve quantities and prices for the real-time market. These decisions are independent of the market uncertainty, i.e. rivals' offering and consumers' bidding prices. At the next stage, the market operator clears the day-ahead market, and the strategic producer is paid for its scheduled power quantities for each representative day and market scenario. Finally, the real-time market is cleared by the system operator, and depending on the wind energy realizations it receives payment for the up- and down-reserve deployment from the flexible units as well as receives income or penalty for the deviations caused by the variability of existing and candidate wind power units.

2.1.5 Modelling assumptions

1. Investment planning is carried out in the framework of multi-stage or dynamic approach

2. The day-ahead market is cleared using a stochastic network-constrained auction anticipating future wind power fluctuations, where the profits of the wind and conventional generation units are ensured in the expectation as described in [46].

3. The marginal pricing scheme is adopted [47]. That is, all energy quantities are priced with locational marginal prices (LMPs). For the day-ahead market, they are obtained as a dual variable of the day-ahead balance constraint. For the real-time market, the price is obtained as a probability removed price as described in [48].

4. Instead of clearing the market for each hour, the formulation assumes a set of typical days that comprehensively describes the operational conditions, i.e. demand and wind capacity factors, within each time stage of the planning horizon. It might be shown that a sufficient number of representative days does not influence the quality of the solution [49].

5. To avoid non-convexity issues, comprehensively studied in [50] and [51], (*i*) power plants are assumed to be fully dispatchable from zero to their maximum capacities and (*ii*) on-off commitment decisions are not modeled.

6. At the day-ahead stage, the load is assumed to be elastic, while at the real-time stage it is inelastic with a large value of lost load. Thus, the day-ahead clearing is represented by the social welfare maximization problem, and the real-time clearing boils down to the minimization of the balancing costs.

7. For the sake of simplicity, supply, demand and investment cost curves are linear functions. However, one might consider alternative stepwise functions [52].

8. For each short- and long-term scenario as well as representative day, the uncertainty affecting the market-clearing process itself is assumed to be solely induced by stochastic producers.

9. Long-term, market and wind power uncertainties are considered through the finite independent sets of scenarios satisfying three Kolmogorov axioms [53].

10. The intertemporal constraints are not included in the problem. However, reserve limits of generations units are considered to differentiate the flexibility of various production technologies.

11. A DC representation of the transmission system is used to account for the system network topology [54].

12. Strategic producer is capable of exercising market power participating in the day-ahead and real-time markets. With fixed energy quantities, it is capable of influencing the market outcomes through strategic price offering, *a la Bertrand* competition employed in [55]. With fixed prices, it could affect the equilibrium through strategic quantity offering, *a la Cournot* competition, as shown in [25]. Finally, it could exercise the market power by strategically deciding on both offering prices and quantities, as illustrated in [56].

13. Strategic producer is assumed to be risk-neutral, i.e. it maximizes the expectation of the profit throughout of the planning horizon. However, it is possible to include a risk-aversion attitude with linear risk measures, such as VaR or CVaR, as described in [57, 58, 59, 60]

2.2 Mathematical formulation

2.2.1 Bilevel formulation

The following two-level formulation represents the considered investment problem. The upper-level objective function is aim at maximizing the expected profit of a strategic producer over the planning horizon and writes as follows:

$$
\begin{aligned}
&\underset{\Delta^{\mathrm{UL}}}{\text{Maximize}} \\
&\sum_{t \in \mathcal{T}} \frac{1}{(1+\mathrm{DR}_t)^t} \cdot \Bigg\{ \sum_{\gamma \in \mathcal{G}} \pi^{\mathrm{L}}_{\gamma} \cdot \Bigg\{ \sum_{h \in \mathcal{H}} N^{\mathrm{MC}}_h \cdot \Bigg[\sum_{k \in \mathcal{K}} \pi^{\mathrm{MS}}_k \cdot \Bigg\langle \sum_{j \in \mathcal{J}^{\mathrm{Own}}} (\lambda^{\mathrm{DA}}_{t\gamma hk(n:j\in\mathcal{J}_n)} - c_{t\gamma j}) \cdot P_{t\gamma hkj} \\
&+ \sum_{\omega \in \mathcal{W}} \pi^{\mathrm{W}}_{\omega} \cdot \Bigg(\sum_{j \in \mathcal{J}^{\mathrm{Own,WP}}} \Big(\sum_{\substack{\tau \in \mathcal{T} \\ \tau \le t}} X_{\tau\gamma j} \cdot K^{\mathrm{CF}}_{hj} \cdot K^{\mathrm{WS}}_{j\omega} - P_{t\gamma hkj} - P^{\mathrm{sp}}_{t\gamma hkj\omega} \Big) \cdot \frac{\lambda^{\mathrm{RT}}_{t\gamma hk(n:j\in\mathcal{J}_n)\omega}}{\pi^{\mathrm{W}}_{\omega}} \Bigg) \\
&+ \sum_{\omega \in \mathcal{W}} \pi^{\mathrm{W}}_{\omega} \cdot \Bigg(\sum_{j \in \mathcal{J}^{\mathrm{Own,Flex}}} \Bigg[\Bigg(\frac{\lambda^{\mathrm{RT}}_{t\gamma hk(n:j\in\mathcal{J}_n)\omega}}{\pi^{\mathrm{W}}_{\omega}} - c_{t\gamma j} \Bigg) \cdot r^{\uparrow}_{t\gamma hkj\omega} - \frac{\lambda^{\mathrm{RT}}_{t\gamma hk(n:j\in\mathcal{J}_n)\omega}}{\pi^{\mathrm{W}}_{\omega}} \cdot r^{\downarrow}_{t\gamma hkj\omega} \Bigg] \Bigg) \Bigg\rangle \Bigg] \\
&- a_t \cdot \sum_{c \in \mathcal{C}} c^{\mathrm{Inv}}_{t\gamma c} \sum_{\substack{\tau \in \mathcal{T} \\ \tau \le t}} X^{\mathrm{C}}_{\tau\gamma c} \Bigg\} \Bigg\}
\end{aligned}
\tag{2.2}
$$

The expected profit comprises of four sets of terms:

1. The first term $\sum_{j\in\mathcal{J}^{\text{Own}}}(\lambda^{\text{DA}}_{t\gamma hk(n:j\in\mathcal{J}_n)} - c_{t\gamma j}) \cdot P_{t\gamma hkj}$ is the profit obtained by selling electricity energy from existing and candidate units at the day-ahead market. It consists of a product of the day-ahead LMP subtracting the generation cost and dispatched power from these units.

2. The second term $\sum_{\omega\in\mathcal{W}}\Bigg(\sum_{j\in\mathcal{J}^{\text{Own,WP}}}\Big(\sum_{\substack{\tau\in\mathcal{T}\\ \tau\le t}} X_{\tau\gamma j}\cdot K^{\text{CF}}_{hj}\cdot K^{\text{WS}}_{j\omega} - P_{t\gamma hkj} - P^{\text{sp}}_{t\gamma hkj\omega}\Big)\cdot \lambda^{\text{RT}}_{t\gamma hk(n:j\in\mathcal{J}_n)\omega}\Bigg)$ is the profit/cost associated with the surplus/shortage of energy production caused by the deviation from the contracted day-ahead quantities from the existing and candidate wind power units. It consists of a product of the real-time LMP and delivered energy quantity, expressed as the actual power production reduced by the scheduled day-ahead quantity and wind power spillage. Notice that the cumulative capacity of the existing units over the planning horizon is a predefined parameter, while it is a decision variable for the candidate wind power units.

3. The third term $\sum_{\omega\in\mathcal{W}}\pi^{\text{W}}_{\omega}\cdot\Bigg(\sum_{j\in\mathcal{J}^{\text{Own,Flex}}}\Bigg[\Big(\frac{\lambda^{\text{RT}}_{t\gamma hk(n:j\in\mathcal{J}_n)\omega}}{\pi^{\text{W}}_{\omega}} - c_{t\gamma j}\Big)\cdot r^{\uparrow}_{t\gamma hkj\omega} - \frac{\lambda^{\text{RT}}_{t\gamma hk(n:j\in\mathcal{J}_n)\omega}}{\pi^{\text{W}}_{\omega}}\cdot r^{\downarrow}_{t\gamma hkj\omega}\Bigg]\Bigg)$ is the real-time profit generated by the existing and candidate flexible units. It is defined by the revenue obtained from the up- and down-reserve deployment and generation costs associated with the up-reserve provision.

4. The last term $a_t \cdot \sum_{c\in\mathcal{C}} c^{\text{Inv}}_{t\gamma c}\sum_{\substack{\tau\in\mathcal{T}\\ \tau\le t}} X^{\text{C}}_{\tau\gamma c}$ is the costs related to the deterioration of candidate units, represented by a product of the amortization rate and investment costs.

The value of the expected profit is subject to the discounting factor, number of long- and short-term scenarios as well as number of representative days considered. It is worth noting that at the day-ahead stage the energy quantities are payed with the day-ahead LMP $\lambda^{\text{DA}}_{t\gamma hk(n:j\in\mathcal{J}_n)}$, while all actions at the real-time stage are priced with the probability removed balancing price $\lambda^{\text{RT}}_{t\gamma hk(n:j\in\mathcal{J}_n)\omega}/\pi^{\text{W}}_{\omega}$.

The upper-level problem constraints are represented by the following set of equations:

$$X^{\text{C}}_{t\gamma c} = \sum_{o\in\mathcal{O}} u^{\text{C}}_{t\gamma co}\cdot \overline{X}^{\text{C}}_{co} \quad \forall t\in\mathcal{T} \quad \forall\gamma\in\mathcal{G} \quad \forall c\in\mathcal{C} \tag{2.3a}$$

$$\sum_{o\in\mathcal{O}} u^{\text{C}}_{t\gamma co} = 1 \quad \forall t\in\mathcal{T} \quad \forall\gamma\in\mathcal{G} \quad \forall c\in\mathcal{C} \tag{2.3b}$$

$$u^{\text{C}}_{t\gamma co}\in\{0;1\} \quad \forall t\in\mathcal{T} \quad \forall\gamma\in\mathcal{G} \quad \forall c\in\mathcal{C} \quad \forall o\in\mathcal{O} \tag{2.3c}$$

$$\sum_{c\in\mathcal{C}} c^{\mathrm{Inv}}_{t\gamma c} \cdot X^{\mathrm{C}}_{t\gamma c} \leq \overline{I}_t \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \tag{2.3d}$$

$$\sum_{j\in\mathcal{J}^{Sys}} \overline{P}_{t\gamma hkj} \geq \chi^{\mathrm{SoS}} \cdot \sum_{d\in\mathcal{D}} \overline{P}^{\mathrm{D}}_{t\gamma d} \cdot K^{\mathrm{DF}}_h \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad \forall k \in \mathcal{K} \tag{2.3e}$$

$$\beta^{\mathrm{E}}_{t\gamma hke}, \beta^{\mathrm{C}}_{t\gamma hkc} \geq 0 \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad \forall k \in \mathcal{K} \quad \forall e \in \mathcal{E} \quad \forall c \in \mathcal{C} \tag{2.3f}$$

$$\beta^{\mathrm{E}\uparrow}_{t\gamma hke}, \beta^{\mathrm{C}\uparrow}_{t\gamma hkc}, \beta^{\mathrm{E}\downarrow}_{t\gamma hke}, \beta^{\mathrm{C}\downarrow}_{t\gamma hkc} \geq 0$$
$$\forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad \forall k \in \mathcal{K} \quad \forall e \in \mathcal{E}^{\mathrm{Flex}} \quad \forall c \in \mathcal{C}^{\mathrm{Flex}} \tag{2.3g}$$

$$0 \leq \overline{P}^{\mathrm{E}}_{t\gamma hke} \leq X^{\mathrm{E}}_e \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad \forall k \in \mathcal{K} \quad \forall e \in \mathcal{E} \tag{2.3h}$$

$$0 \leq \overline{P}^{\mathrm{C}}_{t\gamma hkc} \leq \sum_{\substack{\tau\in\mathcal{T}\\ \tau\leq t}} X^{\mathrm{C}}_{\tau\gamma c} \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad \forall k \in \mathcal{K} \quad \forall c \in \mathcal{C} \tag{2.3i}$$

$$0 \leq \overline{R}^{\mathrm{E}\uparrow}_{t\gamma hke} \leq \alpha^{\mathrm{E}\uparrow}_e \cdot X^{\mathrm{E}}_e \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad \forall k \in \mathcal{K} \quad \forall e \in \mathcal{E}^{\mathrm{Flex}} \tag{2.3j}$$

$$0 \leq \overline{R}^{\mathrm{C}\uparrow}_{t\gamma hkc} \leq \alpha^{\mathrm{C}\uparrow}_c \cdot \sum_{\substack{\tau\in\mathcal{T}\\ \tau\leq t}} X^{\mathrm{C}}_{\tau\gamma c} \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad \forall k \in \mathcal{K} \quad \forall c \in \mathcal{C}^{\mathrm{Flex}} \tag{2.3k}$$

$$0 \leq \overline{R}^{\mathrm{E}\downarrow}_{t\gamma hke} \leq \alpha^{\mathrm{E}\downarrow}_e \cdot X^{\mathrm{E}}_e \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad \forall k \in \mathcal{K} \quad \forall e \in \mathcal{E}^{\mathrm{Flex}} \tag{2.3l}$$

$$0 \leq \overline{R}^{\mathrm{C}\downarrow}_{t\gamma hkc} \leq \alpha^{\mathrm{C}\downarrow}_c \cdot \sum_{\substack{\tau\in\mathcal{T}\\ \tau\leq t}} X^{\mathrm{C}}_{\tau\gamma c} \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad \forall k \in \mathcal{K} \quad \forall c \in \mathcal{C}^{\mathrm{Flex}} \tag{2.3m}$$

$$X^{\mathrm{C}}_{t\gamma c} = X^{\mathrm{C}}_{t\gamma' c} \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall c \in \mathcal{C} : \Theta_{\tau\gamma} = \Theta_{\tau\gamma'}, \quad \forall \tau \leq t \tag{2.3n}$$

A set of constraints (2.3a) - (2.3c) ensures that the producer chooses only one capacity option over the whole variety of options for a particular production technology. This condition is enforced assuming that the unitary cost associated with purchasing, delivering and installation of the particular technology is a linear function of the unit size. Inequality constraint (2.3d) limits the investment expenses for a given period with the available budget for that period. Constraint (2.3e) aims at fulfilling the security of supply conditions declared by TSO. Constraints (2.3f)-(2.3g) maintain the positivity of the offering prices for the existing and candidate units at the day-ahead and real-time floors. Constraints (2.3h) and (2.3i) limit strategic day-ahead offering quantities considering the maximum capacity of existing and candidate units. Constraints (2.3j)-(2.3k) ensure that strategic quantities for the up- and down-reserve provision will not exceed the technical limits for these actions. Finally, (2.3n) is a set of non-anticipativity constraints, i.e. constraints that prevent anticipating information while deciding the investment program at each time stage of the planning horizon.

The set of upper-level decision variables $\Delta^{\mathrm{UL}} \in \{X^{\mathrm{C}}_{t\gamma c}, u^{\mathrm{C}}_{t\gamma co}, \overline{P}^{\mathrm{E}}_{t\gamma hke}, \overline{P}^{\mathrm{C}}_{t\gamma hkc}, \overline{R}^{\mathrm{E}\uparrow}_{t\gamma hke}, \overline{R}^{\mathrm{C}\uparrow}_{t\gamma hkc}, \overline{R}^{\mathrm{E}\downarrow}_{t\gamma hke}, \overline{R}^{\mathrm{C}\downarrow}_{t\gamma hkc}, \beta^{\mathrm{E}}_{t\gamma hke}, \beta^{\mathrm{C}}_{t\gamma hkc}, \beta^{\mathrm{E}\uparrow}_{t\gamma hke}, \beta^{\mathrm{C}\uparrow}_{t\gamma hkc}, \beta^{\mathrm{E}\downarrow}_{t\gamma hke}, \beta^{\mathrm{C}\downarrow}_{t\gamma hkc}, \}$ consists of the investment decisions as well as strategic offering prices and quantities only. The rest of the variables in the upper-level objective function are obtained from the set of lower-level problems associated with the stochastic integrated market clearing in every time period, representative day under all long-term and market uncertainty realizations, as follows:

$$\left(P_{t\gamma hkj}, \lambda^{\mathrm{DA}}_{t\gamma hk}, P^{\mathrm{sp}}_{t\gamma hkj\omega}, r^{\uparrow}_{t\gamma hkj\omega}, r^{\downarrow}_{t\gamma hkj\omega}, \lambda^{\mathrm{RT}}_{t\gamma hk\omega}\right) \in \mathrm{argmax}\Bigg\{ \underset{\Delta^{\mathrm{LL,P}}}{\mathrm{Maximize}}$$

$$\sum_{d\in\mathcal{D}} b^{\mathrm{D}}_{tkd} \cdot P^{\mathrm{D}}_{t\gamma hkd} - \sum_{r\in\mathcal{R}} c^{\mathrm{R}}_{t\gamma kr} \cdot P^{R}_{t\gamma hkr} - \sum_{j\in\mathcal{J}^{\mathrm{Own}}} \beta_{t\gamma hkj} \cdot P_{t\gamma hkj} - \sum_{\omega\in\mathcal{W}} \pi_\omega \cdot \Bigg[$$

$$\sum_{r\in\mathcal{R}^{\mathrm{Flex}}} \left(c^{\mathrm{R}\uparrow}_{t\gamma hkr} \cdot r^{\mathrm{R}\uparrow}_{t\gamma hkr\omega} - c^{\mathrm{R}\downarrow}_{t\gamma hkr} \cdot r^{\mathrm{R}\downarrow}_{t\gamma hkr\omega}\right) + \sum_{j\in\mathcal{J}^{\mathrm{Own,Flex}}} \left(\beta^{\uparrow}_{t\gamma hkj} \cdot r^{\uparrow}_{t\gamma hkj\omega} - \beta^{\downarrow}_{t\gamma hkj} \cdot r^{\downarrow}_{t\gamma hkj\omega}\right) +$$

$$\sum_{d\in\mathcal{D}} c^{\mathrm{VoLL}} \cdot l^{\mathrm{sh}}_{t\gamma hkd\omega}\Bigg] \tag{2.4a}$$

Subject to

$$\sum_{j\in\mathcal{J}^{\mathrm{Sys}}_n} P_{t\gamma hkj} - \sum_{d\in\mathcal{D}_n} P^{\mathrm{D}}_{t\gamma hkd} - \sum_{m\in\mathcal{N}_{nm}} S_{nm} \cdot (\theta^{\mathrm{DA}}_{t\gamma hkn} - \theta^{\mathrm{DA}}_{t\gamma hkm}) = 0 : \lambda^{\mathrm{DA}}_{t\gamma hkn} \quad \forall n \in \mathcal{N} \tag{2.4b}$$

$$\sum_{j\in\mathcal{J}^{\mathrm{Sys,Flex}}_n} (r^{\uparrow}_{t\gamma hkj\omega} - r^{\downarrow}_{t\gamma hkj\omega}) + \sum_{j\in\mathcal{J}^{\mathrm{Sys,WP}}_n} (\sum_{\substack{\tau\in\mathcal{T}\\ \tau\leq t}} X_{\tau\gamma j} \cdot K^{\mathrm{CF}}_{hj} \cdot K^{\mathrm{WS}}_{j\omega} - P_{t\gamma hkj} - P^{\mathrm{sp}}_{t\gamma hkj\omega}) +$$

$$\sum_{d\in\mathcal{D}_n} l^{\mathrm{sh}}_{t\gamma hkd\omega} + \sum_{m\in\mathcal{N}_{nm}} S_{nm} \cdot (\theta^{\mathrm{DA}}_{t\gamma hkn} - \theta^{\mathrm{RT}}_{t\gamma hkn\omega} - \theta^{\mathrm{DA}}_{t\gamma hkm} + \theta^{\mathrm{RT}}_{t\gamma hkm\omega}) = 0 : \lambda^{\mathrm{RT}}_{t\gamma hkn\omega}$$

$$\forall n \in \mathcal{N} \quad \forall \omega \in \mathcal{W} \tag{2.4c}$$

$$S_{nm} \cdot (\theta^{\mathrm{DA}}_{t\gamma hkn} - \theta^{\mathrm{DA}}_{t\gamma hkm}) \leq \overline{F}_{nm} : \mu^{\mathrm{F,DA}}_{t\gamma hknm} \quad \forall (n,m) \in \mathcal{N}_{nm} \tag{2.4d}$$

$$S_{nm} \cdot (\theta^{\mathrm{RT}}_{t\gamma hkn\omega} - \theta^{\mathrm{RT}}_{t\gamma hkm\omega}) \leq \overline{F}_{nm} : \mu^{\mathrm{F,RT}}_{t\gamma hknm\omega} \quad \forall (n,m) \in \mathcal{N}_{nm} \quad \forall \omega \in \mathcal{W} \tag{2.4e}$$

$$\theta^{\mathrm{DA}}_{t\gamma hk(n=1)} = 0 : \mu^{\mathrm{ref,DA}}_{t\gamma hk(n=1)} \tag{2.4f}$$

$$\theta^{\mathrm{RT}}_{t\gamma hk(n=1)\omega} = 0 : \mu^{\mathrm{ref,RT}}_{t\gamma hk(n=1)\omega} \quad \forall \omega \in \mathcal{W} \tag{2.4g}$$

$$0 \leq P^{\mathrm{D}}_{t\gamma hkd} \leq \overline{P}^{\mathrm{D}}_{t\gamma d} \cdot K^{\mathrm{DF}}_{h} : (\underline{\mu}^{\mathrm{D}}_{t\gamma hkd}; \overline{\mu}^{\mathrm{D}}_{t\gamma hkd}) \quad \forall d \in \mathcal{D} \tag{2.4h}$$

$$0 \leq P_{t\gamma hkj} \leq \overline{P}_{t\gamma hkj} : (\underline{\mu}_{t\gamma hkj}; \overline{\mu}_{t\gamma hkj}) \quad \forall j \in \mathcal{J}^{\mathrm{Sys}} \tag{2.4i}$$

$$0 \leq r^{\uparrow}_{t\gamma hkj\omega} \leq \overline{R}^{\uparrow}_{t\gamma hkj} : (\underline{\mu}^{\uparrow}_{t\gamma hkj\omega}; \overline{\mu}^{\uparrow}_{t\gamma hkj\omega}) \quad \forall j \in \mathcal{J}^{\mathrm{Sys,Flex}} \quad \forall \omega \in \mathcal{W} \tag{2.4j}$$

$$0 \leq r^{\downarrow}_{t\gamma hkj\omega} \leq \overline{R}^{\downarrow}_{t\gamma hkj} : (\underline{\mu}^{\downarrow}_{t\gamma hkj\omega}; \overline{\mu}^{\downarrow}_{t\gamma hkj\omega}) \quad \forall j \in \mathcal{J}^{\mathrm{Sys,Flex}} \quad \forall \omega \in \mathcal{W} \tag{2.4k}$$

$$0 \leq P_{t\gamma hlj} + r^{\uparrow}_{t\gamma hkj\omega} - r^{\downarrow}_{t\gamma hkj\omega} \leq \overline{P}_{t\gamma hkj} : (\underline{\mu}^{\uparrow\downarrow}_{t\gamma hkj\omega}; \overline{\mu}^{\uparrow\downarrow}_{t\gamma hkj\omega}) \quad \forall j \in \mathcal{J}^{\mathrm{Sys,Flex}} \quad \forall \omega \in \mathcal{W} \tag{2.4l}$$

$$0 \leq l^{\mathrm{sh}}_{t\gamma hkd\omega} \leq P^{\mathrm{D}}_{t\gamma hkd} : (\underline{\mu}^{\mathrm{D,sh}}_{t\gamma hkd\omega}; \overline{\mu}^{\mathrm{D,sh}}_{t\gamma hkd\omega}) \quad \forall d \in \mathcal{D} \quad \forall \omega \in \mathcal{W} \tag{2.4m}$$

$$0 \leq P^{\mathrm{sp}}_{t\gamma hkj\omega} \leq \sum_{\substack{\tau\in\mathcal{T}\\ \tau\leq t}} X_{\tau\gamma j} \cdot K^{\mathrm{CF}}_{hj} \cdot K^{\mathrm{WS}}_{j\omega} : (\underline{\mu}^{\mathrm{sp}}_{t\gamma hkj\omega}; \overline{\mu}^{\mathrm{sp}}_{t\gamma hkj\omega}) \quad \forall j \in \mathcal{J}^{\mathrm{Sys,WP}} \quad \forall \omega \in \mathcal{W} \tag{2.4n}$$

$$\Bigg\} \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad k \in \mathcal{K}$$

The objective function (2.4a) of each lower-level problem aims at maximizing the social welfare at the day-ahead stage and minimizing the real-time balancing costs. The clearing is performed for each period, long-term scenario, operational day and market scenario. With such objectives, the market operator collects day-ahead prices and quantities from the existing, candidate and rival generation units as well as bidding prices and quantities for each block of demand units. The day-ahead clearing is carried out anticipating the future real-time balancing costs, represented by the up- and down-deployment costs of flexible units and cost of load shedding. The equality constraint (2.4b) ensures the balance between scheduled generation and consumption quantities, as well as scheduled power flows in the network at the day-ahead stage. The real-time balance, controlling the redispatch of the day-ahead scheduled quantities, is enforced by (2.4c). A set of inequality constraints (2.4d) and (2.4e) fulfills the capacity limits of network lines at the day-ahead and real-time, and equality constraints (2.4f) and (2.4g) define a reference node at both stages. Constraints (2.4h) and (2.4i) provide upper and lower bounds for the scheduled quantities of the rival units, existing and candidate units as well as demand units at the day-ahead stage. The real-time deployment of reserves is controlled by the submitted reserve capacities in conditions (2.4j) and (2.4k). The compliance of the day-ahead and real-time dispatch of flexible units is ensured by condition (2.4l). The most expensive balancing actions associated with the load shedding and wind spillage are limited by (2.4m) and (2.4n), respectively.

A set of the primal lower-level decision variables $\Delta^{\mathrm{LL,P}} \in \{P^{\mathrm{D}}_{t\gamma hkd}, P_{t\gamma hkj}, r^{\uparrow}_{t\gamma hkj\omega}, r^{\downarrow}_{t\gamma hkj\omega}, l^{\mathrm{sh}}_{t\gamma hkdb\omega}, P^{\mathrm{sp}}_{t\gamma hkj\omega}, \theta^{\mathrm{DA}}_{t\gamma hkn}, \theta^{\mathrm{RT}}_{t\gamma hkn\omega}\}$ comprises decisions concerning the optimal allocation of the production and consumption quantities at the day-ahead and real-time stages for every time period, representative day, long-term and market uncertainty scenarios. The dual variables of the lower-level problems are listed next to the each constraint after the colon sign and collected in set $\Delta^{\mathrm{LL,D}}$,

$$\Delta^{\mathrm{LL,D}} \in \{\lambda^{\mathrm{DA}}_{t\gamma hkn}, \lambda^{\mathrm{RT}}_{t\gamma hkn\omega}, \mu^{\mathrm{F,DA}}_{t\gamma hknm}, \mu^{\mathrm{F,RT}}_{t\gamma hknm\omega}, \mu^{\mathrm{ref,DA}}_{t\gamma hk(n=1)}, \mu^{\mathrm{ref,RT}}_{t\gamma hk(n=1)\omega}, \underline{\mu}^{\mathrm{D}}_{t\gamma hkd}, \overline{\mu}^{\mathrm{D}}_{t\gamma hkd},$$
$$\underline{\mu}_{t\gamma hkj}, \overline{\mu}_{t\gamma hkj}, \underline{\mu}^{\uparrow}_{t\gamma hkj\omega}, \overline{\mu}^{\uparrow}_{t\gamma hkj\omega}\underline{\mu}^{\downarrow}_{t\gamma hkj\omega}, \overline{\mu}^{\downarrow}_{t\gamma hkj\omega}, \underline{\mu}^{\uparrow\downarrow}_{t\gamma hkj\omega}, \overline{\mu}^{\uparrow\downarrow}_{t\gamma hkj\omega}, \underline{\mu}^{\mathrm{D,sh}}_{t\gamma hkd\omega}, \overline{\mu}^{\mathrm{D,sh}}_{t\gamma hkd\omega},$$
$$\underline{\mu}^{\mathrm{sp}}_{t\gamma hkj\omega}, \overline{\mu}^{\mathrm{sp}}_{t\gamma hkj\omega}\}.$$

2.2.2 MPEC reformulation of the bilevel problem

Standard optimization packages can not handle the bilevel structure of problem (2.2)-(2.4n). This is because the upper-level profit maximization problem is constrained by a set of lower-level optimization problems representing market auctions for different scenarios. However, the objective function of each LL problem is convex and LL constraints are affine functions. Thus a set of LL problems could be replaced with their Karush–Kuhn–Tucker optimally conditions. In such way, the strategic expansion problem is materialized as a single-level mathematical programming with equilibrium constraint (MPEC) problem.

For the sake of clarity, the indexes of time periods, long-term and market uncertainty scenarios, as well as representative days, are omitted. The Lagrangian function for each

LL problem is defined as follows:

$$
\begin{aligned}
&\mathcal{L}_{t\gamma hk} = -\sum_{d\in\mathcal{D}} b_d^{\mathrm{D}} \cdot P_d^{\mathrm{D}} + \sum_{r\in\mathcal{R}} c_r^{\mathrm{R}} \cdot P_r^{R} + \sum_{j\in\mathcal{J}^{\mathrm{Own}}} \beta_j \cdot P_j + \sum_{\omega\in\mathcal{W}} \pi_\omega \cdot \Bigg[\sum_{r\in\mathcal{R}^{\mathrm{Flex}}} \Big(c_r^{\mathrm{R}\uparrow} \cdot r_{r\omega}^{\mathrm{R}\uparrow} \\
&- c_r^{\mathrm{R}\downarrow} \cdot r_{r\omega}^{\mathrm{R}\downarrow} \Big) + \sum_{j\in\mathcal{J}^{\mathrm{Own,Flex}}} \Big(\beta_j^{\uparrow} \cdot r_{j\omega}^{\uparrow} - \beta_j^{\downarrow} \cdot r_{j\omega}^{\downarrow} \Big) + \sum_{d\in\mathcal{D}} c^{\mathrm{VoLL}} \cdot l_{d\omega}^{\mathrm{sh}} \Bigg] - \sum_{n\in\mathcal{N}} \lambda_n^{\mathrm{DA}} \cdot \Big(\sum_{r\in\mathcal{R}_n} P_r^{\mathrm{R}} \\
&+ \sum_{e\in\mathcal{E}_n} P_e^{\mathrm{E}} + \sum_{c\in\mathcal{C}_n} P_c^{\mathrm{C}} - \sum_{d\in\mathcal{D}_n} P_d^{\mathrm{D}} - \sum_{m\in\mathcal{N}_{nm}} S_{nm} \cdot (\theta_n^{\mathrm{DA}} - \theta_m^{\mathrm{DA}}) \Big) - \sum_{n\in\mathcal{N}} \sum_{\omega\in\mathcal{O}} \lambda_{n\omega}^{\mathrm{RT}} \cdot \Bigg(\\
&\sum_{r\in\mathcal{R}_n^{\mathrm{Flex}}} (r_{r\omega}^{\mathrm{R}\uparrow} - r_{r\omega}^{\mathrm{R}\downarrow}) + \sum_{e\in\mathcal{E}_n^{\mathrm{Flex}}} (r_{e\omega}^{\mathrm{E}\uparrow} - r_{e\omega}^{\mathrm{E}\downarrow}) + \sum_{c\in\mathcal{C}_n^{\mathrm{Flex}}} (r_{c\omega}^{\mathrm{C}\uparrow} - r_{c\omega}^{\mathrm{C}\downarrow}) + \sum_{r\in\mathcal{R}_n^{\mathrm{WP}}} (P_{r\omega}^{\mathrm{R,a}} - P_r^{\mathrm{R}} - P_{r\omega}^{\mathrm{R,sp}}) \\
&+ \sum_{e\in\mathcal{E}_n^{\mathrm{WP}}} (P_{e\omega}^{\mathrm{E,a}} - P_e^{\mathrm{E}} - P_{e\omega}^{\mathrm{E,sp}}) + \sum_{c\in\mathcal{C}_n^{\mathrm{WP}}} (P_{c\omega}^{\mathrm{C,a}} - P_c^{\mathrm{C}} - P_{c\omega}^{\mathrm{C,sp}}) + \sum_{d\in\mathcal{D}_n} l_{td\omega}^{\mathrm{sh}} + \sum_{m\in\mathcal{N}_{nm}} S_{nm} \cdot (\theta_n^{\mathrm{DA}} - \theta_{n\omega}^{\mathrm{RT}} \\
&- \theta_m^{\mathrm{DA}} + \theta_{m\omega}^{\mathrm{RT}}) \Bigg) + \sum_{(n,m)\in\mathcal{N}_{nm}} \mu_{nm}^{\mathrm{F,DA}} \cdot (S_{nm} \cdot (\theta_{t\gamma hkn}^{\mathrm{DA}} - \theta_{t\gamma hkm}^{\mathrm{DA}}) - \overline{F}_{nm}) + \sum_{(n,m)\in\mathcal{N}_{nm}} \sum_{\omega\in\mathcal{O}} \mu_{nm\omega}^{\mathrm{F,RT}} \cdot \\
&(S_{nm} \cdot (\theta_{t\gamma hkn\omega}^{\mathrm{RT}} - \theta_{t\gamma hkm\omega}^{\mathrm{RT}}) - \overline{F}_{nm}) + \mu_{(n=1)}^{\mathrm{ref,DA}} \cdot \theta_{(n=1)}^{\mathrm{DA}} + \mu_{(n=1)\omega}^{\mathrm{ref,RT}} \cdot \theta_{(n=1)\omega}^{\mathrm{RT}} \\
&+ \sum_{d\in\mathcal{D}} \Big(\overline{\mu}_d^{\mathrm{D}} \cdot (P_d^{\mathrm{D}} - \overline{P}_d^{\mathrm{D}} \cdot K_h^{\mathrm{DF}}) - \underline{\mu}_d^{\mathrm{D}} \cdot P_d^{\mathrm{D}} \Big) + \sum_{r\in\mathcal{R}} \Big(\overline{\mu}_r^{\mathrm{R}} \cdot (P_r^{\mathrm{R}} - \overline{P}_r^{\mathrm{R}}) - \underline{\mu}_r^{\mathrm{R}} \cdot P_r^{\mathrm{R}} \Big) \\
&+ \sum_{e\in\mathcal{E}} \Big(\overline{\mu}_e^{\mathrm{E}} \cdot (P_e^{\mathrm{E}} - \overline{P}_e^{\mathrm{E}}) - \underline{\mu}_e^{\mathrm{E}} \cdot P_e^{\mathrm{E}} \Big) + \sum_{c\in\mathcal{C}} \Big(\overline{\mu}_c^{\mathrm{C}} \cdot (P_c^{\mathrm{C}} - \overline{P}_c^{\mathrm{C}}) - \underline{\mu}_c^{\mathrm{C}} \cdot P_c^{\mathrm{C}} \Big) \\
&+ \sum_{r\in\mathcal{R}^{\mathrm{Flex}}} \sum_{\omega\in\mathcal{O}} \Big(\overline{\mu}_{r\omega}^{\mathrm{R}\uparrow} \cdot (r_{r\omega}^{\mathrm{R}\uparrow} - \overline{R}_r^{\mathrm{R}\uparrow}) - \underline{\mu}_{r\omega}^{\mathrm{R}\uparrow} \cdot r_{r\omega}^{\mathrm{R}\uparrow} \Big) + \sum_{e\in\mathcal{E}^{\mathrm{Flex}}} \sum_{\omega\in\mathcal{O}} \Big(\overline{\mu}_{e\omega}^{\mathrm{E}\uparrow} \cdot (r_{e\omega}^{\mathrm{E}\uparrow} - \overline{R}_e^{\mathrm{E}\uparrow}) - \underline{\mu}_{e\omega}^{\mathrm{E}\uparrow} \cdot r_{e\omega}^{\mathrm{E}\uparrow} \Big) \\
&+ \sum_{c\in\mathcal{C}^{\mathrm{Flex}}} \sum_{\omega\in\mathcal{O}} \Big(\overline{\mu}_{c\omega}^{\mathrm{C}\uparrow} \cdot (r_{c\omega}^{\mathrm{C}\uparrow} - \overline{R}_c^{\mathrm{C}\uparrow}) - \underline{\mu}_{c\omega}^{\mathrm{C}\uparrow} \cdot r_{c\omega}^{\mathrm{C}\uparrow} \Big) + \sum_{r\in\mathcal{R}^{\mathrm{Flex}}} \sum_{\omega\in\mathcal{O}} \Big(\overline{\mu}_{r\omega}^{\mathrm{R}\downarrow} \cdot (r_{r\omega}^{\mathrm{R}\downarrow} - \overline{R}_r^{\mathrm{R}\downarrow}) - \underline{\mu}_{r\omega}^{\mathrm{R}\downarrow} \cdot r_{r\omega}^{\mathrm{R}\downarrow} \Big) \\
&+ \sum_{e\in\mathcal{E}^{\mathrm{Flex}}} \sum_{\omega\in\mathcal{O}} \Big(\overline{\mu}_{e\omega}^{\mathrm{E}\downarrow} \cdot (r_{e\omega}^{\mathrm{E}\downarrow} - \overline{R}_e^{\mathrm{E}\downarrow}) - \underline{\mu}_{e\omega}^{\mathrm{E}\downarrow} \cdot r_{e\omega}^{\mathrm{E}\downarrow} \Big) + \sum_{c\in\mathcal{C}^{\mathrm{Flex}}} \sum_{\omega\in\mathcal{O}} \Big(\overline{\mu}_{c\omega}^{\mathrm{C}\downarrow} \cdot (r_{c\omega}^{\mathrm{C}\downarrow} - \overline{R}_c^{\mathrm{C}\downarrow}) - \underline{\mu}_{c\omega}^{\mathrm{C}\downarrow} \cdot r_{c\omega}^{\mathrm{C}\downarrow} \Big) \\
&+ \sum_{r\in\mathcal{R}^{\mathrm{Flex}}} \sum_{\omega\in\mathcal{O}} \Big(\overline{\mu}_{r\omega}^{\mathrm{R}\uparrow\downarrow} \cdot (P_r^{\mathrm{R}} + r_{r\omega}^{\mathrm{R}\uparrow} - r_{r\omega}^{\mathrm{R}\downarrow} - \overline{P}_r^{\mathrm{R}}) - \underline{\mu}_{r\omega}^{\mathrm{R}\uparrow\downarrow} \cdot (P_r^{\mathrm{R}} + r_{r\omega}^{\mathrm{R}\uparrow} - r_{r\omega}^{\mathrm{R}\downarrow}) \Big) \\
&+ \sum_{e\in\mathcal{E}^{\mathrm{Flex}}} \sum_{\omega\in\mathcal{O}} \Big(\overline{\mu}_{e\omega}^{\mathrm{E}\uparrow\downarrow} \cdot (P_e^{\mathrm{E}} + r_{e\omega}^{\mathrm{E}\uparrow} - r_{e\omega}^{\mathrm{E}\downarrow} - \overline{P}_e^{\mathrm{E}}) - \underline{\mu}_{e\omega}^{\mathrm{E}\uparrow\downarrow} \cdot (P_e^{\mathrm{E}} + r_{e\omega}^{\mathrm{E}\uparrow} - r_{e\omega}^{\mathrm{E}\downarrow}) \Big) \\
&+ \sum_{c\in\mathcal{C}^{\mathrm{Flex}}} \sum_{\omega\in\mathcal{O}} \Big(\overline{\mu}_{c\omega}^{\mathrm{C}\uparrow\downarrow} \cdot (P_c^{\mathrm{C}} + r_{c\omega}^{\mathrm{C}\uparrow} - r_{c\omega}^{\mathrm{C}\downarrow} - \overline{P}_c^{\mathrm{C}}) - \underline{\mu}_{c\omega}^{\mathrm{C}\uparrow\downarrow} \cdot (P_c^{\mathrm{C}} + r_{c\omega}^{\mathrm{C}\uparrow} - r_{c\omega}^{\mathrm{C}\downarrow}) \Big) \\
&+ \sum_{d\in\mathcal{D}} \sum_{\omega\in\mathcal{O}} \Big(\overline{\mu}_{d\omega}^{\mathrm{D,sh}} \cdot (l_{d\omega}^{\mathrm{sh}} - P_d^{\mathrm{D}}) - \underline{\mu}_{d\omega}^{\mathrm{D,sh}} \cdot l_{d\omega}^{\mathrm{sh}} \Big) + \sum_{r\in\mathcal{R}^{\mathrm{WP}}} \sum_{\omega\in\mathcal{O}} \Big(\overline{\mu}_{r\omega}^{\mathrm{R,sp}} \cdot (P_{r\omega}^{\mathrm{R,sp}} - P_{r\omega}^{\mathrm{R,a}}) \\
&- \underline{\mu}_{r\omega}^{\mathrm{R,sp}} \cdot P_{r\omega}^{\mathrm{R,sp}} \Big) + \sum_{e\in\mathcal{E}^{\mathrm{WP}}} \sum_{\omega\in\mathcal{O}} \Big(\overline{\mu}_{e\omega}^{\mathrm{E,sp}} \cdot (P_{e\omega}^{\mathrm{E,sp}} - P_{e\omega}^{\mathrm{E,a}}) - \underline{\mu}_{e\omega}^{\mathrm{E,sp}} \cdot P_{e\omega}^{\mathrm{E,sp}} \Big)
\end{aligned}
$$

$$+\sum_{c\in\mathcal{C}^{\mathrm{WP}}}\sum_{\omega\in\mathcal{O}}\left(\overline{\mu}^{\mathrm{C,sp}}_{c\omega}\cdot(P^{\mathrm{C,sp}}_{c\omega}-P^{\mathrm{C,a}}_{c\omega})-\underline{\mu}^{\mathrm{C,sp}}_{c\omega}\cdot P^{\mathrm{C,sp}}_{c\omega}\right)$$

$$\forall t\in\mathcal{T}\quad\forall\gamma\in\mathcal{G}\quad\forall h\in\mathcal{H}\quad k\in\mathcal{K}\tag{2.5}$$

KKT conditions comprise of (*i*) stationarity conditions, (*ii*) primal feasibility, (*iii*) dual feasibility and (*iv*) complementary slackness.

Stationarity conditions are defined as private deviates of the Lagrangian function with respect to each primal decision variable, and writes as follows:

$$\begin{cases}\dfrac{\partial\mathcal{L}}{\partial P^{\mathrm{D}}_{d}}=-b^{\mathrm{D}}_{d}+\lambda^{\mathrm{DA}}_{(n:d\in\mathcal{D}_n)}+\overline{\mu}^{\mathrm{D}}_{d}-\underline{\mu}^{\mathrm{D}}_{d}-\sum_{\omega\in\mathcal{O}}\overline{\mu}^{\mathrm{D,sh}}_{d\omega}=0\quad\forall d\in\mathcal{D}\end{cases}\tag{2.6a}$$

$$\frac{\partial\mathcal{L}}{\partial P^{\mathrm{R}}_{r}}=c^{\mathrm{R}}_{r}-\lambda^{\mathrm{DA}}_{(n:r\in\mathcal{R}_n)}+\overline{\mu}^{\mathrm{R}}_{r}-\underline{\mu}^{\mathrm{R}}_{r}+\sum_{\omega\in\mathcal{O}}(\overline{\mu}^{\mathrm{R}\uparrow\downarrow}_{r\omega}-\underline{\mu}^{\mathrm{R}\uparrow\downarrow}_{r\omega})=0\quad\forall r\in\mathcal{R}^{\mathrm{Flex}}\tag{2.6b}$$

$$\frac{\partial\mathcal{L}}{\partial P^{\mathrm{E}}_{e}}=\beta^{\mathrm{E}}_{e}-\lambda^{\mathrm{DA}}_{(n:e\in\mathcal{E}_n)}+\overline{\mu}^{\mathrm{E}}_{e}-\underline{\mu}^{\mathrm{E}}_{e}+\sum_{\omega\in\mathcal{O}}(\overline{\mu}^{\mathrm{E}\uparrow\downarrow}_{e\omega}-\underline{\mu}^{\mathrm{E}\uparrow\downarrow}_{e\omega})=0\quad\forall e\in\mathcal{E}^{\mathrm{Flex}}\tag{2.6c}$$

$$\frac{\partial\mathcal{L}}{\partial P^{\mathrm{C}}_{c}}=\beta^{\mathrm{C}}_{c}-\lambda^{\mathrm{DA}}_{(n:c\in\mathcal{C}_n)}+\overline{\mu}^{\mathrm{C}}_{c}-\underline{\mu}^{\mathrm{C}}_{c}+\sum_{\omega\in\mathcal{O}}(\overline{\mu}^{\mathrm{C}\uparrow\downarrow}_{c\omega}-\underline{\mu}^{\mathrm{C}\uparrow\downarrow}_{c\omega})=0\quad\forall c\in\mathcal{C}^{\mathrm{Flex}}\tag{2.6d}$$

$$\frac{\partial\mathcal{L}}{\partial P^{\mathrm{R}}_{r}}=c^{\mathrm{R}}_{r}-\lambda^{\mathrm{DA}}_{(n:r\in\mathcal{R}_n)}+\sum_{\omega\in\mathcal{O}}\lambda^{\mathrm{RT}}_{(n:r\in\mathcal{R}_n)\omega}+\overline{\mu}^{\mathrm{R}}_{r}-\underline{\mu}^{\mathrm{R}}_{r}=0\quad\forall r\in\mathcal{R}^{\mathrm{WP}}\tag{2.6e}$$

$$\frac{\partial\mathcal{L}}{\partial P^{\mathrm{E}}_{e}}=\beta^{\mathrm{E}}_{e}-\lambda^{\mathrm{DA}}_{(n:e\in\mathcal{E}_n)}+\sum_{\omega\in\mathcal{O}}\lambda^{\mathrm{RT}}_{(n:e\in\mathcal{E}_n)\omega}+\overline{\mu}^{\mathrm{E}}_{e}-\underline{\mu}^{\mathrm{E}}_{e}=0\quad\forall e\in\mathcal{E}^{\mathrm{WP}}\tag{2.6f}$$

$$\frac{\partial\mathcal{L}}{\partial P^{\mathrm{C}}_{c}}=\beta^{\mathrm{C}}_{c}-\lambda^{\mathrm{DA}}_{(n:c\in\mathcal{C}_n)}+\sum_{\omega\in\mathcal{O}}\lambda^{\mathrm{RT}}_{(n:c\in\mathcal{C}_n)\omega}+\overline{\mu}^{\mathrm{C}}_{c}-\underline{\mu}^{\mathrm{C}}_{c}=0\quad\forall c\in\mathcal{C}^{\mathrm{WP}}\tag{2.6g}$$

$$\frac{\partial\mathcal{L}}{\partial r^{\mathrm{R}\uparrow}_{r\omega}}=\pi_\omega\cdot c^{\mathrm{R}\uparrow}_{r}-\lambda^{\mathrm{RT}}_{(n:r\in\mathcal{R}_n)\omega}+\overline{\mu}^{\mathrm{R}\uparrow}_{r\omega}-\underline{\mu}^{\mathrm{R}\uparrow}_{r\omega}+\overline{\mu}^{\mathrm{R}\uparrow\downarrow}_{r\omega}-\underline{\mu}^{\mathrm{R}\uparrow\downarrow}_{r\omega}=0\quad\forall\omega\in\mathcal{O}\quad\forall r\in\mathcal{R}^{\mathrm{Flex}}\tag{2.6h}$$

$$\frac{\partial\mathcal{L}}{\partial r^{\mathrm{E}\uparrow}_{e\omega}}=\pi_\omega\cdot\beta^{\mathrm{E}\uparrow}_{e}-\lambda^{\mathrm{RT}}_{(n:e\in\mathcal{E}_n)\omega}+\overline{\mu}^{\mathrm{E}\uparrow}_{e\omega}-\underline{\mu}^{\mathrm{E}\uparrow}_{e\omega}+\overline{\mu}^{\mathrm{E}\uparrow\downarrow}_{e\omega}-\underline{\mu}^{\mathrm{E}\uparrow\downarrow}_{e\omega}=0\quad\forall\omega\in\mathcal{O}\quad\forall e\in\mathcal{E}^{\mathrm{Flex}}\tag{2.6i}$$

$$\frac{\partial\mathcal{L}}{\partial r^{\mathrm{C}\uparrow}_{c\omega}}=\pi_\omega\cdot\beta^{\mathrm{C}\uparrow}_{c}-\lambda^{\mathrm{RT}}_{(n:c\in\mathcal{C}_n)\omega}+\overline{\mu}^{\mathrm{C}\uparrow}_{c\omega}-\underline{\mu}^{\mathrm{C}\uparrow}_{c\omega}+\overline{\mu}^{\mathrm{C}\uparrow\downarrow}_{c\omega}-\underline{\mu}^{\mathrm{C}\uparrow\downarrow}_{c\omega}=0\quad\forall\omega\in\mathcal{O}\quad\forall c\in\mathcal{C}^{\mathrm{Flex}}\tag{2.6j}$$

$$\frac{\partial\mathcal{L}}{\partial r^{\mathrm{R}\downarrow}_{r\omega}}=-\pi_\omega\cdot c^{\mathrm{R}\downarrow}_{r}+\lambda^{\mathrm{RT}}_{(n:r\in\mathcal{R}_n)\omega}+\overline{\mu}^{\mathrm{R}\downarrow}_{r\omega}-\underline{\mu}^{\mathrm{R}\downarrow}_{r\omega}-\overline{\mu}^{\mathrm{R}\uparrow\downarrow}_{r\omega}+\underline{\mu}^{\mathrm{R}\uparrow\downarrow}_{r\omega}=0\quad\forall\omega\in\mathcal{O}\quad\forall r\in\mathcal{R}^{\mathrm{Flex}}\tag{2.6k}$$

$$\frac{\partial\mathcal{L}}{\partial r^{\mathrm{E}\downarrow}_{e\omega}}=-\pi_\omega\cdot\beta^{\mathrm{E}\downarrow}_{e}+\lambda^{\mathrm{RT}}_{(n:e\in\mathcal{E}_n)\omega}+\overline{\mu}^{\mathrm{E}\downarrow}_{e\omega}-\underline{\mu}^{\mathrm{E}\downarrow}_{e\omega}-\overline{\mu}^{\mathrm{E}\uparrow\downarrow}_{e\omega}+\underline{\mu}^{\mathrm{E}\uparrow\downarrow}_{e\omega}=0\quad\forall\omega\in\mathcal{O}\quad\forall e\in\mathcal{E}^{\mathrm{Flex}}\tag{2.6l}$$

$$\frac{\partial \mathcal{L}}{\partial r^{\mathrm{C}\downarrow}_{c\omega}} = -\pi_\omega \cdot \beta^{\mathrm{C}\downarrow}_c + \lambda^{\mathrm{RT}}_{(n:c\in\mathcal{C}_n)\omega} + \overline{\mu}^{\mathrm{C}\downarrow}_{c\omega} - \underline{\mu}^{\mathrm{C}\downarrow}_{c\omega} - \overline{\mu}^{\mathrm{C}\uparrow\downarrow}_{c\omega} + \underline{\mu}^{\mathrm{C}\uparrow\downarrow}_{c\omega} = 0 \quad \forall \omega \in \mathcal{O} \quad \forall c \in \mathcal{C}^{\mathrm{Flex}} \tag{2.6m}$$

$$\frac{\partial \mathcal{L}}{\partial l^{\mathrm{sh}}_{d\omega}} = \pi_\omega \cdot c^{\mathrm{VoLL}} - \lambda^{\mathrm{RT}}_{(n:d\in\mathcal{D}_n)\omega} + \overline{\mu}^{\mathrm{D,sh}}_{d\omega} - \underline{\mu}^{\mathrm{D,sh}}_{d\omega} = 0 \quad \forall \omega \in \mathcal{O} \quad \forall d \in \mathcal{D} \tag{2.6n}$$

$$\frac{\partial \mathcal{L}}{\partial P^{\mathrm{R,sp}}_{r\omega}} = \lambda^{\mathrm{RT}}_{(n:r\in\mathcal{R}_n)\omega} + \overline{\mu}^{\mathrm{R,sp}}_{r\omega} - \underline{\mu}^{\mathrm{R,sp}}_{r\omega} = 0 \quad \forall \omega \in \mathcal{O} \quad \forall r \in \mathcal{R}^{\mathrm{WP}} \tag{2.6o}$$

$$\frac{\partial \mathcal{L}}{\partial P^{\mathrm{E,sp}}_{e\omega}} = \lambda^{\mathrm{RT}}_{(n:e\in\mathcal{E}_n)\omega} + \overline{\mu}^{\mathrm{E,sp}}_{e\omega} - \underline{\mu}^{\mathrm{E,sp}}_{e\omega} = 0 \quad \forall \omega \in \mathcal{O} \quad \forall e \in \mathcal{E}^{\mathrm{WP}} \tag{2.6p}$$

$$\frac{\partial \mathcal{L}}{\partial P^{\mathrm{C,sp}}_{c\omega}} = \lambda^{\mathrm{RT}}_{(n:c\in\mathcal{C}_n)\omega} + \overline{\mu}^{\mathrm{C,sp}}_{c\omega} - \underline{\mu}^{\mathrm{C,sp}}_{c\omega} = 0 \quad \forall \omega \in \mathcal{O} \quad \forall c \in \mathcal{C}^{\mathrm{WP}} \tag{2.6q}$$

$$\frac{\partial \mathcal{L}}{\partial \theta^{\mathrm{DA}}_{n}} = \sum_{m\in\mathcal{N}_{nm}} S_{nm} \cdot \left[\lambda^{\mathrm{DA}}_n - \sum_{\omega\in\mathcal{O}} \lambda^{\mathrm{RT}}_{n\omega} + \mu^{\mathrm{F,DA}}_{nm}\right] - \sum_{m\in\mathcal{N}_{nm}} S_{mn} \cdot \left[\lambda^{\mathrm{DA}}_m - \sum_{\omega\in\mathcal{O}} \lambda^{\mathrm{RT}}_{m\omega} + \mu^{\mathrm{F,DA}}_{mn}\right] - \mu^{\mathrm{ref,DA}}_{(n=1)} = 0 \quad \forall n \in \mathcal{N} \tag{2.6r}$$

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial \theta^{\mathrm{RT}}_{n\omega}} = \sum_{m\in\mathcal{N}_{nm}} S_{nm} \cdot \left[\lambda^{\mathrm{RT}}_{n\omega} + \mu^{\mathrm{F,RT}}_{nm\omega}\right] - \sum_{m\in\mathcal{N}_{nm}} S_{mn} \cdot \left[\lambda^{\mathrm{RT}}_{m\omega} + \mu^{\mathrm{F,RT}}_{mn\omega}\right] - \mu^{\mathrm{ref,RT}}_{(n=1)\omega} = 0 \\ \forall \omega \in \mathcal{O} \quad \forall n \in \mathcal{N} \end{aligned}\right\} \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad k \in \mathcal{K} \tag{2.6s}$$

The primal feasibility conditions state that the optimal solution to the problem has to fulfill the constraint declared by the primal problem, and thus they are written according to (2.4b)-(2.4n). The dual feasibility conditions imply that the dual variables associated with the primal inequality constraints have to be positive, such that:

$$\mu^{\mathrm{F,DA}}_{m}, \mu^{\mathrm{F,RT}}_{m\omega}, \underline{\mu}^{\mathrm{D}}_{db}, \overline{\mu}^{\mathrm{D}}_{db}, \underline{\mu}_{j}, \overline{\mu}_{j}, \underline{\mu}^{\uparrow}_{j\omega}, \overline{\mu}^{\uparrow}_{j\omega}\underline{\mu}^{\downarrow}_{j\omega}, \overline{\mu}^{\downarrow}_{j\omega}, \underline{\mu}^{\uparrow\downarrow}_{j\omega}, \overline{\mu}^{\uparrow\downarrow}_{j\omega}, \underline{\mu}^{\mathrm{D,sh}}_{db\omega}, \overline{\mu}^{\mathrm{D,sh}}_{db\omega}, \underline{\mu}^{\mathrm{sp}}_{j\omega}, \overline{\mu}^{\mathrm{sp}}_{j\omega} \geq 0 \tag{2.7}$$

The last set of KKT conditions is complementarity slackness constraints. These constraints relate to the primal inequality constraints only and imply that only one state might hold: either the inequality constraint is not binding and the dual variable of this inequality constraint is zero or otherwise. They are defined as follows:

$$\left\{ \left[\overline{F}_{nm} - S_{nm} \cdot (\theta^{\mathrm{DA}}_{n} - \theta^{\mathrm{DA}}_{m})\right] \cdot \mu^{\mathrm{F,DA}}_{nm} = 0 \quad \forall (n,m) \in \mathcal{N}_{nm} \right. \tag{2.8}$$

$$\left[\overline{F}_{nm} - S_{nm} \cdot (\theta^{\mathrm{RT}}_{n\omega} - \theta^{\mathrm{RT}}_{m\omega})\right] \cdot \mu^{\mathrm{F,RT}}_{nm\omega} = 0 \quad \forall (n,m) \in \mathcal{N}_{nm} \quad \forall \omega \in \mathcal{W} \tag{2.9}$$

$$(\overline{P}^{\mathrm{D}}_{d} - P^{\mathrm{D}}_{d}) \cdot \overline{\mu}^{\mathrm{D}}_{d} = 0 \quad \forall d \in \mathcal{D} \tag{2.10}$$

$$P^{\mathrm{D}}_{d} \cdot \underline{\mu}^{\mathrm{D}}_{d} = 0 \quad \forall d \in \mathcal{D} \tag{2.11}$$

$$(\overline{P}^{\mathrm{R}}_{r} - P^{\mathrm{R}}_{r}) \cdot \overline{\mu}^{\mathrm{R}}_{r} = 0 \quad \forall r \in \mathcal{R} \tag{2.12}$$

$$P^{\mathrm{R}}_{r} \cdot \underline{\mu}^{\mathrm{R}}_{r} = 0 \quad \forall r \in \mathcal{R} \tag{2.13}$$

$$(\overline{P}_e^{\mathrm{E}} - P_e^{\mathrm{E}}) \cdot \overline{\mu}_e^{\mathrm{E}} = 0 \quad \forall e \in \mathcal{E} \tag{2.14}$$
$$P_e^{\mathrm{E}} \cdot \underline{\mu}_e^{\mathrm{E}} = 0 \quad \forall e \in \mathcal{E} \tag{2.15}$$
$$(\overline{P}_c^{\mathrm{C}} - P_c^{\mathrm{C}}) \cdot \overline{\mu}_c^{\mathrm{C}} = 0 \quad \forall c \in \mathcal{C} \tag{2.16}$$
$$P_c^{\mathrm{C}} \cdot \underline{\mu}_c^{\mathrm{C}} = 0 \quad \forall c \in \mathcal{C} \tag{2.17}$$
$$(\overline{R}_r^{\mathrm{R}\uparrow} - r_{r\omega}^{\mathrm{R}\uparrow}) \cdot \overline{\mu}_{r\omega}^{\mathrm{R}\uparrow} = 0 \quad \forall r \in \mathcal{R}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.18}$$
$$r_{r\omega}^{\mathrm{R}\uparrow} \cdot \underline{\mu}_{r\omega}^{\mathrm{R}\uparrow} = 0 \quad \forall r \in \mathcal{R}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.19}$$
$$(\overline{R}_e^{\mathrm{E}\uparrow} - r_{e\omega}^{\mathrm{E}\uparrow}) \cdot \overline{\mu}_{e\omega}^{\mathrm{E}\uparrow} = 0 \quad \forall e \in \mathcal{E}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.20}$$
$$r_{e\omega}^{\mathrm{E}\uparrow} \cdot \underline{\mu}_{e\omega}^{\mathrm{E}\uparrow} = 0 \quad \forall e \in \mathcal{E}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.21}$$
$$(\overline{R}_c^{\mathrm{C}\uparrow} - r_{c\omega}^{\mathrm{C}\uparrow}) \cdot \overline{\mu}_{c\omega}^{\mathrm{C}\uparrow} = 0 \quad \forall c \in \mathcal{C}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.22}$$
$$r_{c\omega}^{\mathrm{C}\uparrow} \cdot \underline{\mu}_{c\omega}^{\mathrm{C}\uparrow} = 0 \quad \forall c \in \mathcal{C}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.23}$$
$$(\overline{R}_r^{\mathrm{R}\downarrow} - r_{r\omega}^{\mathrm{R}\downarrow}) \cdot \overline{\mu}_{r\omega}^{\mathrm{R}\downarrow} = 0 \quad \forall r \in \mathcal{R}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.24}$$
$$r_{r\omega}^{\mathrm{R}\downarrow} \cdot \underline{\mu}_{r\omega}^{\mathrm{R}\downarrow} = 0 \quad \forall r \in \mathcal{R}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.25}$$
$$(\overline{R}_e^{\mathrm{E}\downarrow} - r_{e\omega}^{\mathrm{E}\downarrow}) \cdot \overline{\mu}_{e\omega}^{\mathrm{E}\downarrow} = 0 \quad \forall e \in \mathcal{E}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.26}$$
$$r_{e\omega}^{\mathrm{E}\downarrow} \cdot \underline{\mu}_{e\omega}^{\mathrm{E}\downarrow} = 0 \quad \forall e \in \mathcal{E}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.27}$$
$$(\overline{R}_c^{\mathrm{C}\downarrow} - r_{c\omega}^{\mathrm{C}\downarrow}) \cdot \overline{\mu}_{c\omega}^{\mathrm{C}\downarrow} = 0 \quad \forall c \in \mathcal{C}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.28}$$
$$r_{c\omega}^{\mathrm{C}\downarrow} \cdot \underline{\mu}_{c\omega}^{\mathrm{C}\downarrow} = 0 \quad \forall c \in \mathcal{C}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.29}$$
$$(\overline{P}_r^{\mathrm{R}} - P_r^{\mathrm{R}} - r_{r\omega}^{\mathrm{R}\uparrow} + r_{r\omega}^{\mathrm{R}\downarrow}) \cdot \overline{\mu}_{r\omega}^{\mathrm{R}\uparrow\downarrow} = 0 \quad \forall r \in \mathcal{R}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.30}$$
$$(P_r^{\mathrm{R}} + r_{r\omega}^{\mathrm{R}\uparrow} - r_{r\omega}^{\mathrm{R}\downarrow}) \cdot \underline{\mu}_{r\omega}^{\mathrm{R}\uparrow\downarrow} = 0 \quad \forall r \in \mathcal{R}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.31}$$
$$(\overline{P}_e^{\mathrm{E}} - P_e^{\mathrm{E}} - r_{e\omega}^{\mathrm{E}\uparrow} + r_{e\omega}^{\mathrm{E}\downarrow}) \cdot \overline{\mu}_{e\omega}^{\mathrm{E}\uparrow\downarrow} = 0 \quad \forall e \in \mathcal{E}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.32}$$
$$(P_e^{\mathrm{E}} + r_{e\omega}^{\mathrm{E}\uparrow} - r_{e\omega}^{\mathrm{E}\downarrow}) \cdot \underline{\mu}_{e\omega}^{\mathrm{E}\uparrow\downarrow} = 0 \quad \forall e \in \mathcal{E}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.33}$$
$$(\overline{P}_c^{\mathrm{C}} - P_c^{\mathrm{C}} - r_{c\omega}^{\mathrm{C}\uparrow} + r_{c\omega}^{\mathrm{C}\downarrow}) \cdot \overline{\mu}_{c\omega}^{\mathrm{C}\uparrow\downarrow} = 0 \quad \forall c \in \mathcal{C}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.34}$$
$$(P_c^{\mathrm{C}} + r_{c\omega}^{\mathrm{C}\uparrow} - r_{c\omega}^{\mathrm{C}\downarrow}) \cdot \underline{\mu}_{c\omega}^{\mathrm{C}\uparrow\downarrow} = 0 \quad \forall c \in \mathcal{C}^{\mathrm{Flex}} \quad \forall \omega \in \mathcal{O} \tag{2.35}$$
$$(P_d^{\mathrm{D}} - l_{d\omega}^{\mathrm{sh}}) \cdot \overline{\mu}_{d\omega}^{\mathrm{D,sh}} = 0 \quad \forall d \in \mathcal{D} \quad \forall \omega \in \mathcal{O} \tag{2.36}$$
$$l_{d\omega}^{\mathrm{sh}} \cdot \underline{\mu}_{d\omega}^{\mathrm{D,sh}} = 0 \quad \forall d \in \mathcal{D} \quad \forall \omega \in \mathcal{O} \tag{2.37}$$
$$(P_{r\omega}^{\mathrm{R,a}} - P_{r\omega}^{\mathrm{R,sp}}) \cdot \overline{\mu}_{r\omega}^{\mathrm{R,sp}} = 0 \quad \forall r \in \mathcal{R}^{\mathrm{WP}} \quad \forall \omega \in \mathcal{O} \tag{2.38}$$
$$P_{r\omega}^{\mathrm{R,sp}} \cdot \underline{\mu}_{r\omega}^{\mathrm{R,sp}} = 0 \quad \forall r \in \mathcal{R}^{\mathrm{WP}} \quad \forall \omega \in \mathcal{O} \tag{2.39}$$
$$(P_{e\omega}^{\mathrm{E,a}} - P_{e\omega}^{\mathrm{E,sp}}) \cdot \overline{\mu}_{e\omega}^{\mathrm{E,sp}} = 0 \quad \forall e \in \mathcal{E}^{\mathrm{WP}} \quad \forall \omega \in \mathcal{O} \tag{2.40}$$
$$P_{e\omega}^{\mathrm{E,sp}} \cdot \underline{\mu}_{e\omega}^{\mathrm{E,sp}} = 0 \quad \forall e \in \mathcal{E}^{\mathrm{WP}} \quad \forall \omega \in \mathcal{O} \tag{2.41}$$
$$(P_{c\omega}^{\mathrm{C,a}} - P_{c\omega}^{\mathrm{C,sp}}) \cdot \overline{\mu}_{c\omega}^{\mathrm{C,sp}} = 0 \quad \forall c \in \mathcal{C}^{\mathrm{WP}} \quad \forall \omega \in \mathcal{O} \tag{2.42}$$
$$\left. P_{c\omega}^{\mathrm{C,sp}} \cdot \underline{\mu}_{c\omega}^{\mathrm{C,sp}} = 0 \quad \forall c \in \mathcal{C}^{\mathrm{WP}} \quad \forall \omega \in \mathcal{O} \right\} \tag{2.43}$$
$$\forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad k \in \mathcal{K}$$

The bilevel problem could be now recast as the following single-level MPEC problem:

$$\begin{aligned} \underset{\Delta^{\mathrm{UL}} \bigcup \Delta^{\mathrm{LL,P}} \bigcup \Delta^{\mathrm{LL,D}}}{\text{Maximize}} \quad & (2.2) \\ \text{Subject to} \quad & (2.3a) - (2.3n), (2.6a) - (2.6s) \\ & (2.4b) - (2.4n), (2.7), (2.8) - (2.43) \end{aligned}$$

2.2.3 MILP reformulation of the MPEC problem

The resulting single-level equivalent of the bilevel problem is non-linear since it consists of several non-linear terms in the objective function as well as a set of non-linear complementarity slackness constraints (2.8)-(2.43) . Solving this problem directly could lead to sub-optimal investment solutions. In this line, it is of crucial importance to derive a linear equivalent ensuring the global optimum for the expected profit. The non-linear terms in the objective functions are as follows:

1. $P_{t\gamma hkj} \cdot \lambda^{\mathrm{DA}}_{t\gamma hk(n:j\in\mathcal{J}^{\mathrm{Own}})}$ – product of scheduled energy quantities and LMP at the day-ahead market.

2. $\lambda^{\mathrm{RT}}_{t\gamma hk(n:j\in\mathcal{J}^{\mathrm{Own,WP}})\omega} \cdot X_{t\gamma j}$ – product of real-time LMP and investment decisions on candidate wind power units.

3. $\lambda^{\mathrm{RT}}_{t\gamma hk(n:j\in\mathcal{J}^{\mathrm{Own,WP}})\omega} \cdot P_{t\gamma hkj}$ – product of real-time LMP and day-ahead scheduled quantities.

4. $\lambda^{\mathrm{RT}}_{t\gamma hk(n:j\in\mathcal{J}^{\mathrm{Own,WP}})\omega} \cdot P^{\mathrm{sp}}_{t\gamma hkj\omega}$ – product of real-time LMP and wind spillage balancing actions for candidate units.

5. $\lambda^{\mathrm{RT}}_{t\gamma hk(n:j\in\mathcal{J}^{\mathrm{Own,Flex}})\omega} \cdot (r^{\uparrow}_{t\gamma hkj\omega} - r^{\downarrow}_{t\gamma hkj\omega})$ – product of real-time LMP and up- and down-reserve deployment balancing actions for existing and candidate flexible units.

To derive a linear equivalent of the objective function, the strong duality property of the lower-level problem is used. In fact, problem (2.4a)-(2.4n) is convex, thus the duality gap between primal and dual objective functions is equal to zero in optimum. Thus, the following condition holds:

$$\sum_{d\in\mathcal{D}} b^{\mathrm{D}}_d \cdot P^{\mathrm{D}}_d - \sum_{r\in\mathcal{R}} c^{\mathrm{R}}_r \cdot P^{R}_r - \sum_{j\in\mathcal{J}^{\mathrm{Own}}} \beta_j \cdot P_j - \sum_{\omega\in\mathcal{W}} \pi_\omega \cdot \Bigg[\sum_{r\in\mathcal{R}^{\mathrm{Flex}}} \left(c^{\mathrm{R}\uparrow}_r \cdot r^{\mathrm{R}\uparrow}_{r\omega} - c^{\mathrm{R}\downarrow}_r \cdot r^{\mathrm{R}\downarrow}_{r\omega} \right)$$

$$+ \sum_{j\in\mathcal{J}^{\mathrm{Own,Flex}}} \left(\beta^{\uparrow}_j \cdot r^{\uparrow}_{j\omega} - \beta^{\downarrow}_j \cdot r^{\downarrow}_{j\omega} \right) + \sum_{d\in\mathcal{D}} c^{\mathrm{VoLL}} \cdot l^{\mathrm{sh}}_{d\omega} \Bigg] = \sum_{\omega\in\mathcal{O}} \lambda^{\mathrm{RT}}_\omega \cdot \Bigg(\sum_{r\in\mathcal{R}^{\mathrm{WP}}} P^{\mathrm{R,a}}_{r\omega} + \sum_{e\in\mathcal{E}^{\mathrm{WP}}} P^{\mathrm{E,a}}_{e\omega}$$

$$+\sum_{c\in\mathcal{C}^{\mathrm{WP}}} P^{\mathrm{C,a}}_{c\omega}\Big) + \sum_{(n,m)\in N_{nm}} \mu^{\mathrm{F,DA}}_{nm}\cdot\overline{F}_{nm} + \sum_{\omega\in\mathcal{O}}\sum_{(n,m)\in N_{nm}} \mu^{\mathrm{F,RT}}_{nm\omega}\cdot\overline{F}_{nm} + \sum_{d\in\mathcal{D}}\overline{\mu}^{\mathrm{D}}_{d}\cdot\overline{P}^{\mathrm{D}}_{d}$$
$$+\sum_{r\in\mathcal{R}}\overline{\mu}^{\mathrm{R}}_{r}\cdot\overline{P}^{\mathrm{R}}_{r} + \sum_{e\in\mathcal{E}}\overline{\mu}^{\mathrm{E}}_{e}\cdot\overline{P}^{\mathrm{E}}_{e} + \sum_{c\in\mathcal{C}}\overline{\mu}^{\mathrm{C}}_{c}\cdot\overline{P}^{\mathrm{C}}_{c} + \sum_{\omega\in\mathcal{O}}\sum_{r\in\mathcal{R}^{\mathrm{Flex}}}\overline{\mu}^{\mathrm{R}\uparrow}_{tr\omega}\cdot\overline{R}^{\mathrm{R}\uparrow}_{r} + \sum_{\omega\in\mathcal{O}}\sum_{e\in\mathcal{E}^{\mathrm{Flex}}}\overline{\mu}^{\mathrm{E}\uparrow}_{e\omega}\cdot\overline{R}^{\mathrm{E}\uparrow}_{e}$$
$$+\sum_{\omega\in\mathcal{O}}\sum_{c\in\mathcal{C}^{\mathrm{Flex}}}\overline{\mu}^{\mathrm{C}\uparrow}_{c\omega}\cdot\overline{R}^{\mathrm{C}\uparrow}_{c} + \sum_{\omega\in\mathcal{O}}\sum_{r\in\mathcal{R}^{\mathrm{Flex}}}\overline{\mu}^{\mathrm{R}\downarrow}_{r\omega}\cdot\overline{R}^{\mathrm{R}\downarrow}_{r} + \sum_{\omega\in\mathcal{O}}\sum_{e\in\mathcal{E}^{\mathrm{Flex}}}\overline{\mu}^{\mathrm{E}\downarrow}_{e\omega}\cdot\overline{R}^{\mathrm{E}\downarrow}_{e} + \sum_{\omega\in\mathcal{O}}\sum_{c\in\mathcal{C}^{\mathrm{Flex}}}\overline{\mu}^{\mathrm{C}\downarrow}_{c\omega}\cdot\overline{R}^{\mathrm{C}\downarrow}_{c}$$
$$+\sum_{\omega\in\mathcal{O}}\sum_{r\in\mathcal{R}^{\mathrm{Flex}}}\overline{\mu}^{\mathrm{R}\uparrow\downarrow}_{r\omega}\cdot\overline{P}^{\mathrm{R}}_{r} + \sum_{\omega\in\mathcal{O}}\sum_{e\in\mathcal{E}^{\mathrm{Flex}}}\overline{\mu}^{\mathrm{E}\uparrow\downarrow}_{e\omega}\cdot\overline{P}^{\mathrm{E}}_{e} + \sum_{\omega\in\mathcal{O}}\sum_{c\in\mathcal{C}^{\mathrm{Flex}}}\overline{\mu}^{\mathrm{C}\uparrow\downarrow}_{c\omega}\cdot\overline{P}^{\mathrm{C}}_{c} + \sum_{\omega\in\mathcal{O}}\sum_{r\in\mathcal{R}^{\mathrm{WP}}}\overline{\mu}^{\mathrm{R,sp}}_{r\omega}\cdot P^{\mathrm{R,a}}_{r\omega}$$
$$+\sum_{\omega\in\mathcal{O}}\sum_{e\in\mathcal{E}^{\mathrm{WP}}}\overline{\mu}^{\mathrm{E,sp}}_{e\omega}\cdot P^{\mathrm{E,a}}_{e\omega} + \sum_{\omega\in\mathcal{O}}\sum_{c\in\mathcal{C}^{\mathrm{WP}}}\overline{\mu}^{\mathrm{C,sp}}_{c\omega}\cdot P^{\mathrm{C,a}}_{c\omega} \tag{2.45}$$

At the next step, KKT optimality conditions of LL problems are transformed to be later substituted into (2.45). First, the stationarity conditions (2.6c) - (2.6d), (2.6f) - (2.6g), (2.6i) - (2.6j), (2.6l) - (2.6m), (2.6p) - (2.6q) are multiplied by $P^{\mathrm{E}}_{e}, P^{\mathrm{C}}_{c}, P^{\mathrm{E}}_{e}, P^{\mathrm{C}}_{c}, r^{\mathrm{E}\uparrow}_{e\omega}$, $r^{\mathrm{C}\uparrow}_{c\omega}, r^{\mathrm{E}\downarrow}_{e\omega}, r^{\mathrm{C}\downarrow}_{c\omega}, P^{\mathrm{C,sp}}_{c\omega}, P^{\mathrm{E,sp}}_{e\omega}$, respectively. Then, taking into account complementarity conditions (2.14) - (2.17), (2.20) - (2.23), (2.26) - (2.29), (2.32) - (2.35), (2.43), the resulting expressions are substituted into (2.45), such that the linear equivalent of the upper-level objective function writes as follows:

$$\underset{\Delta^{\mathrm{UL}}\cup\Delta^{\mathrm{LL,P}}\cup\Delta^{\mathrm{LL,D}}}{\text{Maximize}}$$
$$\sum_{t\in\mathcal{T}}\frac{1}{(1+\mathrm{DR}_t)^t}\cdot\Bigg\{\sum_{\gamma\in\mathcal{G}}\pi^{\mathrm{L}}_{\gamma}\cdot\Bigg\{\sum_{h\in\mathcal{H}}N^{\mathrm{MC}}_{h}\cdot\Bigg[\sum_{k\in\mathcal{K}}\pi^{\mathrm{MS}}_{k}\cdot\Bigg\langle -\sum_{e\in\mathcal{E}}c^{\mathrm{E}}_{t\gamma e}\cdot P^{\mathrm{E}}_{t\gamma hke} - \sum_{c\in\mathcal{C}}c^{\mathrm{C}}_{t\gamma c}\cdot P^{\mathrm{C}}_{t\gamma hkc}$$
$$-\sum_{\omega\in\mathcal{W}}\pi^{\mathrm{W}}_{\omega}\cdot\Bigg(\sum_{e\in\mathcal{E}^{\mathrm{CCGT}}}c^{E}_{t\gamma e}\cdot r^{\mathrm{E}\uparrow}_{t\gamma hke\omega} + \sum_{c\in\mathcal{C}^{\mathrm{CCGT}}}c^{C}_{t\gamma c}\cdot r^{\mathrm{C}\uparrow}_{t\gamma hkc\omega}\Bigg) + \sum_{d\in\mathcal{D}}\sum_{b\in\mathcal{B}}b^{\mathrm{D}}_{tkdb}\cdot P^{\mathrm{D}}_{t\gamma hkdb}$$
$$-\sum_{r\in\mathcal{R}}c^{\mathrm{R}}_{t\gamma r}\cdot P^{\mathrm{R}}_{t\gamma hkr} - \sum_{d\in\mathcal{D}}\sum_{b\in\mathcal{B}}\overline{\mu}^{\mathrm{D}}_{t\gamma hkdb}\cdot\overline{P}^{\mathrm{D}}_{t\gamma db}\cdot K^{\mathrm{DF}}_{h} - \sum_{r\in\mathcal{R}}\overline{\mu}^{\mathrm{R}}_{t\gamma hkr}\cdot\overline{P}^{\mathrm{R}}_{t\gamma r}$$
$$-\sum_{\omega\in\mathcal{W}}\pi_{\omega}\cdot\Bigg[\sum_{r\in\mathcal{R}^{\mathrm{CCGT}}}\Big(c^{\mathrm{R}\uparrow}_{t\gamma r}\cdot r^{\mathrm{R}\uparrow}_{t\gamma hkr\omega} - c^{\mathrm{R}\downarrow}_{t\gamma r}r^{\mathrm{R}\downarrow}_{t\gamma hkr\omega}\Big) + \sum_{d\in\mathcal{D}}\sum_{b\in\mathcal{B}}c^{\mathrm{VoLL}}\cdot l^{\mathrm{sh}}_{t\gamma hkdb\omega}\Bigg]$$
$$-\sum_{\omega\in\mathcal{W}}\sum_{r\in\mathcal{R}^{\mathrm{WP}}}\lambda^{\mathrm{RT}}_{t\gamma hk(n:r\in\mathcal{R}_n)\omega}\cdot X^{\mathrm{R}}_{t\gamma r}\cdot K^{\mathrm{CF}}_{hr}\cdot K^{\mathrm{WS}}_{r\omega} - \sum_{\omega\in\mathcal{W}}\sum_{r\in\mathcal{R}^{\mathrm{CCGT}}}\overline{\mu}^{\mathrm{R}\uparrow}_{t\gamma hkr\omega}\cdot\overline{R}^{\mathrm{R}\uparrow}_{t\gamma r}$$
$$-\sum_{\omega\in\mathcal{W}}\sum_{r\in\mathcal{R}^{\mathrm{CCGT}}}\overline{\mu}^{\mathrm{R}\downarrow}_{t\gamma hkr\omega}\cdot\overline{R}^{\mathrm{R}\downarrow}_{t\gamma r} - \sum_{\omega\in\mathcal{W}}\sum_{r\in\mathcal{R}^{\mathrm{CCGT}}}\overline{\mu}^{\mathrm{R}\uparrow\downarrow}_{t\gamma hkr\omega}\cdot\overline{P}^{\mathrm{R}}_{t\gamma r} - \sum_{(n,m)\in\mathcal{N}}\mu^{\mathrm{F,DA}}_{t\gamma hknm}\cdot\overline{F}_{nm}$$
$$-\sum_{\omega\in\mathcal{W}}\sum_{(n,m)\in\mathcal{N}}\mu^{\mathrm{F,RT}}_{t\gamma hknm\omega}\cdot\overline{F}_{nm} - \sum_{\omega\in\mathcal{W}}\sum_{r\in\mathcal{R}^{\mathrm{WP}}}\overline{\mu}^{\mathrm{R,sp}}_{t\gamma hkr\omega}\cdot X^{\mathrm{R}}_{t\gamma r}\cdot K^{\mathrm{CF}}_{hr}\cdot K^{\mathrm{WS}}_{r\omega}\Bigg\rangle\Bigg]$$
$$-a_t\cdot\sum_{c\in\mathcal{C}}c^{\mathrm{Inv}}_{t\gamma c}\sum_{\substack{\tau\in\mathcal{T}\\ \tau\le t}}X^{\mathrm{C}}_{\tau\gamma c}\Bigg\}\Bigg\} \tag{2.46}$$

There are two ways to linearize complementarity conditions (2.8)-(2.43). The first one relies on so-called *Big-M* approach [61]. On the one hand, this method is relatively straightforward to implement as two inequality constraints for each complementarity constraint. On the contrary, considering large scale power systems, a wrong choice of M might lead to the violation of KKT optimality conditions. An appropriate selection of M involves the coordinated settings of M and the binary tolerance of optimization solver. Moreover, the scaling of system parameters is also required if M is intolerably high. An alternative solution would be to consider the linearization using special ordered set of type 1 variables, SOS1 [62]. Unlike *Big-M* approach, the linearization through SOS1 variables might increase the computational time, but the KKT conditions would always be satisfied.

For instance, linearization of complementarity condition (2.12) is carried out as following. First, two variables of SOS1 type are introduced, $\{\nu^{+}_{t\gamma hkr}, \nu^{-}_{t\gamma hkr} \in \text{SOS1} \quad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall h \in \mathcal{H} \quad \forall k \in \mathcal{K} \quad \forall r \in \mathcal{R}\}$. These variables are defined so that one of them can take a strictly positive value while another is equal to 0. Then, expression (2.12) is replaced by two equality constraints, as following:

$$\nu^{+}_{t\gamma hkr} + \nu^{-}_{t\gamma hkr} = \overline{\mu}^{\text{R}}_{t\gamma hkr} + (\overline{P}^{\text{R}}_{t\gamma r} - P^{\text{R}}_{t\gamma hkr}) \qquad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall m \in \mathcal{M} \quad \forall \nu \in \mathcal{N} \quad \forall r \in \mathcal{R} \tag{2.47}$$

$$\nu^{+}_{t\gamma hkr} - \nu^{-}_{t\gamma hkr} = \overline{\mu}^{\text{R}}_{t\gamma hkr} - (\overline{P}^{\text{R}}_{t\gamma r} - P^{\text{R}}_{t\gamma hkr}) \qquad \forall t \in \mathcal{T} \quad \forall \gamma \in \mathcal{G} \quad \forall m \in \mathcal{M} \quad \forall \nu \in \mathcal{N} \quad \forall r \in \mathcal{R} \tag{2.48}$$

Linearization of the remaining complementarity conditions is carried out similarly. Finally, the linear MILP equivalent of the original problem writes as the following problem, which could be efficiently tackled using off-the-shelf commercial software:

$$\begin{aligned} \underset{\Delta^{\text{UL}} \cup \Delta^{\text{LL,P}} \cup \Delta^{\text{LL,D}}}{\text{Maximize}} \quad & (2.46) \\ \text{Subject to} \quad & (2.3a) - (2.3n), (2.6a) - (2.6s) \\ & (2.4b) - (2.4n), (2.7), (2.8) - (2.43) \\ & + \text{ set of linearized complementarity constraints} \\ & \text{similar to} \quad (2.47) - (2.48) \end{aligned}$$

CHAPTER 3

Decomposition of the multi-stage strategic expansion problem via progressive hedging

3.1 Motivation behind the decomposition

Over the course of the last two decades, a class of investment planning problems evolved toward large-scale optimization problems. These problems include diverse and complex objectives and constraints, accounting for technical, social and economic perspectives of the problem. Moreover, in the context of increasing operational and financial multi-scale uncertainty, these problems contain a plenty of scenarios enhancing the dimension of the models. Therefore, this class of models includes a large number of discrete variables which affect the computational burden to a great extent. A number of discrete variables in the considered strategic expansion planning is defined by (i) amount of binary variables related to the choice of available investment option, and (ii) amount of SOS1 variables used to linearize the complementarity constraints. Particularly, a number of binary variables is computed as follows:

$$N_{\text{Bin}} = N_t \times N_\gamma \times N_c \times N_o, \tag{3.1}$$

where N_t, N_γ, N_c, N_o are numbers of time stages, long-term scenarios, candidate generation technologies and available capacity options, respectively. A number of SOS1 variables depends on the amount of complementarity constraints involved into a problem and computes as following:

$$\begin{aligned} N_{\text{SOS1}} =& N_t \times N_\gamma \times N_h \times N_k \times 2 \times \Big[2 \times N_n \times (1 + N_\omega) + 2 \times (N_r + N_e + N_c) \\ & + 6 \times N_\omega \times (N_{r^{Flex}} + N_{e^{Flex}} + N_{c^{Flex}}) + 2 \times N_\omega \times (N_{r^{WP}} + N_{e^{WP}} + N_{c^{WP}}) \\ & + 2 \times (N_\omega + 1) \times N_d \Big] \end{aligned} \tag{3.2}$$

where $N_h, N_k, N_n, N_\omega, N_r, N_e, N_c, N_d$ are numbers of representative days, market scenarios, nodes, wind power scenarios, rival, existing units, candidate units, and loads, respectively.

Apart from the discrete variables, the complexity of the problem is enforced by non-anticipativity conditions, preventing dependency of the investment decisions on the future long-term scenario realization. The number of such constraints is defined by a number of time stages within a planning horizon, long-term scenarios, and candidate units.

The decomposition of the original problem into a set of sub-problem governed by a common convergence criterion allows speeding up the simulations. This is due to the fact that the dimension of each sub-problem is generally much smaller than the size of the original model. For example, by decomposing the considered problem per long-term scenarios, the amount of discrete and SOS1 variables will be reduced down to $N_{\text{Bin}}/N_{\gamma}$ and $N_{\text{SOS1}}/N_{\gamma}$, respectively. Thus, solving scenario-specific sub-problems iteratively yields in less computational burden compared to the direct approach to the original problem.

3.2 General background on the progressive hedging decomposition

Progressive Hedging Algorithm (PHA) introduced by [41] belongs to a class of augmented Lagrangian relaxation techniques. Unlike Benders decomposition which separates the investment decisions from the operational ones, the PH approach relies on scenario decomposition of the original problem. In this line, the non-anticipativity conditions are relaxed, and investment decisions are transformed into scenario-dependent decisions, such that the original problem boils down to the set of scenario-specific sub-problems. The non-anticipativity conditions are restored iteratively by penalizing the objective function of each sub-problem with respect to the deviation of the investment solution from the average over the adjacent nodes of the decision tree, as depicted in Figure 3.1.

The figure illustrates the PHA relaxation of the three-stage investment planning problem involving two long-term scenarios at each time stage. At the beginning of the first time stage, the producer has to decide on optimal site and size of available technologies to invest. At the second time stage, the long-term uncertainty partially discloses, and it adjusts its investment planning by building new generation units at the second stage. Then, at the third stage, the uncertainty again discloses and it makes the final investment decisions. The PH algorithm relaxes the non-anticipativity conditions and solves 4 sub-problems independently, one per each long-term scenario. Apparently, in some sense, each sub-problem is deterministic since it does not consider the entire set of long-term scenarios, and thus it is less complicated to solve. The sub-problems are solved iteratively to tighten the nodes of the relaxed tree to fulfill the conditions declared by the original tree. In this line, the PH algorithm penalizes the objective function of each sub-problem with respect to the deviation of the investment solution from the probability-weighted average over the bundle of nodes highlighted by blue dashed lines. Eventually, PHA converges if the solution of each problem results in the same investment decisions for the adjacent nodes.

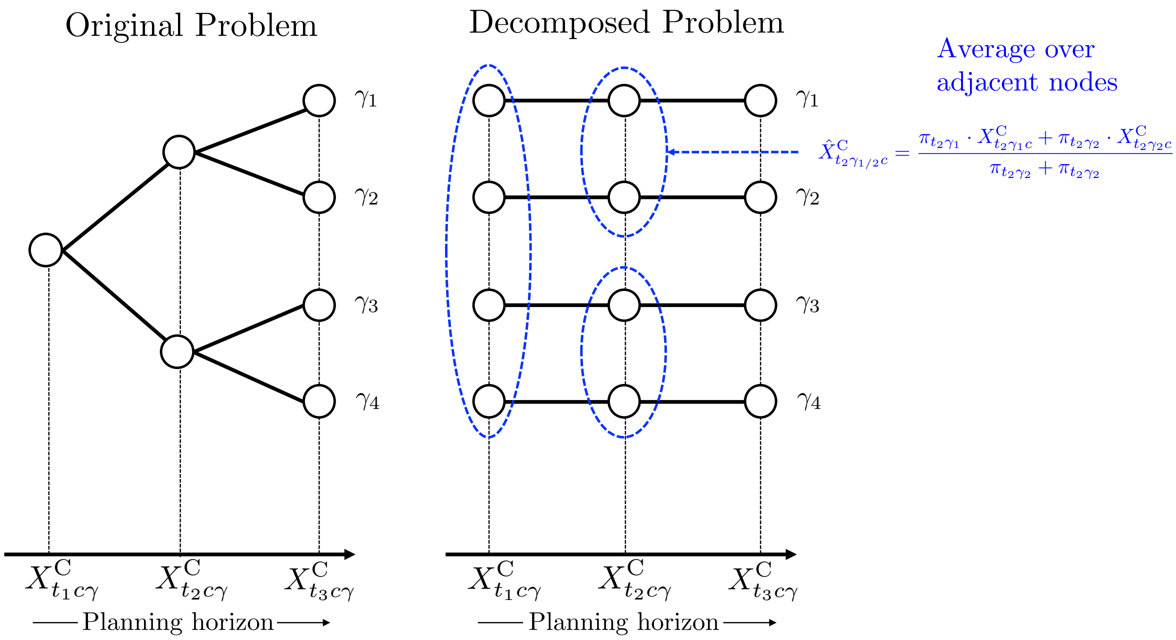


Figure 3.1: PH relaxation of the long-term decision tree.

3.3 Implementation of the progressive hedging algorithm

3.3.1 Decomposition over long-term uncertainty scenarios

To give a formal description of the algorithm, the following compact formulation of the multi-stage stochastic optimization problem is introduced:

$$\underset{x_{t\gamma}}{\text{Minimize}} \quad \sum_{t \in \mathcal{T}} \sum_{\gamma \in \mathcal{G}} \pi^{\mathrm{L}}_{\gamma} \cdot c^{\top}_{t\gamma} \cdot x_{t\gamma} \tag{3.3a}$$

$$\text{Subject to} \quad (x_{t\gamma} \in \mathcal{Q}_{t\gamma}, \forall t \in \mathcal{T}, \forall \gamma \in \mathcal{G}) \tag{3.3b}$$

where $c_{t\gamma}$ is a cost coefficient vector, $x_{t\gamma}$ - a vector of decision variables, $c_{t\gamma}$ and $x_{t\gamma}$ are of the same length; notation $x_{t\gamma} \in \mathcal{Q}_{t\gamma}$ expresses the problem constraints, i.e. to ensure $x_{t\gamma}$ is a feasible solution in the scenario γ at time stage t. Notice, that the original problem is maximization of the expected profit, while the standard form of an optimization problem writes with minimization operator.

To formulate the PHA relaxation of problem (3.3) over the long-term scenarios, first the probability-weighed average of investment solutions among the adjacent nodes of the decision tree at each time stage t for each long-term scenario γ and each candidate unit c $\hat{X}^{\mathrm{C}(i)}_{t\gamma c}$ is introduced, such that:

$$\hat{X}^{\mathrm{C}(i)}_{t\gamma c} = \frac{\sum_{\gamma' \in \overline{\mathcal{G}}_t} \pi^{\mathrm{L}}_{\gamma'} \cdot X^{\mathrm{C}(i)}_{t\gamma' c}}{\sum_{\gamma' \in \overline{\mathcal{G}}_t} \pi^{\mathrm{L}}_{\gamma'}} \tag{3.4}$$

where $X^{\mathrm{C}(i)}_{t\gamma' c}$ is a solution of $X^{\mathrm{C}}_{t\gamma' c}$ at i^{th} iteration of PHA and $\overline{\mathcal{G}}_t$ denotes a set of adjacent scenarios to scenario γ at time stage t, as depicted in Figure 3.1.

At the next step, the Lagrange-multiplier vector of non-anticipativity conditions, or so-called PH multiplier, $m^{\mathrm{PH}(i)}_{t\gamma c}$ is introduced. The superscript (i) stands for an iteration counter of the algorithm. This multiplier is computed as follows:

$$m^{\mathrm{PH}(i)}_{t\gamma c} = m^{\mathrm{PH}(i-1)}_{t\gamma c} + \rho \cdot (X^{\mathrm{C}(i)}_{t\gamma c} - \hat{X}^{\mathrm{C}(i)}_{t\gamma c}) \tag{3.5}$$

where $\rho > 0$ is arbitrary chosen penalization factor. This multiplier is substituted into objective function of each sub-problem, such that the deviation of investment decisions from the relevant probability-weighted average is penalized. Given the introduced notations, the PHA for the long-term scenarios decomposition writes as follows:

Algorithm 1 Multistage PHA for long-term uncertainty scenario decomposition

1: **Initialization:** $i := 0$
2: **Iteration 0:** $\forall \gamma \in \mathcal{G}, \quad X^{\mathrm{C}(i)}_{t\gamma c} \leftarrow \underset{x_t}{\operatorname{argmin}} \Big\{ \sum_{t \in \mathcal{T}} c^{\top}_{t\gamma} \cdot x_t : x_t \in \mathcal{Q}_{t\gamma} \Big\}$
3: **Aggregation:** $\forall t \in \mathcal{T}, \forall \gamma \in \mathcal{G}, \forall c \in \mathcal{C}, \quad \hat{X}^{\mathrm{C}(i)}_{t\gamma c} \leftarrow \frac{\sum_{\gamma' \in \overline{\mathcal{G}}_t} \pi^{\mathrm{L}}_{\gamma'} \cdot X^{\mathrm{C}(i)}_{t\gamma' c}}{\sum_{\gamma' \in \overline{\mathcal{G}}_t} \pi^{\mathrm{L}}_{\gamma'}}$
4: **PH Multiplier:** $\forall t \in \mathcal{T}, \forall \gamma \in \mathcal{G}, \forall c \in \mathcal{C}, \quad m^{\mathrm{PH}(i)}_{t\gamma c} \leftarrow \rho \cdot (X^{\mathrm{C}(i)}_{t\gamma' c} - \hat{X}^{\mathrm{C}(i)}_{t\gamma c})$
5: **Iteration update:** $i \leftarrow i + 1$
6: **Iteration i:** $\forall \gamma \in \mathcal{G}, \quad X^{\mathrm{C}(i)}_{t\gamma c} \leftarrow \underset{x_t}{\operatorname{argmin}} \Big\{ \sum_{t \in \mathcal{T}} c^{\top}_{t\gamma} \cdot x_t + \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}} m^{\mathrm{PH}(i-1)}_{t\gamma c} \cdot X^{\mathrm{C}}_{tc} + \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}} \frac{\rho}{2} \cdot ||X^{\mathrm{C}}_{tc} - \hat{X}^{\mathrm{C}(i-1)}_{t\gamma c}||^2 : x_t \in \mathcal{Q}_{t\gamma} \Big\}$
7: **Aggregation:** $\forall t \in \mathcal{T}, \forall \gamma \in \mathcal{G}, \forall c \in \mathcal{C}, \quad \hat{X}^{\mathrm{C}(i)}_{t\gamma c} \leftarrow \frac{\sum_{\gamma' \in \overline{\mathcal{G}}_t} \pi^{\mathrm{L}}_{\gamma'} \cdot X^{\mathrm{C}(i)}_{t\gamma' c}}{\sum_{\gamma' \in \overline{\mathcal{G}}_t} \pi^{\mathrm{L}}_{\gamma'}}$
8: **PH Multiplier:** $\forall t \in \mathcal{T}, \forall \gamma \in \mathcal{G}, \forall c \in \mathcal{C}, \quad m^{\mathrm{PH}(i)}_{t\gamma c} \leftarrow m^{\mathrm{PH}(i-1)}_{t\gamma c} + \rho \cdot (X^{\mathrm{C}(i)}_{t\gamma c} - \hat{X}^{\mathrm{C}(i)}_{t\gamma c})$
9: **Convergence:** If $\forall t \in \mathcal{T}, \forall \gamma \in \mathcal{G}, \forall c \in \mathcal{C}, \quad g^{(i)}_{t\gamma c} \leftarrow \big(X^{\mathrm{C}(i)}_{t\gamma c} - \hat{X}^{\mathrm{C}(i)}_{t\gamma c}\big) < \epsilon$ - exit, otherwise go to Step 5.

The algorithm consists of several steps. First, the iteration counter is initialized. Second, PHA solves the original problem per each long-term scenario and collects the initial solution for scenario-specific investment decisions $X^{\mathrm{C}(0)}_{t\gamma c}$ for all time stages, long-term scenarios and candidate generation units. Third, it computes the probability-weighted average of investment solutions over the adjacent nodes of the long-term decision tree. At the next step, it calculates the initial value of PH multiplier based on the distance between individual solutions of each sub-problem and their relevant average. Once the iteration counter is updated, the algorithm solves the relaxed version of the original problem, which includes two augmented terms in the objective functions: $\sum_{t\in\mathcal{T}}\sum_{c\in\mathcal{C}} m^{\mathrm{PH}(i-1)}_{t\gamma c}\cdot X^{\mathrm{C}}_{t\gamma c}$ aims at adjusting the investment solutions toward the mean of adjacent nodes, while the square proximal term $\sum_{t\in\mathcal{T}}\sum_{c\in\mathcal{C}} \frac{\varrho}{2}\cdot ||X^{\mathrm{C}}_{t\gamma c} - \hat{X}^{\mathrm{C}(i-1)}_{t\gamma c}||^2$ aims at reaching the convergence criterion. The next step is to recompute the average concerning new investment solution and update PH multiplier including its historical value. Steps 5 to 8 are repeated iteratively till the convergence criterion is satisfied.

Apparently, step 6 requires solving the quadratic mixed-integer problem since the objective function includes the square term. In general, unlike the linear solvers, the quadratic solvers take more computation efforts and time towards the global optimum. In this line, it is reasonable to rewrite the square term in its linear equivalent form as follows:

$$\begin{aligned} ||X^{\mathrm{C}}_{t\gamma c} - \hat{X}^{\mathrm{C}(i-1)}_{t\gamma c}||^2 &= (X^{\mathrm{C}}_{t\gamma c} - \hat{X}^{\mathrm{C}(i-1)}_{t\gamma c})\cdot(X^{\mathrm{C}}_{t\gamma c} - \hat{X}^{\mathrm{C}(i-1)}_{t\gamma c}) \\ &= (X^{\mathrm{C}}_{t\gamma c})^2 - 2\cdot X^{\mathrm{C}}_{t\gamma c}\cdot \hat{X}^{\mathrm{C}(i-1)}_{t\gamma c} + (\hat{X}^{\mathrm{C}(i-1)}_{t\gamma c})^2 \\ &= \sum_{o\in\mathcal{O}} u^{\mathrm{C}}_{t\gamma co}\cdot(\overline{X}^{\mathrm{C}}_{co})^2 - 2\cdot X^{\mathrm{C}}_{t\gamma c}\cdot \hat{X}^{\mathrm{C}(i-1)}_{t\gamma c} + (\hat{X}^{\mathrm{C}(i-1)}_{t\gamma c})^2 \qquad (3.6) \end{aligned}$$

$\overline{X}^{\mathrm{C}}_{co}$ and $\hat{X}^{\mathrm{C}(i-1)}_{t\gamma c}$ are parameters and the square of binary variable $u^{\mathrm{C}}_{t\gamma co}$ is the value of this variable, thus term (3.6) does not involve quadratic variables.

3.3.2 Decomposition over long-term and short-term uncertainty scenarios

Each sub-problem of iterative Algorithm 1 solves the original problem per each long-term scenario only. Thus each problem decides on the optimal investment decisions taking into account market and wind power uncertainty only. These sets of scenarios together could include a significant number of rivals' and consumers' price policies and stochastic generation realizations. Therefore, each sub-problem could be itself difficult to solve.

This observation motivates to decompose the original problem even deeper as depicted in Figure 3.2. During each time stage of the planning horizon, the market participation is modulated with a certain number of representative days N_h. For each h, the producer has to decide on optimal day-ahead and real-time offering strategy, i.e. short-term strategy. The structure of the short-term decision tree includes three stages: (*i*) day-ahead submission of offering prices and quantities, (*ii*) market uncertainty realization and (*iii*) wind power uncertainty realization. This framework suggests decomposing the problem not only per long-term uncertainty scenarios but also per market uncertainty scenarios. In this way, the producer will decide the investment planning anticipating market uncertainty realizations, such that the decision variable $X^{\mathrm{C}}_{t\gamma c}$ rewrites as $X^{\mathrm{C}}_{t\gamma kc}$, where k is an index of market scenarios. Apparently, a new PH algorithm has to tighten both the adjacent nodes of the relaxed long-term decision tree and first-stage nodes of the relaxed short-term decision tree for every representative day inside each time stage of the planning horizon. Therefore, each sub-problem of the proposed solution framework will be less difficult to solve.

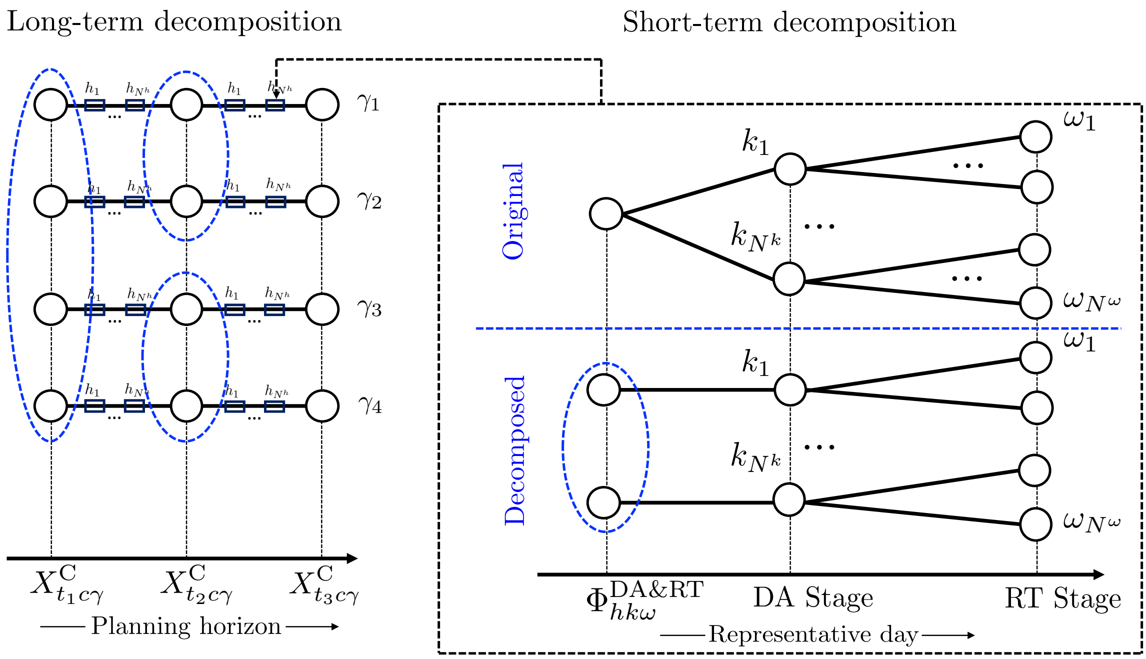


Figure 3.2: PH relaxation over long-term and short-term scenarios.

For the relaxation given in Figure 3.2, the optimal solution will be found if and only if both long-term and short-term non-anticipativity conditions are fulfilled. Thus, the Algorithm 1 has to be modified. First, a new probability-weighted average for the investment solutions is introduced, such that:

$$\hat{X}^{C(i)}_{t\gamma kc} = \frac{\sum_{\gamma' \in \overline{\mathcal{G}}_t} \sum_{k \in \mathcal{K}} \pi^{L}_{\gamma'} \cdot \pi^{MS}_{k} \cdot X^{C(i)}_{t\gamma' kc}}{\sum_{\gamma' \in \overline{\mathcal{G}}_t} \sum_{k \in \mathcal{K}} \pi^{L}_{\gamma'} \cdot \pi^{MS}_{k}} \tag{3.7}$$

Notice, that this formulation is only possible if the market and long-term uncertainty scenarios are independent. Unlike the long run conditions, the short-term non-anticipativity conditions have to be fulfilled at every first-stage node of the short-term decision sequence. Therefore, a new PH multiplier is introduced to comply with this principle, such that:

$$m^{PH(i)}_{t\gamma kc} = m^{PH(i-1)}_{t\gamma kc} + \rho \cdot (X^{C(i)}_{t\gamma kc} - \hat{X}^{C(i)}_{t\gamma kc}) \tag{3.8}$$

Finally, solving the original problem in a decomposed manner, the solution of each sub-problem has to be feasible for the set of constraints $\mathcal{Q}_{t\gamma k}$, declared by both long-term and market uncertainty scenarios. Moreover, the cost coefficient vector in the objective function of each sub-problem also has to account for specifics of each market scenarios, thus $c_{t\gamma} \rightarrow c_{t\gamma k}$. With the introduced definitions, the new PHA relaxing both long-term and short-term decision trees writes as follows:

Algorithm 2 Multistage PHA decomposition over short- and long-term uncertainty scenarios

1: **Initialization:** $i := 0$

2: **Iteration 0:** $\forall \gamma \in \mathcal{G}, \forall k \in \mathcal{K}, \quad X^{\mathrm{C}(i)}_{t\gamma kc} \leftarrow \underset{x_t}{\operatorname{argmin}} \Big\{ \sum_{t\in\mathcal{T}} c^{\top}_{t\gamma k} \cdot x_t : x_t \in \mathcal{Q}_{t\gamma k} \Big\}$

3: **Aggregation:** $\forall t \in \mathcal{T}, \forall \gamma \in \mathcal{G}, \forall k \in \mathcal{K}, \forall c \in \mathcal{C}, \quad \hat{X}^{\mathrm{C}(i)}_{t\gamma kc} = \frac{\sum_{\gamma' \in \overline{\mathcal{G}}_t} \sum_{k\in\mathcal{K}} \pi^{\mathrm{L}}_{\gamma'} \cdot \pi^{\mathrm{MS}}_{k} \cdot X^{\mathrm{C}(i)}_{t\gamma' kc}}{\sum_{\gamma' \in \overline{\mathcal{G}}_t} \sum_{k\in\mathcal{K}} \pi^{\mathrm{L}}_{\gamma'} \cdot \pi^{\mathrm{MS}}_{k}}$

4: **PH Multiplier:** $\forall t \in \mathcal{T}, \forall \gamma \in \mathcal{G}, \forall k \in \mathcal{K}, \forall c \in \mathcal{C}, \quad m^{\mathrm{PH}(i)}_{t\gamma kc} = \rho \cdot (X^{\mathrm{C}(i)}_{t\gamma kc} - \hat{X}^{\mathrm{C}(i)}_{t\gamma kc})$

5: **Iteration update:** $i \leftarrow i + 1$

6: **Iteration i:** $\forall \gamma \in \mathcal{G}, \forall k \in \mathcal{K}, \quad X^{\mathrm{C}(i)}_{t\gamma kc} \leftarrow \underset{x_t}{\operatorname{argmin}} \Big\{ \sum_{t\in\mathcal{T}} c^{\top}_{t\gamma k} \cdot x_t + \sum_{t\in\mathcal{T}} \sum_{c\in\mathcal{C}} m^{\mathrm{PH}(i-1)}_{t\gamma kc} \cdot X^{\mathrm{C}}_{tc} + \sum_{t\in\mathcal{T}} \sum_{c\in\mathcal{C}} \frac{\rho}{2} \cdot ||X^{\mathrm{C}}_{tc} - \hat{X}^{\mathrm{C}(i-1)}_{t\gamma kc}||^2 : x_t \in \mathcal{Q}_{t\gamma} \Big\}$

7: **Aggregation:** $\forall t \in \mathcal{T}, \forall \gamma \in \mathcal{G}, \forall k \in \mathcal{K}, \forall c \in \mathcal{C}, \quad \hat{X}^{\mathrm{C}(i)}_{t\gamma kc} = \frac{\sum_{\gamma' \in \overline{\mathcal{G}}_t} \sum_{k\in\mathcal{K}} \pi^{\mathrm{L}}_{\gamma'} \cdot \pi^{\mathrm{MS}}_{k} \cdot X^{\mathrm{C}(i)}_{t\gamma' kc}}{\sum_{\gamma' \in \overline{\mathcal{G}}_t} \sum_{k\in\mathcal{K}} \pi^{\mathrm{L}}_{\gamma'} \cdot \pi^{\mathrm{MS}}_{k}}$

8: **PH Multiplier:** $\forall t \in \mathcal{T}, \forall \gamma \in \mathcal{G}, \forall k \in \mathcal{K}, \forall c \in \mathcal{C}, \quad m^{\mathrm{PH}(i)}_{t\gamma kc} \leftarrow m^{\mathrm{PH}(i-1)}_{t\gamma kc} + \rho \cdot (X^{\mathrm{C}(i)}_{t\gamma kc} - \hat{X}^{\mathrm{C}(i)}_{t\gamma kc})$

9: **Convergence:** If $\forall t \in \mathcal{T}, \forall \gamma \in \mathcal{G}, \forall k \in \mathcal{K}, \forall c \in \mathcal{C}, \quad g^{(i)}_{t\gamma kc} \leftarrow \big(X^{\mathrm{C}(i)}_{t\gamma kc} - \hat{X}^{\mathrm{C}(i)}_{t\gamma kc}\big) < \epsilon$ - exit, otherwise go to Step 5.

3.3.3 Lower bound for the progressive hedging algorithm

Although PH algorithms 1 and 2 can be successfully applied to solve the multi-stage capacity expansion planning, the solution of each algorithm is limited by the optimal investment decisions only, such that the information on the value of the expected profit is not provided. However, this deficiency of the PH algorithm was solved, for instance in [44] and [22], for the two-stage and multi-stage mixed-integer programs by providing a solution framework for computing the global lower bound for the objective function. This section restates the proof for the solution approach in terms of the considered multi-stage strategic capacity expansion planning decomposed per long-term scenarios.

Proposition 1. By denoting optimal investment decisions by x^*, the following condition holds for each PHA iteration:

$$\sum_{\gamma\in\mathcal{G}} \pi^{\mathrm{L}}_{\gamma} \sum_{t\in\mathcal{T}} m^{\mathrm{PH}(i)\top}_{t\gamma} \cdot x^*_{t\gamma} = 0. \tag{3.9}$$

Proof. It might be proved by induction. Let's first consider iteration 0, in which $m^{\mathrm{PH}(0)}_{t\gamma} = \rho \cdot (x^0_{t\gamma} - \hat{x}^0_{t\gamma})$. Thus, for $\forall \gamma \in \mathcal{G}$ and $\forall t \in \mathcal{T}$

$$\begin{aligned} \sum_{\gamma\in\mathcal{G}} \pi^{\mathrm{L}}_{\gamma} \cdot m^{\mathrm{PH}(0)\top}_{t\gamma} &= \rho \sum_{\gamma\in\mathcal{G}} \pi^{\mathrm{L}}_{\gamma} \cdot (x^0_{t\gamma} - \hat{x}^0_{t\gamma}) \\ &= \rho \sum_{\gamma\in\mathcal{G}} \pi^{\mathrm{L}}_{\gamma} \frac{\sum_{\gamma'\in\overline{\mathcal{G}}_t} \pi^{\mathrm{L}}_{\gamma'} \cdot (x^0_{t\gamma} - x^0_{t\gamma'})}{\sum_{\gamma'\in\overline{\mathcal{G}}_t} \pi^{\mathrm{L}}_{\gamma'}} = 0. \end{aligned} \tag{3.10}$$

By induction, the same is proved for $i \geq 1$.

The following is next introduced:

$$D_{\gamma}^{(i)} = \underset{x_t \in \mathcal{Q}_{t\gamma}}{\text{Minimize}} \left\{ \sum_{t \in \mathcal{T}} \left(c_{t\gamma}^{\top} \cdot x_t + m_{t\gamma}^{\text{PH}(i)\top} \cdot x_t \right) \right\}. \tag{3.11}$$

Theorem 1. By denoting the global minimum of the stochastic mixed-integer program (3.3) as z^*, the following holds:

$$\sum_{\gamma \in \mathcal{G}} \pi_{\gamma}^{\text{L}} \cdot D_{\gamma}^{(i)} \leq z^*. \tag{3.12}$$

where $\sum_{\gamma \in \mathcal{G}} \pi_{\gamma}^{\text{L}} \cdot D_{\gamma}^{(i)}$ is a global lower bound of (3.3) at each PHA iteration.

Proof. From the definitions of D_{γ} and x^*, the following expression holds:

$$D_{\gamma}^{(i)} \leq \sum_{t \in \mathcal{T}} \left(c_{t\gamma}^{\top} \cdot x_t^* + m_{t\gamma}^{\text{PH}(i)\top} \cdot x_t^* \right). \tag{3.13}$$

Taking into account (3.9),

$$\begin{aligned} \sum_{\gamma \in \mathcal{G}} \pi_{\gamma}^{\text{L}} \cdot D_{\gamma}^{(i)} &\leq \sum_{\gamma \in \mathcal{G}} \pi_{\gamma}^{\text{L}} \sum_{t \in \mathcal{T}} \left(c_{t\gamma}^{\top} \cdot x_t^* + m_{t\gamma}^{\text{PH}(i)\top} \cdot x_t^* \right) \\ &= \sum_{\gamma \in \mathcal{G}} \pi_{\gamma}^{\text{L}} \sum_{t \in \mathcal{T}} c_{t\gamma}^{\top} \cdot x_t^* + \sum_{\gamma \in \mathcal{G}} \pi_{\gamma}^{\text{L}} \sum_{t \in \mathcal{T}} m_{t\gamma}^{\text{PH}(i)\top} \cdot x_t^* \\ &= \sum_{\gamma \in \mathcal{G}} \pi_{\gamma}^{\text{L}} \sum_{t \in \mathcal{T}} c_{t\gamma}^{\top} \cdot x_t^* = z^* \end{aligned} \tag{3.14}$$

The lower bound for Algorithm 2 is computed similarly. Notice, the lower bound was defined for a standard form of an optimization problem. The original problem involves the maximization operator, thus (3.12) will define a global upper bound for the expected profit of the strategic producers.

3.3.4 Implementation issues and innovations

3.3.4.1 Cycling behavior of the algorithm

PHA proved to be an efficient tool to solve mixed-integer stochastic programs. However, its heuristic nature involves some level of risk related to the so-called cycling behavior which could lead to non-convergence of the PH algorithm. The cycling behavior is manifested in the fact that some of the hedged integer variables, such as investment decisions, might be repeated in cycles throughout the iteration procedure. In this case, there is a risk that the value of particular decision will not meet the average solution of the adjacent nodes. To fix this issue, a heuristic method was proposed in [63, 64], which detects the cycling focusing on historical values of the PH multiplier. Under this approach, a set of integer values l_γ is generated for each decomposition scenario using a random number generator. Then, at each iteration hash values are computed as $\sum_\gamma l_\gamma \cdot m_\gamma^{\mathrm{PH}(i)}$. By comparing hash values throughout subsequent iterations, the cycling behavior might be detected. To break the cycle in order to achieve the convergence of PHA, the values of hedged variable x_γ are fixed to $\max_{\gamma \in \mathcal{G}} x_\gamma$, ensuring feasibility for one-sided problems.

3.3.4.2 Optimal choice of penalization factor ρ

The choice of penalty factor ρ significantly influences the quality of the solution as well as simulation time. Practically, there is a trade-off between the speed of the algorithm and precision of the lower bound estimate. Using large values of ρ involves a lower number of iteration and fewer computation efforts. However, the lower bound of the objective function could be estimated too far from the optimal objective value [44]. In contrast, small values of ρ result in higher computational complexity, but the quality of the solution is significantly improved.

This issue had been broadly studied in the technical literature. For example, [65] found it beneficial to use ρ updating across different PHA iterations to improve the simulation time of the stochastic unit commitment problem. The adaptive factor ρ could be chosen according to cost coefficient vector of hedged variables [63], shadow prices of hedged variables [66], locational marginal prices [67] and a certain proportion of the objective function's value [65]. Despite that, this thesis considers fixed values of ρ ensuring the smooth profile of the lower bound estimate throughout iterations providing good quality of the solution.

3.3.4.3 Scenario bundling

Dealing with a large number of scenarios, one of the proven ways to accelerate the convergence of PHA is to group a certain amount of scenarios in bundles [44, 68, 69, 70]. Under this approach, each sub-problem of the algorithm is solved for a small number of scenarios satisfying the non-anticipativity constraints on the considered bundle. However, the number of scenarios in the bundle should be balanced with increasing computational complexity of each sub-problem.

Unfortunately, all sub-problems in Algorithms 1 and 2 themselves are complicated to solve, and thus the scenario bundling was not applied.

3.3.5 Programming implementation

Algorithms 1 and 2 include three groups of computation tasks. First, at zero iteration the original problem is solved per each decomposition scenario. Second, starting from the first iteration, a set of PH relaxation sub-problems is solved. Third, once all PH relaxation sub-problems are solved, the lower bound is computed at the same iteration. Each of these groups involves a plenty of computational tasks defined by a number of sub-problems to be solved inside each cluster.

There are two approaches to solve tasks inside each group: sequential and distributed (asynchronous). The first one solves all tasks one by one using all computation power of the processor. The second one considers splitting of the processor into portions solving several tasks simultaneously. Apparently, with the balanced sub-problem complexity and computational power of one processor portion, the computational time significantly reduces.

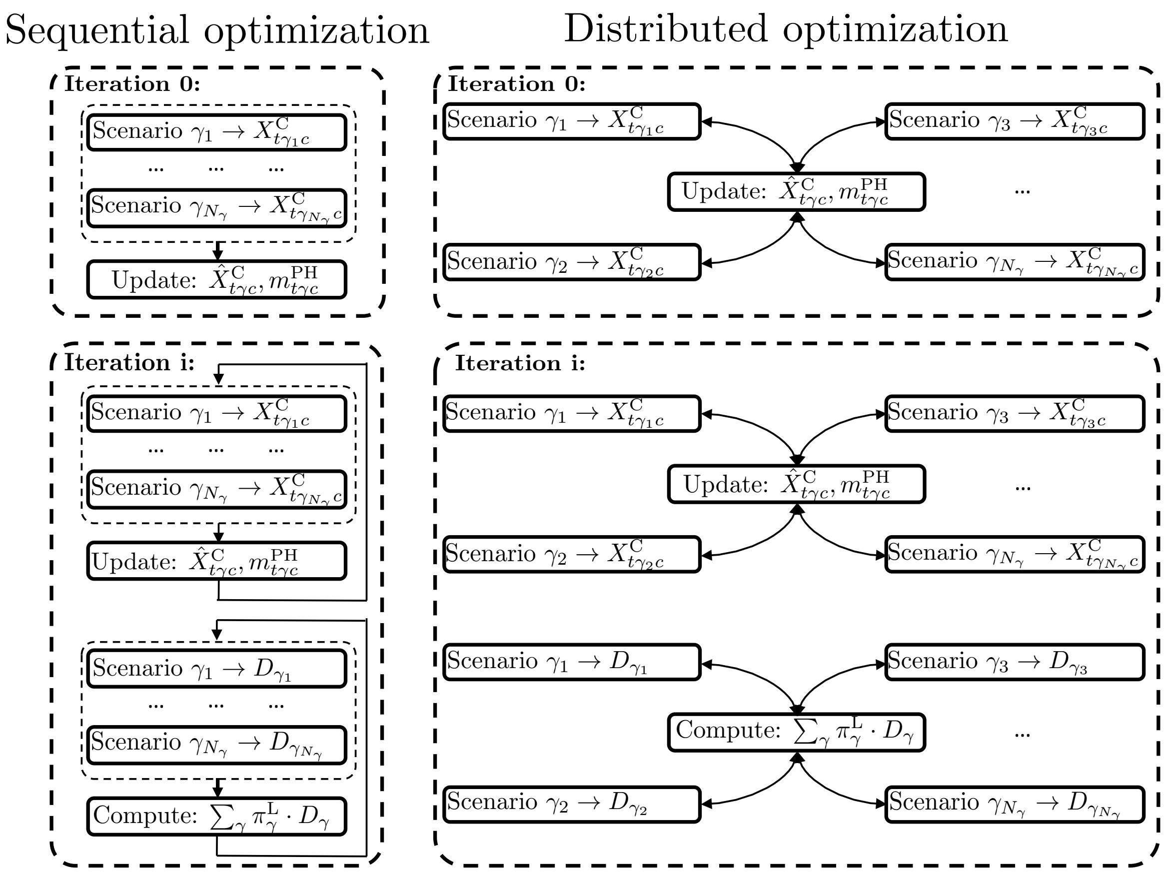


Figure 3.3: Sequential and distributed PHA implementation.

Figure 3.3 depicts these two approaches applied to Algorithm 1. It shows the advantage of the distributed optimization compared to the standard sequential approach in terms of simulation time. The distributed optimization approach was implemented using CPLEX 12.1 under GAMS [71] and grid-enabled framework [72] on Intel Xeon processor E5-2680 v2 with 8 cores clocking at 2.8 GHz and 128 GB of RAM at DTU Computing Center [73].

CHAPTER 4

Case Studies

4.1 Two-bus illustrative example

4.1.1 Case description

In this section, the purposed decision-making tool is applied to a small power system, depicted in Figure 4.1. Generation and demand units are allocated among two nodes, connected through the overhead transmission line of 50MW capacity and 7.7S susceptance. Initially, there are two conventional power plants in the system: existing unit with a total capacity of 300MW, which belongs to a strategic producer, and rival unit of 250MW of overall capacity. The producer has two investment options, i.e. WP and CCGT power plants, to be built in any of the nodes. The capital costs of building 1 unit of CCGT and WP technologies are 0.2 and 0.6 mil.$/MW, respectively. Detailed parameters of generation units are collected in Table 4.2. Notice, it is assumed that generation units enter the real-time market with price bids for up- and down-reserve deployment equal to $1.1 \times c$ and $0.9 \times c$, respectively. System demand is represented by two loads of 200MW and 150MW of peak demand, one per each node. For simplicity, it is assumed that they enter with one block bid with the utility of 50 and 40 $/MW, respectively. For the sake of simplicity, there is only one representative day considered, such that the capacity factor of existing and candidate units is 0.61 and 0.69, respectively, and the demand factor is 0.71 for each load.

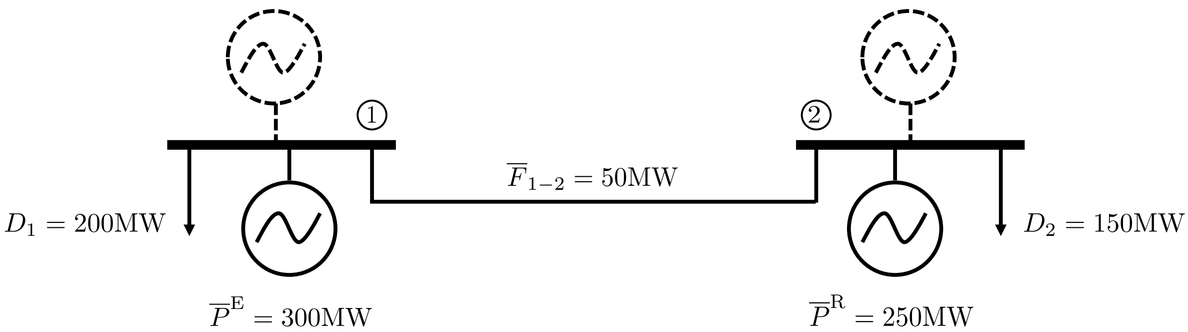


Figure 4.1: Network of the illustrative example.

The model is applied to a two-stage investment planning horizon. In this time-frame, a strategic producer has to decide on optimal siting and sizing of available technologies at the begging of the first and second years.

Table 4.2: Illustrative example: parameters of generation units.

Unit	$\overline{P}$ [MW]	c [\$/MW]	$\overline{R}^{\uparrow}$ [MW]	$\overline{R}^{\downarrow}$ [MW]
P^{E}	300	30	$0.25 \times \overline{P}$	$0.25 \times \overline{P}$
P^{R}	250	25	$0.25 \times \overline{P}$	$0.25 \times \overline{P}$
$P^{\mathrm{C,WP}}$	25,50,75,100	0	-	-
$P^{\mathrm{C,CCGT}}$	25,50,75,100	22	$1.0 \times \overline{P}$	$1.0 \times \overline{P}$

Three types of uncertainty are considered: long-term uncertainty, short-term market uncertainty, and very short-term wind power uncertainty. The first class of uncertainty includes unpredictability of long-lasting macroeconomics indicators, such as demand growth and investment costs. Short-term uncertainty comprises stochasticity of market participation strategies of rival producers and consumers and reveals at the day-ahead stage. The last uncertainty class relates to the stochastic nature of wind power production and discloses at the real-time stage. All of this uncertainty sources are modeled through finite sets of scenarios.

The long-term uncertainty includes three demand growth (DG) scenarios and three investment costs (IC) scenarios. The system demand varies at the second stage with 1.2 (γ_1^{DG}), 1.0 (γ_2^{DG}) and 0.8 (γ_3^{DG}) rates with probability of 0.3, 0.4 and 0.3, respectively. The investment costs for WP technology at the second stage remains the same (γ_1^{IC}), or decrease with 0.8 (γ_2^{IC}) or 0.6 (γ_3^{IC}) rates with probability of 0.2, 0.4 and 0.4, respectively.

Market uncertainty set includes three scenarios: rival generation units offer and consumers bid with 1.1, 1.0 and 0.9 rates of their marginal costs and utilities at each representative days with identical probabilities.

Three plausible scenarios describe wind power uncertainty: wind energy output changes by +20%, 0% and -20% from the average wind capacity factor of a given representative day. The probabilities of these scenarios are 0.25, 0.5 and 0.25, respectively.

Investment budget is 1 bill.$ and assumed to be large enough, such that the budget constraint is not binding. For simplicity, the discount factor is set to zero. Annual amortization rate is 10% of the capital costs. The cost of load shedding is 2000 $/MW. Finally, the security of the supply factor declared by TSO is set to 1.2. Notice that this requirement is not bidding in any scenario of uncertainty realization.

4.1.2 Direct solution

The problem is first solved directly, without any decomposition applied. Table 4.3 collects the value of objective function along with computational time for different cases depending on what sources of uncertainty are considered. The relevant investment decisions are summarized in Table 4.4. The model optimizes the expected value of the stochastic parameter if the uncertainty around it is not explicitly included in the model.

First three cases do not consider any long-term scenarios. The expectation of the profit is the same for these cases since the wind and market uncertainty has symmetric distributions. The investment decisions are quite similar in this case: the power producer finds it profitable to invest in 200 MW of the wind generation and invest only 25MW of the CCGT generation at the first period. At the subsequent period, it invests only in 150 MW of the wind power. It is noteworthy that in the third instance, the producer allocates the candidate units at the second stage slightly differently. In this particular example, the slight reallocation of the units among the buses at the second stage does not significantly influence the profit. Considering the market uncertainty only, and fixing the solution for the second stage to the values obtained in two previous cases, the profit reduces by 20000$ only.

When the long-term uncertainty comes into play, the investment planning is significantly reshaped. Anticipating the future demand fluctuations, the company strategically decides to invest more resources in CCGT technology in the first period. It then adjusts its investment decisions with respect to the different demand realizations: with a decrease of the system demand, it invests in smaller wind power production capacities at the second stage. Unlike demand fluctuations, the variation in the investment costs does not have an impact on the investment decisions at the second stage. Despite that, the producer tends to postpone the investments to the second time period compared to the previous case.

The consideration of the demand and investment costs uncertainty together turns to be a challenging task. The problem includes 23,618 constraints, 27,433 continues variables, 360 discrete variables and 13,824 SOS1 variables. Thus, the problem consumes a lot of computation efforts: the simulation time is 250 times larger compared to the cases, where both sources were considered independently. Second, the solver could only reach 2.48% duality gap.

Table 4.3: Illustrative example: direct solution: objective function value and simulation time.

Uncertainty	Expected profit [mil.$]	CPU Time [s]
-	97.791	2.4
Wind	97.791	3.1
Market	97.791	3.1
Wind + Market	90.897	4.7
Wind + Market + DG	86.945	326.9
Wind + Market + IC	89.217	325.5
Wind + Market + DG + IC	88.987[1]	81386.1

[1] Duality gap is 2.48%

Table 4.4: Illustrative example: direct solution: investment decisions [MW].

Uncertainty	LT scenario	Period 1				Period 2			
		Bus 1		Bus 2		Bus 1		Bus 2	
		WP	CCGT	WP	CCGT	WP	CCGT	WP	CCGT
-	-	100	25	100	0	75	0	75	0
Wind	-	100	25	100	0	75	0	75	0
Market	-	100	25	100	0	50	0	100	0
Wind + Market	-	100	25	100	0	100	0	50	0
Wind + Market+DG	γ_1^{DG}	100	50	100	25	50	0	100	0
	γ_2^{DG}					25	0	50	0
	γ_3^{DG}					0	0	25	0
Wind + Market+IC	γ_1^{IC}	100	25	100	0	75	0	75	0
	γ_2^{IC}					75	0	75	0
	γ_3^{IC}					75	0	75	0
Wind + Market+DG+IC	$\gamma_1^{DG}+\gamma_1^{IC}$	100	50	100	0	100	0	100	0
	$\gamma_1^{DG}+\gamma_2^{IC}$					100	0	100	0
	$\gamma_1^{DG}+\gamma_3^{IC}$					100	0	100	0
	$\gamma_2^{DG}+\gamma_1^{IC}$					75	0	75	0
	$\gamma_2^{DG}+\gamma_2^{IC}$					100	0	50	0
	$\gamma_2^{DG}+\gamma_3^{IC}$					100	0	50	0
	$\gamma_3^{DG}+\gamma_1^{IC}$					25	0	50	0
	$\gamma_3^{DG}+\gamma_2^{IC}$					0	25	25	0
	$\gamma_3^{DG}+\gamma_3^{IC}$					0	0	25	0

4.1.3 Solution via progressive hedging

The progressive hedging decomposition is carried out with two different algorithms:

- **Decomposition over the long-term scenarios only**. Under this framework, there are 9 sub-problems solved in the first iteration, and 18 sub-problems solved at each subsequent interaction.

- **Decomposition over the long-term and market scenarios**. Under this framework, there are 27 sub-problems solved in the first iteration, and 54 sub-problems solved at each subsequent interaction.

Table 4.5 summarizes the solutions for the upper-bound of the expected profit with respect to the algorithm applied and a value of penalization factor defining the accuracy of the objective function estimation. The solution demonstrates that both approaches resulted in similar profit estimates.

Table 4.5: Illustrative example: PH solution: upper bound of the expected profit for different value of penalization factor ρ [mil.$.].

PH Decomposition	$\rho = 500$	$\rho = 100$	$\rho = 50$	$\rho = 30$	$\rho = 20$	$\rho = 10$
Over LT scenarios	89.371	89.301	89.315	89.309	89.310	89.308
Over LT&MS scenarios	89.383	89.340	89.316	89.308	89.305	89.304

Two decomposition algorithms resulted in the same first-stage decisions indicated in Table 4.4. The solution for the second-stage decisions for the highest precision with $\rho = 10$ is collected in Table 4.6. As it illustrated in the table, the decomposition over the long-term scenarios only and decomposition over the long-term and market scenarios provided almost identical results: only 4 out of 36 recourse decisions are different. The small difference is observed in the first and fifth scenarios. However, as it was stated earlier, the slight reallocation of the candidate units among the system buses at the second stage does not have a significant influence on the expected profit estimate. Despite that, the overall wind power capacity to be installed in the second period is perfectly matched.

Table 4.7 describes how the precision of the algorithms affects the computational burden. With lower values of ρ, the solutions tend to converge with a larger number of iterations and time resources.

Figures 4.8 and 4.9 illustrate the evolution of the upper bound of the objective function over the PH algorithm iterations. It is seen that when the investment decisions are closer to the conditions declared by the non-anticipativity constraints, the expected profit logically decreases. The small difference between the first and last iterations testifies that the expected value of perfect information is relatively small for this illustrative example.

It is also observed that the decomposition over the long-term and market scenarios results in a smoother upper bound profile. It is because the bound is computed as a

Table 4.6: Illustrative example: PH solution for the investment decisions in the second period with $\rho = 10$ [MW].

Decomposition	LT only				LT&MS				
Scenario	Bus 1		Bus 2		Bus 1		Bus 2		Match[1]
	WP	CCGT	WP	CCGT	WP	CCGT	WP	CCGT	
$\gamma_1^{DG}+\gamma_1^{IC}$	75	0	75	0	50	0	100	0	-
$\gamma_1^{DG}+\gamma_2^{IC}$	50	0	100	0	50	0	100	0	+
$\gamma_1^{DG}+\gamma_3^{IC}$	75	0	75	0	75	0	75	0	+
$\gamma_2^{DG}+\gamma_1^{IC}$	25	0	50	0	25	0	50	0	+
$\gamma_2^{DG}+\gamma_2^{IC}$	100	0	50	0	75	0	75	0	-
$\gamma_2^{DG}+\gamma_3^{IC}$	75	0	75	0	75	0	75	0	+
$\gamma_3^{DG}+\gamma_1^{IC}$	0	0	25	0	0	0	25	0	+
$\gamma_3^{DG}+\gamma_1^{IC}$	0	0	25	0	0	0	25	0	+
$\gamma_3^{DG}+\gamma_1^{IC}$	0	0	25	0	0	0	25	0	+

[1] Shows whether solutions of two PHA are identical or not.

Table 4.7: Illustrative example: simulation time and number of iterations for different value of ρ.

Penalization factor	$\rho = 500$	$\rho = 100$	$\rho = 50$	$\rho = 30$	$\rho = 20$	$\rho = 10$
Over LT scenarios						
CPU Time [s]	26.9	46.2	69.7	145.5	187.3	374.9
Number of iterations	6	10	15	31	38	69
Over LT&MS scenarios						
CPU Time [s]	35	92.8	222.2	379.8	515.1	1083.7
Number of iterations	9	23	51	83	121	235

probability-weighted average among 27 sub-problems, whereas under the decomposition over the long-term scenarios only, the average is computed out of 9 sub-problems only. This observation suggests that the decomposition over the long-term and market scenarios results in a more accurate estimation of the expected profit.

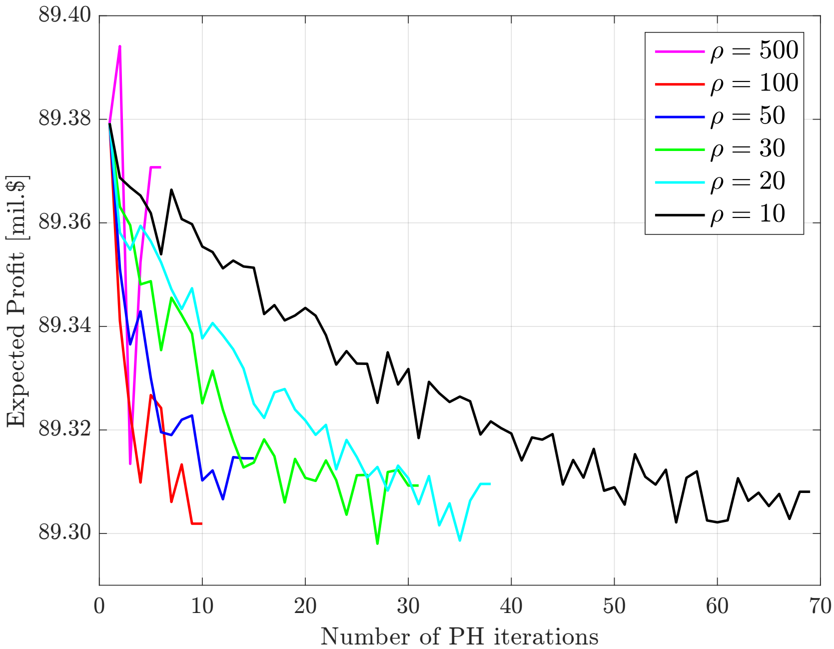


Figure 4.8: PH decomposition over LT scenarios only: evolution of the upper bound of the objective function for different value of ρ.

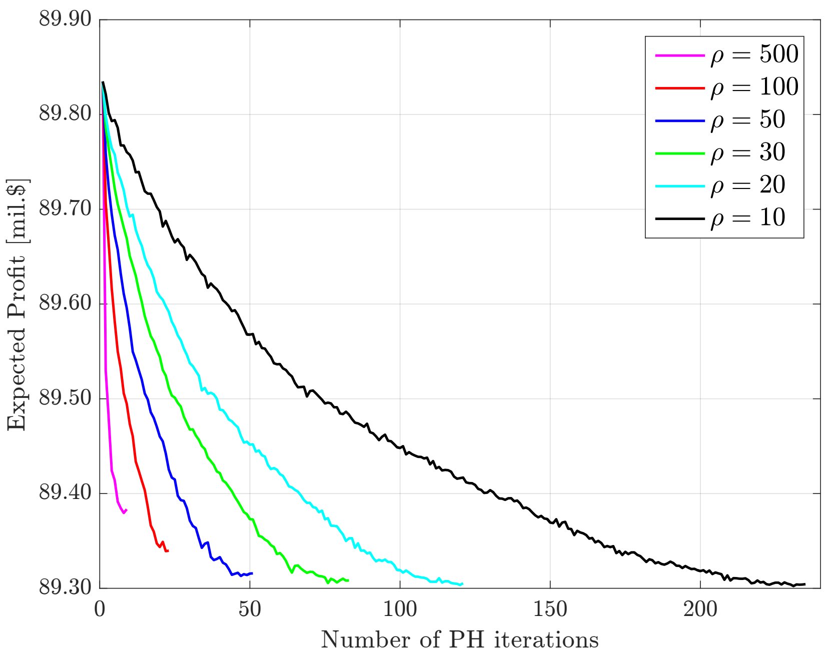


Figure 4.9: PH decomposition over LT&MS scenarios: evolution of the upper bound of the objective function for different value of ρ.

4.1.4 Sequential vs. distributed optimization

Table 4.10 collects the computational time for two PH algorithms when sub-problems are solved sequentially and in the distributed fashion. In this particular example, decomposing the problem over the long-term scenarios only, there are 9 sub-problems to be solved at the first iteration and 18 sub-problems at the subsequent iterations. By decomposing the problem through the long-term and market scenarios, there are 27 sub-problems at the first iteration and 54 sub-problems at the subsequent iterations. Solving the problems sequentially, all eight cores are employed to address each sub-problem. If the problem is solved in the distributed manner, only one available core is used for each sub-problem. The results demonstrate that the distributed approach to the PH algorithm allows improving simulation time vastly.

Table 4.10: Illustrative example: simulation time applying PHA in sequential and distributed fashion [s].

Penalization facor	$\rho = 500$	$\rho = 100$	$\rho = 50$	$\rho = 30$	$\rho = 20$	$\rho = 10$
Decomposition over LT scenarios						
Sequential	55.2	251.5	531.7	892.0	1186.0	1872.7
Distributed	26.9	46.2	69.7	145.5	187.3	374.9
Decomposition over LT&MS scenarios						
Sequential	242.7	1199.5	2926.2	2884.3	4709.0	9714.9
Distributed	35	92.8	222.2	379.8	515.1	1083.7

4.1.5 Decomposition over long-term scenarios only vs. decomposition over long-term and market scenarios

So far, the simulations were performed assuming that the market uncertainty scenarios for the rival offering prices and demand bidding prices are depended: they increase, decrease or remain the same simultaneously. Assuming that they are independent, the number of market scenarios is increased up to 9. The computational performance of both PH algorithms with respect to this assumption is summarized in Table 4.11. For the worst precision with $\rho = 500$, the decomposition over the long-term scenarios converged in 3.5 hours, while the decomposition over the long-term and market scenarios converged in slightly more than 1 minute. In the case of the highest precision, the time difference is also impressive: 12.1 hours against 11.4 minutes.

4.1.6 Impact of the market power on the investment planning

In this section, the impact of the market power of a strategic producer on its investment decisions and expected profit is investigated. The company exercises its market power deciding on the day-ahead energy quantities and prices (energy and reserve quantities) as well as real-time prices for up- and down-reserve deployment for each unit in the

Table 4.11: Illustrative example: computational time and number of iterations involved in two PH algorithms with 9 market scenarios.

Penalization factor	$\rho = 500$	$\rho = 100$	$\rho = 50$
Decomposition over LT scenarios			
CPU Time [s]	12793.7	29005.9	43621.3
Number of iterations	5	13	16
Decomposition over LT&MS scenarios			
CPU Time [s]	80.3	593.1	688.4
Number of iterations	9	52	59

generation portfolio, i.e. the existing and candidate units. There are five cases under consideration:

- Case 1. The company strategically decides on the entire range of decision variables defining the participation strategy, i.e. full market power is studied.
- Case 2. The company strategically decides on the offering prices only, and fixes the energy and reserve quantities to the maximum possible capacity, i.e. *a la Bertrand* competition is studied
- Case 3. The company strategically decides on the offering quantities only, and fixes the energy offering prices to the marginal costs of production and the reserve offering prices to $\pm 10\%$ of the marginal costs of production for the up- and down-reserve deployment, respectively, i.e. *a la Cournot* competition is studied.
- Case 4. The company enters the market with true energy and reserve quantities and prices, i.e. no market power is exercised.
- Case 5. No investments are performed. This instance is used as an auxiliary benchmark to the rest of the cases.

The PH algorithm with the decomposition over the long-term and market scenarios and $\rho = 100$ is applied to solve the first four cases. For the last case, the problem is solved with fixed investment decisions directly. Table 4.12 summarizes the expected profits and simulation time for each considered case. As it turned out, the partial limitation of the market power through the prices or quantities did not result in a significant profit downturn. The PH algorithm accuracy explains the slight difference in the values of the lower bound of the expected profit. However, if the strategic producer is forced to enter the market as a price-taker, the expected profit drops by 41%. Apparently, when the company does not carry out any investment decisions, the lowest expected profit is observed. By investing in new generation units, it increases the profit by 64% as a price-taking market participant, and it increases the profit by 208% acting as a price-maker.

The computation time derived in the first four cases varies significantly. First of all, it is important to mention that the decision variables defining the strategic participation are positive variables, and strategic offering prices are not limited above. Fixing these variables to particular parameters while limiting the market power of the company, the problem becomes more constrained, and thus it is supposed to take more CPU time and efforts towards the optimum. However, solving the problem in the decompose fashion, this proposition is not necessarily held. According to Table 4.12, by fixing the strategic energy and reserve quantities to their maximum capacities, the simulation time increases by 60%. In the case of the fixed offering prices, the time drops by 70%. However, the real difference in the computational time and efforts is observed while considering the company as a price-taker. First of all, it takes 25 times more resources compared to the case of a price-maker. Moreover, the algorithm converged only in 1 iteration meaning that the investment decisions are the same for any long-term and market scenarios. This iteration took 33.5 minutes, while the average iteration time in the case of a full price-maker is 4 seconds.

Table 4.12: Expected profits and simulation time for the different cases of market power exercise.

Case	Expected profit [mil.$]	Iterations	CPU time [s]
Case 1	89.304	23	92.8
Case 2	88.902	9	227.8
Case 3	89.331	31	28.4
Case 4	52.287	1	2007.0
Case 5	32.285	-	1.3

Tables 4.13-4.16 illustrate the optimal investment decisions in the first four cases. The decisions in the first instance had already been discussed above and used as a reference for the following case. Limiting the market power of the strategic producer by forcing it to enter the market with actual power capacity, the company tends to postpone the investments to the second period. Indeed, it reduces the capacity of the CCGT candidate unit at bus 1 to build more wind power capacity later in the most negative demand growth scenario in the most positive investment costs realization. Moreover, if demand increases by 20% and capital costs drop by 40%, it tends to invest more resources into wind power production compared to the reference case. Similarly, if the system demand and investment cost are not changed in the second period, it significantly increases investments in the wind generation. By restricting the ability of the company to exert market power through the strategic prices, the investment planning does not change substantially. Indeed, compared to the first case, it only reallocates the candidate wind energy capacities among the buses at the second stage. In the fourth case, when the company is considered as a price-taker, the investment planning had changed a lot. First, the company no longer considers CCGT technology as a profitable investment option. Second, it considers the second bus, where the rival unit is located, as a most profitable one. Finally, the investment cost scenarios do not influence the decision-making process at the second stage; only the demand growth scenarios shape the investment planning.

Table 4.13: Investment decisions in case 1: the company completely exercises the market power [MW].

Scenario	Period 1				Period 2			
	Bus 1		Bus 2		Bus 1		Bus 2	
	WP	CCGT	WP	CCGT	WP	CCGT	WP	CCGT
$\gamma_1^{\mathrm{DG}}+\gamma_1^{\mathrm{IC}}$	100	50	100	0	50	0	100	0
$\gamma_1^{\mathrm{DG}}+\gamma_2^{\mathrm{IC}}$					50	0	100	0
$\gamma_1^{\mathrm{DG}}+\gamma_3^{\mathrm{IC}}$					75	0	75	0
$\gamma_2^{\mathrm{DG}}+\gamma_1^{\mathrm{IC}}$					25	0	50	0
$\gamma_2^{\mathrm{DG}}+\gamma_2^{\mathrm{IC}}$					75	0	75	0
$\gamma_2^{\mathrm{DG}}+\gamma_3^{\mathrm{IC}}$					75	0	75	0
$\gamma_3^{\mathrm{DG}}+\gamma_1^{\mathrm{IC}}$					0	0	25	0
$\gamma_3^{\mathrm{DG}}+\gamma_2^{\mathrm{IC}}$					0	0	25	0
$\gamma_3^{\mathrm{DG}}+\gamma_3^{\mathrm{IC}}$					0	0	25	0

Table 4.14: Investment decisions in case 2: the company strategically decides on the offering prices only [MW].

Scenario	Period 1				Period 2			
	Bus 1		Bus 2		Bus 1		Bus 2	
	WP	CCGT	WP	CCGT	WP	CCGT	WP	CCGT
$\gamma_1^{\mathrm{DG}}+\gamma_1^{\mathrm{IC}}$	100	25	100	0	100	0	50	0
$\gamma_1^{\mathrm{DG}}+\gamma_2^{\mathrm{IC}}$					100	0	50	0
$\gamma_1^{\mathrm{DG}}+\gamma_3^{\mathrm{IC}}$					100	0	100	0
$\gamma_2^{\mathrm{DG}}+\gamma_1^{\mathrm{IC}}$					100	0	50	0
$\gamma_2^{\mathrm{DG}}+\gamma_2^{\mathrm{IC}}$					75	0	75	0
$\gamma_2^{\mathrm{DG}}+\gamma_3^{\mathrm{IC}}$					75	0	75	0
$\gamma_3^{\mathrm{DG}}+\gamma_1^{\mathrm{IC}}$					0	0	25	0
$\gamma_3^{\mathrm{DG}}+\gamma_2^{\mathrm{IC}}$					0	0	25	0
$\gamma_3^{\mathrm{DG}}+\gamma_3^{\mathrm{IC}}$					25	0	50	0

Table 4.15: Investment decisions in case 3: the company strategically decides on the offering quantities only [MW].

Scenario	Period 1				Period 2			
	Bus 1		Bus 2		Bus 1		Bus 2	
	WP	CCGT	WP	CCGT	WP	CCGT	WP	CCGT
$\gamma_1^{DG}+\gamma_1^{IC}$	100	50	100	0	75	0	75	0
$\gamma_1^{DG}+\gamma_2^{IC}$					75	0	75	0
$\gamma_1^{DG}+\gamma_3^{IC}$					75	0	75	0
$\gamma_2^{DG}+\gamma_1^{IC}$					25	0	50	0
$\gamma_2^{DG}+\gamma_2^{IC}$					75	0	75	0
$\gamma_2^{DG}+\gamma_3^{IC}$					75	0	75	0
$\gamma_3^{DG}+\gamma_1^{IC}$					0	0	25	0
$\gamma_3^{DG}+\gamma_2^{IC}$					0	0	25	0
$\gamma_3^{DG}+\gamma_3^{IC}$					0	0	25	0

Table 4.16: Investment decisions in case 4: the company does not exercise the market power [MW].

Scenario	Period 1				Period 2			
	Bus 1		Bus 2		Bus 1		Bus 2	
	WP	CCGT	WP	CCGT	WP	CCGT	WP	CCGT
$\gamma_1^{DG}+\gamma_1^{IC}$	100	0	100	0	50	0	100	0
$\gamma_1^{DG}+\gamma_2^{IC}$					50	0	100	0
$\gamma_1^{DG}+\gamma_3^{IC}$					50	0	100	0
$\gamma_2^{DG}+\gamma_1^{IC}$					25	0	50	0
$\gamma_2^{DG}+\gamma_2^{IC}$					25	0	50	0
$\gamma_2^{DG}+\gamma_3^{IC}$					25	0	50	0
$\gamma_3^{DG}+\gamma_1^{IC}$					0	0	25	0
$\gamma_3^{DG}+\gamma_2^{IC}$					0	0	25	0
$\gamma_3^{DG}+\gamma_3^{IC}$					0	0	25	0

4.2 Two-stage wind and CCGT expansion planning with modified two-area version of IEEE 24-Bus RTS

The aim of this case study is to estimate the ability of the proposed decision-making tool to handle larger power systems and number of uncertainty realization scenarios. In this line, two-period strategic CCGT and WP expansion is considered in the framework of the modified IEEE 24-Bus RTS [74].

4.2.1 Case description

The network configuration of this case study is depicted in Figure 4.17 and comprises of two areas. The capacity of the internal network lines is assumed to be enough, such that there is no congestion inside each zone. However, the aggregated capacity of the tie-lines connecting two areas is set 700MW to cause bottlenecks in the system. The susceptance of the transmission lines is 40S.

Initially, there are 13 generating units in the system; their parameters are collected in Table 4.18. 9 out of 12 conventional generators are flexible with various ramping characteristics. It is assumed that they enter the real-time market with price bids for up- and down-reserve deployment equal to $1.1 \times c_j$ and $0.9 \times c_j$, respectively. Units 8, 9 and 10 are inflexible power plants but with relatively small marginal costs of production. Only the last unit is a stochastic wind power producer. The strategic producer possesses the first two generating units while the rest of power plants belong to rivals. The loads in each area are aggregated to a single demand unit with parameters given in Table 4.19.

To account for demand and wind power variability within each year of a planning horizon, five representative days are considered. Their parameters are summarized in Table 4.20.

Planning its future power portfolio, the strategic producer is capable of building up to 200MW of wind power and CCGT technologies in any area per year. The generation parameters of the investment options are summarized in Table 4.21. CCGT technology is characterized by relatively small installation costs and nearly average production costs. Notice, that these units are fully dispatchable and able to enter the real-time market with full capacities for up- and down-reserve. Although the production cost of WP is zero, they are four times more expensive than the conventional generators.

Three plausible scenarios describe the real-time WP deviation: it is 20% higher, the same or 20% lower than the day-ahead contracted quantities with equal probabilities. Market uncertainty comprises three scenarios for rivals' offering price policies: they are 10% higher, the same or 10% lower than their marginal costs of production.

Long-term uncertainty is only related to the second time stage and consists of five independent stochastic processes: demand growth, investment costs, fuel costs, rival investments in CCGT and WP production technologies. Their scenario description is provided in table 4.22.

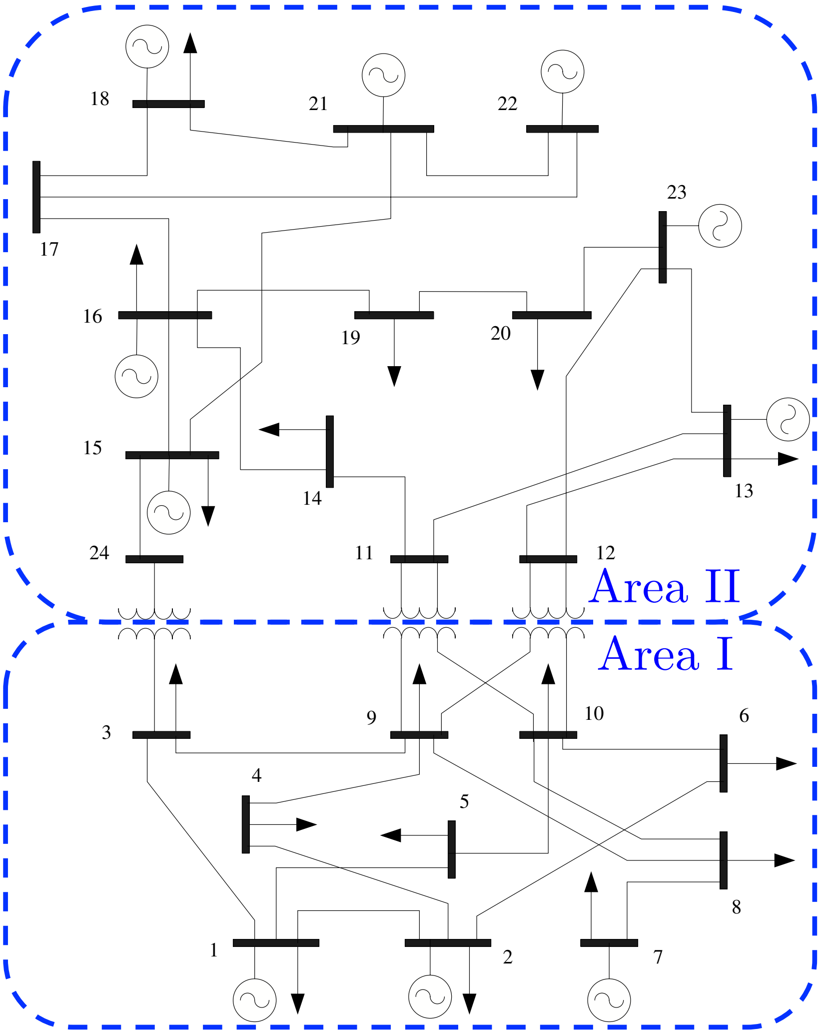


Figure 4.17: Network of two-area version of IEEE 24-Bus RTS.

Investment budget is assumed to be large enough so that strategic producer is capable of expanding its portfolio up to the maximum capacity of the candidate units in both periods. The producer assumes a low risk of future cash flows in the following two years and chooses a relatively small discount factor of 3%. The lifetime of candidate units is ten year, thus linear amortization factor of 10% is considered. TSO requires supply to be at least 10% higher than the expected demand for each representative day in the planning horizon. Although this requirement affects participation strategy of the strategic producer, it does not provoke the investments in excess generation. The tolerance of PHA ϵ is 0MW

Table 4.18: Two-stage WP and CCGT expansion planning: parameters of initial generation units.

Unit	Area	$\overline{P}_j$ [MW]	$\overline{R}_j^{\uparrow}$ [MW]	$\overline{R}_j^{\downarrow}$ [MW]	c_j [$/MW]
1	I	152	40	40	13.32
2	I	152	40	40	13.32
3	I	350	70	70	20.7
4	II	591	180	180	20.93
5	II	60	60	60	26.11
6	II	155	30	30	10.52
7	II	155	30	30	10.52
8	II	400	-	-	6.02
9	II	400	-	-	5.47
10	II	300	-	-	0
11	II	310	60	60	10.52
12	II	350	40	40	10.89
13	II	200	-	-	0

Table 4.19: Two-stage WP and CCGT expansion planning: demand data.

Area	$\overline{P}_d^{\mathrm{D}}$, [MW]	b_d^{D}, [$/MW]
I	1332	25
II	1518	35

Table 4.20: Two-stage WP and CCGT expansion planning: representative days data.

Representative day	Demand Factor [p.u.]	Wind Power Capacity factor [p.u.]	Weight [hour]
h_1	0.6920	0.1223	1036
h_2	0.7107	0.1415	4306
h_3	0.7093	0.6968	443
h_4	0.7292	0.1307	955
h_5	0.7300	0.7671	2020

Table 4.21: Two-stage WP and CCGT expansion planning: parameters of candidate generation units.

Technology	Area	$\overline{P}_c$ [MW]	$\overline{R}_c^{\uparrow}$ [MW]	$\overline{R}_c^{\downarrow}$ [MW]	C_c [\$/MW]	c_c^{Inv} [\$/MW]
CCGT	I,II	20,40,60,80,100, 120,140,160,180,200	Full capacity		15	100000
WP	I,II	20,40,60,80,100, 120,140,160,180,200	-		0	400000

Table 4.22: Two-stage WP and CCGT expansion planning: scenario description of the long-term uncertainty.

Uncertainty source	Scenario	Probability	Rate of change
Demand growth	γ_1^{DG}	30%	+5%
	γ_2^{DG}	40%	0%
	γ_3^{DG}	30%	-5%
Investment costs	γ_1^{IC}	20%	0%
	γ_2^{IC}	30%	-5%
	γ_3^{IC}	50%	-10%
Fuel costs	γ_1^{FC}	33.3%	+5%
	γ_2^{FC}	33.3%	0%
	γ_3^{FC}	33.3%	-5%
Rival investment in CCGT in area I	$\gamma_1^{\text{RI, CCGT}}$	50%	400MW
	$\gamma_2^{\text{RI, CCGT}}$	50%	0MW
Rival investment in WP in area II	$\gamma_1^{\text{RI, WP}}$	50%	200MW
	$\gamma_2^{\text{RI, WP}}$	50%	0MW

4.2.2 Results

PH with decomposition over LT and MS scenarios is applied. At the first iteration, there are 324 scenario specific problems to be solved, formed by 108 long-term scenarios and 3 market scenarios. At the second iteration, there are 648 sub-problems to be solved, half of them for PH relaxation, and another half for the upper-bound estimation. The complexity of each sub-problem in PH relaxation section of the algorithm is defined by 8,989 constraints, 10,347 continues variables, 88 discrete variables, and 844 SOS1 variables. The solution to the two-stage strategic capacity investment planning is found with 18 PHA iterations taking 50.7 hours of computational time with penalization factor $\rho = 1000$. The algorithm estimates the upper bound on the expected profit over the next two years at 11.96 mil.$, as depicted in Figure 4.23.

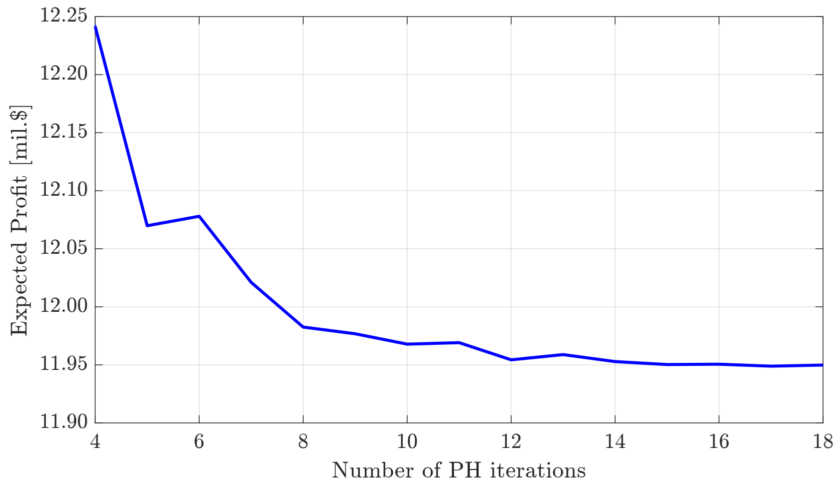


Figure 4.23: PH decomposition over LT scenarios only: evolution of the upper bound of the objective function for different value of ρ.

This level of profit is expected with building 140MW of WP units in area I and 60MW of the same technology in area II in the first year of the planning horizon. The flexible CCGT units are not financially attractive to invest at this stage. The information on the further capacity expansion is given in Tables A.1-A.4. In fact, new reserve providers are built in the second year in two scenarios only. Disregarding the demand growth dynamic, they are built if and only if fuel prices are increased by 5% and rivals introduce 200MW of new wind power generation. This observation suggests that (*i*) the given power system is sufficiently saturated with flexible generation and (*ii*) investments in CCGT technology are caused rather by rival wind energy enlargement than by own wind power expansion.

For the sake of clear results interpretation, the second stage decisions on wind power investments are studied from the perspective of concrete realizations of the demand growth, capital costs, fuel costs, and rival investments. These outcomes are collected in Table 4.24. The rate of load turned out to have a substantial impact on the second-stage expansion decisions, which are aligned with demand dynamic. As it shown in the table, a 5% demand growth requires nearly 60MW of new wind power units. Moreover, with a decreasing demand rate, the producer gives preference to the candidate units located in the second area, reducing the wind expansion in the domestic region, i.e. negative demand development stimulates to increase market share in the neighbouring area.

Table 4.24: Two-stage WP and CCGT expansion planning: wind power investment decisions for the second time period for specific long-term uncertainty realizations.

Scenario	Probability	Rate	Installed capacity [MW]			Share [%]	
			Total	Area I	Area II	Area I	Area II
Demand growth							
γ_1^{DG}	30%	+5%	187	143	44	76	24
γ_2^{DG}	40%	0%	120	84	36	70	30
γ_3^{DG}	30%	-5%	69	19	50	28	72
Investment costs							
γ_1^{IC}	20%	0%	53	46	8	86	14
γ_2^{IC}	30%	-5%	110	75	35	68	32
γ_3^{IC}	50%	-10%	162	101	61	62	38
Fuel costs							
γ_1^{FC}	33.3%	+5%	167	96	71	58	42
γ_2^{FC}	33.3%	0%	126	81	44	65	35
γ_3^{FC}	33.3%	-5%	49	36	13	74	26
Rival investments in CCGT in Area I							
$\gamma_1^{RI,\ CCGT}$	50%	400MW	75	66	9	88	12
$\gamma_2^{RI,\ CCGT}$	50%	0MW	174	98	76	56	44
Rival investments in WP in Area II							
$\gamma_1^{RI,\ WP}$	50%	200MW	125	81	44	65	35
$\gamma_2^{RI,\ WP}$	50%	0MW	125	83	42	67	33

Investment costs scenarios also demonstrate their impact on recourse actions. The aggregated capacity of new wind installations significantly increases even with 5% decrease in capital costs. Allocation of the wind units among areas shows that in all cases the strategic producer carries out the planning favoring the residential area for any realization of investment costs.

According to the initial expectations, the fuel cost uncertainty was included in an attempt to provoke or correct the investments in flexible but expensive CCGT technology. However, the fuel prices turned to have an impact rather on the wind power units with practically zero marginal costs of production. In fact, the strategic producer permanently offers a slightly lower price than the cost of marginal generation unit in the system. Thus, with increasing generation costs of marginal producers, the company receives greater profits, which are sufficient to cover high investment costs of wind power units.

Despite the fact that in some uncertainty realizations investment decisions are indifferent to rival expansion, in average it proves to have a substantial impact on the strategic development planning. This influence entirely depends on the area of interest. On the one hand, if rival producers introduce 400MW in the domestic, it decides to compensate the market share concentrating new wind installations in that area. Otherwise, it nearly equality allocates new capacity among the areas but in greater quantities. In contrary, if rivals build new 200MW of wind power in the second area, it practically does not affect any recourse decision.

To sum up, the investment decisions resulting from the PHA application are quite aligned with a long-term dynamic of power system development providing a fair response to the future challenges, highlighting the practical interest of the proposed decision-making tool.

4.3 Three-stage wind power investment planning in a pool-based electricity market

The aim of this case study is to validate the ability of the proposed decision-making tool to carry out the investment planning for a multi-stage planning horizon. In this line, a three-year strategic wind power investment planning in a pool-based electricity market from a private investor point of view is studied. Initially, there are no existing units in the investor's generation portfolio. In addition, the impact of different long-term uncertainty sources on the investment decisions and expected profit is investigated.

4.3.1 Case description

The pool is represented by generation units given in Table 4.18 and 2700MW of system load with the utility of 40$/MW. For the sake of simplicity, there is only one representative day for each year of the planning horizon: demand factor is equal to 1, and wind power factor is a weighted average of the values collected in Table 4.20.

Market uncertainty is portrayed by a set of three scenarios for rival power producers: they offer energy quantities with 1.1, 1.0 and 0.9 rates of their marginal costs with identical probabilities. Wind power uncertainty is given with a set of three scenarios: wind power is equal to, 20% higher or 20% lower than the average wind capacity factor with equal probabilities.

The long-term uncertainty involves variability of system load, investment costs and rival investments at the second and third years of the planning horizon. Variability of the system demand is described by two scenarios: demand is 5% higher than ($\gamma_t^{\text{DG-I}}$) or equal to ($\gamma_t^{\text{DG-II}}$) demand at the previous period with probabilities of 0.6 and 0.4, respectively. Investment cost uncertainty is given by two scenarios: they remain the same ($\gamma_t^{\text{IC-I}}$) or 5% less than the values at the previous time stage ($\gamma_t^{\text{IC-II}}$) with probability of 0.6 and 0.4, respectively. Finally, the investment uncertainty is described by two equiprobable scenarios: rival producers build ($\gamma_t^{\text{RI-I}}$) or not build ($\gamma_t^{\text{RI-II}}$) the wind power units of aggregated capacity of 600MW. In total there are 8 possible long-term uncertainty realizations at the beginning of the second year, and 64 realizations at the beginning of the third year.

Capital costs are assumed to be 0.93 mil.$/MW. The lifetime of wind power units is set to 15 years, such that with a linear amortization rate the investor loses 6.7% of the initial asset value. The investor is capable of building up to 300MW of wind power generation every year with a step of 1MW. The tolerance of PHA ϵ is 1MW. The rest of model's parameters are the same as in the previous case.

4.3.2 Results

The simulations are carried out by applying PHA decomposition for both long-term and short-term decisions tree. To validate the ability of the algorithm to tackle a multi-stage strategic investment planning, it was applied to several cases. First, the problem is solved with no long-term uncertainty considered. Then, the impact of demand growth, investment costs, and rival investment uncertainty is studied independently. Finally, all uncertainty sources are considered simultaneously. The complexity of each PH relaxation sub-problem is defined by 1,504 constraints, 1,681 continues variables, 903 binary variables, and 496 SOS1 variables.

As it was shown in [28], if a decision-maker ignores uncertainty of specific parameters and solves the problem in a deterministic fashion, the cost of this negligence is of the same magnitude as the investment costs. In this line, the present case study estimates the opportunity costs caused by misjudgment of uncertainty. Notice, if demand growth, investment costs or rival investments are not explicitly considered as stochastic parameters, they are set to expected values.

Table 4.25 summarizes the optimal investment decisions with respect to different uncertainty sources engaged. In the deterministic case, where no variability of long-term indicators is taken into account, the investor finds it profitable to invest in all available wind power capacity at all three time periods. This solution is used as a benchmark for other cases.

When demand growth uncertainty is taken into consideration, the solution slightly changes. Strategic investor keeps the same investment decisions for the first and second stages, and then his decisions are strongly dependent on the demand realizations. If demand increases at the second stage, the investor decides to reduce investments if demand further increases and continue expansion if it remains the same. The motivation behind this is the fact that under certain circumstances, it strategically reduces wind power penetration to keep energy prices high. In contrary, if demand remains the same at the second stage, further expansion decisions are quite aligned with demand trajectories.

Capital costs scenarios tuned to have a larger impact on the investment planning. Building all available capacity in the first period, the subsequent investment dynamic is shaped by investment costs variation. On the one hand, with diminishing capital expenses, producer always choices entire capacity to build. On the other hand, it significantly cuts the expansion if investment costs remain the same: at the second stage by 43% and at the third stage by 85% of the deterministic solution.

Rival investment decisions also have a substantial impact on the strategic investment planning. If rivals do not expand their wind assets, the strategic producer decides on the largest capacity option. However, if rivals build 600MW of wind power at the second

Table 4.25: Three-stage WP investment planning: investment decisions and expected profit for different uncertainty cases.

Uncertainty	Scenario	Inv. decisions [MW]			Expected profit [mil.$]
		t_1	t_2	t_3	
No	-	300	300	300	2.194
DG	$\gamma_{t_2}^{\text{DG-I}} + \gamma_{t_3}^{\text{DG-I}}$	300	300	284	2.359
	$\gamma_{t_2}^{\text{DG-I}} + \gamma_{t_3}^{\text{DG-II}}$			300	
	$\gamma_{t_2}^{\text{DG-II}} + \gamma_{t_3}^{\text{DG-I}}$		300	300	
	$\gamma_{t_2}^{\text{DG-II}} + \gamma_{t_3}^{\text{DG-II}}$			293	
IC	$\gamma_{t_2}^{\text{IC-I}} + \gamma_{t_3}^{\text{IC-I}}$	300	170	46	2.401
	$\gamma_{t_2}^{\text{IC-I}} + \gamma_{t_3}^{\text{IC-II}}$			300	
	$\gamma_{t_2}^{\text{IC-II}} + \gamma_{t_3}^{\text{IC-I}}$		300	300	
	$\gamma_{t_2}^{\text{IC-II}} + \gamma_{t_3}^{\text{IC-II}}$			300	
RI	$\gamma_{t_2}^{\text{RI-I}} + \gamma_{t_3}^{\text{RI-I}}$	300	246	189	2.238
	$\gamma_{t_2}^{\text{RI-I}} + \gamma_{t_3}^{\text{RI-II}}$			225	
	$\gamma_{t_2}^{\text{RI-II}} + \gamma_{t_3}^{\text{RI-I}}$		300	300	
	$\gamma_{t_2}^{\text{RI-II}} + \gamma_{t_3}^{\text{RI-II}}$			300	
DG+IC+RI	Expectation	300	240	201	2.295

stage, further investor's expansion is reduced. In fact, if all producers realize their investments to the full extent of available options, it leads to a sensible price reduction, such that capital costs are not ensured. In this line, investor strategically decides to interrupt expansion to keep high prices in the market, enhancing capital cost recovery.

In the last experiment, all uncertainty sources are considered together. In this case, strategic investor builds all available generation capacity at the first stage. Later, the expansion is gradually decreasing in the expectation and kept at much lower level than it is proposed by the deterministic solution. Table 4.27 summarizes the detailed investment plan for the strategic investor. As it turns out, in some certain scenarios the most profitable option is do not invest at all. For example, with unaltered demand growth and investment cost at the second stage, and if rivals introduce new wind power capacities, the best response is to postpone expansion further till, for example, system load will be increased.

Although investment decisions significantly differ from one case to another, the expectation of the profit over planning horizon is estimated at nearly the same level. Despite that, solving the model considering the entire set of uncertainty allows deriving an informed investment planning anticipating the inherent dynamic of power system development.

Finally, Table 4.26 collects PHA performance for all considered cases. It is noteworthy that even for the deterministic case several iterations are required to find the solution. This is explained by the fact that solutions for different market uncertainty scenarios, comprising stochasticity of rival offering price policies, might be different at first iteration, such that several steps are required to tighten short-term decision tree. In the last case, the algorithm only converges with a tolerance of 5MW due to cycling issue occurred for the second stage investment decisions.

Table 4.26: Three-stage WP investment planning: computational complexity.

LT uncertainty	CPU time [s]	Iterations
-	22614	21
DG	7717	12
IC	12274	18
RI	9835	16
DG+IC+RI	27680	100

Table 4.27: Three-stage WP investment planning: detailed investment decisions with full uncertainty included [MW].

t_2			t_3			t_1	t_2	t_3	t_2			t_3			t_1	t_2	t_3
Scenario									Scenario								
DG	IC	RI	DG	IC	RI				DG	IC	RI	DG	IC	RI			
5%	0%	600	5%	0%	600	300	205	0	0%	0%	600	5%	0%	600	300	0	0
					0			193						0			15
				-5%	600			300					-5%	600			300
					0			300						0			300
			0%	0%	600			0				0%	0%	600			0
					0			0						0			0
				-5%	600			153					-5%	600			0
					0			300						0			300
		0	5%	0%	600		300	96			0	5%	0%	600		252	0
					0			300						0			300
				-5%	600			284					-5%	600			300
					0			300						0			300
			0%	0%	600			0				0%	0%	600			0
					0			300						0			0
				-5%	600			300					-5%	600			300
					0			300						0			300
	-5%	600	5%	0%	600		300	300		-5%	600	5%	0%	600		300	56
					0			284						0			300
				-5%	600			300					-5%	600			56
					0			284						0			300
			0%	0%	600			56				0%	0%	600			0
					0			300						0			295
				-5%	600			56					-5%	600			0
					0			300						0			300
		0	5%	0%	600		300	284			0	5%	0%	600		300	300
					0			300						0			300
				-5%	600			284					-5%	600			300
					0			300						0			300
			0%	0%	600			300				0%	0%	600			293
					0			300						0			300
				-5%	600			300					-5%	600			293
					0			300						0			300

CHAPTER 5

Conclusions

The aim of this thesis is to build a decision-making tool which supports strategic multi-stage investment planning for a generation company for years ahead in the presence of uncertainty. As a result, the main outcome of this work is a generic equilibrium model allowing to smartly allocate investment decisions in the given time-frame and network topology, optimally deciding on an appropriate timing, siting, and sizing of CCGT and wind power production units.

The initial foundation of the proposed tool is formalized in a framework of the bilevel optimization problem. The upper-level problem aims at maximizing the expected profit of a strategic producer over the planning horizon, defining technical and financial boundaries on company's operations. The purpose of a set of lower-level problems is to approximate market participation strategy anticipating various short- and long-term uncertainty. To achieve an equivalent and tractable single-level linear formulation, a set of KKT conditions of lower-level problems is applied. As a result, the problem is recast as a mixed-integer linear programming problem. Despite that, the tractability of the model is still limited for large-scale applications.

To achieve scalability of the proposed tool, progressive hedging decomposition is employed. This approach belongs to a class of augmented Lagrangian relaxation techniques and decomposes the problem per scenarios. Treating the problem in an iterative manner, the algorithm solves a set of scenario-specific problems restoring the non-anticipativy conditions on investment variables declared by original problem. Specifically, two algorithms were built. The first one is based on the relaxation of the long-term decision tree comprising investment decisions throughout a multi-stage planning horizon. The second one aims at more deeper decomposition based on the simultaneous relaxation of the long-term decision tree and short-term decision tree bound up with market participation decisions. Despite the heuristical nature of this method, this work develops a generic framework that allows obtaining a reasonable trade-off between quality of the solution and simulation time.

A series of case studies is performed to highlight a practical interest in the proposed solution framework. A small two-bus system is used to estimate the quality of the solution provided by two PHA algorithms. As it turned out, both approaches resulted in nearly the same solution as the one given by a direct approach. More specifically, due to computational limitation, a direct solution was obtained for a duality gap of 2.5% and required almost one day of computational time. PHA application provided the solution for a duality gap of 2% in a matter of minutes.

To perform a more realistic investment planning, two larger case studies were performed. In the first one, a two-area version of IEEE 24-Bus RTS was used to carry out two-stage

CCGT and wind power expansion planning for a strategic company. The second one was focused on the three-year wind-power investment planning for a strategic investor in a pool-based market. In both cases, PHA relaxation over long- and short-term uncertainty scenarios proved to be a useful tool to generate informed decisions. To validate the quality of the solution, the impact of different uncertainty sources was investigated. As a result, the obtained investment solutions are properly aligned with demand, investment costs, fuel costs and rival expansion dynamics.

Despite that, the simulation results suggest several research directions for further enhancement of the proposed model. First, large cases studies were performed with a small number of representative days and wind power real-time outcome scenarios. Even with simultaneous relaxation of the short- and long-term decision trees, the resulting sub-problems are complicated to solve. Thus, even deeper decomposition might be necessary to solve larger instances. In this line, the future work will be focused on a multi-scale decomposition per long-term scenarios, market scenarios and representative days. Second, a small case study indicated that price-making strategy might significantly affect investment decisions compared to a passive price-taking policy. However, in real operations generation companies do not always exercise their market power, and thus solution provided by the model might be too optimistic. In this way, in order to control a degree of market power exertion, it might be interesting to include linear risk measures such as CVaR into a problem. An alternative way to limit market power of a strategic company might be a consideration of long-term bilateral contracts. Finally, the study revealed that rivals' expansion has a substantial impact on company's long-term planning, rising concerns on the fairness of performing the analysis from a perspective of one firm only. In this line, another direction would be to consider a co-planning of capacity expansion of several strategic firms. This analysis could be efficiently tackled in the framework of equilibrium programming with equilibrium constraints problem.

APPENDIX A

Investment decisions for the two-stage CCGT and WP expansion planning

Table A.1: Two-stage WP and CCGT expansion planning: investment decisions for the second time period [MW].

Demand growth	Investment costs	Fuel costs	Rival inv. in CCGT	Rival inv. in WP	CCGT Area I	CCGT Area II	WP Area I	WP Area II
+5%	0%	+5%	400	200	0	0	200	0
+5%	0%	+5%	400	0	0	0	100	40
+5%	0%	+5%	0	200	0	0	180	100
+5%	0%	+5%	0	0	0	0	200	0
+5%	0%	0%	400	200	0	0	0	0
+5%	0%	0%	400	0	0	0	0	0
+5%	0%	0%	0	200	0	0	200	0
+5%	0%	0%	0	0	0	0	0	0
+5%	0%	-5%	400	200	0	0	0	0
+5%	0%	-5%	400	0	0	0	0	0
+5%	0%	-5%	0	200	0	0	0	0
+5%	0%	-5%	0	0	0	0	0	0
+5%	-5%	+5%	400	200	0	0	160	0
+5%	-5%	+5%	400	0	0	0	180	0
+5%	-5%	+5%	0	200	20	0	120	140
+5%	-5%	+5%	0	0	0	0	160	100
+5%	-5%	0%	400	200	0	0	180	0
+5%	-5%	0%	400	0	0	0	180	0
+5%	-5%	0%	0	200	0	0	120	100
+5%	-5%	0%	0	0	0	0	200	40
+5%	-5%	-5%	400	200	0	0	0	0
+5%	-5%	-5%	400	0	0	0	20	0
+5%	-5%	-5%	0	200	0	0	180	0
+5%	-5%	-5%	0	0	0	0	200	0

Table A.2: Two-stage WP and CCGT expansion planning: investment decisions for the second time period [MW] (continuation).

Demand	Investment	Fuel	Rival inv.	Rival inv.	CCGT		WP	
growth	costs	costs	in CCGT	in WP	Area I	Area II	Area I	Area II
+5%	-10%	+5%	400	200	0	0	180	20
+5%	-10%	+5%	400	0	0	0	160	80
+5%	-10%	+5%	0	200	0	0	160	160
+5%	-10%	+5%	0	0	0	0	160	160
+5%	-10%	0%	400	200	0	0	160	0
+5%	-10%	0%	400	0	0	0	180	0
+5%	-10%	0%	0	200	0	0	160	140
+5%	-10%	0%	0	0	0	0	160	140
+5%	-10%	-5%	400	200	0	0	180	0
+5%	-10%	-5%	400	0	0	0	200	0
+5%	-10%	-5%	0	200	0	0	160	80
+5%	-10%	-5%	0	0	0	0	200	0
0%	0%	+5%	400	200	0	0	60	0
0%	0%	+5%	400	0	0	0	60	0
0%	0%	+5%	0	200	0	0	180	0
0%	0%	+5%	0	0	0	0	120	80
0%	0%	0%	400	200	0	0	20	0
0%	0%	0%	400	0	0	0	0	0
0%	0%	0%	0	200	0	0	20	0
0%	0%	0%	0	0	0	0	160	0
0%	0%	-5%	400	200	0	0	0	0
0%	0%	-5%	400	0	0	0	0	0
0%	0%	-5%	0	200	0	0	0	0
0%	0%	-5%	0	0	0	0	0	0
0%	-5%	+5%	400	200	0	0	60	20
0%	-5%	+5%	400	0	0	0	80	20
0%	-5%	+5%	0	200	0	0	140	60
0%	-5%	+5%	0	0	0	0	80	100
0%	-5%	0%	400	200	0	0	60	0
0%	-5%	0%	400	0	0	0	20	40
0%	-5%	0%	0	200	0	0	80	80
0%	-5%	0%	0	0	0	0	80	0
0%	-5%	-5%	400	200	0	0	0	0
0%	-5%	-5%	400	0	0	0	20	0
0%	-5%	-5%	0	200	0	0	60	0
0%	-5%	-5%	0	0	0	0	180	0

Table A.3: Two-stage WP and CCGT expansion planning: investment decisions for the second time period [MW] (continuation).

Demand growth	Investment costs	Fuel costs	Rival inv. in CCGT	Rival inv. in WP	CCGT Area I	CCGT Area II	WP Area I	WP Area II
0%	-10%	+5%	400	200	0	0	80	20
0%	-10%	+5%	400	0	0	0	80	20
0%	-10%	+5%	0	200	0	0	180	100
0%	-10%	+5%	0	0	0	0	80	180
0%	-10%	0%	400	200	0	0	60	20
0%	-10%	0%	400	0	0	0	80	0
0%	-10%	0%	0	200	0	0	160	120
0%	-10%	0%	0	0	0	0	120	100
0%	-10%	-5%	400	200	0	0	60	0
0%	-10%	-5%	400	0	0	0	60	0
0%	-10%	-5%	0	200	0	0	140	40
0%	-10%	-5%	0	0	0	0	140	40
-5%	0%	+5%	400	200	0	0	20	20
-5%	0%	+5%	400	0	0	0	20	0
-5%	0%	+5%	0	200	0	0	20	0
-5%	0%	+5%	0	0	0	0	40	40
-5%	0%	0%	400	200	0	0	0	0
-5%	0%	0%	400	0	0	0	0	0
-5%	0%	0%	0	200	0	0	0	0
-5%	0%	0%	0	0	0	0	20	0
-5%	0%	-5%	400	200	0	0	0	0
-5%	0%	-5%	400	0	0	0	0	0
-5%	0%	-5%	0	200	0	0	0	0
-5%	0%	-5%	0	0	0	0	0	0
-5%	-5%	+5%	400	200	0	0	20	20
-5%	-5%	+5%	400	0	0	0	0	20
-5%	-5%	+5%	0	200	0	0	20	180
-5%	-5%	+5%	0	0	0	0	40	140
-5%	-5%	0%	400	200	0	0	20	0
-5%	-5%	0%	400	0	0	0	20	20
-5%	-5%	0%	0	200	0	0	20	60
-5%	-5%	0%	0	0	0	0	20	60
-5%	-5%	-5%	400	200	0	0	0	0
-5%	-5%	-5%	400	0	0	0	0	0
-5%	-5%	-5%	0	200	0	0	0	80
-5%	-5%	-5%	0	0	0	0	0	0
-5%	-10%	+5%	400	200	0	0	20	20
-5%	-10%	+5%	400	0	0	0	20	20
-5%	-10%	+5%	0	200	20	0	20	180
-5%	-10%	+5%	0	0	0	0	40	180

Table A.4: Two-stage WP and CCGT expansion planning: investment decisions for the second time period [MW] (continuation).

Demand growth	Investment costs	Fuel costs	Rival inv. in CCGT	Rival inv. in WP	CCGT Area I	CCGT Area II	WP Area I	WP Area II
-5%	-10%	0%	400	200	0	0	20	20
-5%	-10%	0%	400	0	0	0	40	0
-5%	-10%	0%	0	200	0	0	20	160
-5%	-10%	0%	0	0	0	0	20	140
-5%	-10%	-5%	400	200	0	0	20	0
-5%	-10%	-5%	400	0	0	0	40	0
-5%	-10%	-5%	0	200	0	0	20	40
-5%	-10%	-5%	0	0	0	0	40	60

Bibliography

[1] Pierre Pinson et al. "Towards fully renewable energy systems: Experience and trends in Denmark". In: *CSEE Journal of Power and Energy Systems* 3.1 (2017), pages 26–35.

[2] Tryggvi Jónsson, Pierre Pinson, and Henrik Madsen. "On the market impact of wind energy forecasts". In: *Energy Economics* 32.2 (2010), pages 313–320.

[3] *Henrik Poulsen Dong Energy A/S Presentations: Strategic Progress.* http://assets.dongenergy.com/DONGEnergyDocuments/com/Investor/Financial_Presentations/MTM-2017-HENPO-presentation_EN.pdf. Accessed: 2017-06-14.

[4] David Pozo, Enzo Sauma, and Javier Contreras. "When doing nothing may be the best investment action: Pessimistic anticipative power transmission planning". In: *Applied Energy* 200 (2017), pages 383–398. ISSN: 0306-2619. DOI: https://doi.org/10.1016/j.apenergy.2017.05.030. URL: http://www.sciencedirect.com/science/article/pii/S0306261917305329.

[5] Antonio J. Conejo et al. "Investment in production capacity". In: *Investment in Electricity Generation and Transmission: Decision Making under Uncertainty.* Cham: Springer International Publishing, 2016, pages 169–227. ISBN: 978-3-319-29501-5. DOI: 10.1007/978-3-319-29501-5_5. URL: http://dx.doi.org/10.1007/978-3-319-29501-5_5.

[6] I. De J. Silva et al. "Transmission network expansion planning considering uncertainty in Demand". In: *IEEE Transactions on Power Systems* 21.4 (November 2006), pages 1565–1573. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2006.881159.

[7] Z. Xu, Z. Y. Dong, and K. P. Wong. "Transmission planning in a deregulated environment". In: *IEE Proceedings - Generation, Transmission and Distribution* 153.3 (May 2006), pages 326–334. ISSN: 1350-2360. DOI: 10.1049/ip-gtd:20050201.

[8] Adelino JC Pereira and João Tomé Saraiva. "Generation expansion planning in competitive electricity markets". In: *Power Tech, 2007 IEEE Lausanne*. IEEE. 2007, pages 897–902.

[9] E. Spyrou et al. "What are the benefits of co-optimizing transmission and generation investment? Eastern interconnection case study". In: *IEEE Transactions on Power Systems* PP.99 (2017), pages 1–1. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2017.2660249.

[10] Y. Dvorkin et al. "Co-planning of investments in transmission and merchant energy storage". In: *IEEE Transactions on Power Systems* PP.99 (2017), pages 1–1. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2017.2705187.

[11] Severin Borenstein and James Bushnell. "An empirical analysis of the potential for market power in California's electricity industry". In: *The Journal of Industrial Economics* 47.3 (1999), pages 285–323. ISSN: 1467-6451. DOI: 10.1111/1467-6451.00102. URL: http://dx.doi.org/10.1111/1467-6451.00102.

[12] B. F. Hobbs, C. B. Metzler, and J. S. Pang. "Strategic gaming analysis for electric power systems: an MPEC approach". In: *IEEE Transactions on Power Systems* 15.2 (May 2000), pages 638–645. ISSN: 0885-8950. DOI: 10.1109/59.867153.

[13] Stefanos Delikaraoglou et al. "Price-maker wind power producer participating in a joint day-ahead and real-time market". In: *European Energy Market (EEM), 2015 12th International Conference on the*. IEEE. 2015, pages 1–5.

[14] S. J. Kazempour, A. J. Conejo, and C. Ruiz. "Strategic bidding for a large consumer". In: *IEEE Transactions on Power Systems* 30.2 (March 2015), pages 848–856. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2014.2332540.

[15] L. Baringo and A. J. Conejo. "Strategic wind power investment". In: *IEEE Transactions on Power Systems* 29.3 (May 2014), pages 1250–1260. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2013.2292859.

[16] E. Nasrolahpour et al. "Strategic sizing of energy storage facilities in electricity markets". In: *IEEE Transactions on Sustainable Energy* 7.4 (October 2016), pages 1462–1472. ISSN: 1949-3029. DOI: 10.1109/TSTE.2016.2555289.

[17] Seyyed Jalal Kazempour. "Strategic generation investment and equilibria in oligopolistic electricity markets". In: *Ph.D. Thesis* (2013).

[18] S. J. Kazempour, A. J. Conejo, and C. Ruiz. "Generation investment equilibria with strategic producers. Part I: Formulation". In: *IEEE Transactions on Power Systems* 28.3 (August 2013), pages 2613–2622. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2012.2235467.

[19] S. J. Kazempour, A. J. Conejo, and C. Ruiz. "Generation Investment Equilibria With Strategic Producers. Part II: Case Studies". In: *IEEE Transactions on Power Systems* 28.3 (August 2013), pages 2623–2631. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2012.2235468.

[20] S. Jin and S. M. Ryan. "A tri-Level model of centralized transmission and decentralized generation expansion planning for an electricity market; Part I". In: *IEEE Transactions on Power Systems* 29.1 (January 2014), pages 132–141. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2013.2280085.

[21] S. Jin and S. M. Ryan. "A tri-Level model of centralized transmission and decentralized generation expansion planning for an electricity market; Part II". In: *IEEE Transactions on Power Systems* 29.1 (January 2014), pages 142–148. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2013.2280082.

[22] Y. Liu, R. Sioshansi, and A. Conejo. “Multistage stochastic investment planning with multiscale representation of uncertainties and decisions”. In: *IEEE Transactions on Power Systems* PP.99 (2017), pages 1–1. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2017.2694612.

[23] S. J. Kazempour, A. J. Conejo, and C. Ruiz. “Strategic generation investment using a complementarity approach”. In: *IEEE Transactions on Power Systems* 26.2 (May 2011), pages 940–948. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2010.2069573.

[24] A. Moreira et al. “A five-level MILP model for flexible transmission network planning under uncertainty: a min-max regret approach”. In: *IEEE Transactions on Power Systems* PP.99 (2017), pages 1–1. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2017.2710637.

[25] S. Delikaraoglou et al. “Price-maker wind power producer participating in a joint day-ahead and real-time market”. In: *2015 12th International Conference on the European Energy Market (EEM)*. May 2015, pages 1–5. DOI: 10.1109/EEM.2015.7216701.

[26] Antonio J. Conejo et al. “Transmission expansion planning”. In: *Investment in electricity generation and transmission: decision making under uncertainty*. Cham: Springer International Publishing, 2016, pages 21–59. ISBN: 978-3-319-29501-5. DOI: 10.1007/978-3-319-29501-5_2. URL: http://dx.doi.org/10.1007/978-3-319-29501-5_2.

[27] Ansgar Geiger. *Strategic power plant investment planning under fuel and carbon price uncertainty*. KIT Scientific Publishing, 2011.

[28] F. D. Munoz et al. “An engineering-economic approach to transmission planning under market and regulatory uncertainties: WECC case study”. In: *IEEE Transactions on Power Systems* 29.1 (January 2014), pages 307–317. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2013.2279654.

[29] J. Li et al. “Robust coordinated transmission and generation expansion planning considering ramping requirements and construction periods”. In: *IEEE Transactions on Power Systems* PP.99 (2017), pages 1–1. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2017.2687318.

[30] R. A. Jabr, I. Džafić, and B. C. Pal. “Robust optimization of storage investment on transmission networks”. In: *IEEE Transactions on Power Systems* 30.1 (January 2015), pages 531–539. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2014.2326557.

[31] Juan M. Morales et al. “Clearing the day-ahead market with a high penetration of stochastic production”. In: *Integrating Renewables in Electricity Markets: Operational Problems*. Boston, MA: Springer US, 2014, pages 57–100. ISBN: 978-1-4614-9411-9. DOI: 10.1007/978-1-4614-9411-9_3. URL: http://dx.doi.org/10.1007/978-1-4614-9411-9_3.

[32] J. F. Benders. “Partitioning procedures for solving mixed-variables programming problems”. In: *Numer. Math.* 4.1 (December 1962), pages 238–252. ISSN: 0029-599X. DOI: 10.1007/BF01386316. URL: http://dx.doi.org/10.1007/BF01386316.

[33] S. Binato, M. V. F. Pereira, and S. Granville. “A new benders decomposition approach to solve power transmission network design problems”. In: *IEEE Transactions on Power Systems* 16.2 (May 2001), pages 235–240. ISSN: 0885-8950. DOI: 10.1109/59.918292.

[34] S. J. Kazempour and A. J. Conejo. “Strategic generation investment under uncertainty via benders decomposition”. In: *IEEE Transactions on Power Systems* 27.1 (February 2012), pages 424–432. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2011.2159251.

[35] Francisco D. Munoz and Jean-Paul Watson. “A scalable solution framework for stochastic transmission and generation planning problems”. In: *Computational Management Science* 12.4 (2015), pages 491–518. ISSN: 1619-6988. DOI: 10.1007/s10287-015-0229-y. URL: http://dx.doi.org/10.1007/s10287-015-0229-y.

[36] S. Binato, G. C. de Oliveira, and J. L. de Araujo. “A greedy randomized adaptive search procedure for transmission expansion planning”. In: *IEEE Transactions on Power Systems* 16.2 (May 2001), pages 247–253. ISSN: 0885-8950. DOI: 10.1109/59.918294.

[37] G. C. Oliveira, A. P. C. Costa, and S. Binato. “Large scale transmission network planning using optimization and heuristic techniques”. In: *IEEE Transactions on Power Systems* 10.4 (November 1995), pages 1828–1834. ISSN: 0885-8950. DOI: 10.1109/59.476047.

[38] G. Latorre-Bayona and I. J. Perez-Arriaga. “CHOPIN, a heuristic model for long term transmission expansion planning”. In: *IEEE Transactions on Power Systems* 9.4 (November 1994), pages 1886–1894. ISSN: 0885-8950. DOI: 10.1109/59.331446.

[39] E. L. da Silva, H. A. Gil, and J. M. Areiza. “Transmission network expansion planning under an improved genetic algorithm”. In: *Proceedings of the 21st International Conference on Power Industry Computer Applications. Connecting Utilities. PICA 99. To the Millennium and Beyond (Cat. No.99CH36351)*. July 1999, pages 315–321. DOI: 10.1109/PICA.1999.779513.

[40] H. Park and R. Baldick. “Transmission planning under uncertainties of wind and load: sequential approximation approach”. In: *IEEE Transactions on Power Systems* 28.3 (August 2013), pages 2395–2402. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2013.2251481.

[41] R Tyrrell Rockafellar and Roger J-B Wets. “Scenarios and policy aggregation in optimization under uncertainty”. In: *Mathematics of operations research* 16.1 (1991), pages 119–147.

[42] F. S. Reis, P. M. S. Carvalho, and L. A. F. M. Ferreira. “Reinforcement scheduling convergence in power systems transmission planning”. In: *IEEE Transactions on Power Systems* 20.2 (May 2005), pages 1151–1157. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2005.846073.

[43] Shan Jin et al. "Modeling and solving a large-scale generation expansion planning problem under uncertainty". In: *Energy Systems* 2.3 (2011), pages 209–242. ISSN: 1868-3975. DOI: 10.1007/s12667-011-0042-9. URL: http://dx.doi.org/10.1007/s12667-011-0042-9.

[44] Dinakar Gade et al. "Obtaining lower bounds from the progressive hedging algorithm for stochastic mixed-integer programs". In: *Mathematical Programming* 157.1 (2016), pages 47–67. ISSN: 1436-4646. DOI: 10.1007/s10107-016-1000-z. URL: http://dx.doi.org/10.1007/s10107-016-1000-z.

[45] Heinrich von Stackelberg. "Chapter 3. Analysis of the relationship between dual markets and multiple markets". In: *Market structure and equilibrium*. Berlin, Heidelberg: Springer Berlin Heidelberg, 2011, pages 27–39. ISBN: 978-3-642-12586-7. DOI: 10.1007/978-3-642-12586-7_3. URL: http://dx.doi.org/10.1007/978-3-642-12586-7_3.

[46] J. M. Morales et al. "Pricing electricity in pools with wind producers". In: *IEEE Transactions on Power Systems* 27.3 (August 2012), pages 1366–1376. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2011.2182622.

[47] Fred C. Schweppe et al. "Behavior of hourly spot prices". In: *Spot Pricing of Electricity*. Boston, MA: Springer US, 1988, pages 31–53. ISBN: 978-1-4613-1683-1. DOI: 10.1007/978-1-4613-1683-1_2. URL: http://dx.doi.org/10.1007/978-1-4613-1683-1_2.

[48] S. Wong and J. D. Fuller. "Pricing energy and reserves using stochastic optimization in an alternative electricity market". In: *IEEE Transactions on Power Systems* 22.2 (May 2007), pages 631–638. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2007.894867.

[49] Yixian Liu. "Electricity capacity investments and cost Recovery with renewables". PhD thesis. The Ohio State University, 2016.

[50] Veronica Castillo Araoz. "Pricing in non-convex electricity markets". In: (2012).

[51] C. Ruiz, A. J. Conejo, and S. A. Gabriel. "Pricing non-convexities in an electricity pool". In: *IEEE Transactions on Power Systems* 27.3 (August 2012), pages 1334–1342. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2012.2184562.

[52] Antonio J. Conejo, Miguel Carrión, and Juan M. Morales. "Producer pool trading". In: *Decision making under uncertainty in electricity markets*. Boston, MA: Springer US, 2010, pages 157–194. ISBN: 978-1-4419-7421-1. DOI: 10.1007/978-1-4419-7421-1_5. URL: http://dx.doi.org/10.1007/978-1-4419-7421-1_5.

[53] Alan Hájek. "Interpretations of Probability". In: *The Stanford Encyclopedia of Philosophy*. Edited by Edward N. Zalta. Winter 2012. Metaphysics Research Lab, Stanford University, 2012.

[54] Antonio Gómez Expósito et al. *Electric energy systems: analysis and operation*. CRC Press, 2016.

[55] A. Daraeepour et al. "Strategic demand-side response to wind power integration". In: *IEEE Transactions on Power Systems* 31.5 (September 2016), pages 3495–3505. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2015.2493984.

[56] M. Shafie-khah et al. "Strategic offering for a price-maker wind power producer in oligopoly markets considering demand response exchange". In: *IEEE Transactions on Industrial Informatics* 11.6 (December 2015), pages 1542–1553. ISSN: 1551-3203. DOI: 10.1109/TII.2015.2472339.

[57] Antonio J. Conejo, Miguel Carrión, and Juan M. Morales. "Risk management". In: *Decision making under uncertainty in electricity markets*. Boston, MA: Springer US, 2010, pages 121–156. ISBN: 978-1-4419-7421-1. DOI: 10.1007/978-1-4419-7421-1_4. URL: http://dx.doi.org/10.1007/978-1-4419-7421-1_4.

[58] S. J. Kazempour and P. Pinson. "Effects of risk aversion on market outcomes: A stochastic two-stage equilibrium model". In: *2016 International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*. October 2016, pages 1–6. DOI: 10.1109/PMAPS.2016.7764200.

[59] L. Baringo and A. J. Conejo. "Risk-constrained multi-stage wind power investment". In: *IEEE Transactions on Power Systems* 28.1 (February 2013), pages 401–411. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2012.2205411.

[60] L. Baringo and A. J. Conejo. "Offering strategy of wind-Power Producer: a multi-stage risk-constrained approach". In: *IEEE Transactions on Power Systems* 31.2 (March 2016), pages 1420–1429. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2015.2411332.

[61] José Fortuny-Amat and Bruce McCarl. "A representation and economic interpretation of a two-level programming problem". In: *Journal of the Operational Research Society* 32.9 (1981), pages 783–792. ISSN: 1476-9360. DOI: 10.1057/jors.1981.156. URL: http://dx.doi.org/10.1057/jors.1981.156.

[62] Steven A. Gabriel et al. "Optimization problems constrained by complementarity and other optimization problems". In: *Complementarity Modeling in Energy Markets*. New York, NY: Springer New York, 2013, pages 221–262. ISBN: 978-1-4419-6123-5. DOI: 10.1007/978-1-4419-6123-5_6. URL: http://dx.doi.org/10.1007/978-1-4419-6123-5_6.

[63] Jean-Paul Watson and David L. Woodruff. "Progressive hedging innovations for a class of stochastic mixed-integer resource allocation problems". In: *Computational Management Science* 8.4 (2011), pages 355–370. ISSN: 1619-6988. DOI: 10.1007/s10287-010-0125-4. URL: http://dx.doi.org/10.1007/s10287-010-0125-4.

[64] Arne Løkketangen and David L. Woodruff. "Progressive hedging and tabu search applied to mixed integer (0,1) multistage stochastic programming". In: *Journal of Heuristics* 2.2 (1996), pages 111–128. ISSN: 1572-9397. DOI: 10.1007/BF00247208. URL: http://dx.doi.org/10.1007/BF00247208.

[65] C. Ordoudis et al. "Stochastic unit commitment via Progressive Hedging: extensive analysis of solution methods". In: *2015 IEEE Eindhoven PowerTech*. June 2015, pages 1–6. DOI: 10.1109/PTC.2015.7232629.

[66] Chao Li, Muhong Zhang, and Kory W Hedman. "N-1 reliable unit commitment via progressive hedging". In: *Journal of Energy Engineering* 141.1 (2014), B4014004.

[67] S. M. Ryan et al. "Toward scalable, parallel progressive hedging for stochastic unit commitment". In: *2013 IEEE Power Energy Society General Meeting*. July 2013, pages 1–5. DOI: 10.1109/PESMG.2013.6673013.

[68] Teodor Gabriel Crainic, Mike Hewitt, and Walter Rei. "Scenario grouping in a progressive hedging-based meta-heuristic for stochastic network design". In: *Computers and Operations Research* 43 (2014), pages 90–99. ISSN: 0305-0548. DOI: https://doi.org/10.1016/j.cor.2013.08.020. URL: http://www.sciencedirect.com/science/article/pii/S0305054813002268.

[69] Laureano F. Escudero et al. "Scenario cluster decomposition of the Lagrangian dual in two-stage stochastic mixed 0–1 optimization". In: *Computers and Operations Research* 40.1 (2013), pages 362–377. ISSN: 0305-0548. DOI: https://doi.org/10.1016/j.cor.2012.07.009. URL: http://www.sciencedirect.com/science/article/pii/S0305054812001542.

[70] Roger J-B Wets. "The aggregation principle in scenario analysis and stochastic optimization". In: *Algorithms and model formulations in mathematical programming*. Edited by Stein W. Wallace. Berlin, Heidelberg: Springer Berlin Heidelberg, 1989, pages 91–113. ISBN: 978-3-642-83724-1. DOI: 10.1007/978-3-642-83724-1_4. URL: http://dx.doi.org/10.1007/978-3-642-83724-1_4.

[71] A Brooke, D Kendrick, and A Meeraus. "Gams/cplex 12". In: *User notes. GAMS Development Corporation* (2010).

[72] Michael R Bussieck, Michael C Ferris, and Alexander Meeraus. "Grid-enabled optimization with GAMS". In: *INFORMS Journal on Computing* 21.3 (2009), pages 349–362.

[73] *DTU Computing Center (DCC)*. http://www.cc.dtu.dk/. Accessed: 2017-06-10.

[74] Christos Ordoudis et al. "An updated version of the IEEE RTS 24-Bus system for electricity market and power system operation studies". In: *Technical University of Denmark* (2016).