Emission-Constrained Optimization of Gas Systems with Input-Convex Neural Networks

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ICLR 2023 Workshop: Tackling Climate Change with Machine Learning

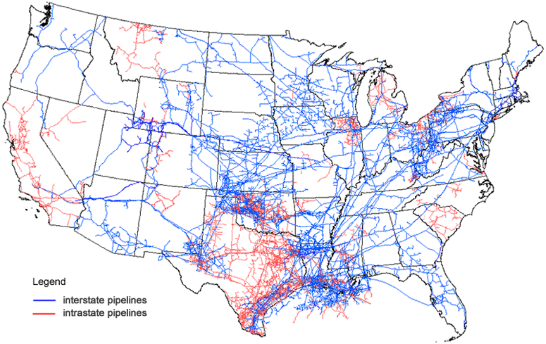
MARIE CURIE ACTIONS

Fundación IBERDROLA ESPAÑA

Background

- Large physical infrastructure
- Diversity of gas supply
- Complex physics of gas flows
- Large emission footprint

Map of U.S. interstate and intrastate natural gas pipelines



Source: U.S. Energy Information Administration, *About U.S. Natural Gas Pipelines*

Emission-constrained optimal gas flow problem

- Minimize the cost of gas injections ϑ
- Satisfying nodal gas demands δ
- and non-convex Weymouth equation, which couples flows φ and pressures π

$$\begin{aligned} \underset{\varphi,\vartheta,\pi\in\mathcal{F}}{\text{minimize}} \quad & c^\top \vartheta \\ \text{subject to} \quad & A\varphi = \vartheta - \delta \\ & \varphi \circ |\varphi| = \text{diag}[\omega] A^\top \pi \end{aligned}$$

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Two options to control emission footprint

Emission-constrained optimal gas flow problem

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- Satisfying nodal gas demands δ
- and non-convex Weymouth equation, which couples flows φ and pressures π

$$\begin{aligned} \underset{\varphi,\vartheta,\pi\in\mathcal{F}}{\text{minimize}} \quad & c^\top\vartheta + t^\top\vartheta \\ \text{subject to} \quad & A\varphi = \vartheta - \delta \\ & \varphi \circ |\varphi| = \text{diag}[\omega]A^\top\pi \end{aligned}$$

Two options to control emission footprint

emission tax

- Soft penalty for emission intensity
- Local optimality guarantee

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Two options to control emission footprint

emission tax

- Soft penalty for emission intensity
- Local optimality guarantee

emission cap

- Hard constraint on emissions
- Sensitive to initialization point

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Two options for deep learning applications

end-to-end learning

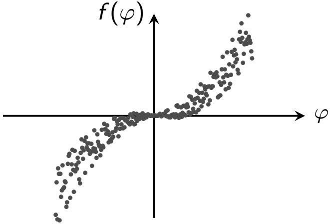
- Replaces optimization *per se*
- Directly predicts OGF solution
- May face mistrust in industry

learning-aided optimization

- Only assist OGF optimization
- Predicts the non-convex part only
- Less barriers for implementation

Optimal gas flow problem

$$\begin{aligned} \underset{\vartheta,\varphi,\pi\in\mathcal{F}}{\text{minimize}} \quad & c(\vartheta) \\ \text{subject to} \quad & \underbrace{\text{diag}[w]^{-1}\varphi \circ |\varphi|}_{\text{non-convex function } f(\varphi)} = A^{\top}\pi \end{aligned}$$



Optimal gas flow problem

$$\underset{\vartheta,\varphi,\pi\in\mathcal{F}}{\text{minimize}} \quad c(\vartheta)$$

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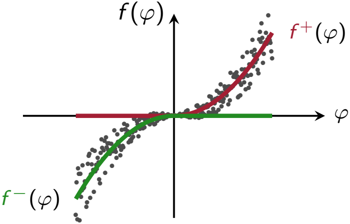
$$\downarrow\downarrow\downarrow$$

$$f^{-}(\varphi) + f^{+}(\varphi) = A^{\top}\pi$$



Aiding short-term planning with input-convex neural networks

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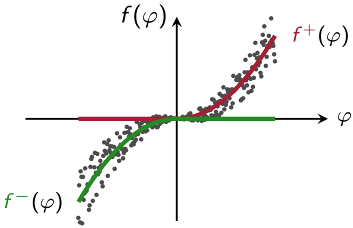
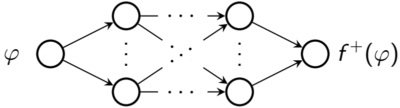


Input-convex neural network

minimize loss $\|f^{+}(\varphi) - A^{\top}\pi\|$



subject to: $f^{+}(\varphi)$ is **convex** in φ

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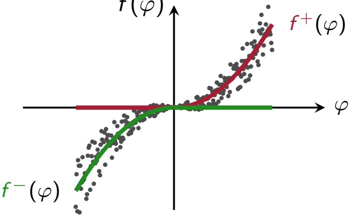
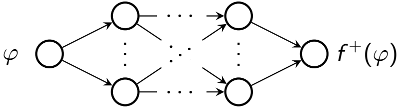


Input-convex neural network

minimize loss $\|f^+(\varphi) - A^\top \pi\|$



subject to: $f^+(\varphi)$ is **convex** in φ

A LP from the trained NN

$$\begin{aligned} \boldsymbol{f}^+(\varphi) := \underset{z^1,\dots,z^k\geqslant 0}{\text{minimize}} \quad & z^k \\ \text{subject to} \quad & z^1 \geqslant \boldsymbol{W}^0\varphi + \boldsymbol{b}^0 \\ & z^{i+1} \geqslant \boldsymbol{W}^i z^i + \boldsymbol{b}^i \end{aligned}$$

$\boldsymbol{W}$ and $\boldsymbol{b}$ are optimized weights and biases

From non-convex to equivalent bilevel optimization

Non-convex optimization

$$\begin{aligned} \underset{\vartheta,\varphi,\pi\in\mathcal{F}}{\text{minimize}} \quad & c(\vartheta) \\ \text{subject to} \quad & e^\top \vartheta \leqslant \overline{e} \\ & A\varphi = \vartheta - \delta \\ & \text{diag}[w]^{-1}\varphi \circ |\varphi| = A^\top \pi \end{aligned}$$

Equivalent bilevel optimization

$$\begin{aligned} \underset{\vartheta,\varphi,\pi\in\mathcal{F}}{\text{minimize}} \quad & c(\vartheta) \\ \text{subject to} \quad & e^\top \vartheta \leqslant \overline{e} \\ & A\varphi = \vartheta - \delta \\ & \boldsymbol{f}^+ + \boldsymbol{f}^- = A^\top \pi \\ & \boldsymbol{f}^+ \in \text{input-convex LP}(\varphi) \\ & \boldsymbol{f}^- \in \text{input-concave LP}(\varphi) \end{aligned}$$

Non-convex optimization

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Equivalent bilevel optimization

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& \boldsymbol{f}^+ + \boldsymbol{f}^- = A^\top \pi \\
& \boldsymbol{f}^+ \in \text{input-convex LP}(\varphi) \\
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$$

Aiding network design planning with neural networks

- Accommodating emission via network expansion
- Weymouth equation with variable pipeline **diameter**

$$\text{diag}[\boldsymbol{d}]^{-1}\varphi \circ |\varphi| = \text{diag}[\hat{\omega}]A^{\top}\pi$$

- A growing **diameter** increases the flow of gas mass and eases pressure congestion

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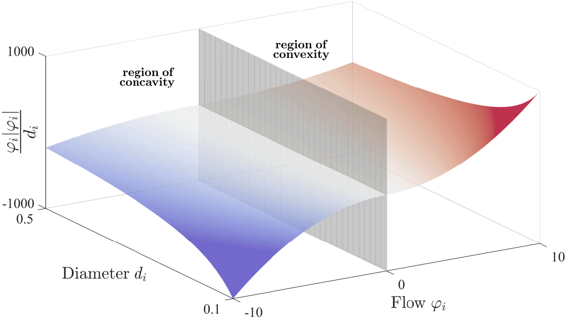
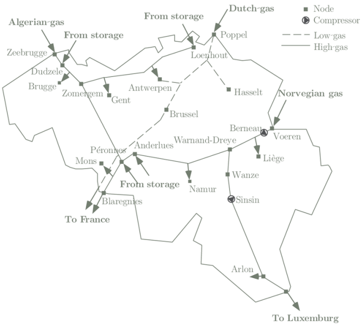


Allocation to the Belgium gas network

We compare three methods:

- Interior point solver IPOPT
- A mixed-integer quadratic programming (MIQP) relaxation
- Proposed ICNN-aided optimization



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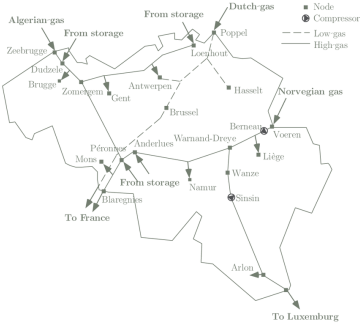

Results for emission-constrained short-term operational planning

Emission cap, kT	1,000 random IPOPT initializations				MIQP relaxation	ICNN-aided solution
	min	mean	max	prob. of failure		
∞	1,923.3	1,927.2	1,929.2	**16.6%**		
100	2,225.1	2,235.1	2,256.2	**16.0%**		
48.9	4,344.6	4,344.6	4,344.6	**39.0%**		

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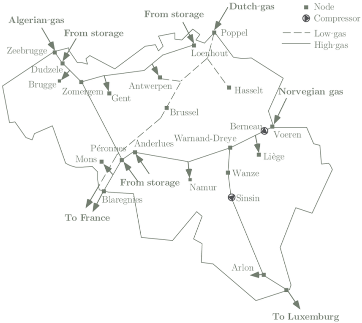

Results for emission-constrained short-term operational planning

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∞	1,923.3	1,927.2	1,929.2	**16.6%**	1,540.8	1,929.2	
100	2,225.1	2,235.1	2,256.2	**16.0%**	2,137.2	2,225.1	
48.9	4,344.6	4,344.6	4,344.6	**39.0%**	4,200.8	4,344.6	

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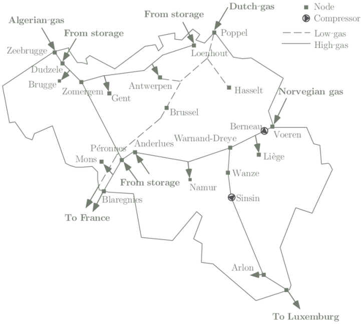

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100	2,225.1	2,235.1	2,256.2	**16.0%**	2,137.2	2,225.1	2,241.3	**2,225.1**
48.9	4,344.6	4,344.6	4,344.6	**39.0%**	4,200.8	4,344.6	4,290.1	**4,291.2**

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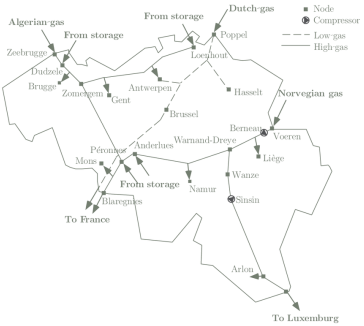

Results for emission-constrained long-term design planning

Emission cap, kT	1,000 random IPOPT initializations				ICNN-aided solution
	min	mean	max	prob. of failure	
∞	2,671.7	2,701.8	2,829.5	**28.6%**	
100	3,057.8	3,090.2	3,191.9	**30.3%**	
48.9	5,079.1	5,138.7	5,247.9	**41.4%**	

Allocation to the Belgium gas network

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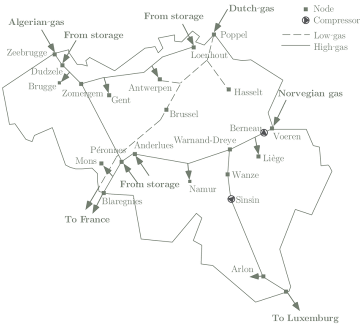

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	min	mean	max	prob. of failure	optimal	warm start for IPOPT
∞	2,671.7	2,701.8	2,829.5	**28.6%**	2,666.4	2,671.6
100	3,057.8	3,090.2	3,191.9	**30.3%**	3,056.6	3,057.8
48.9	5,079.1	5,138.7	5,247.9	**41.4%**	5,079.9	5,079.1

Conclusions

- New method for emission-constrained planning of natural gas networks
- Our neural network-based `ICNN` solver outperforms `IPOPT` and `MIQP` solvers
- Savings up to 1.2% of operating costs in short-term emission-constrained planning
- Savings up to 5.9% of emission-constrained investment planning costs in a long run

Thank you for your attention!

https://arxiv.org/pdf/2209.08645.pdf