Regression Equilibrium in Electricity Markets

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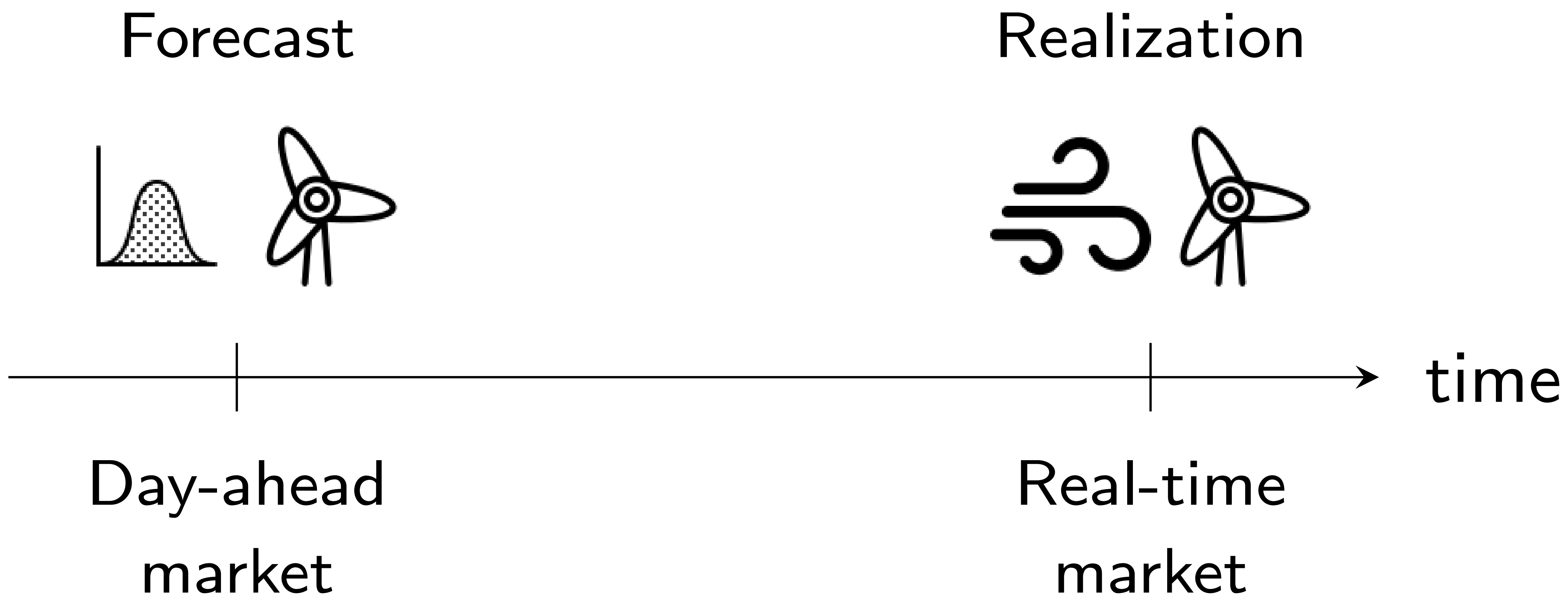
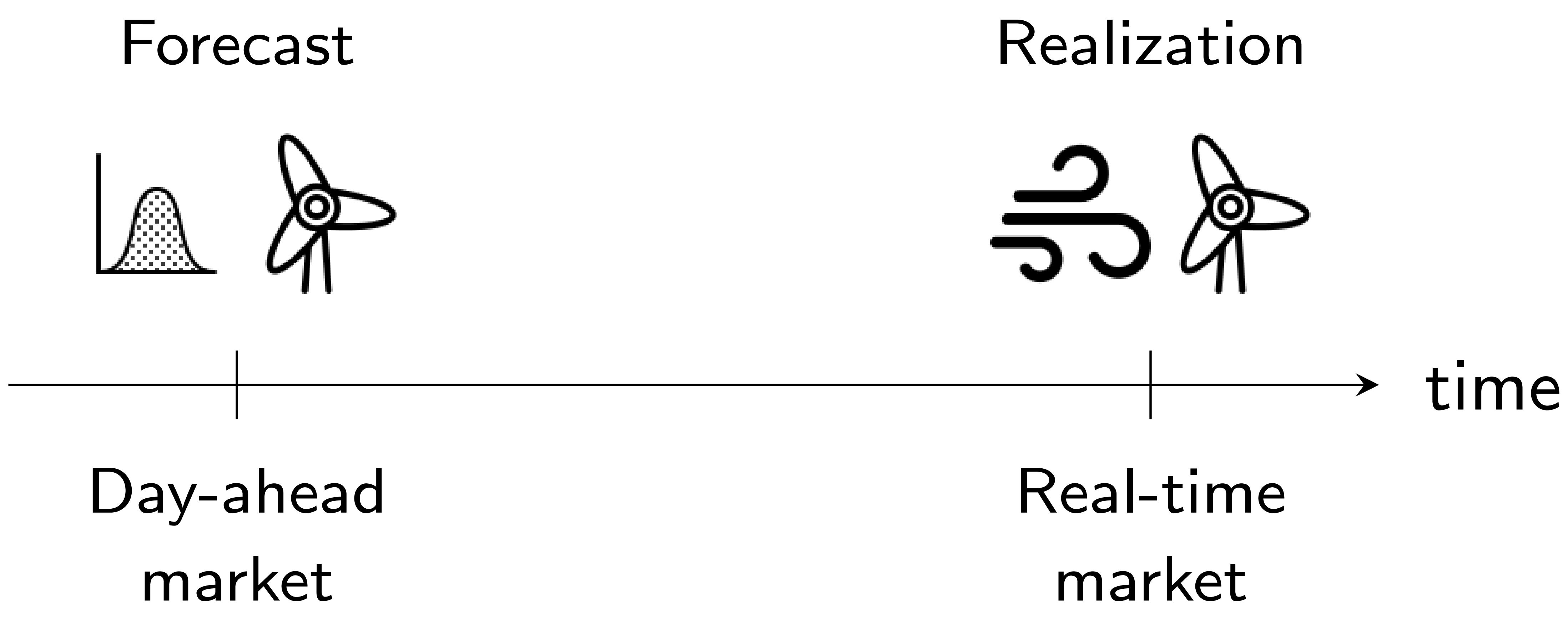
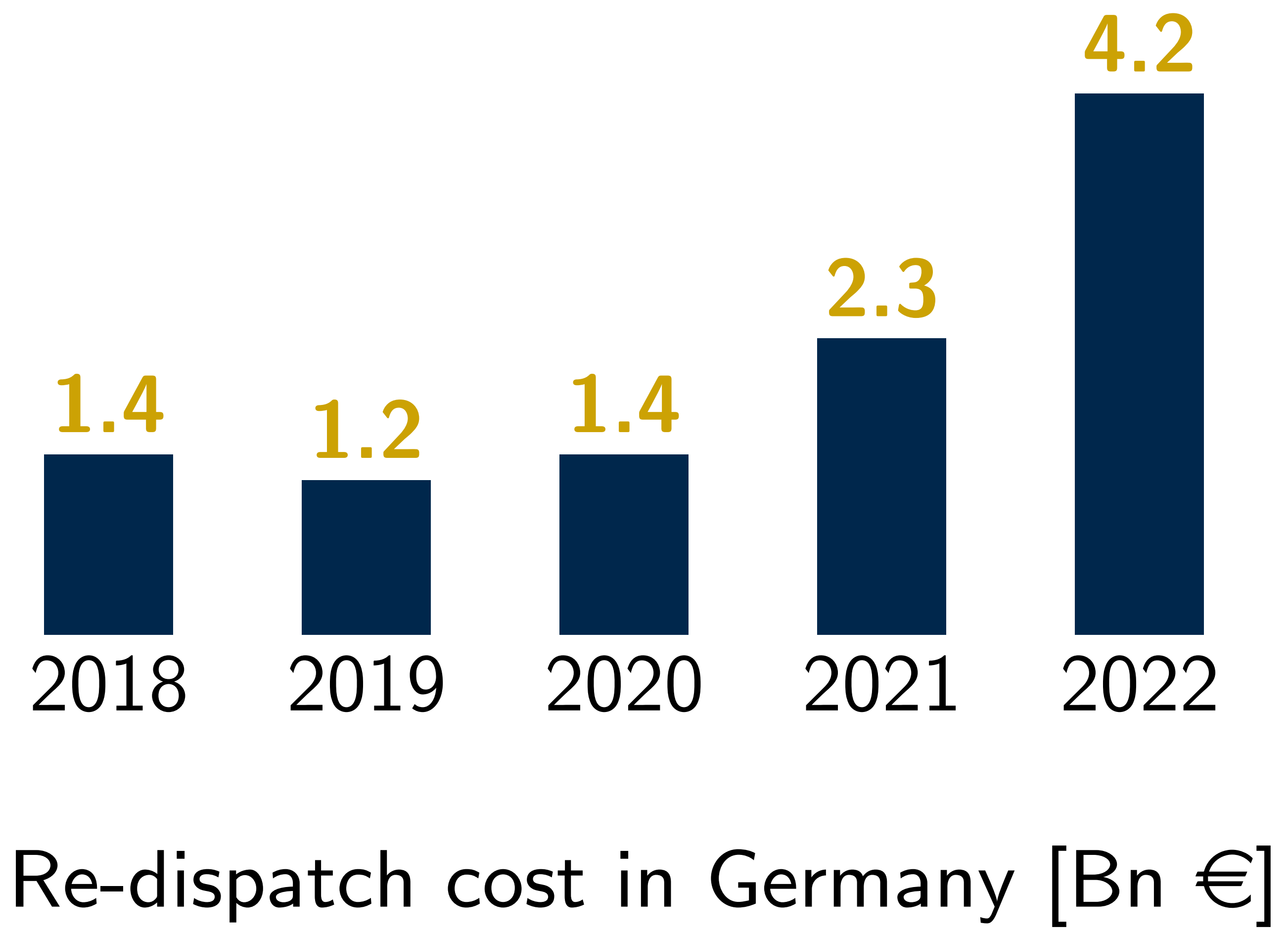


- Two-stage electricity markets to manage uncertainty of renewables:
  - Day-ahead market: minimize the cost of power supply w.r.t. forecast
  - Real-time market: least-cost re-dispatch to accommodate forecast errors

- As renewable penetration increases, the cost of real-time re-dispatch also increases





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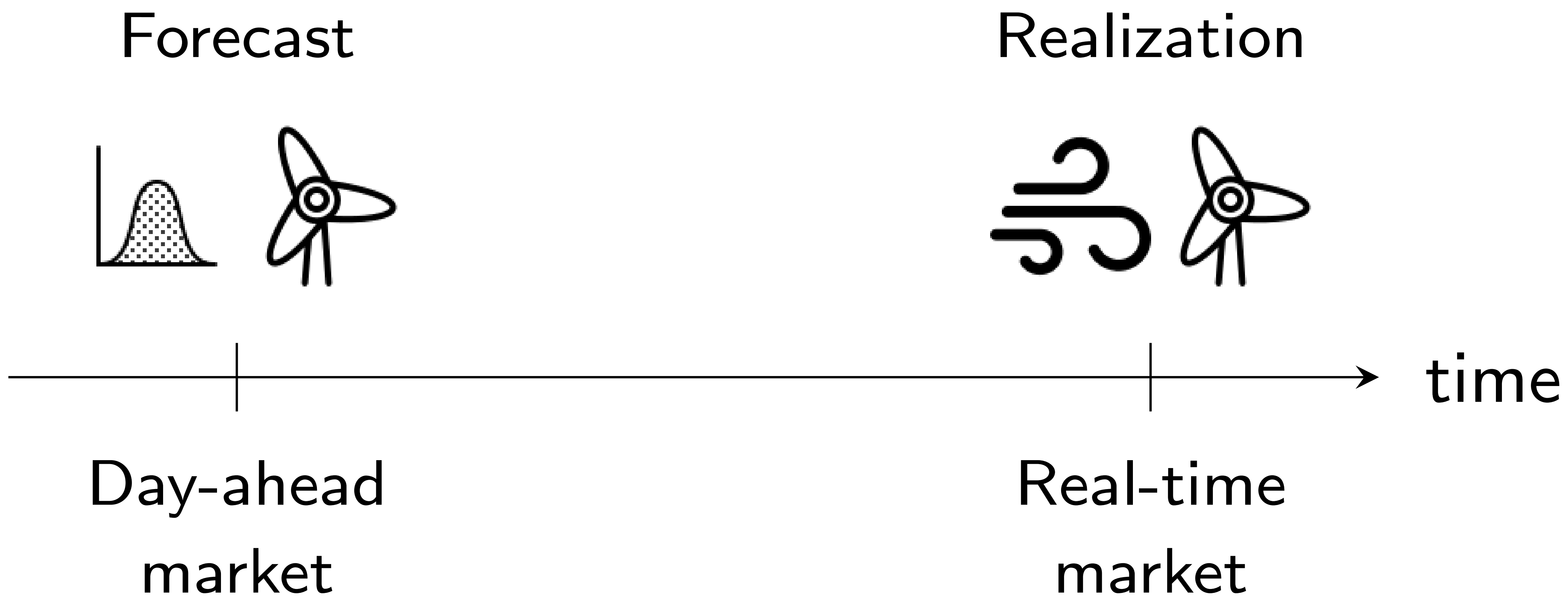
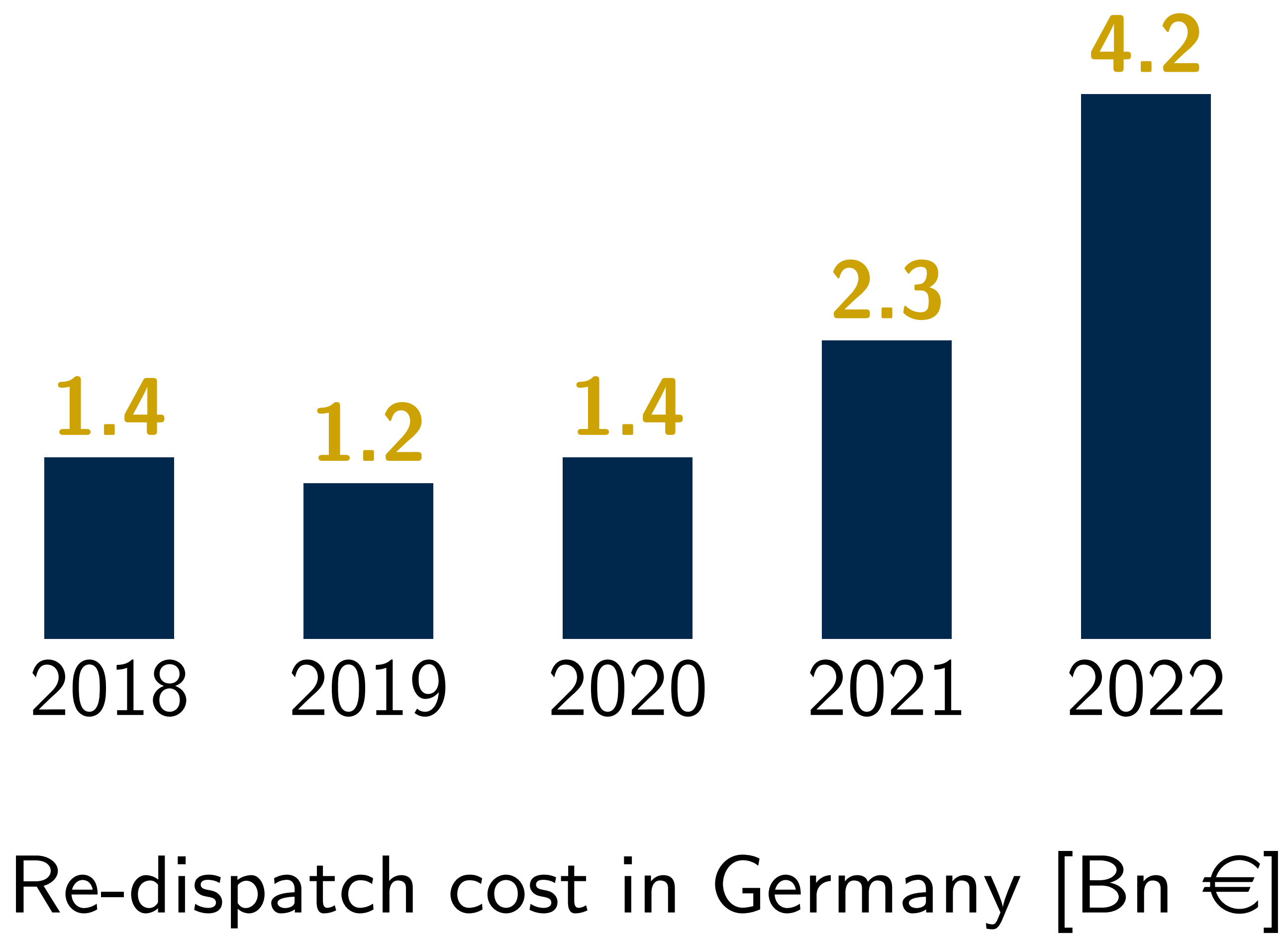
Two-stage electricity markets





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How to make renewable power generation less expensive for the system?

Two-stage electricity markets (continued)

day-ahead market

min dispatch cost
s.t. grid limits ($\widehat{w}$)

fixed dispatch

real-time market

min re-dispatch cost
s.t. grid limits ($\Delta\widehat{w}$)

Two-stage electricity markets (continued)

day-ahead market

$$\boxed{\begin{array}{ll}\min & \text{dispatch cost} \\ \text{s.t.} & \text{grid limits } (\widehat{w})\end{array}} \xrightarrow{\text{fixed dispatch}} \boxed{\begin{array}{ll}\min & \text{re-dispatch cost} \\ \text{s.t.} & \text{grid limits } (\boldsymbol{\Delta}\widehat{w})\end{array}}$$

real-time market

To improve cost efficiency across day-ahead and real-time markets:

- **Stochastic electricity market design** [PZP10, M+12, Dvo19]:
  - + Co-optimization of dispatch and re-dispatch decisions
  - + Least-cost solution in *expectation*
  - – Market properties only hold in *expectation*

$$\boxed{\begin{array}{ll}\min & \text{dispatch cost} + \mathbb{E}_{\mathbb{P}_{\boldsymbol{\Delta}\widehat{w}}}[\text{re-dispatch cost}] \\ \text{s.t.} & \text{grid limits } (\widehat{w}, \boldsymbol{\Delta}\widehat{w}) \text{ for all } \boldsymbol{\Delta}\widehat{w} \sim \mathbb{P}_{\boldsymbol{\Delta}\widehat{w}}\end{array}}$$

day-ahead market

$$\begin{aligned} \min \quad & \text{dispatch cost} \\ \text{s.t.} \quad & \text{grid limits } (\widehat{w}) \end{aligned}$$

$\xrightarrow{\text{fixed dispatch}}$

real-time market

$$\begin{aligned} \min \quad & \text{re-dispatch cost} \\ \text{s.t.} \quad & \text{grid limits } (\boldsymbol{\Delta}\widehat{w}) \end{aligned}$$

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- **Tuning deterministic market parameters to approximate stochastic efficiency:**
  - Improved scheduling of renewbales [M+14]
  - Cost-aware reserve requirements and transmission allocation [DDM18, MP24, JKP17, DP19]

Two-stage electricity markets (continued)

day-ahead market | real-time market

day-ahead market		real-time market
min dispatch cost	fixed dispatch $\longrightarrow$	min re-dispatch cost
s.t. grid limits ($\widehat{w}$)		s.t. grid limits ($\Delta\widehat{w}$)

To improve cost efficiency across day-ahead and real-time markets:

- **Stochastic electricity market design** [PZP10, M$^+$12, Dvo19]:
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- **This paper** proves that markets incentivize tuning private forecasts to minimize total system costs, yielding a socially optimal **regression equilibrium**.

Baseline approach to wind power forecasting:

- Collect a training dataset $\mathcal{D} = \{(\varphi_1, \mathbf{w}_1), \dots, \varphi_n, \mathbf{w}_n)\}$
- Machine learning model $\mathbb{W}_{\boldsymbol{\theta}} : \mathcal{F} \mapsto \mathcal{W}$ with parameter $\boldsymbol{\theta}$
- Learn optimal parameter $\boldsymbol{\theta}^\star$ by minimizing a prediction loss

$$\min_{\|\boldsymbol{\theta}\|_1 \leqslant \tau} \quad \mathcal{L}(\boldsymbol{\theta} \,|\, \mathcal{D}) = \frac{1}{n} \sum_{i=1}^{n} \|\mathbb{W}_{\boldsymbol{\theta}}(\varphi_i) - \mathbf{w}_i\|_2^2$$

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Revenue-optimal forecasting [PCK07, CK19, WSC23]:

1. Day-ahead stage: LMP λ_1 pricing the forecast of wind power
2. Real-time stage: LMP λ_2 pricing any forecast deviation

$$\max_{\|\theta\|_1 \leqslant \tau} \quad \mathcal{R}^{W}(\theta \,|\, \mathcal{D}, \lambda_1, \lambda_2) = \frac{1}{n}\sum_{i=1}^{n} \Big(\underbrace{\lambda_{1i}\mathbb{W}_{\theta}(\varphi_i)}_{\text{day-ahead revenue}} + \underbrace{\lambda_{2i}(\mathbf{w}_i - \mathbb{W}_{\theta}(\varphi_i))}_{\text{real-time revenue}} \Big)$$

Optimization of wind power producers

$$\max_{\|\boldsymbol{\theta}_j\|\leqslant\tau_j} \quad \mathcal{R}^W(\boldsymbol{\theta}_j|\mathcal{D}_j,\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2) - \gamma\cdot\mathcal{L}(\boldsymbol{\theta}_j|\mathcal{D}_j)$$

for all producers $j \in 1,\dots,b$

Optimization of wind power producers

$$\underbrace{\max_{\|\boldsymbol{\theta}_j\|\leqslant\tau_j}}_{\text{feature selection}} \quad \underbrace{\mathcal{R}^W(\boldsymbol{\theta}_j|\mathcal{D}_j,\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2)}_{\text{expected revenue}} - \underbrace{\gamma\cdot\mathcal{L}(\boldsymbol{\theta}_j|\mathcal{D}_j)}_{\text{regularization by prediction loss}}$$

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Optimization of controllable generators

$$\max_{\mathbf{p}_i,\mathbf{r}_i\in\mathcal{G}} \quad \mathcal{R}^G(\mathbf{p}_i,\mathbf{r}_i \mid \boldsymbol{\lambda}_{1i},\boldsymbol{\lambda}_{2i}) - c(\mathbf{p}_i,\mathbf{r}_i)$$

for all training samples $i \in \mathcal{D}_{1:b}$

Regression equilibrium in two-stage electricity markets [Dvo24]

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Market-clearing conditions at the 1st and 2nd stages

$$\mathbf{0}\leqslant\boldsymbol{\lambda}_{1i}\perp\mathbf{p}_i+\sum_{j=1}^{b}\mathbb{W}_{\boldsymbol{\theta}_j}(\boldsymbol{\varphi}_i)-\mathbf{d}\geqslant\mathbf{0},\quad \mathbf{0}\leqslant\boldsymbol{\lambda}_{2i}\perp\mathbf{r}_i+\sum_{j=1}^{b}\mathbf{w}_{ji}-\sum_{j=1}^{b}\mathbb{W}_{\boldsymbol{\theta}_j}(\boldsymbol{\varphi}_i)\geqslant\mathbf{0}$$

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Assumptions:

- Class of ML models $\mathbb{W}_{\boldsymbol{\theta}}$ is **convex** in $\boldsymbol{\theta}$, e.g., kernel regression
- Training datasets are such that $n \gg \text{card}[\boldsymbol{\varphi}]$ (unique regression solution)
- The intersection of private feasible regions is compact (at least one feasible dispatch $\forall i \in \mathcal{D}_{1:b}$)

Main result: regression equilibrium exists and is unique!

Optimization of wind power producers

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Equilibrium regression profile $\Theta^\star = (\theta_1^\star, \ldots, \theta_b^\star)$, such that:

- Feasible operation of the power gird and markets
- Maximized wind power profits, with no incentives to deviate
- Minimized expected dispatch costs across the two markets

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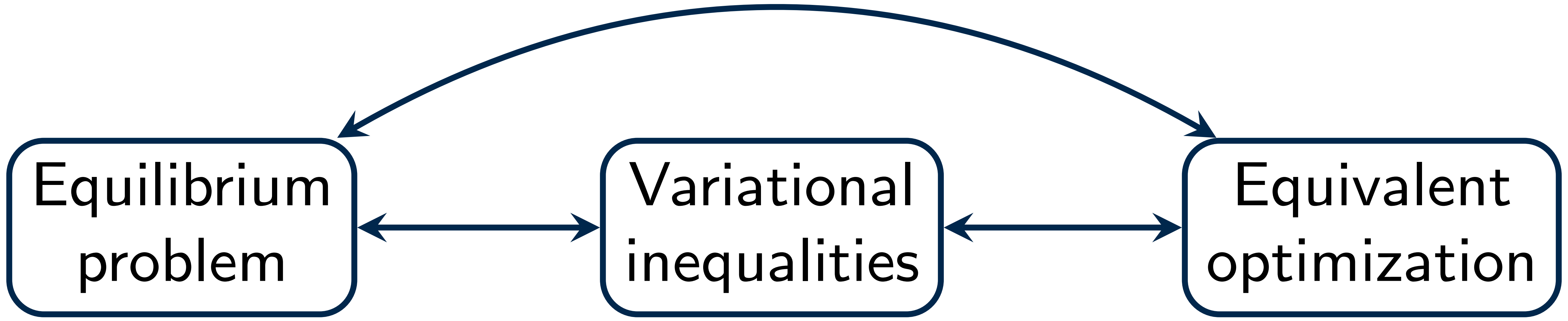
How to compute equilibrium regression $\boldsymbol{\Theta}^\star$?

Connection to variational inequalities theory [S+10]



- Equilibrium problem: stacks many private optimization problems
- Variational inequalities (VI): analyzes the interaction between private optimization problems
- In some special cases (like ours), VI connects equilibrium to a centralized optimization

For more details visit Appenix A in: https://arxiv.org/pdf/2405.17753

Computing regression equilibrium: centralized optimization

- By the Symmetry Principle Theorem [FP03], there exists an equivalent opt solving the equilibrium
- ... which happens to minimizes the expected generation and regulation costs
- ... thus enhancing the temporal coordination of day-ahead and real-time markets

$$
\begin{aligned}
\min_{\boldsymbol{\Theta},\mathbf{p},\mathbf{r}} \quad & \frac{1}{n}\sum_{i=1}^{n} c\left(\mathbf{p}_i,\mathbf{r}_i\right) + \gamma \left\|\boldsymbol{\Theta}\varphi_i - \mathbf{w}_i\right\|_2^2 && \text{regularized expected cost} \\
\text{s.t.} \quad & \mathbf{1}^\top(\mathbf{p}_i + \boldsymbol{\Theta}\varphi_i - \mathbf{d}) = 0, && \text{day-ahead power balance} \\
& \mathbf{1}^\top(\mathbf{r}_i - \boldsymbol{\Theta}\varphi_i + \mathbf{w}_i) = 0, && \text{real-time power balance} \\
& |\mathbf{F}(\mathbf{p}_i + \boldsymbol{\Theta}\varphi_i - \mathbf{d})| \leqslant \overline{\mathbf{f}}, && \text{day-ahead power flow limit} \\
& |\mathbf{F}(\mathbf{p}_i + \boldsymbol{\Theta}\varphi_i - \mathbf{d}) \\
& \quad + \mathbf{F}(\mathbf{r}_i - \boldsymbol{\Theta}\varphi_i + \mathbf{w}_i)| \leqslant \overline{\mathbf{f}}, && \text{real-time power flow limit} \\
& \underline{\mathbf{p}} \leqslant \mathbf{p}_i + \mathbf{r}_i \leqslant \overline{\mathbf{p}}, && \text{generation limit} \\
& |\mathbf{r}_i| \leqslant \overline{\mathbf{r}}, \quad \forall i = 1,\dots,n, && \text{regulation limit} \\
& |\boldsymbol{\Theta}| \leqslant \tau && \text{equilibrium feature selection}
\end{aligned}
$$

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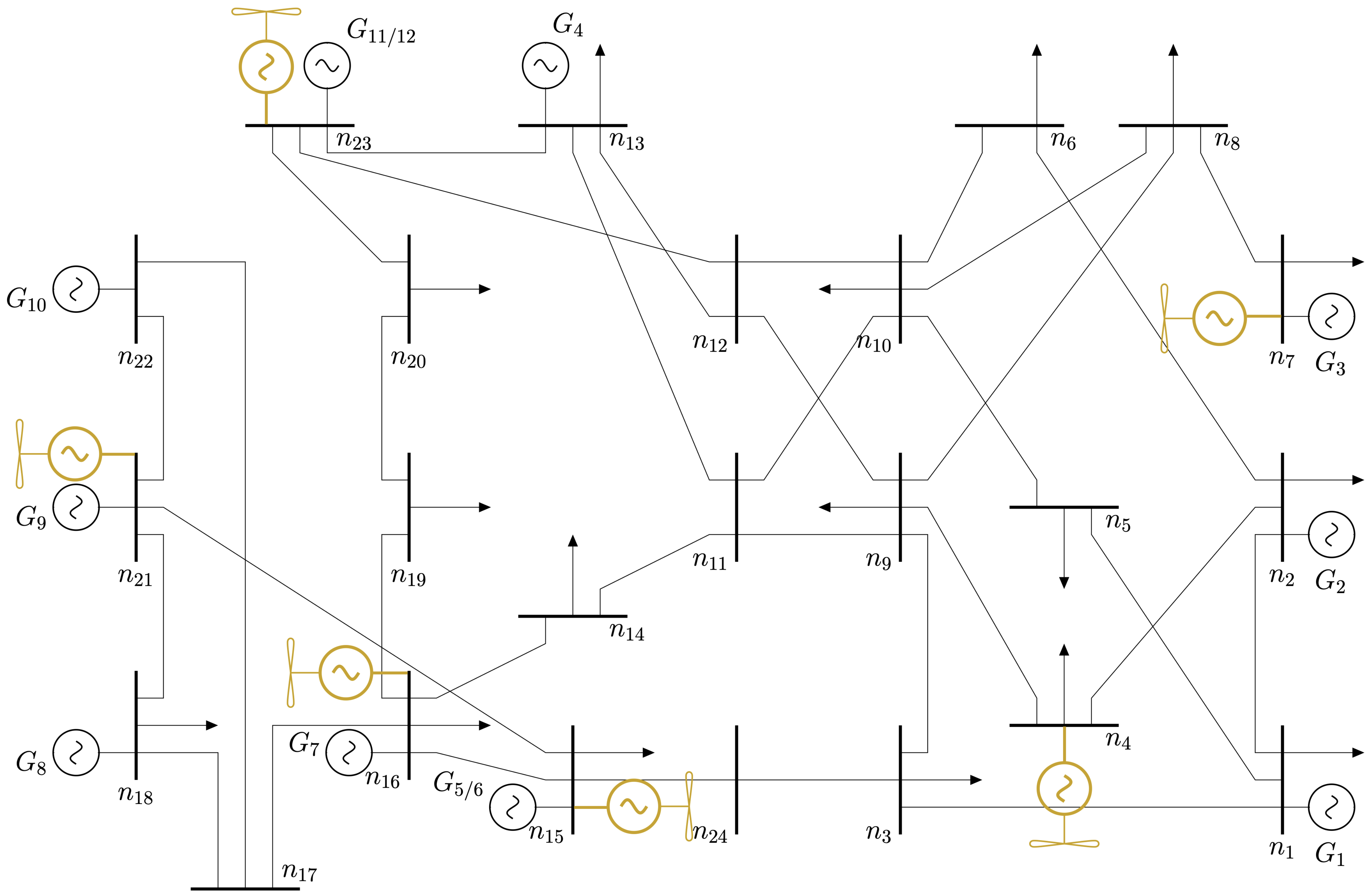
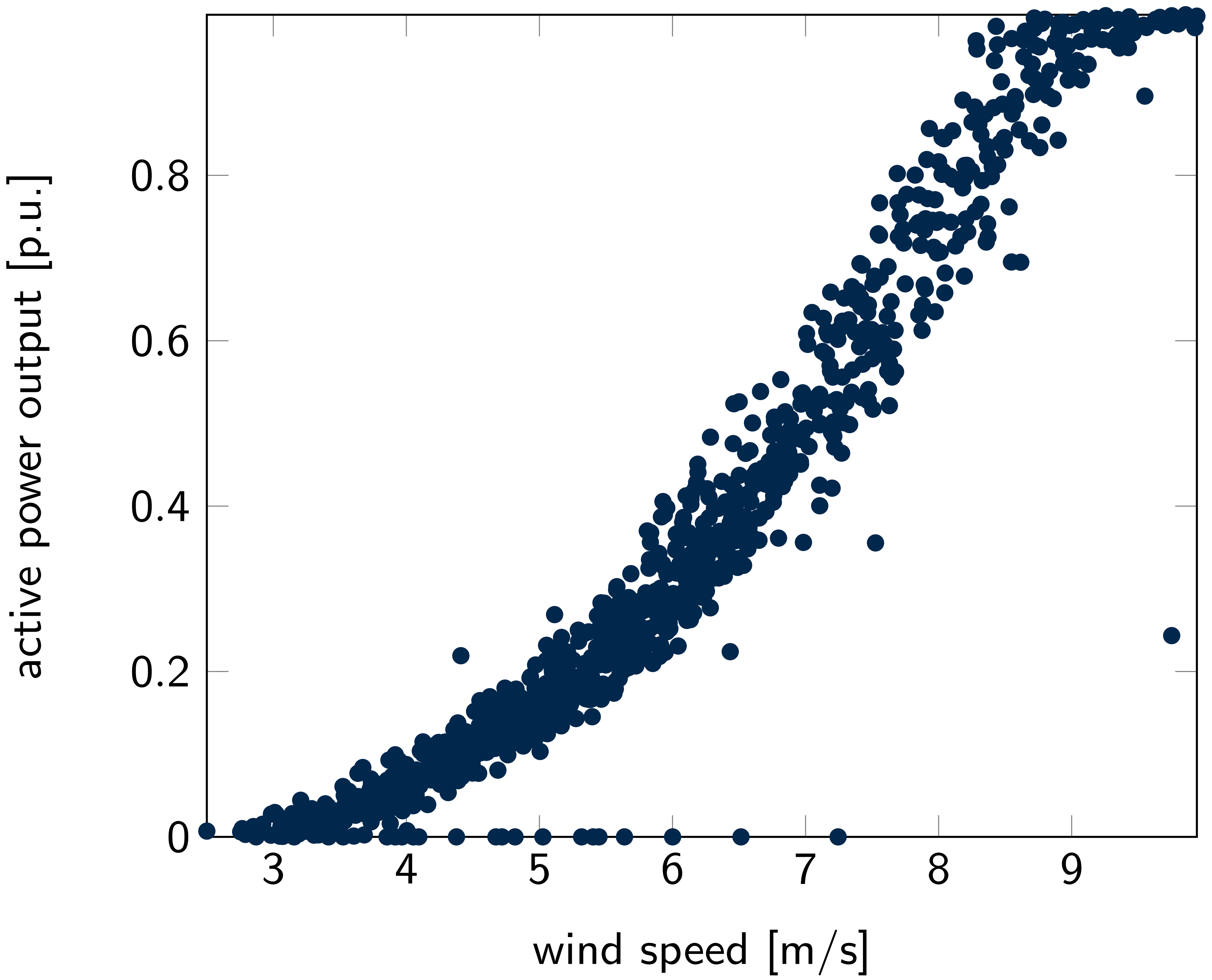
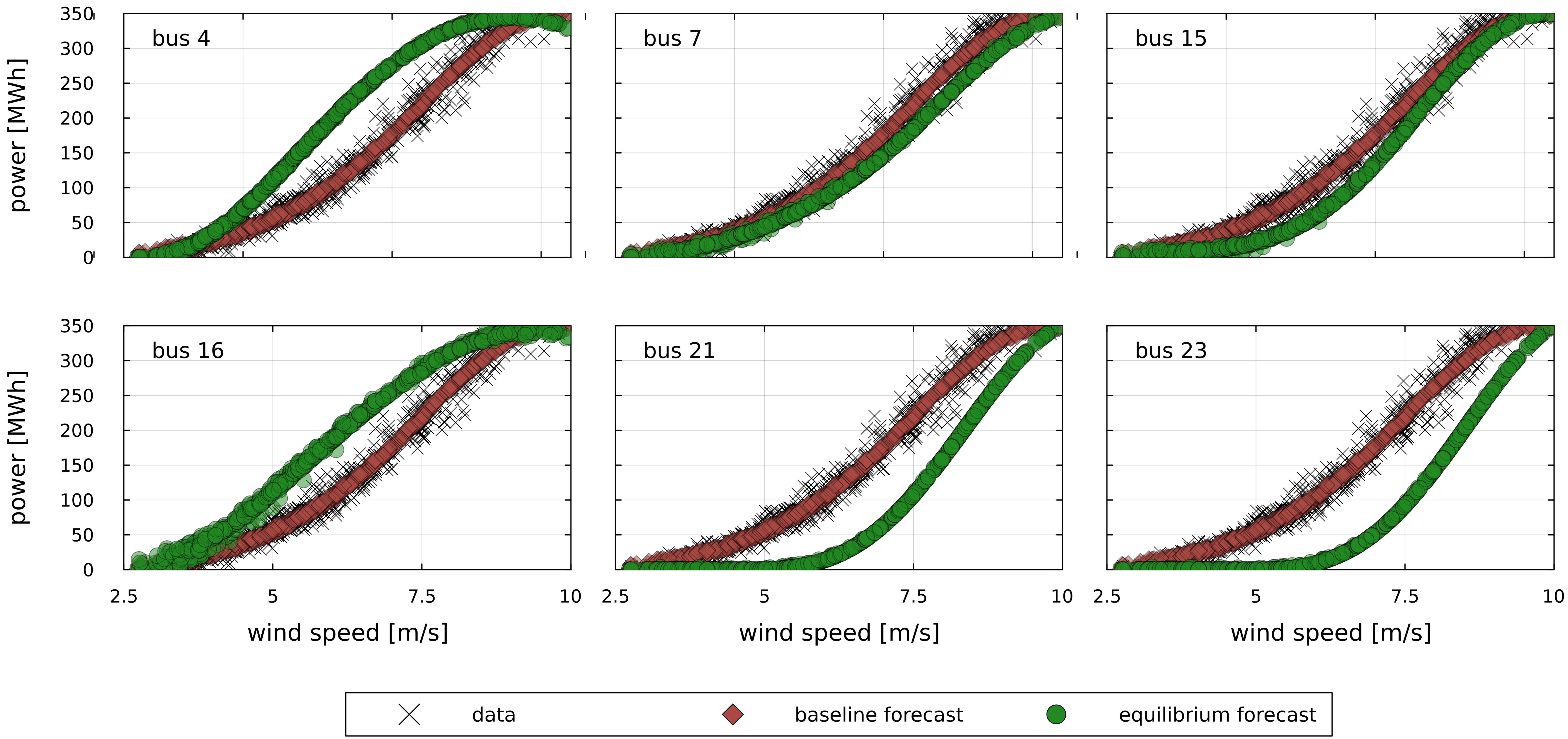




- 6 with farms with identical data and features
- Cover 38.4% of load at peak generation
- Kernel regression with 30 transformed features
- 5,000 training and 10,000 testing samples

- Although data is the same, how do equilibrium forecasts depend on the wind farm location in the grid?
- What are the equilibrium benefits in terms profits (any incentives to deviate?) and cost of electricity?

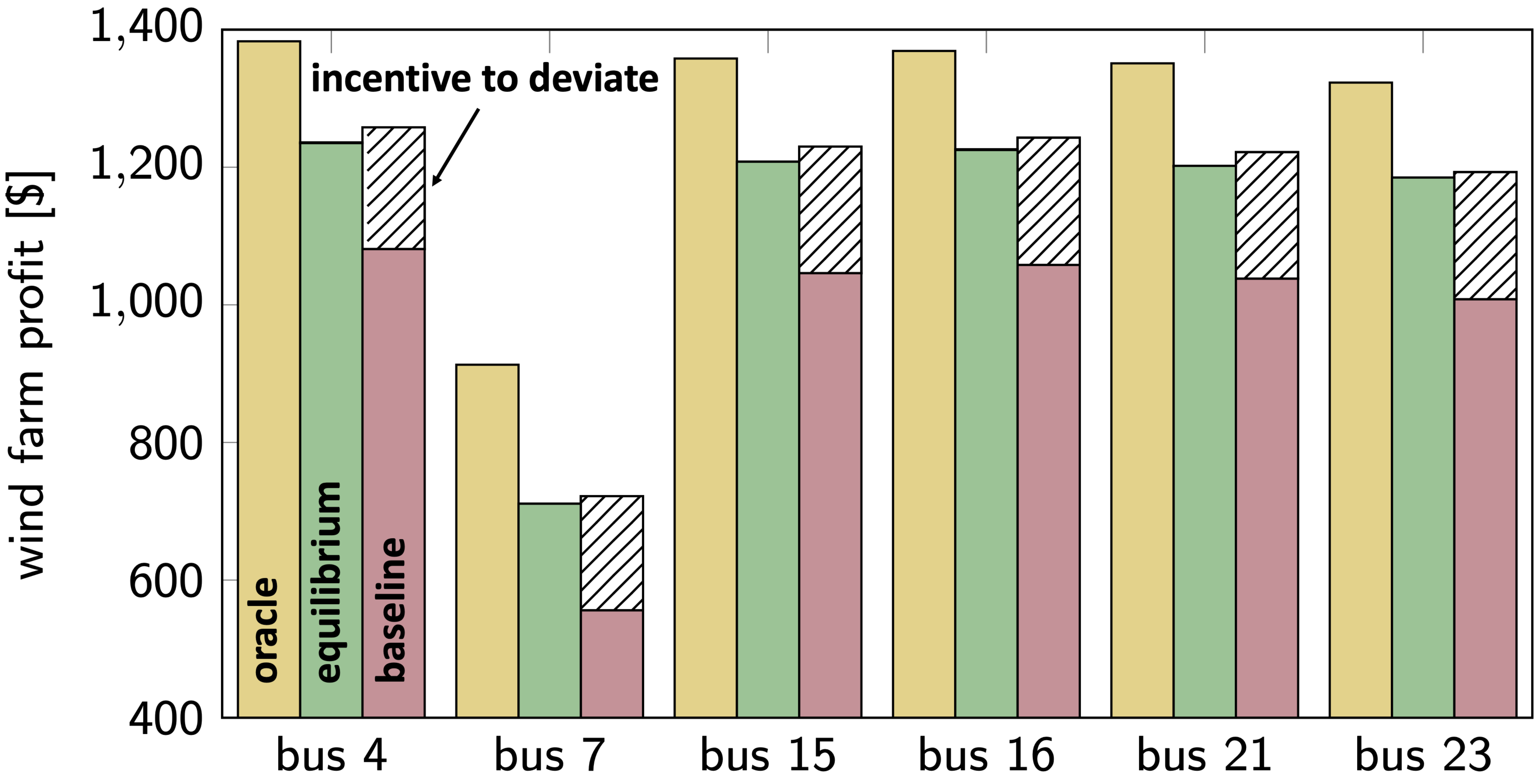
Baseline versus Equilibrium forecasts



- **Baseline:** minimizes a prediction error
- **Equilibrium:** maximizes wind farm profits

Systematic over- or under-prediction depending on the wind farm's location in the grid

Wind farm profits and incentives to deviate



- Equilibrium regression yields larger profits for all wind farms
- There are large profit incentives to unilaterally deviate from the baseline regression
- And (almost) no incentives to deviate from the equilibrium regression

Impact of regression equilibrium on dispatch costs

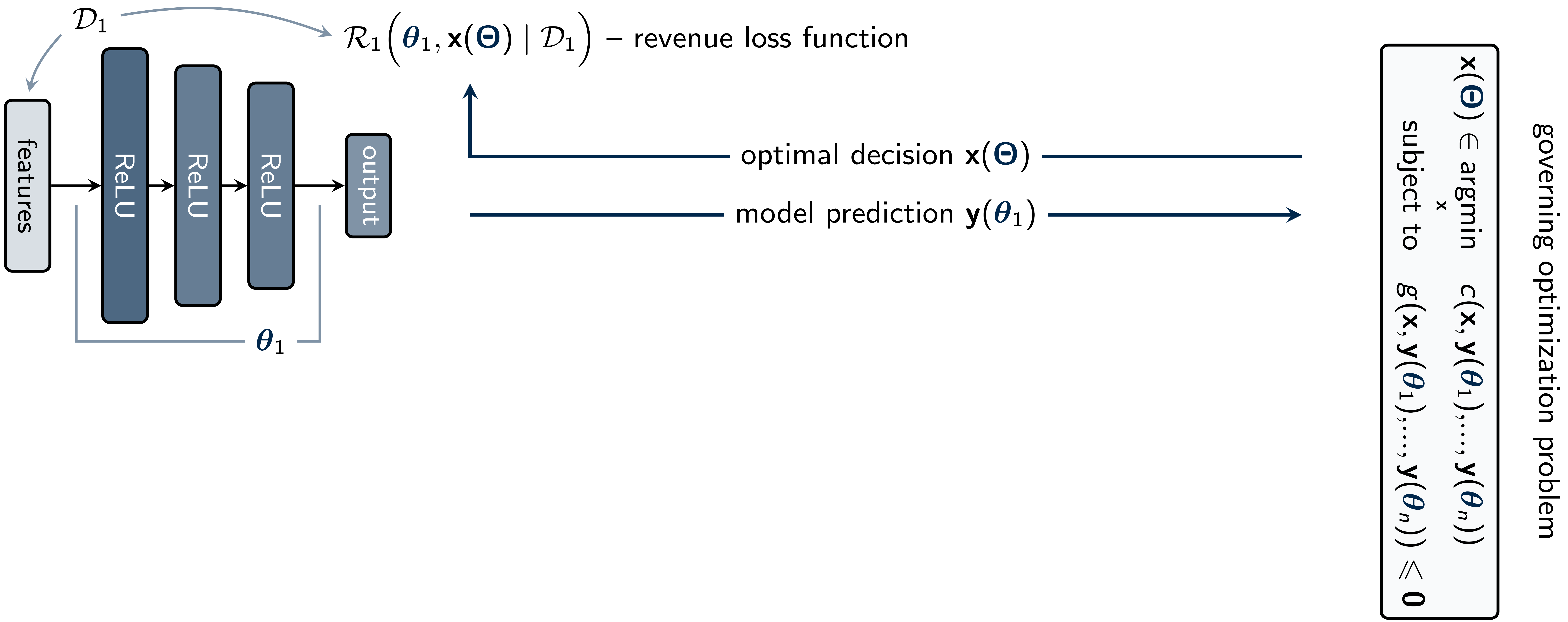
Regression	RMSE, MWh	Average dispatch cost, \$			Total dispatch cost error, \$	
		total	day-ahead	real-time	average	$\text{CVaR}_{10\%}$
Oracle	——	$37,246$	$37,246$	——	——	——
Equilibrium	**395**	**38,326**	**38,154**	**172**	**1,080**	**3,555**
Baseline	88	$39,223$	$37,459$	$1,764$	$1,977$	$8,626$

- Baseline regression: minimal forecast error, yet results in large real-time cost
- Equilibrium regression: large forecast errors, withholds cheap generation from the day-ahead market; yet, results in very cheap real-time re-dispatch
- Saving of 2.4% on average, and 13.6% on average across 10% of the worst-case scenarios

- Network coupling of private ML models (ripple effect on the entire electricity market)
- Nash regression equilibrium syncs private models and yields maximum profits
- It implicitly minimizes the cost across day-ahead and real-time markets ...
- ...thus delivering the benefits of stochastic market design in the existing deterministic markets

Regression Equilibrium in Electricity Markets

What is next? AI equilibrium in systems governed by optimization programs



- Many systems governed by optimization (pricing, scheduling, social planning, optimal control,...)
- Arbitrary AI models (regression, deep learning, reinforcement learning, trees)
- From Nash to Generalized Nash Equilibrium: more insights, algorithms, etc.
- Does the design of the governing optimization steer AI models to operational and economic equilibrium?

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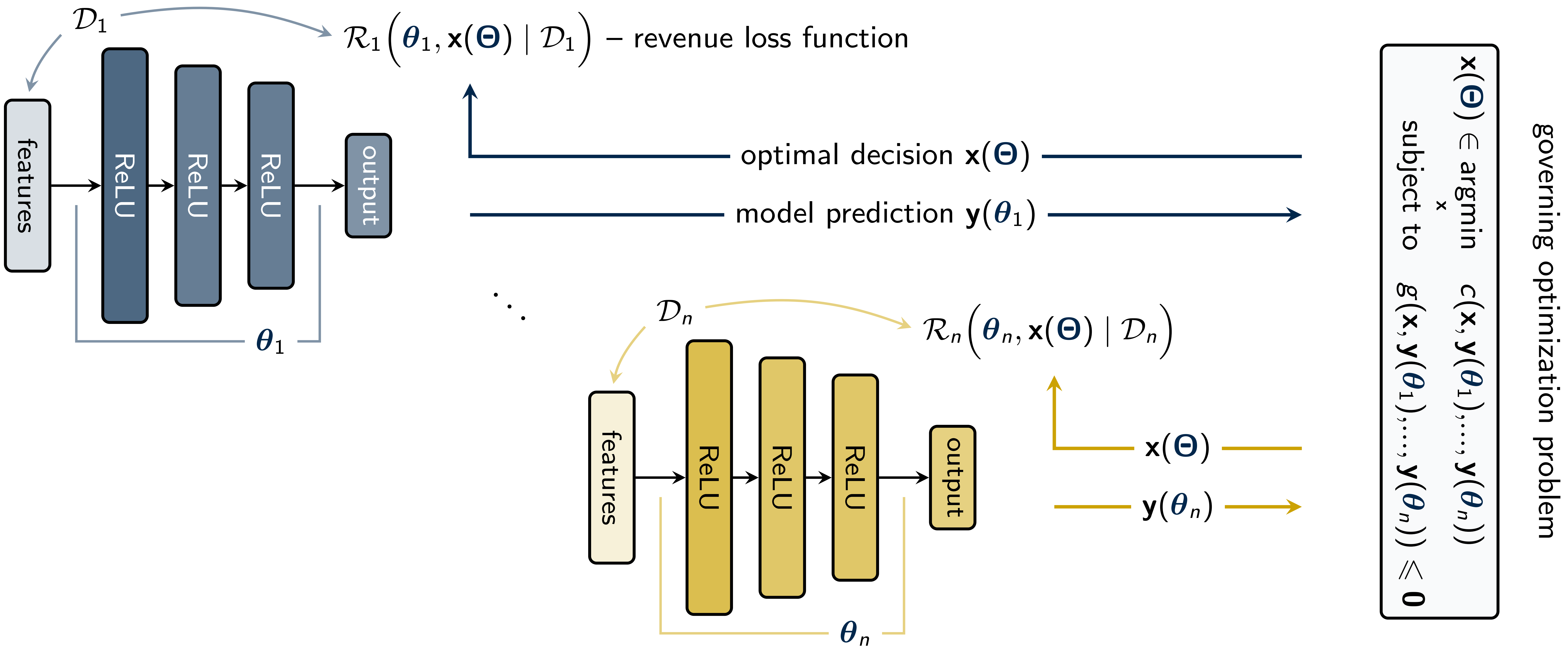

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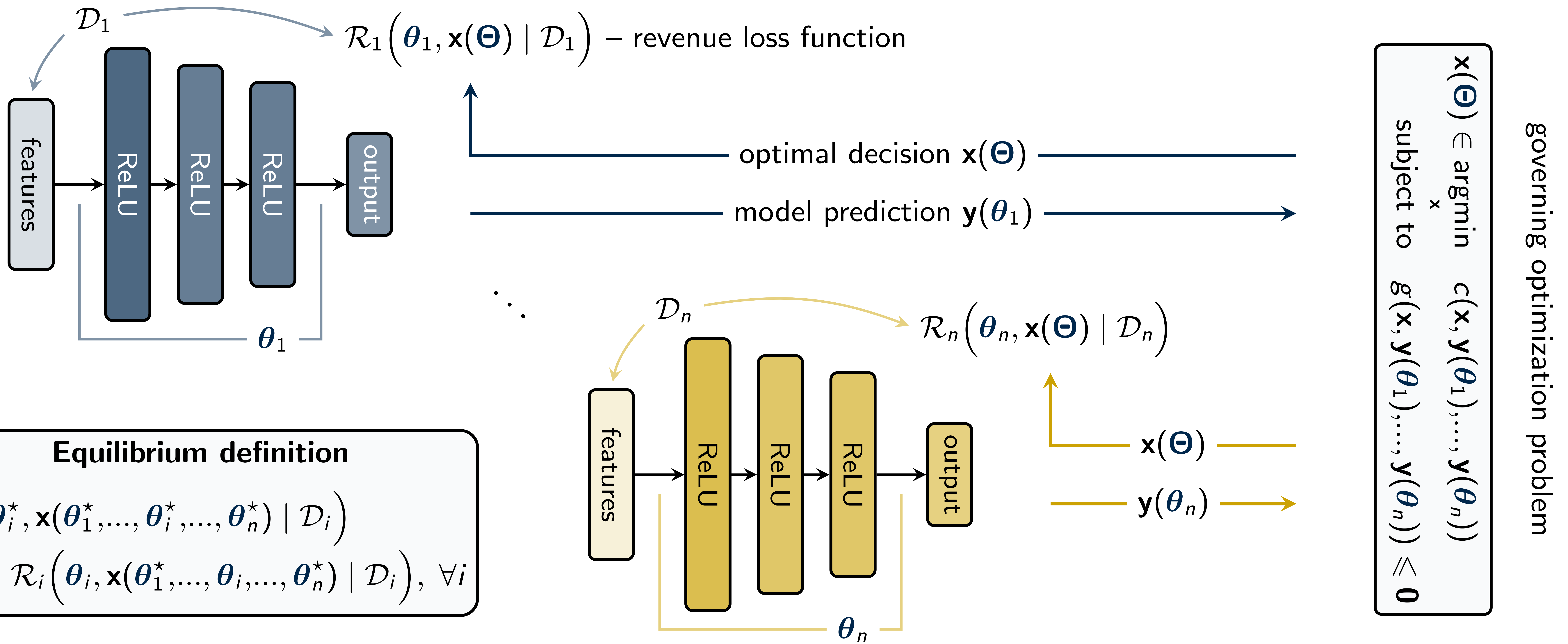

Equilibrium definition

$$\mathcal{R}_i\Big(\boldsymbol{\theta}_i^\star, \mathbf{x}(\boldsymbol{\theta}_1^\star, \dots, \boldsymbol{\theta}_i^\star, \dots, \boldsymbol{\theta}_n^\star) \mid \mathcal{D}_i\Big) \leqslant \mathcal{R}_i\Big(\boldsymbol{\theta}_i, \mathbf{x}(\boldsymbol{\theta}_1^\star, \dots, \boldsymbol{\theta}_i, \dots, \boldsymbol{\theta}_n^\star) \mid \mathcal{D}_i\Big), \ \forall i$$

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