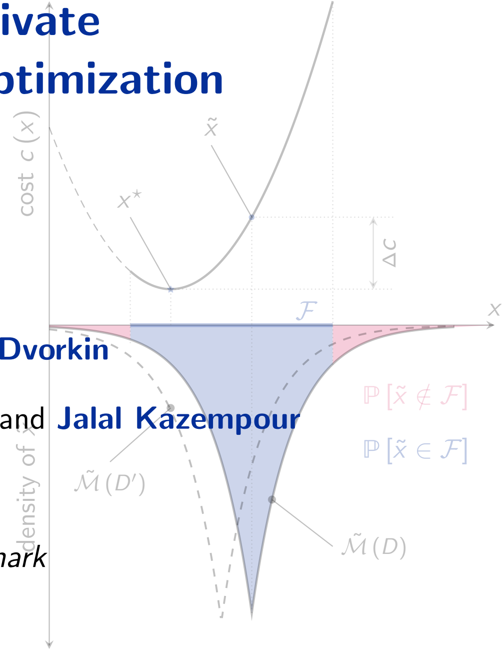
Stochastic and Private Energy System Optimization

Ph.D. Defense



Ph.D. Candidate: **Vladimir Dvorkin**

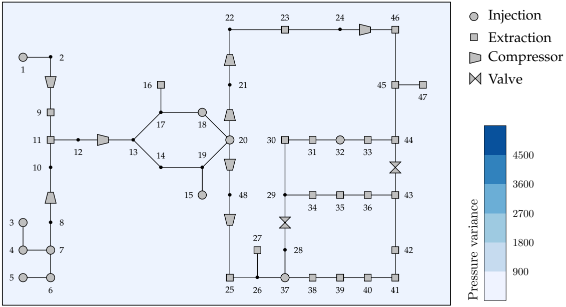
Supervisors: **Pierre Pinson** and **Jalal Kazempour**

March 8, 2021

Technical University of Denmark

Motivation - Energy systems under uncertainty

- From fuel-based to **renewable** energy production
- Less controllability and more **uncertainty**
- Failure to **optimize against uncertainty** leads to catastrophic consequences



mapping uncertainty to system state

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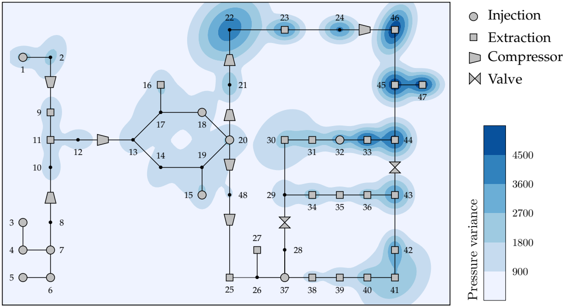
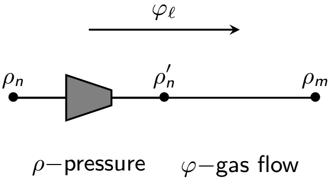


mapping uncertainty to system state

Motivation - Optimizing uncertainty

- Energy flow equations are **non-convex**, e.g, in natural gas systems:



$$\varphi_\ell|\varphi_\ell| = w_\ell(\rho_n^2 + \rho_{n'}^2 - \rho_m^2)$$

Weymouth equation

- Computational **tractability** is achieved through simplifications
- Robust and competitive energy **pricing** under uncertainty

Objective #1: To develop stochastic control models with robust operational & market performance guarantees for energy systems under uncertainty.

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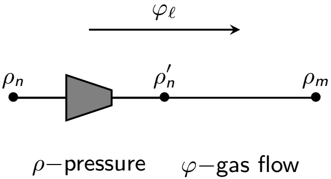

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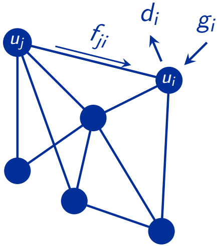
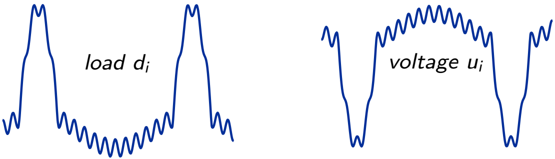
Motivation - Energy data and privacy concerns

- Real-time data transfers enables **cost-** and **security-optimal** operations
- **Energy data tells us more** than what we imagine
  - Contains private user data
  - Data can be **reverse-engineered**
  - Real examples of real-time surveillance



f — flow, d —load,

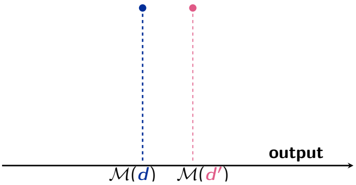
g —generation, u —voltage



Load data leaks through voltage measurements

Motivation - Privacy limits in energy optimization

- **Differential privacy** - is the privacy standard
  - Privacy guarantees through **randomization**
  - Outputs are **stat. similar** on diff. datasets
  - Two strategies: **input** or **output** perturbation



$\mathcal{M} : \mathbb{R} \mapsto \mathbb{R}$

- The two strategies **do not** apply to energy:

Input: no guarantee of solution existence
Output: no guarantee of solution feasibility

- **Unknown** implications to energy supply security and economics

Objective #2: To develop privacy-preserving optimization with formal privacy guarantees for data owners and performance guarantees for system operators

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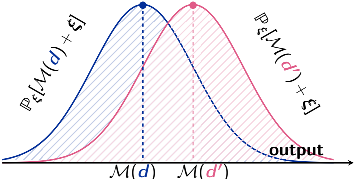

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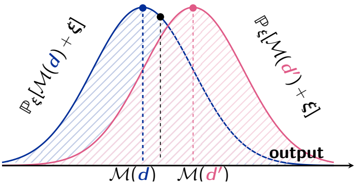

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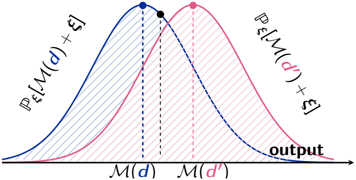

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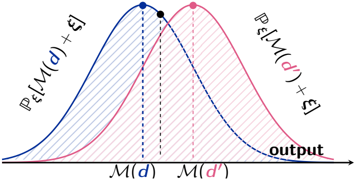

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Thesis contributions

Objective #1: To develop stochastic control models with robust operational & market performance guarantees for energy systems under uncertainty.

Thesis contributions:

1. Stochastic **control policies** to govern non-convex operations and markets under uncertainty and variability of renewable energy resources
   - Feasibility guarantees for secure real-time operations
   - From LP to SOCP duality to price uncertainty and variability
2. Hierarchical stochastic optimization to **ensure the satisfaction** of the basic **market properties irrespective** of uncertainty realizations
   - Trade-offs between market efficiency, cost recovery and revenue adequacy
3. Stochastic energy **market equilibrium under information asymmetry**:
   - Robust market design under information asymmetry

Thesis contributions (cont'd)

Objective #2: To develop privacy-preserving optimization with formal privacy guarantees for data owners and performance guarantees for system operators

Thesis contributions:

1. Adversarial models **to reveal** sensitive **data** from optimization outcomes:
   - Constrained empirical risk minimization
2. **Rigorous a priori privacy guarantees** for energy optimization data
   - At the interface of stochastic programming and privacy
   - For both distributed and centralized computations
3. **Performance guarantees** for the privacy-preserving optimization results
   - Feasibility guarantees
   - Minimal variability of optimization results
   - Expected vs. worst-case optimality loss trade-offs

Thesis publications

Stochastic optimization of energy systems:

A V. Dvorkin, A. Ratha, P. Pinson. and J. Kazempour. "**Stochastic control and pricing for natural gas networks.**" *Conditionally accepted for publication in the IEEE Transactions on Control of Network Systems*, 2020.

B V. Dvorkin, S. Delikaraoglou and J. M. Morales. "**Setting reserve requirements to approximate the efficiency of the stochastic dispatch.**" in *IEEE Transactions on Power Systems*, 2019.

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DTU

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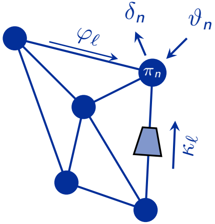
Optimization of natural gas network operations



$$\min_{\vartheta,\kappa,\varphi,\pi} \quad c_1^\top \vartheta + \vartheta^\top \text{diag}[c_2]\vartheta \qquad \textit{gas injection costs}$$

$$\text{s.t.} \quad A\varphi = \vartheta - B\kappa - \delta \qquad \textit{gas conservation law}$$

$$\varphi \circ |\varphi| = \text{diag}[w](A^\top \pi + \kappa) \qquad \textit{Weymouth equation}$$

$$\underline{\pi} \leqslant \pi \leqslant \overline{\pi},\ \underline{\vartheta} \leqslant \vartheta \leqslant \overline{\vartheta} \qquad \textit{network limits}$$

$$\underline{\kappa} \leqslant \kappa \leqslant \overline{\kappa},\ \varphi_\ell \geqslant 0,\ \forall \ell \in \mathcal{E}_a.$$

- Non-convex problem, yet solvable when parameters are certain
- To improve tractability under uncertainty, consider the **linearization**

$$\mathcal{W}(\varphi,\pi,\kappa) \approx \mathcal{J}(\mathring{\pi})(\pi - \mathring{\pi}) + \mathcal{J}(\mathring{\varphi})(\varphi - \mathring{\varphi}) + \mathcal{J}(\mathring{\kappa})(\kappa - \mathring{\kappa}) = \mathbb{0}$$

of the Weymouth equation around stationary point $(\mathring{\varphi}, \mathring{\pi}, \mathring{\kappa})$

- After rearranging the terms, we have

$$\varphi = \varsigma_1(\mathring{\varphi}, \mathring{\pi}, \mathring{\kappa}) + \varsigma_2(\mathring{\varphi}, \mathring{\pi})\pi + \varsigma_3(\mathring{\varphi}, \mathring{\kappa})\kappa, \quad \pi_r = \mathring{\pi}_r$$

where $\varsigma_1, \varsigma_2, \varsigma_3$ denote linear sensitivities

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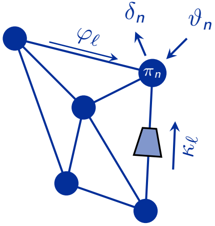

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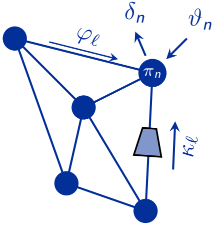

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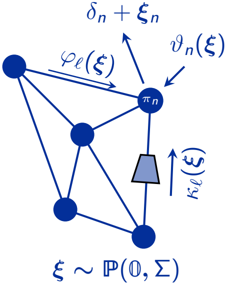


$$\min_{\tilde{\vartheta},\tilde{\kappa},\tilde{\varphi},\tilde{\pi}} \quad \mathbb{E}^{\mathbb{P}}[c_1^\top \tilde{\vartheta}(\boldsymbol{\xi}) + \tilde{\vartheta}(\boldsymbol{\xi})^\top \text{diag}[c_2]\tilde{\vartheta}(\boldsymbol{\xi})]$$

subject to **stochastic gas flow equations**

$$A\tilde{\varphi}(\boldsymbol{\xi}) = \tilde{\vartheta}(\boldsymbol{\xi}) - B\tilde{\kappa}(\boldsymbol{\xi}) - \tilde{\delta}(\boldsymbol{\xi})$$

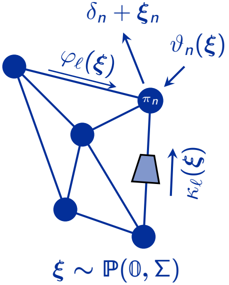
$$\tilde{\varphi}(\boldsymbol{\xi}) = \varsigma_1 + \varsigma_2\tilde{\pi}(\boldsymbol{\xi}) + \varsigma_3\tilde{\kappa}(\boldsymbol{\xi})$$

and a joint chance constraint

$$\mathbb{P}\begin{bmatrix}\underline{\pi} \leqslant \tilde{\pi}(\boldsymbol{\xi}) \leqslant \overline{\pi},\ \underline{\vartheta} \leqslant \tilde{\vartheta}(\boldsymbol{\xi}) \leqslant \overline{\vartheta}, \\ \underline{\kappa} \leqslant \tilde{\kappa}(\boldsymbol{\xi}) \leqslant \overline{\kappa},\ \tilde{\varphi}_\ell(\boldsymbol{\xi}) \geqslant 0,\ \forall \ell \in \mathcal{E}_a\end{bmatrix} \geqslant 1 - \varepsilon$$

where ε is a small prescribed parameter

Optimization of natural gas network operations



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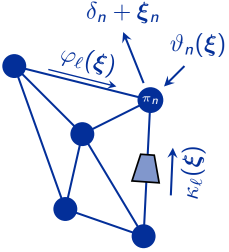

$\boldsymbol{\xi} \sim \mathbb{P}(\mathbb{0}, \Sigma)$

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- Affine **control policies** for gas injection $\tilde{\vartheta}(\boldsymbol{\xi})$ and pressure regulation $\tilde{\kappa}(\boldsymbol{\xi})$:

$$\tilde{\vartheta}(\boldsymbol{\xi}) = \vartheta + \alpha\boldsymbol{\xi}, \quad \tilde{\kappa}(\boldsymbol{\xi}) = \kappa + \beta\boldsymbol{\xi}$$

ϑ, κ – nominal (mean) control inputs

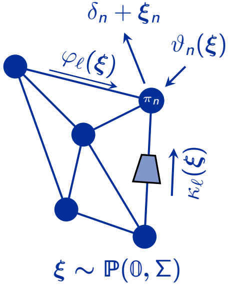
α, β – variable recourse decisions

- State variables then express as

$$\tilde{\pi}(\boldsymbol{\xi}) = \pi + \breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])\boldsymbol{\xi}$$
$$\tilde{\varphi}(\boldsymbol{\xi}) = \varphi + (\grave{\varsigma}_2(\alpha - \text{diag}[\mathbb{1}]) - \grave{\varsigma}_3\beta)\boldsymbol{\xi}$$

π, φ – nominal (mean) values, followed by recourse

Optimization of natural gas network operations



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- Chance constraint reformulation

$$\mathbb{P}\left[\tilde{\pi}_n(\xi) \leqslant \overline{\pi}_n\right] \geqslant 1 - \hat{\varepsilon}$$

is equivalent to

$$z_{\hat{\varepsilon}} \underbrace{\|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])]_n^\top\|}_{\text{pressure standard deviation}} \leqslant \overline{\pi}_n - \pi_n$$

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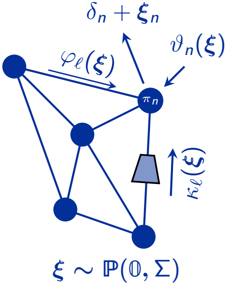

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$$z_{\hat{\varepsilon}} \underbrace{\|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])]_n^\top\|}_{\text{pressure standard deviation}} \leqslant \overline{\pi}_n - \pi_n$$

- Nodal pressure **variability** ...

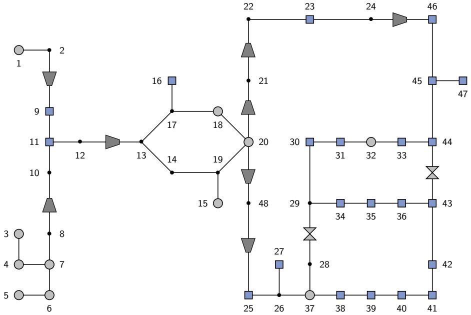
$$\|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])]_n^\top\| \leqslant \boldsymbol{s}_{\boldsymbol{n}}^{\boldsymbol{\pi}}$$

... is penalized by a factor of $\psi_n \geqslant 0$

- **SOCP duality** to price uncertainty&variability

Numerical experiments: Case of 48-node network

- 48-node natural gas network
- 22 stochastic gas extractions ◼
- 8 comp. ▶ and 2 valves ⋈
- 11 gas injections ●
- $\xi \sim N(0, \sigma)$, $\sigma \to 10\%$ of δ
- Violation probability $\varepsilon = 1\%$



Numerical experiments: Case of 48-node network

Parameter	Unit	Deterministic control policy	Chance-constrained control policies						
			Variance-agnostic	Pressure variance-aware, ψ^{π}			Flow variance-aware, ψ^{φ}		
				10^{-3}	10^{-2}	10^{-1}	1	10^{1}	10^{2}
Expected cost	\$1000	80.9	82.5 (100%)	**100.5%**	**105.6%**	**113.8%**	**100.1%**	**102.5%**	**112.6%**
Pressure variance	MPa^2	217.5	63.4 (100%)	**44.2%**	**18.9%**	**12.8%**			
Flow variance	$BMSCFD^2$	26.1	58.0 (100%)				**93.4%**	**44.8%**	**25.9%**
Compression	kPa	1939	3914	3570	3734	3661	3914	4030	3888
Valve regulation	kPa	0	0	0	**150**	**576**	0	**1**	**500**
Infeas. ($\varepsilon = 1\%$)	%	**53.7**	**0.04**	0.02	0.02	0.02	0.03	0.02	0.03

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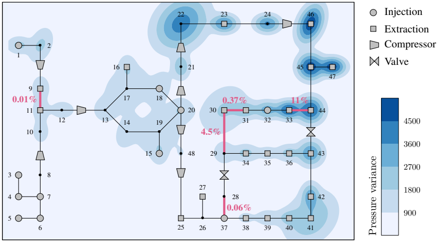
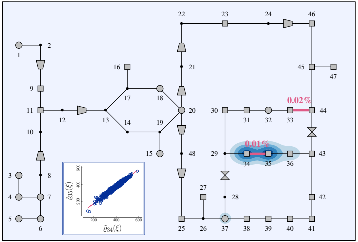
Variance-agnostic policy



$\psi^{\pi} = 0,\ \psi^{\varphi} = 0$

Variance-aware policy



$\psi^{\pi} = 0.1,\ \psi^{\varphi} = 10$

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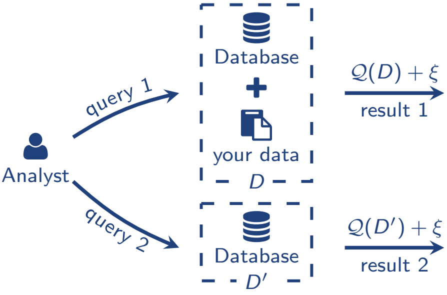
Differential privacy (definition)

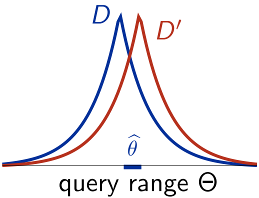


- Query (function) $\mathcal{Q} : \mathcal{D} \mapsto \Theta$
- Random perturbation ξ
- $\mathcal{Q}(D)$ and $\mathcal{Q}(D')$ can be **distinguished**
- $\mathcal{Q}(D)+\xi$ and $\mathcal{Q}(D')+\xi$ are stat. **indistinguishable**

$(\varepsilon, 0)-$differential privacy

A random function $\tilde{\mathcal{Q}} : \mathcal{D} \mapsto \Theta$ with domain $\mathcal{D}$ and range Θ is $(\varepsilon, 0)-$differentially private if for some $\widehat{\theta} \in \Theta$ and all neighboring datasets $D, D' \in \mathcal{D}$,

$$\mathbb{P}\left[\tilde{\mathcal{Q}}(D) \in \widehat{\theta}\right] \leqslant \mathbb{P}\left[\tilde{\mathcal{Q}}(D') \in \widehat{\theta}\right] \exp(\varepsilon)$$



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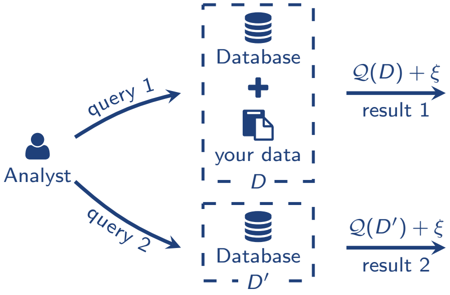

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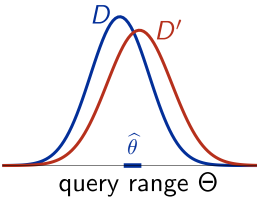
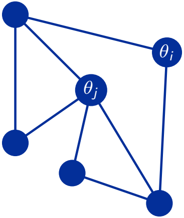


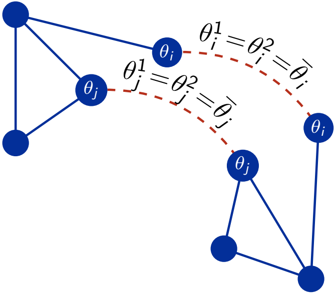
Distributed optimization of energy networks

Centralized optimization



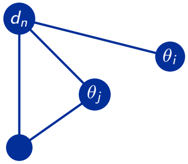
- Solved by a central entity
- All data must be shared
- Solved in a single run

Distributed optimization



- Solved by distributed agents
- Only voltage variables are shared
- Solved over iterations

Privacy breaches in distributed optimization



- Voltage variables implicitly **depend on data**
- Local optimization as a **single**-valued mapping

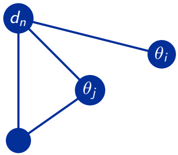
$$\mathcal{M} : \mathcal{D} \mapsto \Theta \times \Theta,\ \mathcal{M}(d_n) = \begin{bmatrix} \theta_i \\ \theta_j \end{bmatrix}$$

  from load domain to voltage domain
- An adversary executes a privacy attack

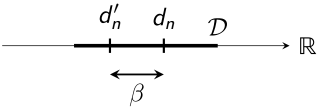
$$\mathcal{A} : \Theta \times \Theta \mapsto \mathcal{D},\ \mathcal{A}\left(\begin{bmatrix} \theta_i \\ \theta_j \end{bmatrix}\right) = d_n,$$

  which is the **opposite** of $\mathcal{M}$.
- Two privacy **attack models** proposed:
  - Tracing attack (based on repeated observations)
  - Reconstruction attack (optimization-based)

Privacy guarantees for distributed optimization



- Load d_n must be **indistinguishable** from any other $\beta-$**adjacent** load $d_n' \in [d_n - \beta, d_n + \beta]$ for some $\beta > 0$ in the release of voltage variables, i.e.,



- To make d_n indistinguishable from $\beta-$adjacent load d_n',

$$\text{let} \quad \tilde{\mathcal{M}}(d_n) = \mathcal{M}(d_n) + \xi = \theta + \xi$$

be a randomized response with perturbation $\xi \in \mathbb{R}^2$

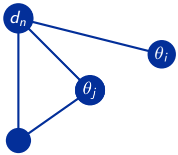
voltage domain Θ

Key result: differential privacy of $\tilde{\mathcal{M}}(d_n)$

When $\xi \sim \text{Lap}(\beta/\varepsilon)^2$, $\tilde{\mathcal{M}}$ is $\varepsilon-$differentially private on $\beta-$adjacent load datasets, i.e.,

$$\mathbb{P}[\tilde{\mathcal{M}}(d_n) \in \widehat{\theta}] \leqslant \mathbb{P}[\tilde{\mathcal{M}}(d_n') \in \widehat{\theta}]\exp(\varepsilon)$$

Privacy guarantees for distributed optimization



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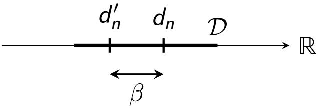

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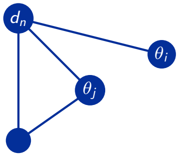

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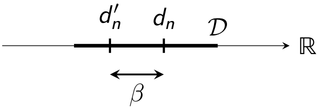

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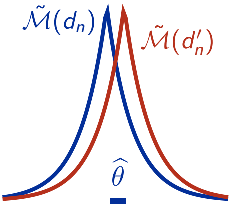

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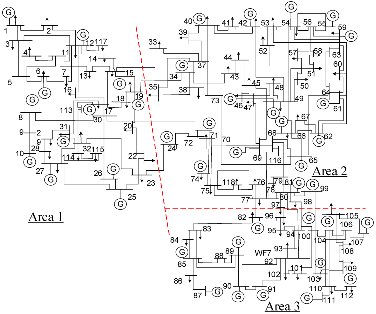
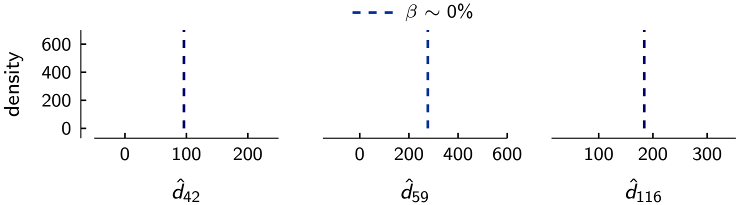
Private distributed optimization: case of IEEE 118-node network

- The network is split into three zones
- An adversary infers individual loads
- Privacy loss is fixed $\varepsilon = 1$
- Adjacency coefficient β varies



Adversarial load inference



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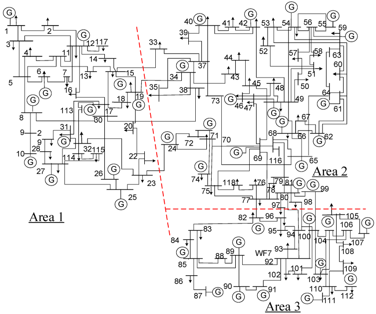

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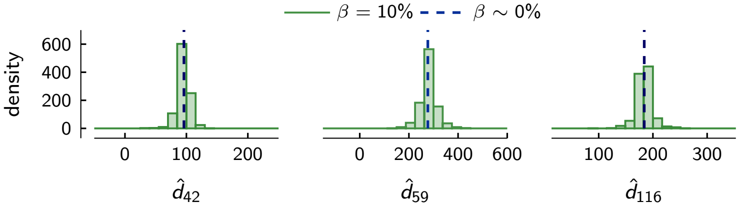

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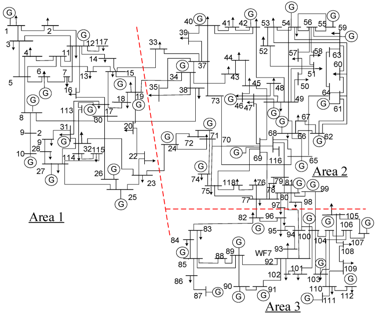

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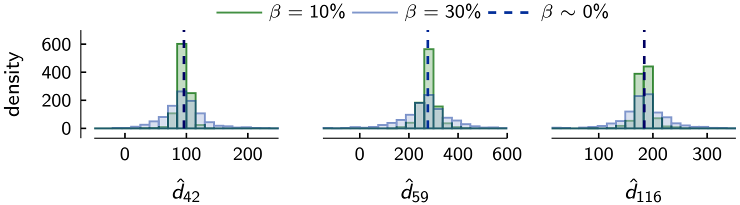

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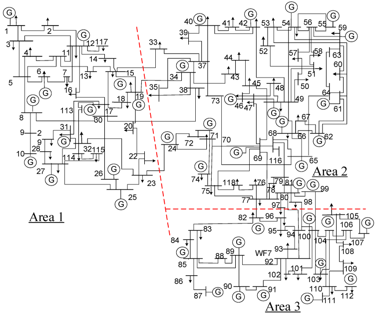

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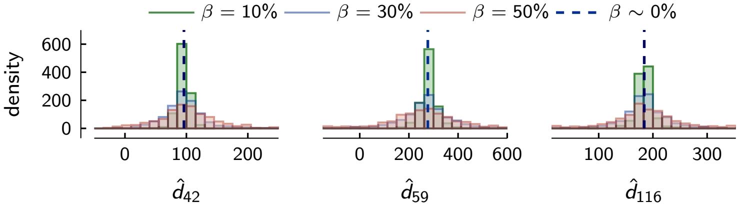

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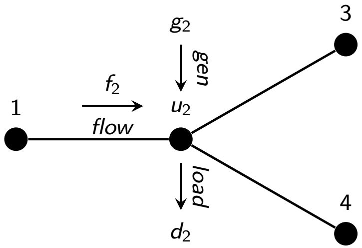

- Distribution OPF problem as a **mapping**

$$\mathcal{M} : \mathcal{D} \mapsto \mathcal{F}$$

from load domain $\mathcal{D}$ to flow domain $\mathcal{F}$

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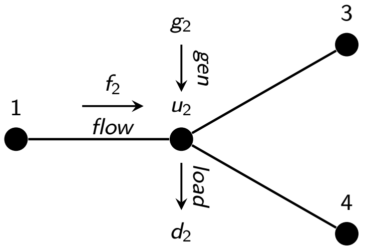

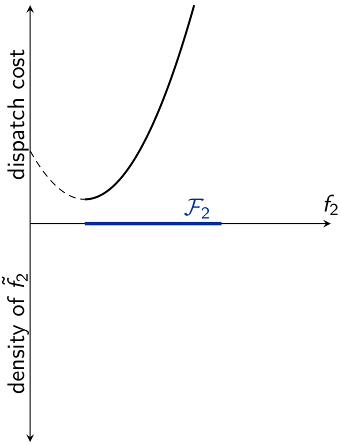

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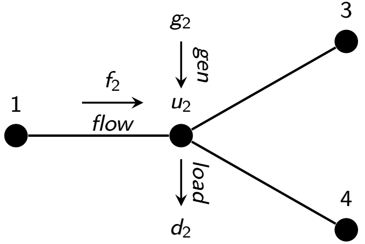

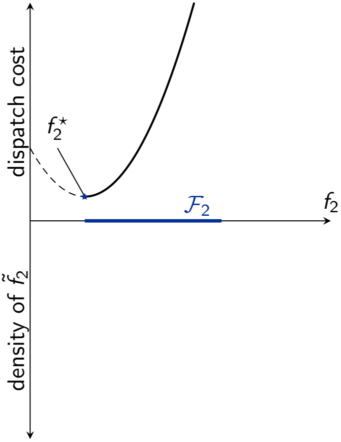

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$$\mathcal{M} : \mathcal{D} \mapsto \mathcal{F}$$

  from load domain $\mathcal{D}$ to flow domain $\mathcal{F}$

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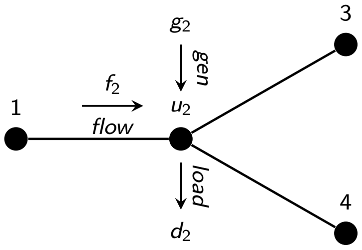
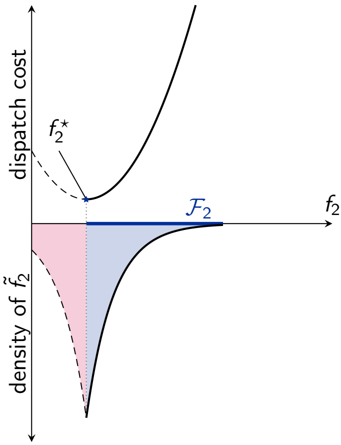
Private distribution OPF: Problem statement





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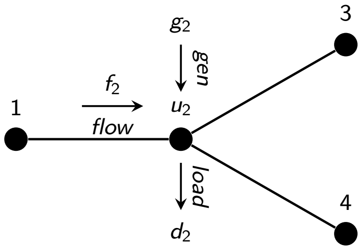

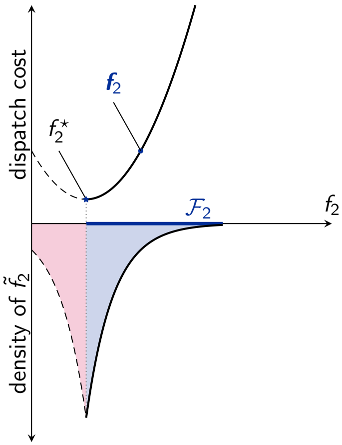

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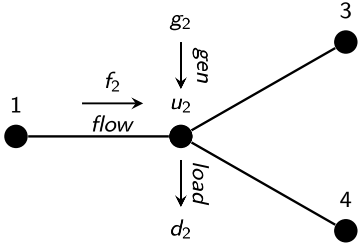

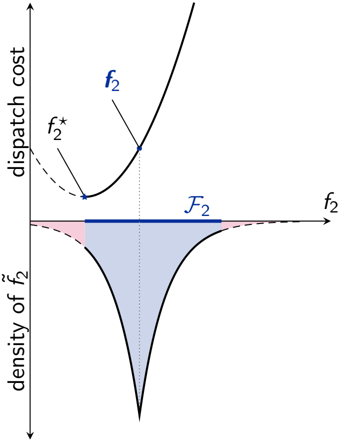

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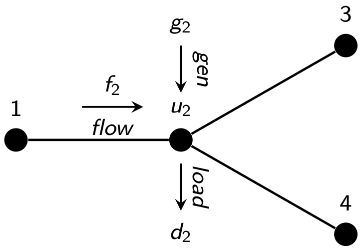

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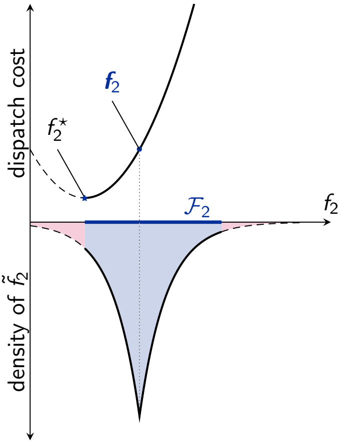
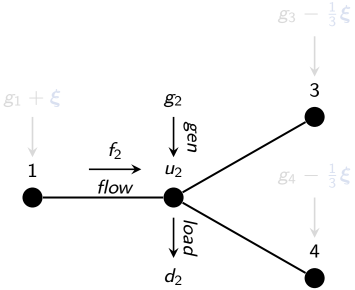


- How to find the **nominal** solution $\boldsymbol{f}_2$?

Private distribution OPF: perturbation strategy



- Generator randomization policy:

$$\tilde{g}_i(\xi) = \underbrace{g_i}_{\text{nom.}} + \underbrace{\left[T_i \circ \alpha_i\right]\xi}_{\text{random component}}$$

$$\sum_{i \in \mathcal{U}_\ell} \alpha_{i\ell} = 1, \ \sum_{i \in \mathcal{D}_\ell} \alpha_{i\ell} = 1, \ \forall \ell \in \mathcal{L},$$

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$\min\limits_{\tilde{g}(\xi), \tilde{f}(\xi), \tilde{u}(\xi)}$	$\mathbb{E}^{\mathbb{P}_\xi}\left[c^\top \tilde{g}(\xi)\right]$	minimum of expected cost
s.t.	$\mathbb{P}_\xi\left[h(\tilde{g}(\xi), \tilde{f}(\xi), \tilde{u}(\xi)) = d\right] = 1$	power balance holds with prob. 1
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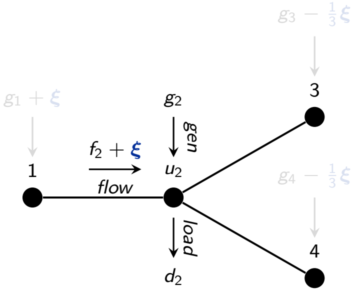

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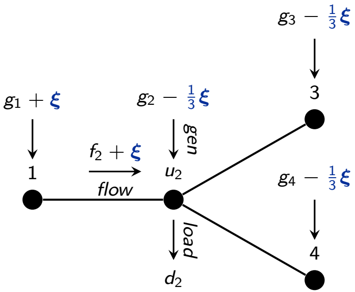

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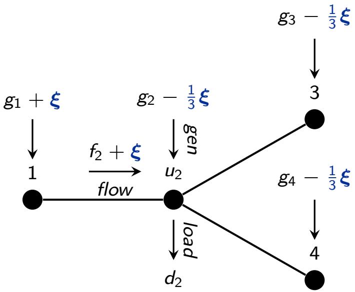

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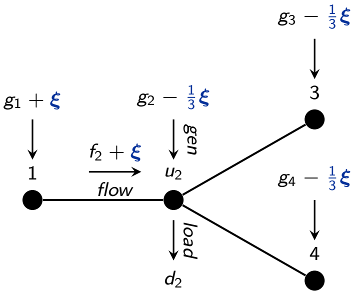

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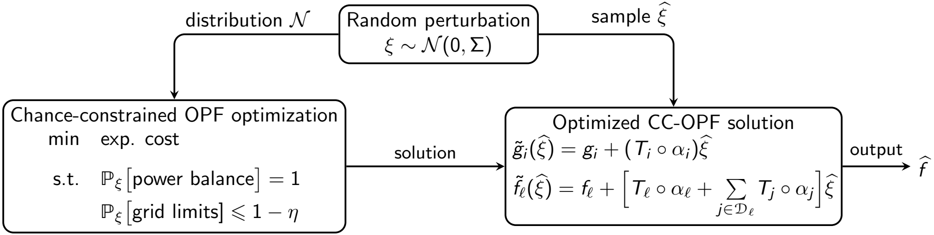
Private distribution OPF: Performance guarantees

Optimize → sample → implement



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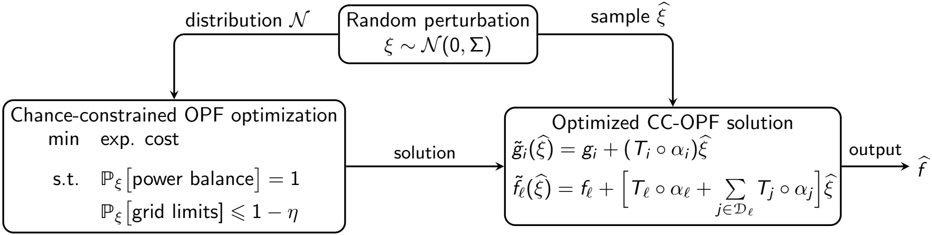


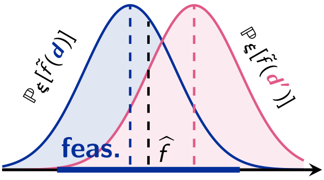
Privacy of $\beta-$adjacent load vectors

$(\varepsilon, \delta)-$differential privacy

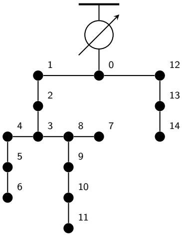
Let $\xi_i \in \mathcal{N}(0, \sigma_i)$ and $\sigma_i \geqslant \beta_i \sqrt{2\ln(1.25/\delta)}/\varepsilon,\ \forall i \in \mathcal{L}$.
Then, for β-adjacent load vectors d and d' :

$$\mathbb{P}_{\xi}[\tilde{f}(\boldsymbol{d'}) \in \widehat{f}] \leqslant \exp(\varepsilon)\mathbb{P}_{\xi}[\tilde{f}(\boldsymbol{d'}) \in \widehat{f}] + \delta,$$

for any flow value $\widehat{f}$

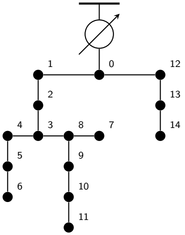


Private distribution OPF: Case of 15-bus feeder



- 15-bus radial distribution network
- 14 customers with DERs, 1 substation
- Full grid observability

Private distribution OPF: Case of 15-bus feeder

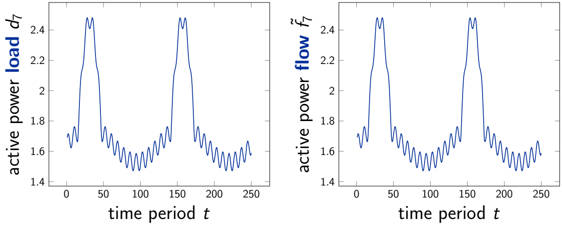


- Customer at node 7 with a load pattern

$$\underbrace{\max\left\{\sin\tfrac{5}{10^2}t, \tfrac{7}{10}\right\}}_{\text{large component}} + \underbrace{\frac{5}{10^2}\sin\frac{5}{10^2}t}_{\text{medium component}} + \underbrace{\frac{25}{10^3}\sin\frac{75}{10^2}t}_{\text{small component}}$$

- The load must be **indistinguishable** from any other **β—adjacent** load
- **Sampled** private power flow trajectories:

Adjacency $\beta_7 = 0$ MW



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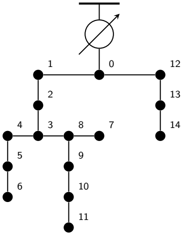

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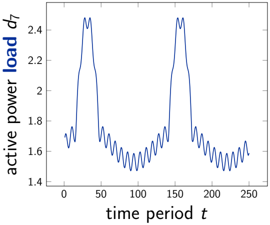
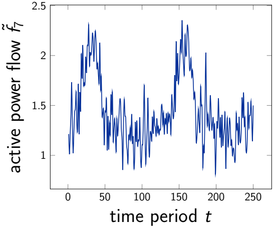


Adjacency $\beta_7 = 0.07$ MW



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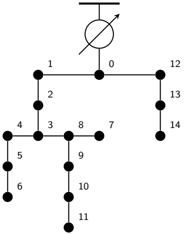

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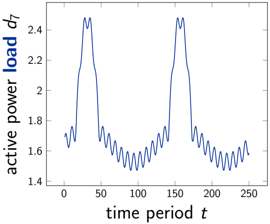

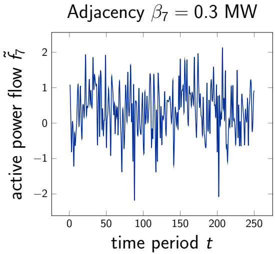

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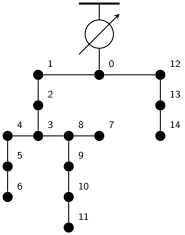

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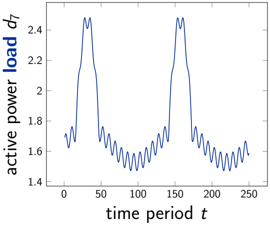

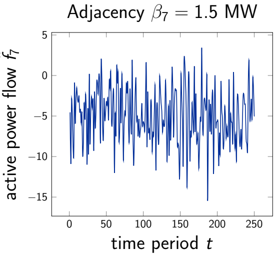

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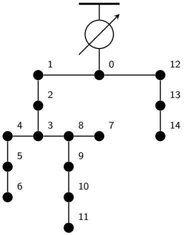

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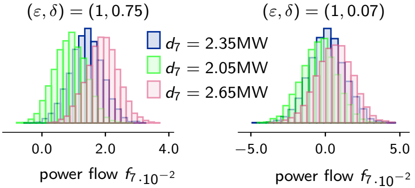
- **$(\varepsilon, \delta)-$differential privacy** guarantees



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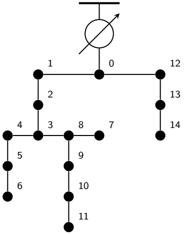

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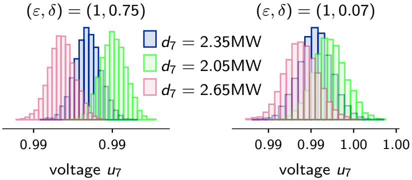


Conclusions

Stochastic policies for uncertainty management in energy systems

- Secure real-time operations with guarantees
- Offering extensions to the stochastic energy pricing

Privacy guarantees for optimization datasets

- For distributed and centralized computations
- At the interface of stochastic programming and differential privacy
- Affine dependency of the solution on the perturbations enabled privacy and feasibility guarantees, minimal variability and worst-case optimality loss

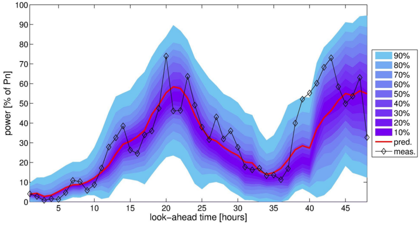
Resolved privacy concerns encourage

- OPF-based dispatch practices
- Engagement of privacy-cognizant energy users
- Safe market and operational transparency

Current research

Multi-stage control policy optimization

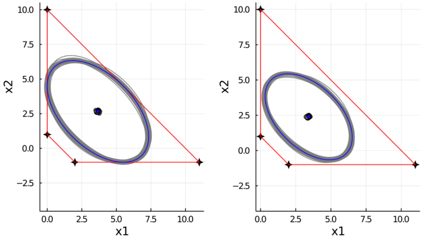
- Intraday actuation of control policies
- More cost-variance trade-offs available



from pierrepinson.com

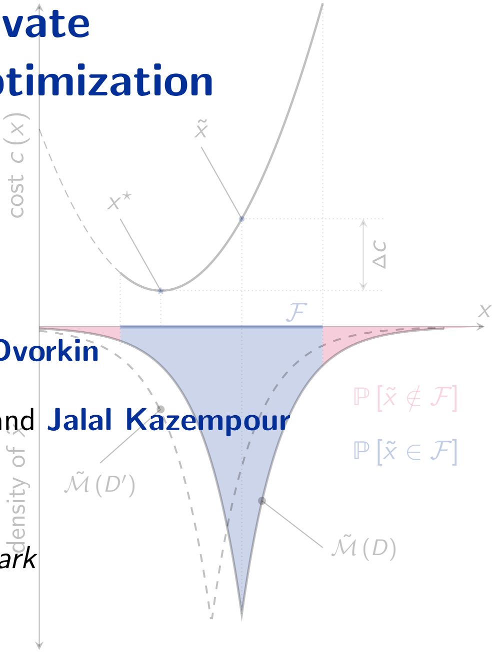
Convex private optimization

- From LP to SOCP and SDP



Stochastic and Private Energy System Optimization

Ph.D. Defense



Ph.D. Candidate: **Vladimir Dvorkin**

Supervisors: **Pierre Pinson** and **Jalal Kazempour**

March 8, 2021

Technical University of Denmark