Stochastic Control and Market Design for Natural Gas Networks

Vladimir Dvorkin, Anubhav Ratha, Jalal Kazempour, Pierre Pinson

Massachusetts Institute of Technology

Technical University of Denmark

dvorkin@mit.edu

MIT Group Meeting, August 2022

Why Stochastic?

- Uncertainty of gas extractions arises from:
  - Imperfect forecasts
  - Imbalances in power grid
  - Renewable gas supply
- Risks are induced by uncertainty:
  - Operational risks (operations at (or beyond) the limits)
  - Market risks (contract violations, variable returns, etc.)
- 2014 North American cold wave is an excellent example
  - Sudden peak in natural gas demand
  - Congestions in the natural gas network
  - Record-high spot prices from Jan 1 to Feb 18
  - Network expansion is prohibitive.
- From deterministic to stochastic control & market practices

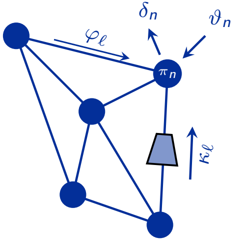
Towards stochastic control polices

- Stochastic control policy:
  - Mathematically strict rule (function)
  - Optimized ahead of real-time operations
  - To produce control inputs in real-time
- We develop control policies that internalize
  - Gas extraction uncertainty
  - Operational and market risk criteria
  - Intra-day variability of network variables
- Stochastic control policies enable
  - Distinguish network assets contributions into uncertainty control
  - Distinguish gas extraction contributions into overall uncertainty
  - Price uncertainty and variability (variance) of network operations

Outline

Outline

Natural gas network equations



- Network as a graph $\Gamma(\mathcal{N}, \mathcal{E})$
  - Set of nodes $\mathcal{N} = \{1, \ldots, N\}$
  - Set of edges $\mathcal{E} = \{1, \ldots, E\}$
- Gas network variables
  - $\vartheta \in \mathbb{R}^N$ – gas injection rates
  - $\delta \in \mathbb{R}^N$ – gas extraction rates
  - $\varphi \in \mathbb{R}^E$ – gas flow rates
  - $\pi \in \mathbb{R}^N$ – squared gas pressures
  - $\kappa \in \mathbb{R}^E$ – squared pressure regulation
- Gas conservation law

$$A\varphi = \vartheta - B\kappa - \delta \ \in \mathbb{R}^N$$

- Weymouth equation

$$\varphi \circ |\varphi| = \text{diag}[w](A^\top \pi + \kappa) \ \in \mathbb{R}^E$$
$$\varphi_\ell \geqslant 0, \ \forall \ell \in \mathcal{E}_a.$$

Optimization of natural gas networks

$$\begin{aligned}
\min_{\vartheta,\kappa,\varphi,\pi} \quad & c_1^\top \vartheta + \vartheta^\top \mathrm{diag}[c_2]\vartheta && \text{Gas injection costs} \\
\text{s.t.} \quad & A\varphi = \vartheta - B\kappa - \delta, && \text{Gas conservation law} \\
& \varphi \circ |\varphi| = \mathrm{diag}[w](A^\top \pi + \kappa), && \text{Weymouth equation} \\
& \underline{\pi} \leqslant \pi \leqslant \overline{\pi},\ \underline{\vartheta} \leqslant \vartheta \leqslant \overline{\vartheta}, && \text{Network limits} \\
& \underline{\kappa} \leqslant \kappa \leqslant \overline{\kappa},\ \varphi_\ell \geqslant 0,\ \forall \ell \in \mathcal{E}_a. &&
\end{aligned}$$

Optimization of natural gas networks

$$\begin{aligned}
\min_{\vartheta,\kappa,\varphi,\pi} \quad & c_1^\top \vartheta + \vartheta^\top \text{diag}[c_2]\vartheta && \text{Gas injection costs} \\
\text{s.t.} \quad & A\varphi = \vartheta - B\kappa - \delta, && \text{Gas conservation law} \\
& \varphi \circ |\varphi| = \text{diag}[w](A^\top \pi + \kappa), && \text{Weymouth equation} \\
& \underline{\pi} \leqslant \pi \leqslant \overline{\pi},\ \underline{\vartheta} \leqslant \vartheta \leqslant \overline{\vartheta}, && \text{Network limits} \\
& \underline{\kappa} \leqslant \kappa \leqslant \overline{\kappa},\ \varphi_\ell \geqslant 0,\ \forall \ell \in \mathcal{E}_a.
\end{aligned}$$

- Non-convex problem
- Solvable when parameters are know
- When parameters are uncertain:
  - Network response is unknown
  - Difficult feasibility guarantees
  - Simplifications or relaxations

Optimization of natural gas networks

$$\begin{aligned}
\min_{\vartheta,\kappa,\varphi,\pi} \quad & c_1^\top \vartheta + \vartheta^\top \mathrm{diag}[c_2]\vartheta && \text{Gas injection costs} \\
\text{s.t.} \quad & A\varphi = \vartheta - B\kappa - \delta, && \text{Gas conservation law} \\
& \varphi \circ |\varphi| = \mathrm{diag}[w](A^\top \pi + \kappa), && \text{Weymouth equation} \\
& \underline{\pi} \leqslant \pi \leqslant \overline{\pi},\ \underline{\vartheta} \leqslant \vartheta \leqslant \overline{\vartheta}, && \text{Network limits} \\
& \underline{\kappa} \leqslant \kappa \leqslant \overline{\kappa},\ \varphi_\ell \geqslant 0,\ \forall \ell \in \mathcal{E}_a. &&
\end{aligned}$$

- Non-convex problem
- Solvable when parameters are know
- When parameters are uncertain:
  - Network response is unknown
  - Difficult feasibility guarantees
  - Simplifications or relaxations

- Linearization of the Weymouth equation

$$\begin{aligned}
\mathcal{W}(\varphi,\pi,\kappa) \approx & \mathcal{J}(\mathring{\pi})(\pi - \mathring{\pi}) + \mathcal{J}(\mathring{\varphi})(\varphi - \mathring{\varphi}) \\
& + \mathcal{J}(\mathring{\kappa})(\kappa - \mathring{\kappa}) = \mathbb{0}
\end{aligned}$$

  around stationary point $(\mathring{\varphi}, \mathring{\pi}, \mathring{\kappa})$
- Linearized Weymouth equation:

$$\begin{aligned}
\varphi &= \varsigma_1(\mathring{\varphi}, \mathring{\pi}, \mathring{\kappa}) + \varsigma_2(\mathring{\varphi}, \mathring{\pi})\pi + \varsigma_3(\mathring{\varphi}, \mathring{\kappa})\kappa \\
\pi_\mathrm{r} &= \mathring{\pi}_\mathrm{r}
\end{aligned}$$

  where $\varsigma_1, \varsigma_2, \varsigma_3$ denote linear sensitivities

Optimization of natural gas networks

$$\begin{aligned}
\min_{\vartheta,\kappa,\varphi,\pi} \quad & c_1^\top \vartheta + \vartheta^\top \text{diag}[c_2]\vartheta && \text{Gas injection costs} \\
\text{s.t.} \quad & A\varphi = \vartheta - B\kappa - \delta, && \text{Gas conservation law} \\
& \boldsymbol{\varphi = \varsigma_1 + \varsigma_2\pi + \varsigma_3\kappa,\ \pi_r = \mathring{\pi}_r,} && \text{Lin. Weymouth equation} \\
& \underline{\pi} \leqslant \pi \leqslant \overline{\pi},\ \underline{\vartheta} \leqslant \vartheta \leqslant \overline{\vartheta}, && \text{Network limits} \\
& \underline{\kappa} \leqslant \kappa \leqslant \overline{\kappa},\ \varphi_\ell \geqslant 0,\ \forall \ell \in \mathcal{E}_a.
\end{aligned}$$

- Non-convex problem
- Solvable when parameters are know
- When parameters are uncertain:
  - Network response is unknown
  - Difficult feasibility guarantees
  - Simplifications or relaxations

- Linearization of the Weymouth equation

$$\mathcal{W}(\varphi,\pi,\kappa) \approx \mathcal{J}(\mathring{\pi})(\pi - \mathring{\pi}) + \mathcal{J}(\mathring{\varphi})(\varphi - \mathring{\varphi}) + \mathcal{J}(\mathring{\kappa})(\kappa - \mathring{\kappa}) = \mathbb{0}$$

around stationary point $(\mathring{\varphi}, \mathring{\pi}, \mathring{\kappa})$

- Linearized Weymouth equation:

$$\varphi = \varsigma_1(\mathring{\varphi}, \mathring{\pi}, \mathring{\kappa}) + \varsigma_2(\mathring{\varphi}, \mathring{\pi})\pi + \varsigma_3(\mathring{\varphi}, \mathring{\kappa})\kappa$$
$$\pi_r = \mathring{\pi}_r$$

where $\varsigma_1, \varsigma_2, \varsigma_3$ denote linear sensitivities

Outline

Chance-constrained gas network optimization

- Stochastic gas extraction rates:

$$\tilde{\delta}(\xi) = \delta + \xi$$

$\xi-$ 0-mean with know covariance Σ

- Chance-constrained network optimization:

$$\min_{\tilde{\vartheta},\tilde{\kappa},\tilde{\varphi},\tilde{\pi}} \quad \mathbb{E}^{\mathbb{P}_\xi}[c_1^\top \tilde{\vartheta}(\xi) + \tilde{\vartheta}(\xi)^\top \mathrm{diag}[c_2]\tilde{\vartheta}(\xi)]$$

subject to

$$\mathbb{P}_\xi \begin{bmatrix} A\tilde{\varphi}(\xi) = \tilde{\vartheta}(\xi) - B\tilde{\kappa}(\xi) - \tilde{\delta}(\xi), \\ \tilde{\varphi}(\xi) = \varsigma_1 + \varsigma_2\tilde{\pi}(\xi) + \varsigma_3\tilde{\kappa}(\xi), \\ \tilde{\pi}_{\mathsf{r}}(\xi) = \mathring{\pi}_{\mathsf{r}} \end{bmatrix} \overset{\text{a.s.}}{=} 1$$

$$\mathbb{P}_\xi \begin{bmatrix} \underline{\pi} \leqslant \tilde{\pi}(\xi) \leqslant \overline{\pi}, \ \underline{\vartheta} \leqslant \tilde{\vartheta}(\xi) \leqslant \overline{\vartheta}, \\ \underline{\kappa} \leqslant \tilde{\kappa}(\xi) \leqslant \overline{\kappa}, \ \tilde{\varphi}_\ell(\xi) \geqslant 0, \ \forall \ell \in \mathcal{E}_a \end{bmatrix} \geqslant 1 - \varepsilon$$

- Infinite-dimensional problem

- Affine control policies resolve tractability

- Controls for injections and pressure regulations:

$$\tilde{\vartheta}(\xi) = \vartheta + \alpha\xi$$
$$\tilde{\kappa}(\xi) = \kappa + \beta\xi$$

$(\vartheta, \kappa)-$ nominal (average) control input
$(\alpha, \beta)-$ variable recourse (adjustment)

Core result: State variable response ...

... is affine in control inputs

$$\tilde{\pi}(\xi) = \pi + \check{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \mathrm{diag}[\mathbb{1}])\xi$$
$$\tilde{\varphi}(\xi) = \varphi + (\grave{\varsigma}_2(\alpha - \mathrm{diag}[\mathbb{1}]) - \grave{\varsigma}_3\beta)\xi$$

with nominal & recourse components

! We can thus control & predict the uncertain network states, i.e., $\tilde{\vartheta}(\xi), \tilde{\kappa}(\xi), \tilde{\pi}(\xi), \tilde{\varphi}(\xi)$

Chance-constrained gas network optimization

- Stochastic gas extraction rates:

$$\tilde{\delta}(\xi) = \delta + \xi$$

$\xi-$ 0-mean with know covariance Σ

- Chance-constrained network optimization:

$$\min_{\tilde{\vartheta},\tilde{\kappa},\tilde{\varphi},\tilde{\pi}} \quad \mathbb{E}^{\mathbb{P}_\xi}[c_1^\top \tilde{\vartheta}(\xi) + \tilde{\vartheta}(\xi)^\top \text{diag}[c_2]\tilde{\vartheta}(\xi)]$$

subject to

$$\mathbb{P}_\xi \begin{bmatrix} A\tilde{\varphi}(\xi) = \tilde{\vartheta}(\xi) - B\tilde{\kappa}(\xi) - \tilde{\delta}(\xi), \\ \tilde{\varphi}(\xi) = \varsigma_1 + \varsigma_2\tilde{\pi}(\xi) + \varsigma_3\tilde{\kappa}(\xi), \\ \tilde{\pi}_\text{r}(\xi) = \mathring{\pi}_\text{r} \end{bmatrix} \overset{\text{a.s.}}{=} 1$$

$$\mathbb{P}_\xi \begin{bmatrix} \underline{\pi} \leqslant \tilde{\pi}(\xi) \leqslant \overline{\pi}, \ \underline{\vartheta} \leqslant \tilde{\vartheta}(\xi) \leqslant \overline{\vartheta}, \\ \underline{\kappa} \leqslant \tilde{\kappa}(\xi) \leqslant \overline{\kappa}, \ \tilde{\varphi}_\ell(\xi) \geqslant 0, \ \forall \ell \in \mathcal{E}_a \end{bmatrix} \geqslant 1 - \varepsilon$$

- Infinite-dimensional problem

- Affine control policies resolve tractability

- Controls for injections and pressure regulations:

$$\tilde{\vartheta}(\xi) = \vartheta + \alpha\xi$$
$$\tilde{\kappa}(\xi) = \kappa + \beta\xi$$

$(\vartheta, \kappa)-$ nominal (average) control input
$(\alpha, \beta)-$ variable recourse (adjustment)

Core result: State variable response ...

... is affine in control inputs

$$\tilde{\pi}(\xi) = \pi + \breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])\xi$$
$$\tilde{\varphi}(\xi) = \varphi + (\grave{\varsigma}_2(\alpha - \text{diag}[\mathbb{1}]) - \grave{\varsigma}_3\beta)\xi$$

with nominal & recourse components

! We can thus control & predict the uncertain network states, i.e., $\tilde{\vartheta}(\xi), \tilde{\kappa}(\xi), \tilde{\pi}(\xi), \tilde{\varphi}(\xi)$

Chance-constrained gas network optimization

- Stochastic gas extraction rates:

$$\tilde{\delta}(\xi) = \delta + \xi$$

$\xi-$ 0-mean with know covariance Σ

- Chance-constrained network optimization:

$$\min_{\tilde{\vartheta},\tilde{\kappa},\tilde{\varphi},\tilde{\pi}} \quad \mathbb{E}^{\mathbb{P}_\xi}[c_1^\top \tilde{\vartheta}(\xi) + \tilde{\vartheta}(\xi)^\top \text{diag}[c_2]\tilde{\vartheta}(\xi)]$$

subject to

$$\mathbb{P}_\xi \begin{bmatrix} A\tilde{\varphi}(\xi) = \tilde{\vartheta}(\xi) - B\tilde{\kappa}(\xi) - \tilde{\delta}(\xi), \\ \tilde{\varphi}(\xi) = \varsigma_1 + \varsigma_2\tilde{\pi}(\xi) + \varsigma_3\tilde{\kappa}(\xi), \\ \tilde{\pi}_\text{r}(\xi) = \mathring{\pi}_\text{r} \end{bmatrix} \overset{\text{a.s.}}{=} 1$$

$$\mathbb{P}_\xi \begin{bmatrix} \underline{\pi} \leqslant \tilde{\pi}(\xi) \leqslant \overline{\pi},\ \underline{\vartheta} \leqslant \tilde{\vartheta}(\xi) \leqslant \overline{\vartheta}, \\ \underline{\kappa} \leqslant \tilde{\kappa}(\xi) \leqslant \overline{\kappa},\ \tilde{\varphi}_\ell(\xi) \geqslant 0,\ \forall \ell \in \mathcal{E}_a \end{bmatrix} \geqslant 1 - \varepsilon$$

- Infinite-dimensional problem

- Affine control policies resolve tractability

- Controls for injections and pressure regulations:

$$\tilde{\vartheta}(\xi) = \vartheta + \alpha\xi$$
$$\tilde{\kappa}(\xi) = \kappa + \beta\xi$$

$(\vartheta, \kappa)-$ nominal (average) control input
$(\alpha, \beta)-$ variable recourse (adjustment)

Core result: State variable response ...

... is affine in control inputs

$$\tilde{\pi}(\xi) = \pi + \breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])\xi$$
$$\tilde{\varphi}(\xi) = \varphi + (\grave{\varsigma}_2(\alpha - \text{diag}[\mathbb{1}]) - \grave{\varsigma}_3\beta)\xi$$

with nominal & recourse components

! We can thus control & predict the uncertain network states, i.e., $\tilde{\vartheta}(\xi), \tilde{\kappa}(\xi), \tilde{\pi}(\xi), \tilde{\varphi}(\xi)$

Chance-constrained gas network optimization

- Stochastic gas extraction rates:

$$\tilde{\delta}(\xi) = \delta + \xi$$

$\xi-$ 0-mean with know covariance Σ

- Chance-constrained network optimization:

$$\min_{\tilde{\vartheta},\tilde{\kappa},\tilde{\varphi},\tilde{\pi}} \quad \mathbb{E}^{\mathbb{P}_\xi}[c_1^\top \tilde{\vartheta}(\xi) + \tilde{\vartheta}(\xi)^\top \text{diag}[c_2]\tilde{\vartheta}(\xi)]$$

subject to

$$\mathbb{P}_\xi \begin{bmatrix} A\tilde{\varphi}(\xi) = \tilde{\vartheta}(\xi) - B\tilde{\kappa}(\xi) - \tilde{\delta}(\xi), \\ \tilde{\varphi}(\xi) = \varsigma_1 + \varsigma_2\tilde{\pi}(\xi) + \varsigma_3\tilde{\kappa}(\xi), \\ \tilde{\pi}_\text{r}(\xi) = \mathring{\pi}_\text{r} \end{bmatrix} \overset{\text{a.s.}}{=} 1$$

$$\mathbb{P}_\xi \begin{bmatrix} \underline{\pi} \leqslant \tilde{\pi}(\xi) \leqslant \overline{\pi},\ \underline{\vartheta} \leqslant \tilde{\vartheta}(\xi) \leqslant \overline{\vartheta}, \\ \underline{\kappa} \leqslant \tilde{\kappa}(\xi) \leqslant \overline{\kappa},\ \tilde{\varphi}_\ell(\xi) \geqslant 0,\ \forall \ell \in \mathcal{E}_a \end{bmatrix} \geqslant 1 - \varepsilon$$

- Infinite-dimensional problem

- Affine control policies resolve tractability

- Controls for injections and pressure regulations:

$$\tilde{\vartheta}(\xi) = \vartheta + \alpha\xi$$
$$\tilde{\kappa}(\xi) = \kappa + \beta\xi$$

$(\vartheta, \kappa)-$ nominal (average) control input
$(\alpha, \beta)-$ variable recourse (adjustment)

Core result: State variable response ...

... is affine in control inputs

$$\tilde{\pi}(\xi) = \pi + \breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])\xi$$
$$\tilde{\varphi}(\xi) = \varphi + (\grave{\varsigma}_2(\alpha - \text{diag}[\mathbb{1}]) - \grave{\varsigma}_3\beta)\xi$$

with nominal & recourse components

! We can thus control & predict the uncertain network states, i.e., $\tilde{\vartheta}(\xi), \tilde{\kappa}(\xi), \tilde{\pi}(\xi), \tilde{\varphi}(\xi)$

Uncertainty- and variance-aware control polices

- Joint feasibility guarantee through individual chance constraints:

- Example for pressure chance constraints:

$$\mathbb{P}_{\xi}\left[\tilde{\pi}_n(\xi) \leqslant \overline{\pi}_n\right] \geqslant 1 - \hat{\varepsilon}, \quad \forall n = 1, \dots, N$$

is equivalent to

$$z_{\hat{\varepsilon}} \underbrace{\|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])]_n^\top\|}_{\text{pressure standard deviation}} \leqslant \overline{\pi}_n - \pi_n, \quad \forall n = 1, \dots, N$$

$z_{\hat{\varepsilon}_i}$ − safety parameter. $z_{\hat{\varepsilon}_i} = 0 \rightarrow$ deterministic solution
F − decomposition of Σ, i.e., $\Sigma = FF^\top$

- Minimization of pressure variance

$$\min_{s_n^\pi} \quad \psi_n^\pi s_n^\pi \quad \text{s.t.} \quad \|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])]_n^\top\| \leqslant s_n^\pi, \quad \forall n = 1, \dots, N$$

s_n^π − pressure standard deviation, ψ_n^π − variance penalty.

- We can thus optimize both **uncertainty**- and **variance-aware** control policies

Uncertainty- and variance-aware control polices

- Joint feasibility guarantee through individual chance constraints:

- Example for pressure chance constraints:

$$\mathbb{P}_{\xi}\left[\tilde{\pi}_n(\xi) \leqslant \overline{\pi}_n\right] \geqslant 1-\hat{\varepsilon}, \quad \forall n=1,\dots,N$$

is equivalent to

$$z_{\hat{\varepsilon}} \underbrace{\|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])]_n^\top\|}_{\text{pressure standard deviation}} \leqslant \overline{\pi}_n - \pi_n, \quad \forall n=1,\dots,N$$

$z_{\hat{\varepsilon}_i}-$ safety parameter. $z_{\hat{\varepsilon}_i}=0 \rightarrow$ deterministic solution
$F-$ decomposition of Σ, i.e., $\Sigma = FF^\top$

- Minimization of pressure variance

$$\min_{s_n^\pi} \quad \psi_n^\pi s_n^\pi \quad \text{s.t.} \quad \|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])]_n^\top\| \leqslant s_n^\pi, \quad \forall n=1,\dots,N$$

$s_n^\pi-$ pressure standard deviation, $\psi_n^\pi-$ variance penalty.

- We can thus optimize both **uncertainty-** and **variance-aware** control policies

Uncertainty- and variance-aware optimization

- Control polices and network response are optimized using the SOCP problem

$$\min_{\mathcal{P}} \; c_1^\top \vartheta + \mathbb{1}^\top c^\vartheta + \mathbb{1}^\top c^\alpha + \psi_\pi^\top s^\pi + \psi_\varphi^\top s^\varphi \qquad \textit{Exp. cost + variance penalty}$$

$$\text{s.t. } A\varphi = \vartheta - B\kappa - \delta, \qquad \textit{Gas conservation law}$$

$$(\alpha - B\beta)^\top \mathbb{1} = \mathbb{1}, \qquad \textit{Recourse balance}$$

$$\varphi = \varsigma_1 + \varsigma_2\pi + \varsigma_3\kappa,\; \pi_{\mathrm{r}} = \mathring{\pi}_{\mathrm{r}}, \qquad \textit{Lin. Weymouth equation}$$

$$\|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])]_n^\top\| \leqslant s_n^\pi \qquad \textit{Pressure Std}$$

$$\|F[\grave{\varsigma}_2(\alpha - \text{diag}[\mathbb{1}]) - \grave{\varsigma}_3\beta]_\ell^\top\| \leqslant s_\ell^\varphi \qquad \textit{Flow Std}$$

$$z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])]_n^\top\| \leqslant \overline{\pi}_n - \pi_n \qquad \textit{Max pressure limit}$$

$$z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])]_n^\top\| \leqslant \pi_n - \underline{\pi}_n \qquad \textit{Min pressure limit}$$

$$z_{\hat{\varepsilon}}\|F[\grave{\varsigma}_2(\alpha - \text{diag}[\mathbb{1}]) - \grave{\varsigma}_3\beta]_\ell^\top\| \leqslant \varphi_\ell \qquad \textit{Flow limit for compressors and valves}$$

+ *constraints on injection and pressure regulation ...*

- Which optimizes the network response ...

$$\tilde{\vartheta}(\xi) = \vartheta + \alpha\xi, \quad \tilde{\kappa}(\xi) = \kappa + \beta\xi \qquad \textit{control actions}$$

$$\tilde{\pi}(\xi) = \pi + \breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \text{diag}[\mathbb{1}])\xi \qquad \textit{pressure response}$$

$$\tilde{\varphi}(\xi) = \varphi + (\grave{\varsigma}_2(\alpha - \text{diag}[\mathbb{1}]) - \grave{\varsigma}_3\beta)\xi \qquad \textit{flow response}$$

- ... to meet the risk ($z_{\hat{\varepsilon}}$) and variance (ψ^π, ψ^φ) criteria of the network operator

Uncertainty- and variance-aware optimization

- Control polices and network response are optimized using the SOCP problem

$$\min_{\mathcal{P}} \; c_1^\top \vartheta + \mathbb{1}^\top c^{\vartheta} + \mathbb{1}^\top c^{\alpha} + \psi_\pi^\top s^\pi + \psi_\varphi^\top s^\varphi \qquad \textit{Exp. cost + variance penalty}$$

$$\begin{aligned}
\text{s.t. } & A\varphi = \vartheta - B\kappa - \delta, && \textit{Gas conservation law}\\
& (\alpha - B\beta)^\top \mathbb{1} = \mathbb{1}, && \textit{Recourse balance}\\
& \varphi = \varsigma_1 + \varsigma_2\pi + \varsigma_3\kappa,\; \pi_{\mathrm{r}} = \mathring{\pi}_{\mathrm{r}}, && \textit{Lin. Weymouth equation}\\
& \|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \mathrm{diag}[\mathbb{1}])]_n^\top\| \leqslant s_n^\pi && \textit{Pressure Std}\\
& \|F[\grave{\varsigma}_2(\alpha - \mathrm{diag}[\mathbb{1}]) - \grave{\varsigma}_3\beta]_\ell^\top\| \leqslant s_\ell^\varphi && \textit{Flow Std}\\
& z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \mathrm{diag}[\mathbb{1}])]_n^\top\| \leqslant \overline{\pi}_n - \pi_n && \textit{Max pressure limit}\\
& z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \mathrm{diag}[\mathbb{1}])]_n^\top\| \leqslant \pi_n - \underline{\pi}_n && \textit{Min pressure limit}\\
& z_{\hat{\varepsilon}}\|F[\grave{\varsigma}_2(\alpha - \mathrm{diag}[\mathbb{1}]) - \grave{\varsigma}_3\beta]_\ell^\top\| \leqslant \varphi_\ell && \textit{Flow limit for compressors and valves}\\
& + \textit{ constraints on injection and pressure regulation ...}
\end{aligned}$$

- Which optimizes the network response ...

$$\begin{aligned}
& \tilde{\vartheta}(\xi) = \vartheta + \alpha\xi, \quad \tilde{\kappa}(\xi) = \kappa + \beta\xi && \textit{control actions}\\
& \tilde{\pi}(\xi) = \pi + \breve{\varsigma}_2(\alpha - \hat{\varsigma}_3\beta - \mathrm{diag}[\mathbb{1}])\xi && \textit{pressure response}\\
& \tilde{\varphi}(\xi) = \varphi + (\grave{\varsigma}_2(\alpha - \mathrm{diag}[\mathbb{1}]) - \grave{\varsigma}_3\beta)\xi && \textit{flow response}
\end{aligned}$$

- ... to meet the risk ($z_{\hat{\varepsilon}}$) and variance (ψ^π, ψ^φ) criteria of the network operator

Outline

Deterministic versus Stochastic Market Design

- Deterministic markets are ignorant to the contributions of network assets to uncertainty management
- ... and to the contributions of stochastic extractions to the network uncertainty and variability
- The proposed polices extend the deterministic payments to price uncertainty and variability

Network equilibrium conditions:

- Stochastic gas flow equations

$$\lambda^{c}: A\varphi = \boldsymbol{\vartheta} - B\boldsymbol{\kappa} - \delta,$$
$$\lambda^{r}: (\boldsymbol{\alpha} - B\boldsymbol{\beta})^{\top}\mathbb{1} = \mathbb{1},$$
$$\lambda^{w}: \varphi = \varsigma_1 + \varsigma_2\pi + \varsigma_3\boldsymbol{\kappa},\ \pi_{\mathrm{r}} = \mathring{\pi}_{\mathrm{r}},$$

- Chance-constrained limits

$$\lambda_n^{\overline{\pi}}: z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}])]_n^{\top}\| \leqslant \overline{\pi}_n - \pi_n$$
$$\lambda_n^{\underline{\pi}}: z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}])]_n^{\top}\| \leqslant \pi_n - \underline{\pi}_n$$
$$\lambda_\ell^{\varphi}: z_{\hat{\varepsilon}}\|F[\grave{\varsigma}_2(\boldsymbol{\alpha} - \mathrm{diag}[\mathbb{1}]) - \grave{\varsigma}_3\boldsymbol{\beta}]_\ell^{\top}\| \leqslant \varphi_\ell$$

- Variance constraints

$$\lambda_n^{\pi}: \|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}])]_n^{\top}\| \leqslant s_n^{\pi}$$
$$\lambda_\ell^{\varphi}: \|F[\grave{\varsigma}_2(\boldsymbol{\alpha} - \mathrm{diag}[\mathbb{1}]) - \grave{\varsigma}_3\boldsymbol{\beta}]_\ell^{\top}\| \leqslant s_\ell^{\varphi}$$

- Pricing based on the combination of linear and conic duality
- Unlike linear, conic constraints are not separable

- SOCP duality distinguishes the effort of network assets w.r.t. each component of forecast errors in $F \in \mathbb{R}^{N\times N}$, e.g.,

$$\lambda_n^{\overline{\pi}} \leqslant \|u_n^{\overline{\pi}}\|,\ u_n^{\overline{\pi}} \in \mathbb{R}^N,$$

where $\lambda_n^{\overline{\pi}}, u_{n1}^{\overline{\pi}}, \dots, u_{nN}^{\overline{\pi}}$ are prices for max. press. control

Revenue		Deterministic	Chance-constrained		
			Uncertainty		Variance
		Nominal balance	Regulation balance	Network limits	State variance
Injection	$R_n^{inj} =$	$f(\lambda_n^c, \vartheta_n)$	$+f(\lambda^r, \alpha_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\varphi}, \alpha_n)$	$+f(u^{\pi}, u^{\varphi}, \alpha_n)$
Comp/valve	$R_\ell^{act} =$	$f(\lambda_{s(\ell)}^c, \lambda_\ell^w, \kappa_\ell)$	$+f(\lambda^r, \beta_\ell)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\varphi}, \beta_\ell)$	$+f(u^{\pi}, u^{\varphi}, \beta_\ell)$
Consumer	$R_n^{con} =$	$f(\lambda_n^c, \delta_n)$	$+f(\lambda_n^r, \mathbb{1}_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\varphi}, [F]_n)$	$+f(u^{\pi}, u^{\varphi}, [F]_n)$

Deterministic versus Stochastic Market Design

- Deterministic markets are ignorant to the contributions of network assets to uncertainty management
- ... and to the contributions of stochastic extractions to the network uncertainty and variability
- The proposed polices extend the deterministic payments to price uncertainty and variability

Network equilibrium conditions:

- Stochastic gas flow equations

$$\lambda^{c}: A\varphi = \boldsymbol{\vartheta} - B\boldsymbol{\kappa} - \delta,$$
$$\lambda^{r}: (\boldsymbol{\alpha} - B\boldsymbol{\beta})^{\top}\mathbb{1} = \mathbb{1},$$
$$\lambda^{w}: \varphi = \varsigma_1 + \varsigma_2\pi + \varsigma_3\boldsymbol{\kappa},\ \pi_{\mathrm{r}} = \mathring{\pi}_{\mathrm{r}},$$

- Chance-constrained limits

$$\lambda_n^{\overline{\pi}}: z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}])]_n^{\top}\| \leqslant \overline{\pi}_n - \pi_n$$
$$\lambda_n^{\underline{\pi}}: z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}])]_n^{\top}\| \leqslant \pi_n - \underline{\pi}_n$$
$$\lambda_\ell^{\varphi}: z_{\hat{\varepsilon}}\|F[\grave{\varsigma}_2(\boldsymbol{\alpha} - \mathrm{diag}[\mathbb{1}]) - \grave{\varsigma}_3\boldsymbol{\beta}]_\ell^{\top}\| \leqslant \varphi_\ell$$

- Variance constraints

$$\lambda_n^{\pi}: \|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}])]_n^{\top}\| \leqslant s_n^{\pi}$$
$$\lambda_\ell^{\varphi}: \|F[\grave{\varsigma}_2(\boldsymbol{\alpha} - \mathrm{diag}[\mathbb{1}]) - \grave{\varsigma}_3\boldsymbol{\beta}]_\ell^{\top}\| \leqslant s_\ell^{\varphi}$$

- Pricing based on the combination of linear and conic duality
- Unlike linear, conic constraints are not separable

- SOCP duality distinguishes the effort of network assets w.r.t. each component of forecast errors in $F \in \mathbb{R}^{N\times N}$, e.g.,

$$\lambda_n^{\overline{\pi}} \leqslant \|u_n^{\overline{\pi}}\|,\ u_n^{\overline{\pi}} \in \mathbb{R}^N,$$

where $\lambda_n^{\overline{\pi}}, u_{n1}^{\overline{\pi}}, \dots, u_{nN}^{\overline{\pi}}$ are prices for max. press. control

Revenue		Deterministic	Chance-constrained		
			Uncertainty		Variance
		Nominal balance	Regulation balance	Network limits	State variance
Injection	$R_n^{inj} =$	$f(\lambda_n^c, \vartheta_n)$	$+f(\lambda^r, \alpha_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\varphi}, \alpha_n)$	$+f(u^{\pi}, u^{\varphi}, \alpha_n)$
Comp/valve	$R_\ell^{act} =$	$f(\lambda_{s(\ell)}^c, \lambda_\ell^w, \kappa_\ell)$	$+f(\lambda^r, \beta_\ell)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\varphi}, \beta_\ell)$	$+f(u^{\pi}, u^{\varphi}, \beta_\ell)$
Consumer	$R_n^{con} =$	$f(\lambda_n^c, \delta_n)$	$+f(\lambda_n^r, \mathbb{1}_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\varphi}, [F]_n)$	$+f(u^{\pi}, u^{\varphi}, [F]_n)$

Deterministic versus Stochastic Market Design

- Deterministic markets are ignorant to the contributions of network assets to uncertainty management
- ... and to the contributions of stochastic extractions to the network uncertainty and variability
- The proposed polices extend the deterministic payments to price uncertainty and variability

Network equilibrium conditions:

- Stochastic gas flow equations

$$\lambda^{c}\colon A\varphi = \boldsymbol{\vartheta} - B\boldsymbol{\kappa} - \delta,$$
$$\lambda^{r}\colon (\boldsymbol{\alpha} - B\boldsymbol{\beta})^{\top}\mathbb{1} = \mathbb{1},$$
$$\lambda^{w}\colon \varphi = \varsigma_1 + \varsigma_2\pi + \varsigma_3\boldsymbol{\kappa},\ \pi_{\mathrm{r}} = \mathring{\pi}_{\mathrm{r}},$$

- Chance-constrained limits

$$\lambda_n^{\overline{\pi}}\colon z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}])]_n^{\top}\| \leqslant \overline{\pi}_n - \pi_n$$
$$\lambda_n^{\underline{\pi}}\colon z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}])]_n^{\top}\| \leqslant \pi_n - \underline{\pi}_n$$
$$\lambda_\ell^{\overline{\varphi}}\colon z_{\hat{\varepsilon}}\|F[\grave{\varsigma}_2(\boldsymbol{\alpha} - \mathrm{diag}[\mathbb{1}]) - \grave{\varsigma}_3\boldsymbol{\beta}]_\ell^{\top}\| \leqslant \varphi_\ell$$

- Variance constraints

$$\lambda_n^{\pi}\colon \|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}])]_n^{\top}\| \leqslant s_n^{\pi}$$
$$\lambda_\ell^{\varphi}\colon \|F[\grave{\varsigma}_2(\boldsymbol{\alpha} - \mathrm{diag}[\mathbb{1}]) - \grave{\varsigma}_3\boldsymbol{\beta}]_\ell^{\top}\| \leqslant s_\ell^{\varphi}$$

- Pricing based on the combination of linear and conic duality
- Unlike linear, conic constraints are not separable

- SOCP duality distinguishes the effort of network assets w.r.t. each component of forecast errors in $F \in \mathbb{R}^{N\times N}$, e.g.,

$$\lambda_n^{\overline{\pi}} \leqslant \|u_n^{\overline{\pi}}\|,\ u_n^{\overline{\pi}} \in \mathbb{R}^N,$$

where $\lambda_n^{\overline{\pi}}, u_{n1}^{\overline{\pi}}, \ldots, u_{nN}^{\overline{\pi}}$ are prices for max. press. control

Revenue		Deterministic	Chance-constrained		
			Uncertainty		Variance
		Nominal balance	Regulation balance	Network limits	State variance
Injection	$R_n^{inj} =$	$f(\lambda_n^c, \vartheta_n)$	$+f(\lambda^r, \alpha_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\overline{\varphi}}, \alpha_n)$	$+f(u^{\pi}, u^{\varphi}, \alpha_n)$
Comp/valve	$R_\ell^{act} =$	$f(\lambda_{s(\ell)}^c, \lambda_\ell^w, \kappa_\ell)$	$+f(\lambda^r, \beta_\ell)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\overline{\varphi}}, \beta_\ell)$	$+f(u^{\pi}, u^{\varphi}, \beta_\ell)$
Consumer	$R_n^{con} =$	$f(\lambda_n^c, \delta_n)$	$+f(\lambda_n^r, \mathbb{1}_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\overline{\varphi}}, [F]_n)$	$+f(u^{\pi}, u^{\varphi}, [F]_n)$

Deterministic versus Stochastic Market Design

- Deterministic markets are ignorant to the contributions of network assets to uncertainty management
- ... and to the contributions of stochastic extractions to the network uncertainty and variability
- The proposed polices extend the deterministic payments to price uncertainty and variability

Network equilibrium conditions:

- Stochastic gas flow equations

$$\lambda^{c}: A\varphi = \boldsymbol{\vartheta} - B\boldsymbol{\kappa} - \delta,$$
$$\lambda^{r}: (\boldsymbol{\alpha} - B\boldsymbol{\beta})^{\top}\mathbb{1} = \mathbb{1},$$
$$\lambda^{w}: \varphi = \varsigma_1 + \varsigma_2\pi + \varsigma_3\boldsymbol{\kappa},\ \pi_{\mathrm{r}} = \mathring{\pi}_{\mathrm{r}},$$

- Chance-constrained limits

$$\lambda_n^{\overline{\pi}}: z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}])]_n^{\top}\| \leqslant \overline{\pi}_n - \pi_n$$
$$\lambda_n^{\underline{\pi}}: z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}])]_n^{\top}\| \leqslant \pi_n - \underline{\pi}_n$$
$$\lambda_\ell^{\varphi}: z_{\hat{\varepsilon}}\|F[\grave{\varsigma}_2(\boldsymbol{\alpha} - \mathrm{diag}[\mathbb{1}]) - \grave{\varsigma}_3\boldsymbol{\beta}]_\ell^{\top}\| \leqslant \varphi_\ell$$

- Variance constraints

$$\lambda_n^{\pi}: \|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}])]_n^{\top}\| \leqslant s_n^{\pi}$$
$$\lambda_\ell^{\varphi}: \|F[\grave{\varsigma}_2(\boldsymbol{\alpha} - \mathrm{diag}[\mathbb{1}]) - \grave{\varsigma}_3\boldsymbol{\beta}]_\ell^{\top}\| \leqslant s_\ell^{\varphi}$$

- Pricing based on the combination of linear and conic duality
- Unlike linear, conic constraints are not separable

- SOCP duality distinguishes the effort of network assets w.r.t. each component of forecast errors in $F \in \mathbb{R}^{N\times N}$, e.g.,

$$\lambda_n^{\overline{\pi}} \leqslant \|u_n^{\overline{\pi}}\|,\ u_n^{\overline{\pi}} \in \mathbb{R}^N,$$

where $\lambda_n^{\overline{\pi}}, u_{n1}^{\overline{\pi}}, \dots, u_{nN}^{\overline{\pi}}$ are prices for max. press. control

Revenue		Deterministic	Chance-constrained		
			Uncertainty		Variance
		Nominal balance	Regulation balance	Network limits	State variance
Injection	$R_n^{inj} =$	$f(\lambda_n^c, \vartheta_n)$	$+f(\lambda^r, \alpha_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\varphi}, \alpha_n)$	$+f(u^{\pi}, u^{\varphi}, \alpha_n)$
Comp/valve	$R_\ell^{act} =$	$f(\lambda_{s(\ell)}^c, \lambda_\ell^w, \kappa_\ell)$	$+f(\lambda^r, \beta_\ell)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\varphi}, \beta_\ell)$	$+f(u^{\pi}, u^{\varphi}, \beta_\ell)$
Consumer	$R_n^{con} =$	$f(\lambda_n^c, \delta_n)$	$+f(\lambda_n^r, \mathbb{1}_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\varphi}, [F]_n)$	$+f(u^{\pi}, u^{\varphi}, [F]_n)$

Deterministic versus Stochastic Market Design

- Deterministic markets are ignorant to the contributions of network assets to uncertainty management
- ... and to the contributions of stochastic extractions to the network uncertainty and variability
- The proposed polices extend the deterministic payments to price uncertainty and variability

Network equilibrium conditions:

- Stochastic gas flow equations

$$\lambda^{c}\colon A\varphi = \boldsymbol{\vartheta} - B\boldsymbol{\kappa} - \delta,$$
$$\lambda^{r}\colon (\boldsymbol{\alpha} - B\boldsymbol{\beta})^{\top}\mathbb{1} = \mathbb{1},$$
$$\lambda^{w}\colon \varphi = \varsigma_1 + \varsigma_2\pi + \varsigma_3\boldsymbol{\kappa},\ \pi_{\text{r}} = \mathring{\pi}_{\text{r}},$$

- Chance-constrained limits

$$\lambda_n^{\overline{\pi}}\colon z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \text{diag}[\mathbb{1}])]_n^{\top}\| \leqslant \overline{\pi}_n - \pi_n$$
$$\lambda_n^{\underline{\pi}}\colon z_{\hat{\varepsilon}}\|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \text{diag}[\mathbb{1}])]_n^{\top}\| \leqslant \pi_n - \underline{\pi}_n$$
$$\lambda_\ell^{\overline{\varphi}}\colon z_{\hat{\varepsilon}}\|F[\grave{\varsigma}_2(\boldsymbol{\alpha} - \text{diag}[\mathbb{1}]) - \grave{\varsigma}_3\boldsymbol{\beta}]_\ell^{\top}\| \leqslant \varphi_\ell$$

- Variance constraints

$$\lambda_n^{\pi}\colon \|F[\breve{\varsigma}_2(\boldsymbol{\alpha} - \hat{\varsigma}_3\boldsymbol{\beta} - \text{diag}[\mathbb{1}])]_n^{\top}\| \leqslant s_n^{\pi}$$
$$\lambda_\ell^{\varphi}\colon \|F[\grave{\varsigma}_2(\boldsymbol{\alpha} - \text{diag}[\mathbb{1}]) - \grave{\varsigma}_3\boldsymbol{\beta}]_\ell^{\top}\| \leqslant s_\ell^{\varphi}$$

- Pricing based on the combination of linear and conic duality
- Unlike linear, conic constraints are not separable

- SOCP duality distinguishes the effort of network assets w.r.t. each component of forecast errors in $F \in \mathbb{R}^{N\times N}$, e.g.,

$$\lambda_n^{\overline{\pi}} \leqslant \|u_n^{\overline{\pi}}\|,\ u_n^{\overline{\pi}} \in \mathbb{R}^N,$$

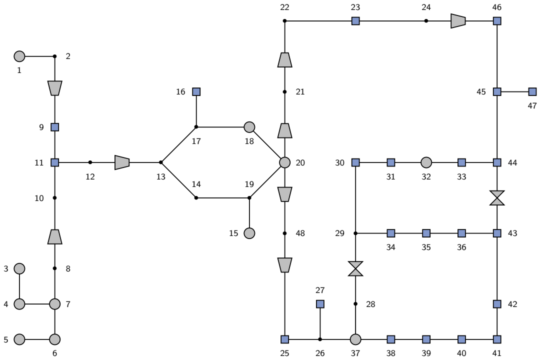
where $\lambda_n^{\overline{\pi}}, u_{n1}^{\overline{\pi}}, \dots, u_{nN}^{\overline{\pi}}$ are prices for max. press. control

Revenue		Deterministic	Chance-constrained		
			Uncertainty		Variance
		Nominal balance	Regulation balance	Network limits	State variance
Injection	$R_n^{inj} =$	$f(\lambda_n^c, \vartheta_n)$	$+f(\lambda^r, \alpha_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\overline{\varphi}}, \alpha_n)$	$+f(u^{\pi}, u^{\varphi}, \alpha_n)$
Comp/valve	$R_\ell^{act} =$	$f(\lambda_{s(\ell)}^c, \lambda_\ell^w, \kappa_\ell)$	$+f(\lambda^r, \beta_\ell)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\overline{\varphi}}, \beta_\ell)$	$+f(u^{\pi}, u^{\varphi}, \beta_\ell)$
Consumer	$R_n^{con} =$	$f(\lambda_n^c, \delta_n)$	$+f(\lambda_n^r, \mathbb{1}_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\overline{\varphi}}, [F]_n)$	$+f(u^{\pi}, u^{\varphi}, [F]_n)$

Outline

Experiments setup

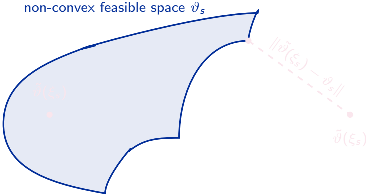
- 48-node natural gas network
- 22 stochastic gas extractions ◻
- 6 comp. and 2 valves
- 11 gas injections ○
- $\xi \sim N(0, \sigma)$, $\sigma \to 10\%$ of δ
- Violation probability $\varepsilon = 1\%$
- Ipopt + JuMP $\to$ sensitivities



Experiment goals

- We compare the optimized network response under
  - Deterministic polices (uncertainty- and variance-agnostic)
  - Chance-constrained polices (uncertainty-aware but variance-agnostic)
  - Chance-constrained variance-aware polices
- ... in terms of operational costs and state variance
- ... in terms of feasibility:
  - count violations of network limits
  - compute the real-time re-dispatch effort by computing the projection

$$
\begin{aligned}
\min_{\vartheta_s,\kappa_s,\varphi_s,\pi_s} \quad & \|\tilde{\vartheta}(\xi_s) - \vartheta_s\| + \|\tilde{\kappa}(\xi_s) - \kappa_s\| \\
\text{s.t.} \quad & A\varphi_s = \vartheta_s - B\kappa_s - \delta_s - \xi_s, \\
& \varphi_s \circ |\varphi_s| = \text{diag}[w](A^\top \pi_s + \kappa_s)
\end{aligned}
$$



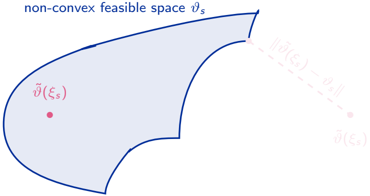
  of control inputs onto the non-convex gas equations for $s = 1 \ldots 1000$ samples of forecast errors.

- ... and in terms of revenues/charges of network components.

Experiment goals

- We compare the optimized network response under
  - Deterministic polices (uncertainty- and variance-agnostic)
  - Chance-constrained polices (uncertainty-aware but variance-agnostic)
  - Chance-constrained variance-aware polices
- . . . in terms of operational costs and state variance
- . . . in terms of feasibility:
  - count violations of network limits
  - compute the real-time re-dispatch effort by computing the projection

$$\begin{aligned} \min_{\vartheta_s,\kappa_s,\varphi_s,\pi_s} \quad & \|\tilde{\vartheta}(\xi_s)-\vartheta_s\| + \|\tilde{\kappa}(\xi_s)-\kappa_s\| \\ \text{s.t.} \quad & A\varphi_s = \vartheta_s - B\kappa_s - \delta_s - \xi_s, \\ & \varphi_s \circ |\varphi_s| = \text{diag}[w](A^\top \pi_s + \kappa_s) \end{aligned}$$



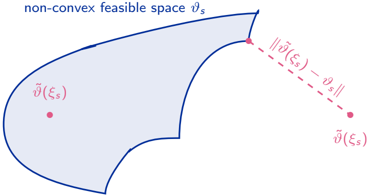
of control inputs onto the non-convex gas equations for $s = 1 \dots 1000$ samples of forecast errors.

- . . . and in terms of revenues/charges of network components.

Experiment goals

- We compare the optimized network response under
  - Deterministic polices (uncertainty- and variance-agnostic)
  - Chance-constrained polices (uncertainty-aware but variance-agnostic)
  - Chance-constrained variance-aware polices
- ... in terms of operational costs and state variance
- ... in terms of feasibility:
  - count violations of network limits
  - compute the real-time re-dispatch effort by computing the projection

$$\begin{aligned}\min_{\vartheta_s,\kappa_s,\varphi_s,\pi_s} \quad & \|\tilde{\vartheta}(\xi_s) - \vartheta_s\| + \|\tilde{\kappa}(\xi_s) - \kappa_s\| \\ \text{s.t.} \quad & A\varphi_s = \vartheta_s - B\kappa_s - \delta_s - \xi_s, \\ & \varphi_s \circ |\varphi_s| = \text{diag}[w](A^\top \pi_s + \kappa_s)\end{aligned}$$



of control inputs onto the non-convex gas equations for $s = 1 \dots 1000$ samples of forecast errors.

- ... and in terms of revenues/charges of network components.

Analysis of the Optimized Network Response

Parameter	Unit	Deterministic control policy	Chance-constrained control policies						
			Variance-agnostic	Pressure variance-aware, ψ^{π}			Flow variance-aware, ψ^{φ}		
				10^{-3}	10^{-2}	10^{-1}	1	10^{1}	10^{2}
Expected cost	\$1000	80.9	82.5 (100%)	**100.5%**	**105.6%**	**113.8%**	**100.1%**	**102.5%**	**112.6%**
Pressure variance	MPa^2	217.5	63.4 (100%)	**44.2%**	**18.9%**	**12.8%**	92.8%	46.7%	24.7%
Flow variance	$BMSCFD^2$	26.1	58.0 (100%)	83.4%	64.1%	59.2%	**93.4%**	**44.8%**	**25.9%**
Compression	kPa	1939	3914	3570	3734	3661	3914	4030	3888
Valve regulation	kPa	0	0	0	**150**	**576**	0	**1**	**500**
Infeas. ($\varepsilon = 1\%$)	%	53.7	0.04	0.02	0.02	0.02	0.03	0.02	0.03
Injection re-disp.	MMSCFD	960.91 **(31.3%)**	0.01	0.03	0.02	0.02	0.02	0.04	0.04
Comp/valve re-disp.	kPa	121.68 **(12.7%)**	0.19	0.08	0.10	0.05	0.28	0.04	0.04

Analysis of the Optimized Network Response

Parameter	Unit	Deterministic control policy	Chance-constrained control policies						
			Variance-agnostic	Pressure variance-aware, ψ^{π}			Flow variance-aware, ψ^{φ}		
				10^{-3}	10^{-2}	10^{-1}	1	10^{1}	10^{2}
Expected cost	\$1000	80.9	82.5 (100%)	**100.5%**	**105.6%**	**113.8%**	**100.1%**	**102.5%**	**112.6%**
Pressure variance	MPa^2	217.5	63.4 (100%)	**44.2%**	**18.9%**	**12.8%**	92.8%	46.7%	24.7%
Flow variance	$BMSCFD^2$	26.1	58.0 (100%)	83.4%	64.1%	59.2%	**93.4%**	**44.8%**	**25.9%**
Compression	kPa	1939	3914	3570	3734	3661	3914	4030	3888
Valve regulation	kPa	0	0	0	**150**	**576**	0	**1**	**500**
Infeas. ($\varepsilon = 1\%$)	%	53.7	0.04	0.02	0.02	0.02	0.03	0.02	0.03
Injection re-disp.	MMSCFD	960.91 **(31.3%)**	0.01	0.03	0.02	0.02	0.02	0.04	0.04
Comp/valve re-disp.	kPa	121.68 **(12.7%)**	0.19	0.08	0.10	0.05	0.28	0.04	0.04

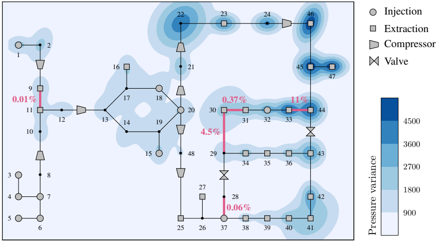
Analysis of the Optimized Network Response

Parameter	Unit	Deterministic control policy	Chance-constrained control policies						
			Variance-agnostic	Pressure variance-aware, ψ^{π}			Flow variance-aware, ψ^{φ}		
				10^{-3}	10^{-2}	10^{-1}	1	10^{1}	10^{2}
Expected cost	\$1000	80.9	82.5 (100%)	**100.5%**	**105.6%**	**113.8%**	**100.1%**	**102.5%**	**112.6%**
Pressure variance	MPa^2	217.5	63.4 (100%)	**44.2%**	**18.9%**	**12.8%**	92.8%	46.7%	24.7%
Flow variance	$BMSCFD^2$	26.1	58.0 (100%)	83.4%	64.1%	59.2%	**93.4%**	**44.8%**	**25.9%**
Compression	kPa	1939	3914	3570	3734	3661	3914	4030	3888
Valve regulation	kPa	0	0	0	**150**	**576**	0	**1**	**500**
Infeas. ($\varepsilon = 1\%$)	%	53.7	0.04	0.02	0.02	0.02	0.03	0.02	0.03
Injection re-disp.	MMSCFD	960.91 **(31.3%)**	0.01	0.03	0.02	0.02	0.02	0.04	0.04
Comp/valve re-disp.	kPa	121.68 **(12.7%)**	0.19	0.08	0.10	0.05	0.28	0.04	0.04

Analysis of the Optimized Network Response

Parameter	Unit	Deterministic control policy	Chance-constrained control policies						
			Variance-agnostic	Pressure variance-aware, ψ^{π}			Flow variance-aware, ψ^{φ}		
				10^{-3}	10^{-2}	10^{-1}	1	10^{1}	10^{2}
Expected cost	\$1000	80.9	82.5 (100%)	**100.5%**	**105.6%**	**113.8%**	**100.1%**	**102.5%**	**112.6%**
Pressure variance	MPa^2	217.5	63.4 (100%)	**44.2%**	**18.9%**	**12.8%**	92.8%	46.7%	24.7%
Flow variance	$BMSCFD^2$	26.1	58.0 (100%)	83.4%	64.1%	59.2%	**93.4%**	**44.8%**	**25.9%**
Compression	kPa	1939	3914	3570	3734	3661	3914	4030	3888
Valve regulation	kPa	0	0	0	**150**	**576**	0	**1**	**500**
Infeas. ($\varepsilon = 1\%$)	%	53.7	0.04	0.02	0.02	0.02	0.03	0.02	0.03
Injection re-disp.	MMSCFD	960.91 **(31.3%)**	0.01	0.03	0.02	0.02	0.02	0.04	0.04
Comp/valve re-disp.	kPa	121.68 **(12.7%)**	0.19	0.08	0.10	0.05	0.28	0.04	0.04

Variance-agnostic policy

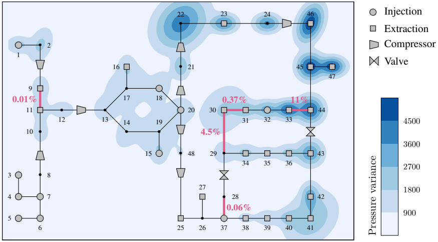


$\psi^{\pi} = 0, \ \psi^{\varphi} = 0$

Analysis of the Optimized Network Response

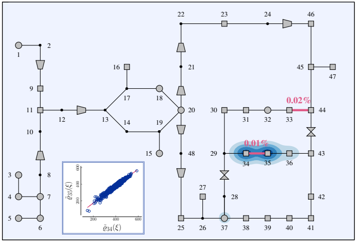
Parameter	Unit	Deterministic control policy	Chance-constrained control policies						
			Variance-agnostic	Pressure variance-aware, ψ^{π}			Flow variance-aware, ψ^{φ}		
				10^{-3}	10^{-2}	10^{-1}	1	10^{1}	10^{2}
Expected cost	\$1000	80.9	82.5 (100%)	**100.5%**	**105.6%**	**113.8%**	**100.1%**	**102.5%**	**112.6%**
Pressure variance	MPa^2	217.5	63.4 (100%)	**44.2%**	**18.9%**	**12.8%**	92.8%	46.7%	24.7%
Flow variance	$BMSCFD^2$	26.1	58.0 (100%)	83.4%	64.1%	59.2%	**93.4%**	**44.8%**	**25.9%**
Compression	kPa	1939	3914	3570	3734	3661	3914	4030	3888
Valve regulation	kPa	0	0	0	**150**	**576**	0	**1**	**500**
Infeas. ($\varepsilon = 1\%$)	%	53.7	0.04	0.02	0.02	0.02	0.03	0.02	0.03
Injection re-disp.	MMSCFD	960.91 **(31.3%)**	0.01	0.03	0.02	0.02	0.02	0.04	0.04
Comp/valve re-disp.	kPa	121.68 **(12.7%)**	0.19	0.08	0.10	0.05	0.28	0.04	0.04

Variance-agnostic policy



$\psi^{\pi} = 0,\ \psi^{\varphi} = 0$

Variance-aware policy



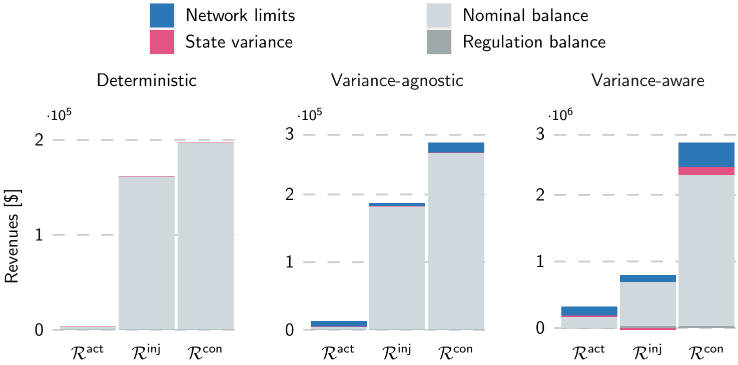
$\psi^{\pi} = 0.1,\ \psi^{\varphi} = 10$

Revenue Analysis

Revenue		Deterministic	Chance-constrained		
			Uncertainty		Variance
		Nominal balance	Regulation balance	Network limits	State variance
Injection	$R_n^{inj} =$	$f(\lambda_n^c, \vartheta_n)$	$+f(\lambda^r, \alpha_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\underline{\varphi}}, \alpha_n)$	$+f(u^{\pi}, u^{\varphi}, \alpha_n)$
Comp/valve	$R_\ell^{act} =$	$f(\lambda_{s(\ell)}^c, \lambda_\ell^w, \kappa_\ell)$	$+f(\lambda^r, \beta_\ell)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\underline{\varphi}}, \beta_\ell)$	$+f(u^{\pi}, u^{\varphi}, \beta_\ell)$
Consumer	$R_n^{con} =$	$f(\lambda_n^c, \delta_n)$	$+f(\lambda_n^r, \mathbb{1}_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\underline{\varphi}}, [F]_n)$	$+f(u^{\pi}, u^{\varphi}, [F]_n)$

Revenue Analysis

Revenue		Deterministic	Chance-constrained		
			Uncertainty		Variance
		Nominal balance	Regulation balance	Network limits	State variance
Injection	$R_n^{inj} =$	$f(\lambda_n^c, \vartheta_n)$	$+f(\lambda^r, \alpha_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\underline{\varphi}}, \alpha_n)$	$+f(u^{\pi}, u^{\varphi}, \alpha_n)$
Comp/valve	$R_\ell^{act} =$	$f(\lambda_{s(\ell)}^c, \lambda_\ell^w, \kappa_\ell)$	$+f(\lambda^r, \beta_\ell)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\underline{\varphi}}, \beta_\ell)$	$+f(u^{\pi}, u^{\varphi}, \beta_\ell)$
Consumer	$R_n^{con} =$	$f(\lambda_n^c, \delta_n)$	$+f(\lambda_n^r, \mathbb{1}_n)$	$+f(u^{\overline{\pi}}, u^{\underline{\pi}}, u^{\underline{\varphi}}, [F]_n)$	$+f(u^{\pi}, u^{\varphi}, [F]_n)$



Conclusions & Outlook

- Chance-constrained polices allow for an explicit uncertainty management while:
  - Internalizing probabilistic forecast and risk criteria of network operators
  - Producing uncertainty- and variance-aware network controls with guarantees
  - Offering the foundation for the stochastic gas market-clearing settlement
- Future extensions:
  - Coordination of power and gas networks
  - Contracts for TSO and NGO for the stochastic renewables, voltage control, etc.
  - More controls to exploit and price the flexibility of natural gas extractions
- arXiv paper & GitHub repository are coming soon

Tak for i dag!