

Multi-Stage Linear Decision Rules for Stochastic Control of Natural Gas Networks with Linepack

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XXII Power Systems Computation Conference

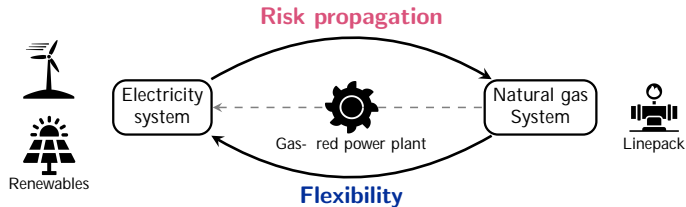
Porto, June 2022

Uncertainty and risks in natural gas networks

- | Uncertainty in natural gas networks arises from:
 - | Imperfect forecasts
 - | Imbalances in power grid
 - | Renewable gas supply
 - | Extreme weather events

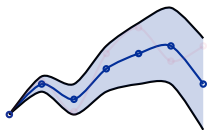
- | Risks in natural gas networks include:
 - | Operational risks (operations at (or beyond) the limits)
 - | Market risks (contract violations, variable returns, etc.)

- | Operations in power systems produce risks in natural gas systems, and vice versa



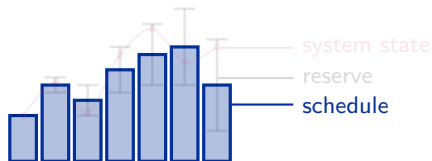
Scheduling under uncertainty meets stochastic control

gas demand uncertainty



scheduling
→
optimization

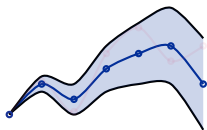
gas injection



- | Look-ahead scheduling problems under uncertainty:
 - | Optimal nominal set-points for gas injections
 - | Optimal reserve margins to hedge against uncertainty
- | Scheduling does not answer how to **control the system state** as uncertainty gradually realizes
- | Hence, it is often assumed that there will be some real-time re-optimization to control the system state
- | **Contribution in a nutshell:** We extended the scheduling under uncertainty to include stochastic control component within operational planning of natural gas systems

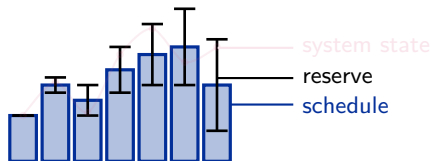
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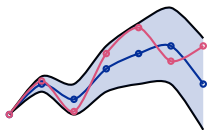
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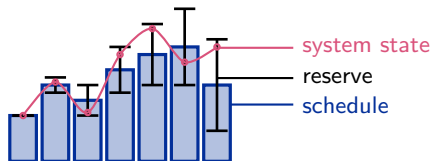
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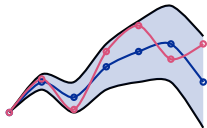
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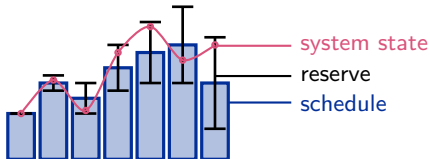
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Contributions

#1 Multi-stage stochastic control policies to guide natural gas networks

- | Optimized ahead of time using linear decision rules (LDRs)
- | Produce real-time control inputs

#2 Multi-stage chance-constrained optimization

- | Distributionally robust LDR optimization
- | Scalable conic formulation

#3 Three stochastic control applications

- | Linepack as a sole flexibility resource
- | Variability & Variance-aware optimization
- | Network topology optimization

Outline

Motivation and contributions

Modeling natural gas networks

LDR-based control policies for network assets

Network control applications

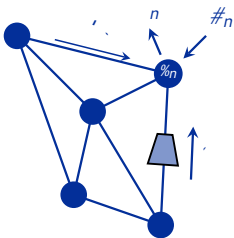
Numerical experiments

Conclusions

Natural gas network equations under uncertainty

- At any stage t , network operations are perturbed by renewable power forecast errors $\mathbf{t} = (t_1; \dots; t_n)$. Stochastic gas extraction $\mathbf{t} = \mathbf{t}(t)$.
- Conservation of natural gas mass under uncertainty

$$\underbrace{A' \mathbf{t}(t)}_{\text{gas flow}} = \underbrace{\#_t(t)}_{\text{gas injection}} - \underbrace{B \mathbf{t}(t) - t(t)}_{\text{gas extraction}}$$



- From non-convex stochastic Weymouth equation of gas flow

$$|\mathbf{t}(t)|' |\mathbf{t}(t)| = W \cdot \left(\#_n(t) + \underbrace{(\mathbf{t}(t))}^{\text{compression}} \right)^2 - \#_m(t)^2$$

quadratic pressure drop

... to its linearized counterpart $\mathbf{t}(t) = W_{0t} + W_{1t} \#_t(t) + W_{2t} \mathbf{t}(t)$

Stochastic Control and Pricing for Natural Gas Networks

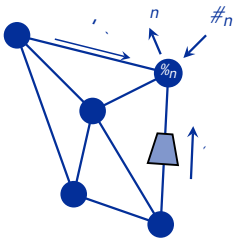
Vladimir Dvorkin¹, Member, IEEE, Anubhav Raiha², Student Member, IEEE,
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Performance guarantees of linearization [DRPK21]

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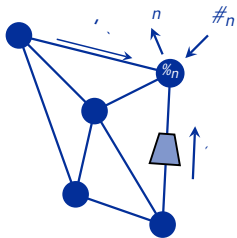
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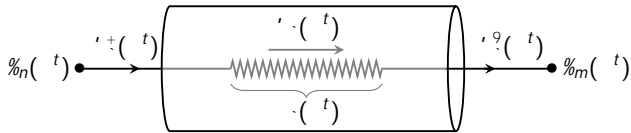
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- Linepack (gas storage) equations under uncertainty^a

$$\mathbf{t}(t) = \frac{1}{2} \text{dg}[s] \quad \mathbf{t}(t) + |\mathbf{A}|^T \mathbf{\%}_t(t) - \text{linepack} \propto \text{to gas pressures}$$

$$\mathbf{t}(t) = \mathbf{t}^{91}(t^{91}) + \mathbf{\%}'_t(t) - \mathbf{\%}^9_t(t) - \text{dynamic state of charge}$$



^arefer to [SOKP19] for original, deterministic equations

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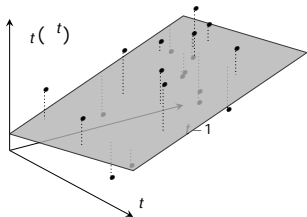
Chance-constrained gas network optimization in linear decision rules

$$\begin{aligned}
 \min_{\#_t, \%_t} & \quad E \left[\sum_{t=1}^T c_1^\top \#_t(\omega^t) + \#_t(\omega^t)^\top C_2 \#_t(\omega^t) \right] && \text{expected quadratic cost} \\
 \text{s.to} & \quad A_t^\# \#_t(\omega^t) + A_t^\% \%_t(\omega^t) = A_t \#_t(\omega^t); && \text{gas network equations} \\
 & \quad P_t \left(\begin{array}{c} \underline{\#} \leq \#_t(\omega^t) \leq \bar{\#} \\ \underline{\%} \leq \%_t(\omega^t) \leq \bar{\%} \end{array} \right) > \underbrace{1 - \epsilon}_{\text{reliability level}}; \forall t && \text{chance constraint on control and state var.}
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 & && \text{control and state var.} \\
 & && \underbrace{> 1 - \epsilon}_{\text{reliability level}}; \forall t
 \end{aligned}$$

- scenario approx.  LDR



- | Scenario approximation is computationally hard
- | Linear decision rule approximation [KWG11]:

$$\#_t(\omega^t) = \Theta_t \omega^t; \quad \%_t(\omega^t) = P_t \omega^t$$

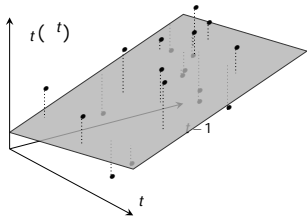
with matrices Θ_t and P_t to be optimized

- | Refer to [DMB22] for optimality guarantees of LDR

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Distributionally robust reformulation of chance constraints

A chance constraint on nodal pressure limits

$$P_t \in \mathcal{P}_{\theta} \left(\frac{1}{\alpha} \right) \Rightarrow \mathbb{P}_{\theta} \left(p_{tn} \leq p_{\max} \right) \geq 1 - \alpha$$

reformulates into 1 conic and 4 linear ineq. [XA17]:

$$P_t \in \mathcal{P}_{\theta} \left(\frac{1}{\alpha} \right) \Rightarrow \mathbb{P}_{\theta} \left(p_{tn} \leq p_{\max} \right) \geq 1 - \alpha$$

$$[P_t S_t]_n \leq \frac{1}{2} \sigma_{\theta} \frac{\sigma_{\theta}}{\alpha} \leq y_{tn} + x_{tn}$$

$$\frac{1}{2} \sigma_{\theta} \frac{\sigma_{\theta}}{\alpha} > x_{tn} > 0; y_{tn} > 0$$

using mean μ and covariance Σ of forecast errors

Pressure LDR is optimized to stay within technical limits with probability $(1 - \alpha)$

Key features:

- | Fits any distribution with given mean and covariance
- | Pareto-dominates single-sided chance constraints

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Application 1: Linepack as a sole exhibity resource

- | Real-time supply corrections are costly
- | How accommodate uncertainty otherwise?
- | Actively deploy linepack by regulating pressure!

- | Chance-constrained optimization of compressor and valve LDR policies, additionally subject to

$$\underbrace{\mathbb{P}[\xi_t] \leq \sigma}_{\text{std of gas injection}} \leq \underbrace{\mu}_{\text{mean gas injection}} \leq \mu + \beta \sigma; \quad \beta \geq 0$$

where β is a control parameter, e.g., $\beta = 0$ sets any supply adjustment

6 compressors, 2 valves and 51 pipeline (storage units)

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$$\underbrace{\sigma[\Delta p]_n}_{\text{std of gas injection}} \leq \beta \underbrace{[\Delta p]_n}_{\text{mean gas injection}} + \beta \Delta p_{\text{max}}; \quad \beta \in [0, 1];$$

where β is a control parameter, e.g., $\beta = 0$ sets any supply adjustment

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Application 2: Variance- and variability-aware operations

- Variable and uncertain renewable generation translates into variable and uncertain network state
- LDRs provide the measure of variability and variance of network state variables
- Pressure variability and uncertainty is penalized by minimizing the following term:

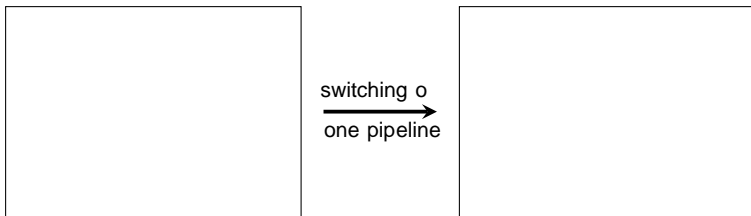
$$E_P = \sum_{t=2}^T \alpha^t \text{Tr} \left(P_t + P_{t-1} + \beta (P_t - P_{t-1})^T \right)$$

where P_t and P_{t-1} are pressure LDR matrices at two subsequent stages, and β is the forecast error covariance

- By varying penalty factor α , we more intensively penalize pressure variability and variance

Application 3: Network topology optimization

- We use LDRs to optimize topology to decouple network parts and reduce uncertainty propagation:



Color density displays nodal pressure variance. Topology switching is an additional flexibility resource [DRPK21]

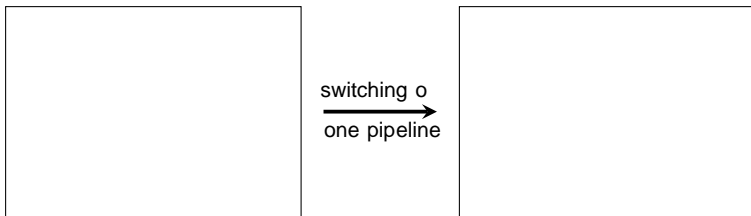
- Given V binary valves, we have a set; $1::; 2^V$ of possible network topologies
- Binary variable $v_c = 1$ if topology c is selected, and $v_c = 0$ otherwise
- Then, the following binary logic is used for topology selection:

$$\underbrace{p_t(t) = w_{0tc} + W_{1tc} p(t) + W_{2tc} p_t(t)}_{\text{Linearized Weymouth equation for each topology } c}; \quad \forall c = 1, \dots, 2^V$$

$$\underbrace{p_t(t) = \sum_{c=1}^{2^V} v_c p_t(t); \quad \sum_{c=1}^{2^V} v_c = 1}_{\text{Select gas flows from one topology only}}$$

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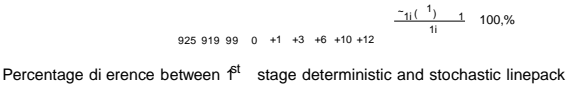
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Experiments on 48-node natural gas network

Uncertain renewable generation from the IEEE 118-node network translates into uncertain gas extraction by CCGTs



Acting on the same uncertainty data, variability-aware LDR solution produces more stable pressure profiles

Analyzing optimal network response

Parameter	Unit	Deterministic control policy	Stochastic control policy				
			Base	Linepack-agnostic	Variability-aware		
					% = 10	% = 50	% = 100
Expected gas injection cost	\$1000	644.8 (94.6%)	681.7 (100.0%)	752.1 (110.3%)	694.4 (101.9%)	701.2 (102.9%)	703.5 (103.2%)
Pressure variability	MPa	72.01 (89.7%)	38.0 (100.0%)	66.0 (173.6%)	7.8 (20.5%)	7.3 (19.2%)	7.2 (19.1%)
Expected / worst-case magnitude of pressure constraint violations	MPa	77.42 / 147.76	0.00 / 0.00	0.00 / 0.00	0.00 / 0.00	0.00 / 0.00	0.00 / 0.00
Expected / worst-case magnitude of gas mass constraint violations	MMSCFD	26.86 / 146.91	0.01 / 0.02	0.13 / 0.13	0.01 / 0.02	0.02 / 0.02	0.02 / 0.02
First-stage gas injection	MMSCFD	2924.8	3229.0	3203.8	3233.2	3219.5	3219.1
Expected compressor deployment	kPa	7127.9	10225.3	10912.7	12464.8	12451.1	12459.8
Expected valve deployment	kPa	0.0	714.8	1251.27	2281.0	2604.2	2647.6

Out-of-sample feasibility guaranteed at a small increase in expected cost by 5.4%

Linepack exibility saves up to 10.3% of expected cost

With only 3.2% increase in expected cost, we achieve > 80% reduction in total pressure variability

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Variability-aware gas network topology optimization

- | Increasing penalty %, we reduce pressure variability but incur additional cost
- | It makes sense to switch topology (valves) to unlock better cost-variability trade-offs
- | Here, we have 4 possible topologies. For a given%, only one of them is optimal

Conclusions & Outlook

- | Chance-constrained LDR-based policies manage uncertainty in gas networks:
 - | Internalize forecast and risk criteria of network operators
 - | Produce uncertainty- and variance-aware network control with guarantees
 - | Unlock linepack cost-saving potential of up to 10.3% of the expected cost
 - | Identify variability- and variance-optimal gas network topology

- | Future work:
 - | Remuneration of linepack exibility
 - | Transient gas ow models

Dvorkin, V., Mallapragada, D., Botterud, A., Kazempour, J., & Pinson, P.
[Multi-Stage Linear Decision Rules for Stochastic Control of Natural Gas Networks with Linepack.](#)
arXiv preprint arXiv:2110.02824.

‡ https://github.com/wdvorkin/LDR_for_gas_network_control

Thank you for your attention!

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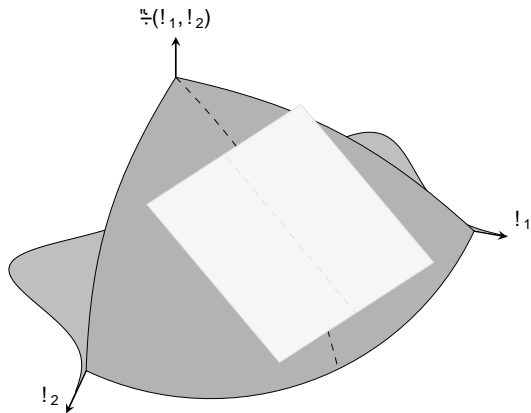
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On the linearization of the Weymouth equation



Stochastic Control and Pricing for Natural Gas Networks

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