Online Electricity Pricing from Frequency Measurements

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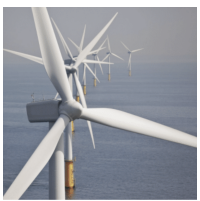
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The problem: markets are slow, physics is fast

- Power imbalance shows up instantly as **frequency deviation**.
- Electricity is priced *offline*: day-ahead (every hour) and real-time (every 5–15 min).

$\Rightarrow$ Prices fixed for minutes cannot see sub-minute renewable and load variability.



Renewable power generation



Datacenter loads

Contribution

Derive an *online* electricity price that updates on frequency timescales, and show it recovers the theoretical properties of offline markets in real time.

Price electricity on frequency timescales

Real-time economic dispatch

$$\begin{aligned} \min_{\mathbf{r}\in\mathcal{P}} \quad & c(\mathbf{r}) \\ \text{s.t.} \quad & \mathbf{1}^\mathsf{T}\mathbf{r} = \delta \quad : \lambda \end{aligned}$$

Primal-dual dynamics:

$$\dot{\mathbf{r}} = \Pi_{\mathcal{P}}[\lambda\mathbf{1} - \nabla c(\mathbf{r})] \qquad \text{Regulation Dynamics,}$$

$$\dot{\lambda} = \delta - \mathbf{1}^\mathsf{T}\mathbf{r} \qquad \text{Price Dynamics.}$$

Linearized swing equation

$$\dot{\omega} = -\tfrac{1}{M}\left(\delta - \mathbf{1}^\mathsf{T}\mathbf{r} + D\omega\right) \qquad \text{Frequency Dynamics.}$$

Dynamics via Lagrangian composition

$$\dot{\mathbf{r}} = \Pi_{\mathcal{P}}\left[\left(\lambda - \tfrac{1}{D}\dot{\omega}\right)\mathbf{1} - \nabla_{\mathbf{r}} c(\mathbf{r})\right],$$

$$\dot{\omega} = -\tfrac{1}{M}\left(\delta - \mathbf{1}^\mathsf{T}\mathbf{r} + D\omega\right),$$

$$\dot{\lambda} = \delta - \mathbf{1}^\mathsf{T}\mathbf{r}.$$

Price update

$$\dot{\lambda} = -D\omega - M\dot{\omega}.$$

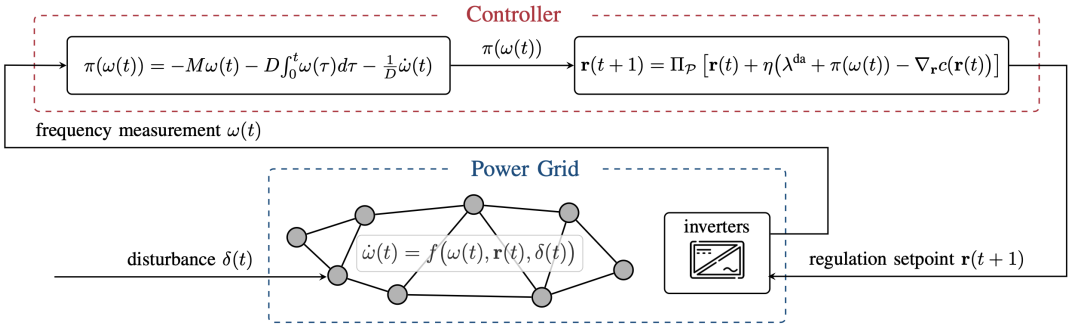
With $\lambda(0) = \lambda^{\mathrm{da}}$ and $\omega(0) = 0$:

$$\lambda(t) = \lambda^{\mathrm{da}} \quad \underbrace{-M\omega(t) - D\int_0^t \omega(\tau)\, d\tau}_{\text{correction (local frequency measurement)}}.$$

Control implementation of fast pricing

Frequency-dependent price adjustment has a PID structure: Distributed control

$$\pi(\omega(t)) = \underbrace{-M\omega(t)}_{\text{P}} \underbrace{-D\int_0^t \omega(\tau)\,d\tau}_{\text{I}} \underbrace{-\frac{1}{D}\dot{\omega}(t)}_{\text{D}}.$$



Market properties preservation

Theorem 1 (Social optimum)

The dynamical system above converges to the solution of

$$\begin{aligned} \underset{\mathbf{r}\in\mathcal{P},\,\omega}{\text{minimize}} \quad & c(\mathbf{r}) \\ \text{subject to} \quad & \mathbf{1}^{\mathsf{T}}\mathbf{r} - \delta = 0, \\ & D\omega - \mathbf{1}^{\mathsf{T}}\mathbf{r} + \delta = 0, \end{aligned}$$

i.e. **minimum-cost generation**, **power balance**, and **zero frequency deviation**, simultaneously.

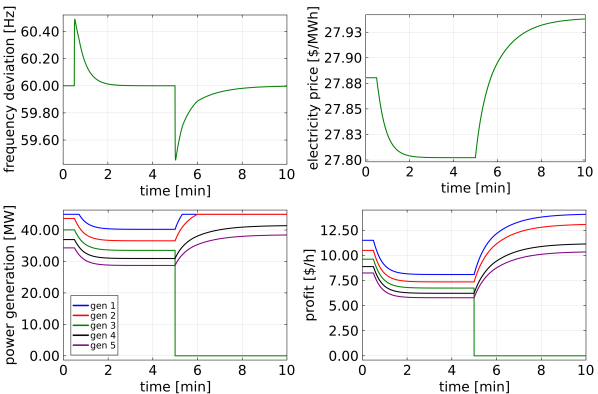
Theorem 2 (Cost recovery)

Discrete-time update, step size η_i per generator i:

$$\begin{aligned} \hat{\mathbf{r}}_i(t+1) &= \mathbf{r}_i(t) + \eta_i\Big(\lambda^{\mathrm{da}} + \pi(\omega(t)) \\ &\qquad - \nabla_{\mathbf{r}_i} c_i(\mathbf{r}_i(t))\Big), \\ \mathbf{r}_i(t+1) &= \Pi_{\mathcal{P}_i}\big[\hat{\mathbf{r}}_i(t+1)\big]. \end{aligned}$$

Choosing $\eta_i = \mathbf{C}_{ii}^{-1}$ (inverse of the quadratic cost coefficient) guarantees **nonnegative profit** for generator i at *every* time step.

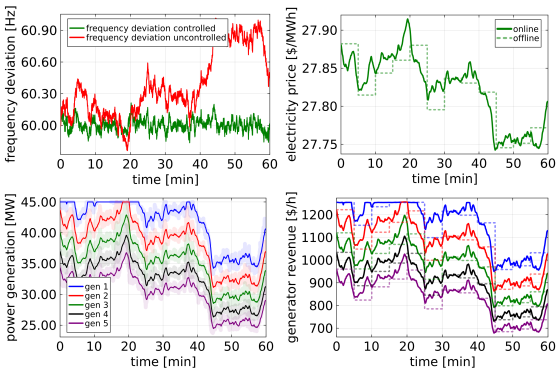
Constant-disturbance experiment



Observations

- Frequency returns to **nominal**.
- Dispatch converges after both events.
- Online profit remains **nonnegative**.

Time-varying stochastic demand



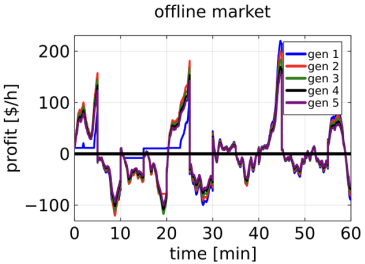
Disturbance model

- Zero-drift Wiener process
- 1 MW standard deviation

Observations

- The controlled frequency near **nominal**.
- Online prices resolve hidden variation.
- **Generation tracks** economic-dispatch.

Frequency-based settlement stabilizes generator profit



Five-minute settlement

- Prices ignored subinterval frequency response.
- Profit is **volatile** and frequently **negative**.

Online settlement

- Prices remunerate the physical response.
- Profit is **smoother** and remains **nonnegative**.

Conclusions

1. Integrated market–frequency dynamics give online electricity pricing directly from local frequency measurements.
2. The resulting controller is a local PID: day-ahead price plus a frequency-derived correction.
3. Provable properties: convergence to the social optimum, cost recovery at every step.