

# Regression Nash Equilibrium in Electricity Market

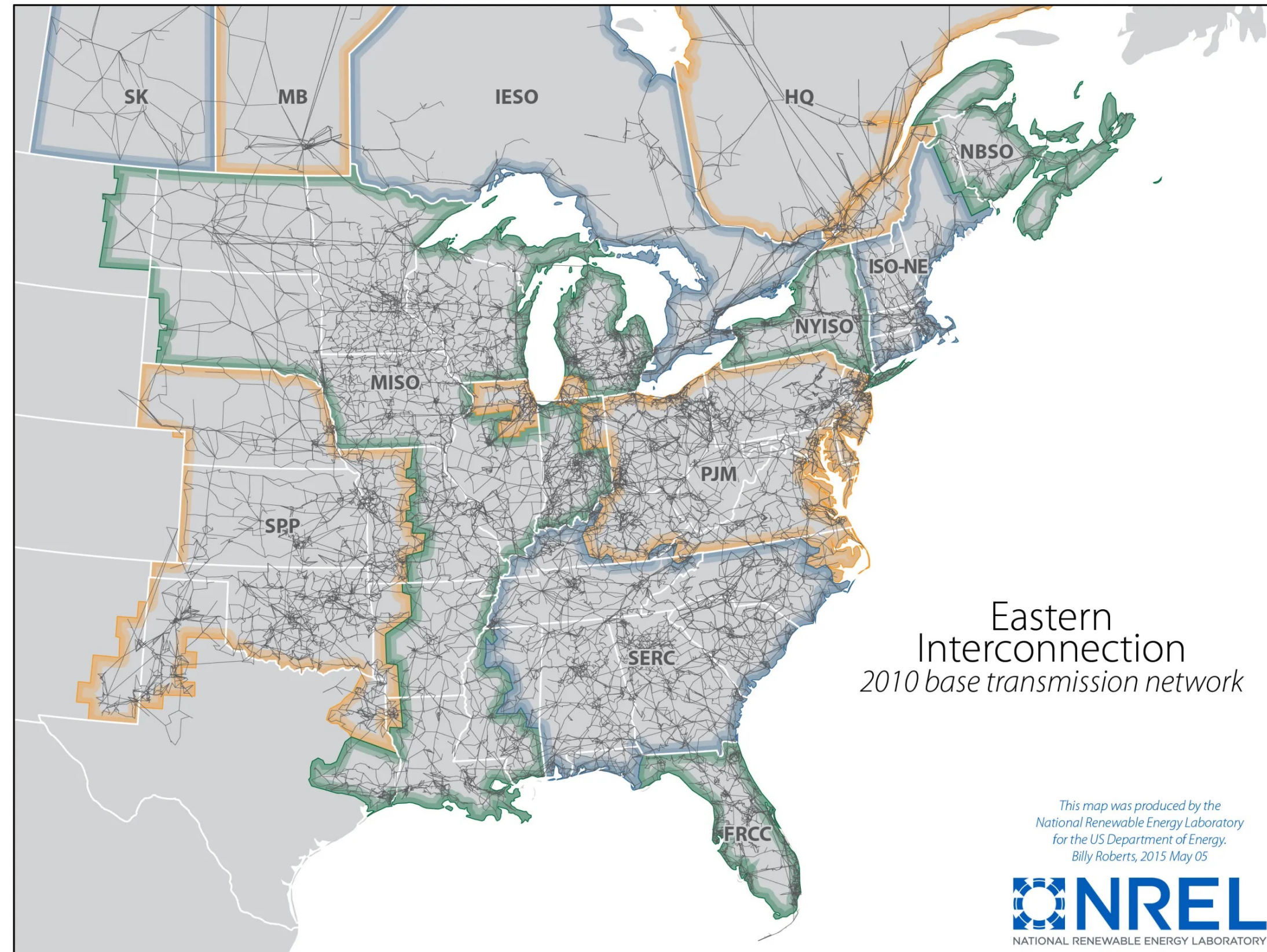
**Vladimir Dvorkin**

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University of Michigan

CONTROL SEMINAR

Ann Arbor, MI

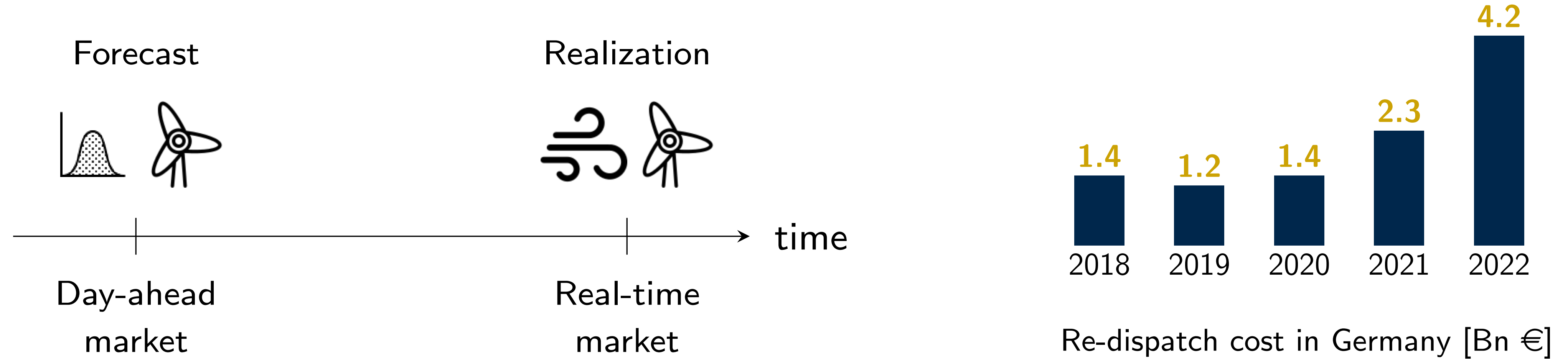
November 1, 2024



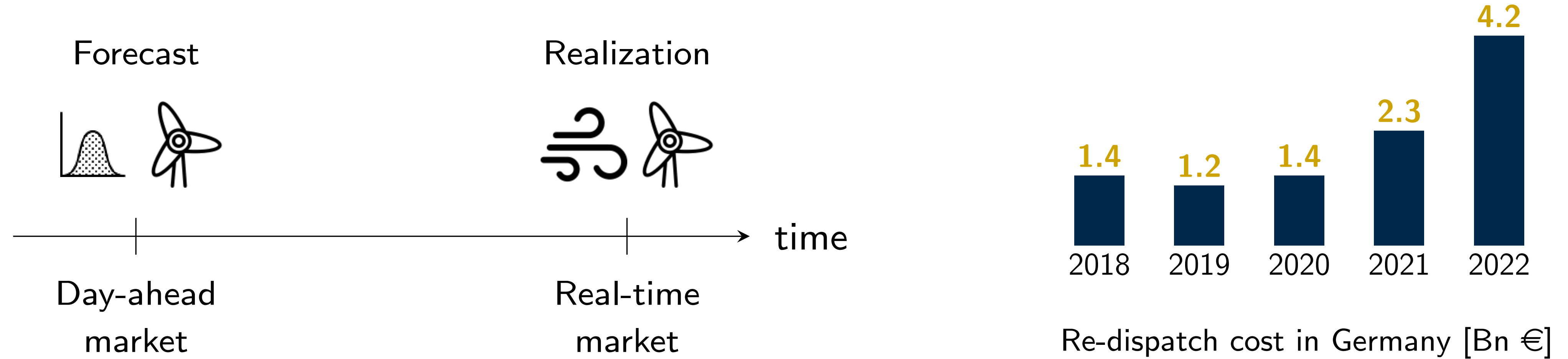
- ▶ The world's largest machine
- ▶ Mathematical programming (optimization) is a major computational tool for power grids:
  - ▶ Operational and long-term planning
  - ▶ Electricity market-clearing auctions
  - ▶ ED, UC, SCUC, PF, OPF, ...
- ▶ What makes electricity such a special commodity?
  - + Homogeneous good with instantaneous delivery
  - Requires very sophisticated infrastructure
  - Limited storage capacity → balance at all times
  - Only marginal % of electricity is traded in real-time; the majority – well ahead of operations.



- ▶ Two-stage electricity markets to manage uncertainty of renewables:
  - ▶ Day-ahead market: minimize the cost of power supply w.r.t. forecast
  - ▶ Real-time market: least-cost re-dispatch to accommodate forecast errors
  
- ▶ As renewable penetration increases, the cost of real-time re-dispatch also increases

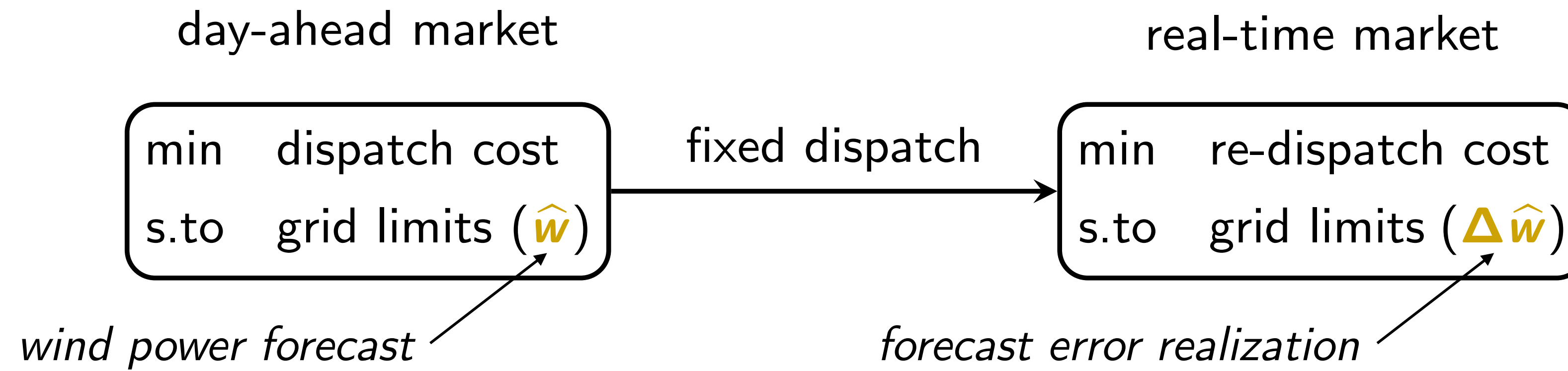


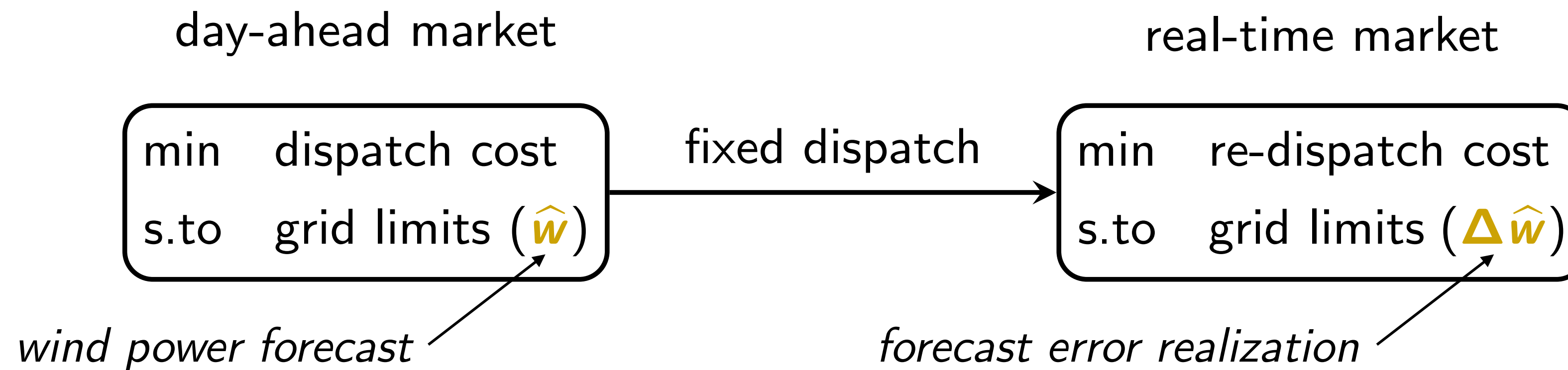
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**How to make renewable power generation less expensive for the system?**



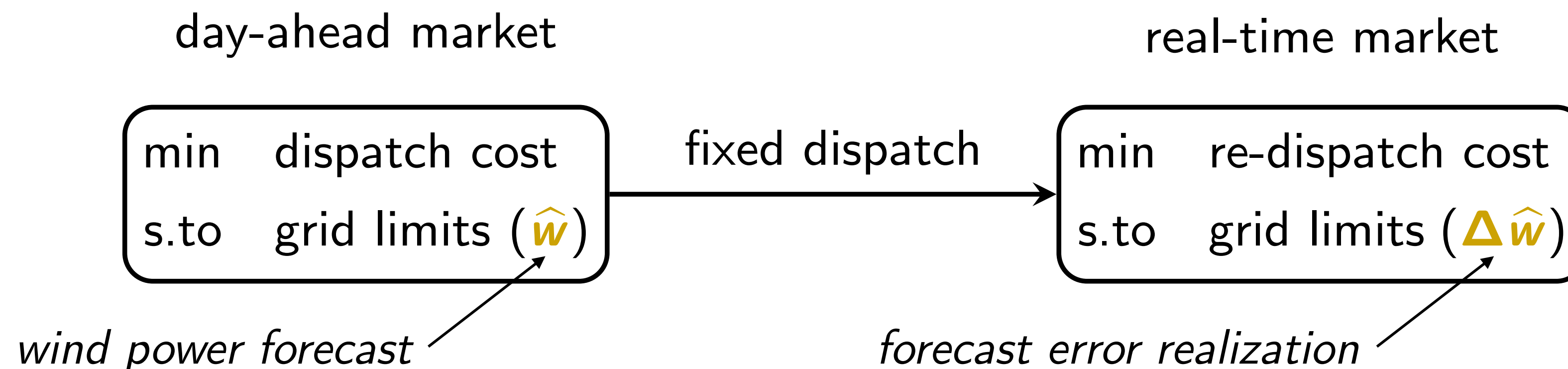


## Improving cost efficiency across day-ahead and real-time markets:

► Stochastic electricity market design [PZP10, M<sup>+</sup>12, Dvo19]:

- + Co-optimization of dispatch and re-dispatch decisions
- + Least-cost solution in *expectation*
- Market properties only hold in *expectation*

$$\begin{array}{l} \min \quad \text{dispatch cost} + \mathbb{E}_{\mathbb{P}_{\Delta \hat{w}}} [\text{re-dispatch cost}] \\ \text{s.to} \quad \text{grid limits } (\hat{w}, \Delta \hat{w}) \text{ for all } \Delta \hat{w} \sim \mathbb{P}_{\Delta \hat{w}} \end{array}$$



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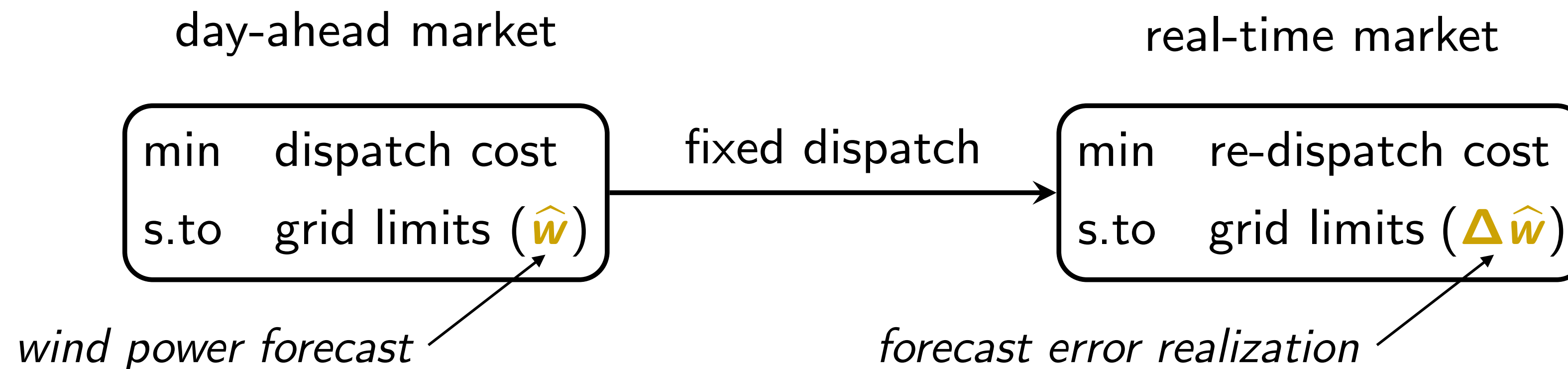
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### ► Approximating stochastic market efficiency within deterministic markets:

- Improved scheduling of renewables [M<sup>+</sup>14]
- Cost-aware reserve requirements [DDM18]
- Cost-aware transmission allocation [JKP17, DP19]





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**In our work, we follow this path with  
focused on advanced data analytics  
(machine learning)**

Typical grid optimization problem:

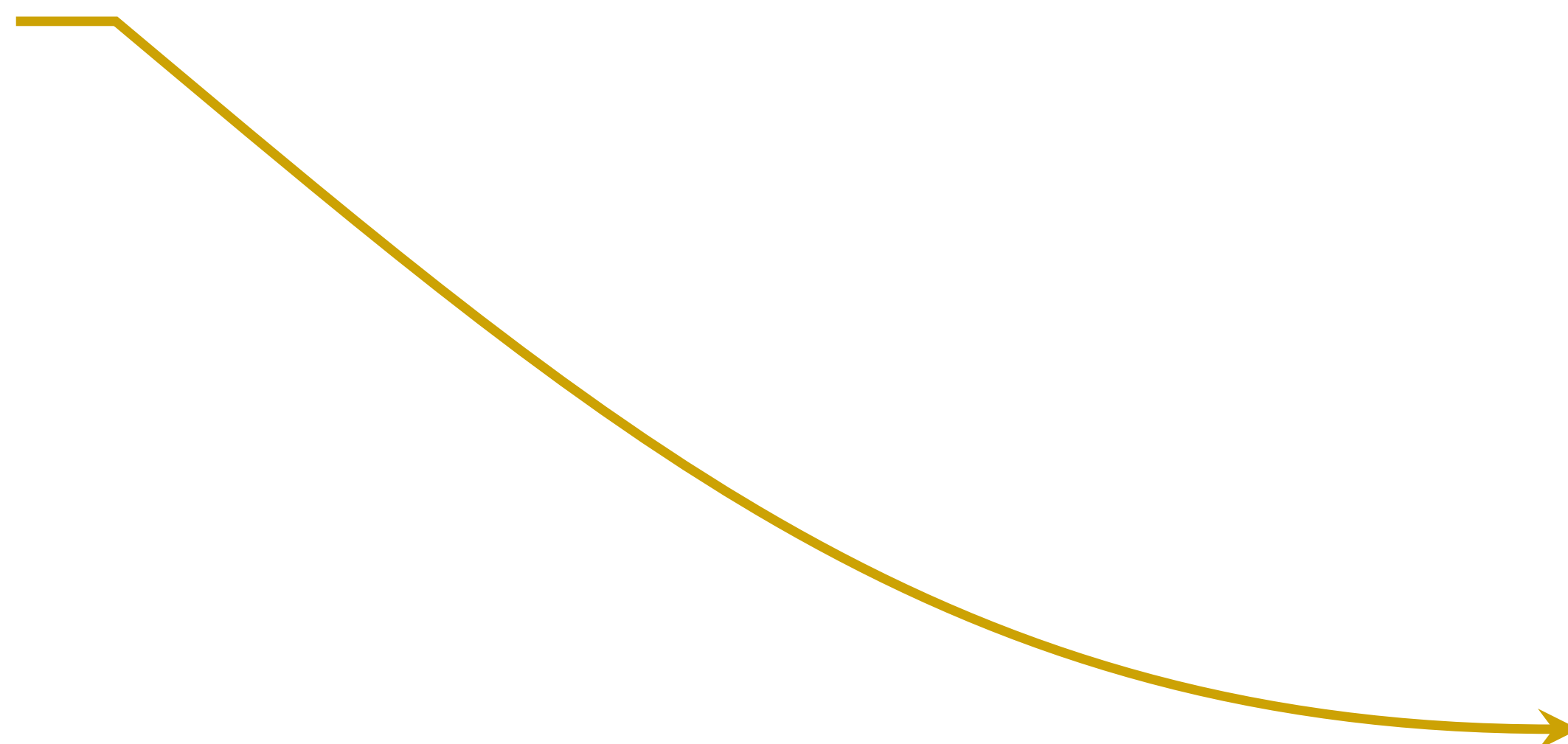
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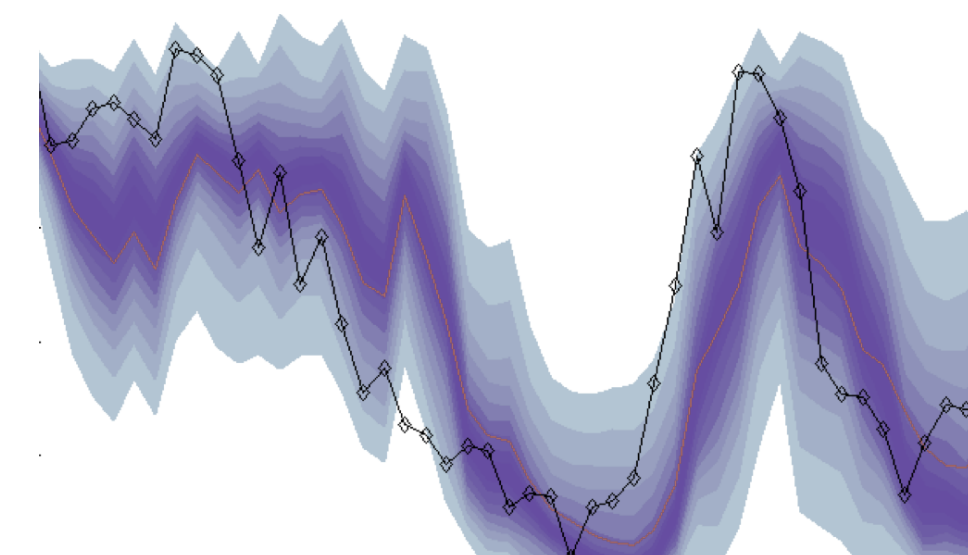
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Net load forecasting [PCK07, ZHS24]



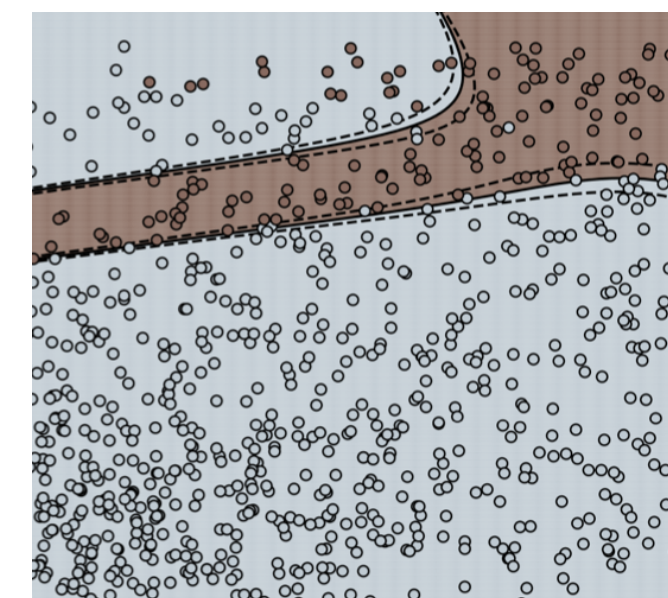
Forecast minimizing the cost of re-dispatch

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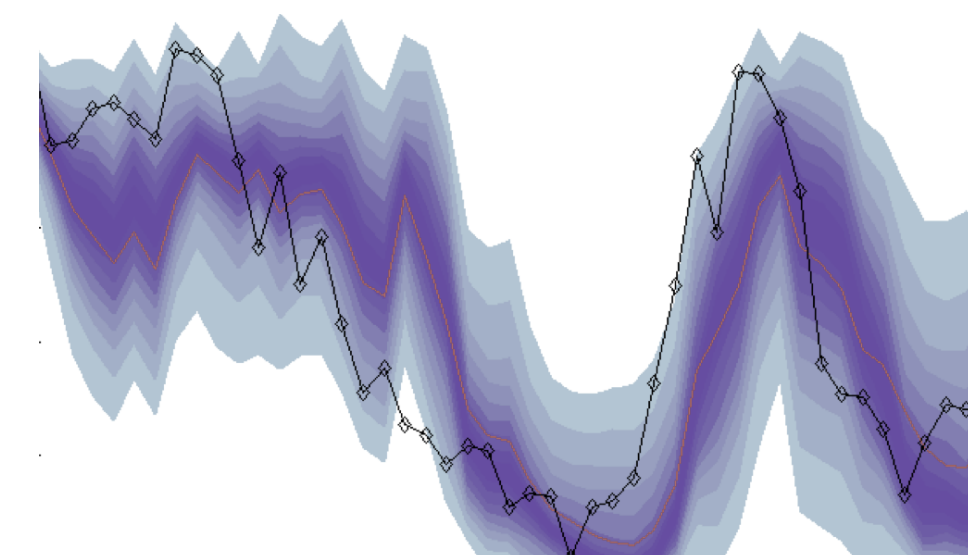
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SVM-based prediction of on/off gen. status [PK24]

Efficient warm start for MIP solvers



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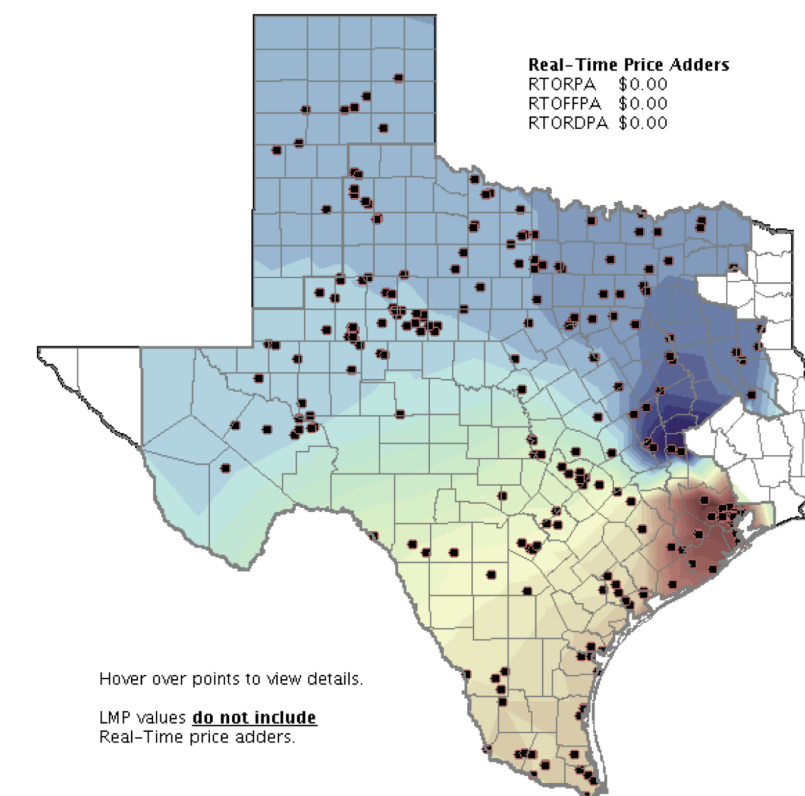
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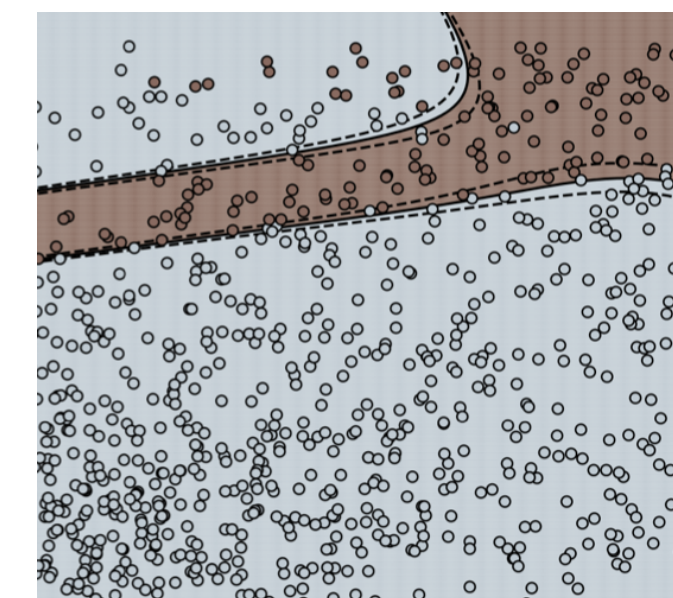
Real-time electricity pricing via GNNs [LWZ21]

Real-time pricing at ultrafast time scales

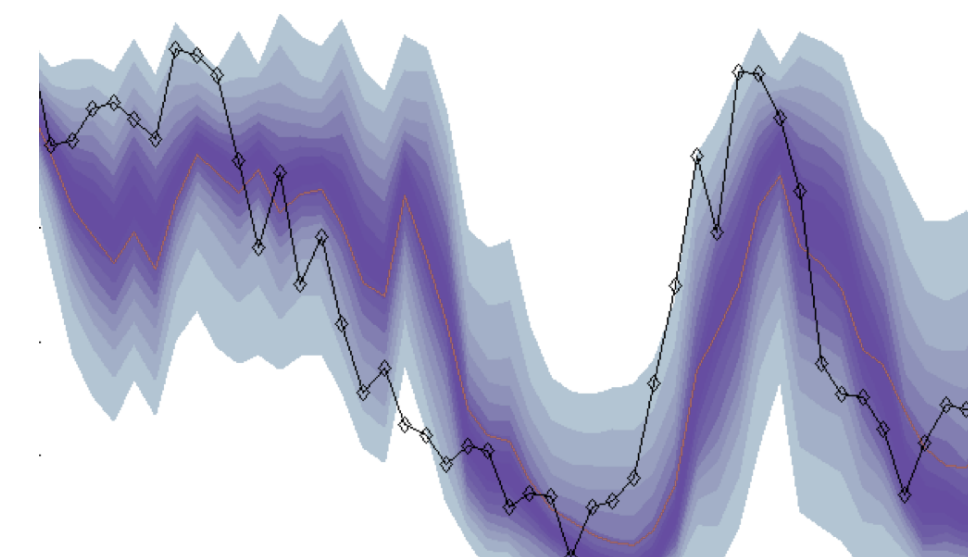


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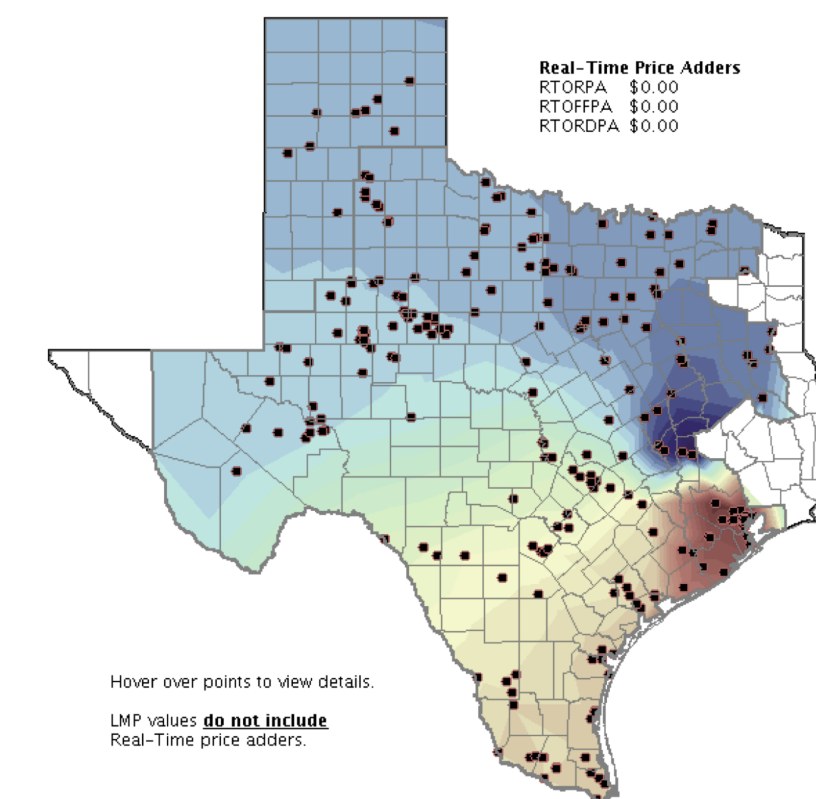
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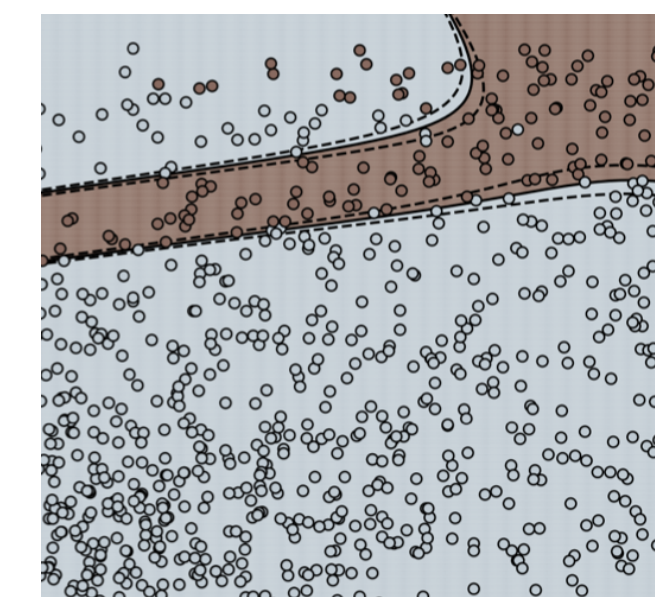
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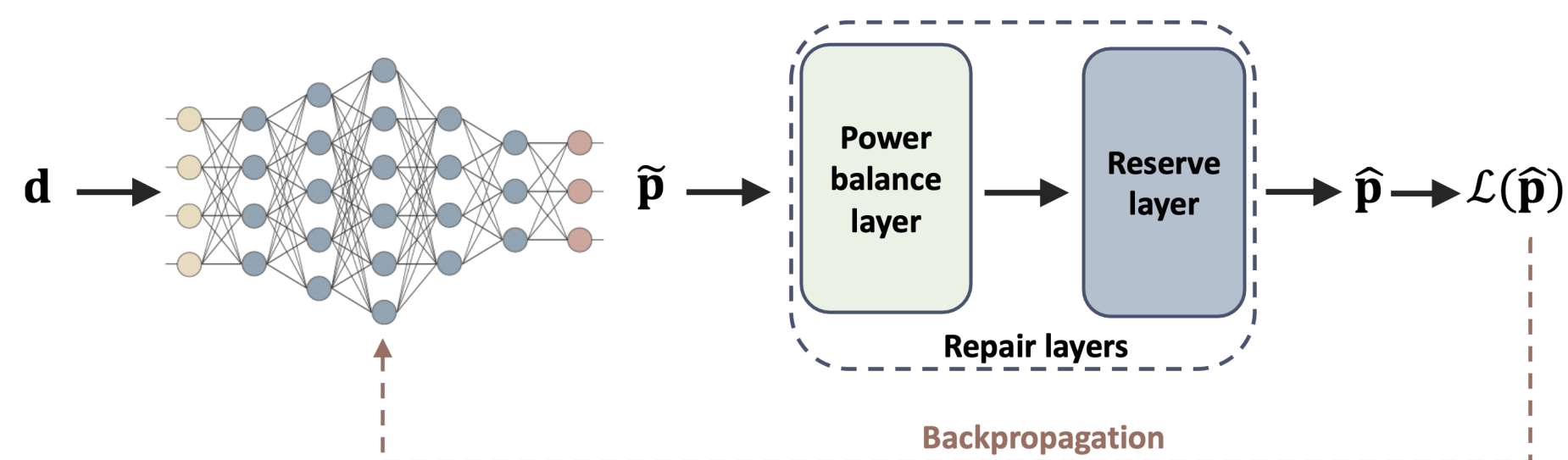
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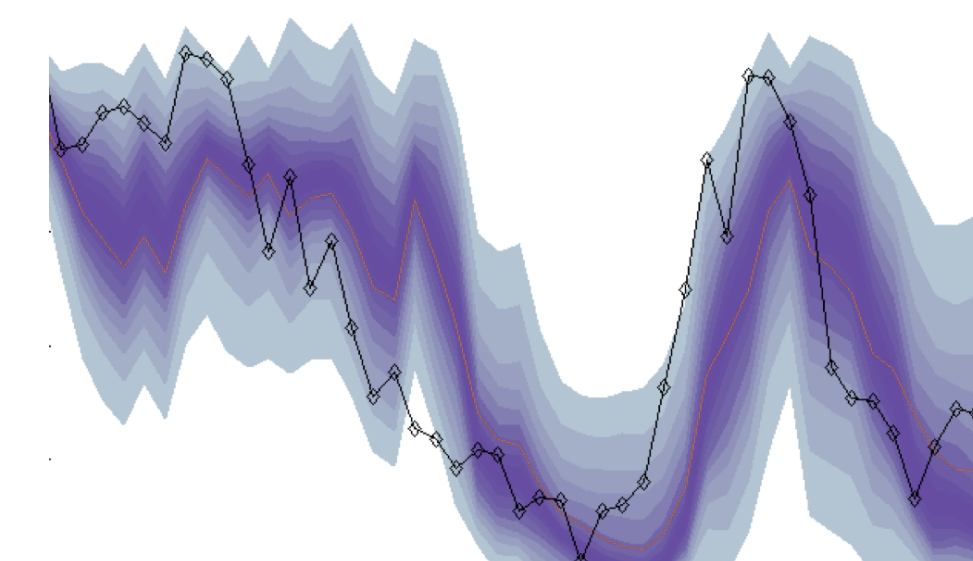


Optimal generation dispatch prediction using optimization proxies [CTVH23]

Net load forecasting [PCK07, ZHS24]



Extremely fast contingency screening



Forecast minimizing the cost of re-dispatch

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## This talk is on the dual role of machine learning in grid operations:

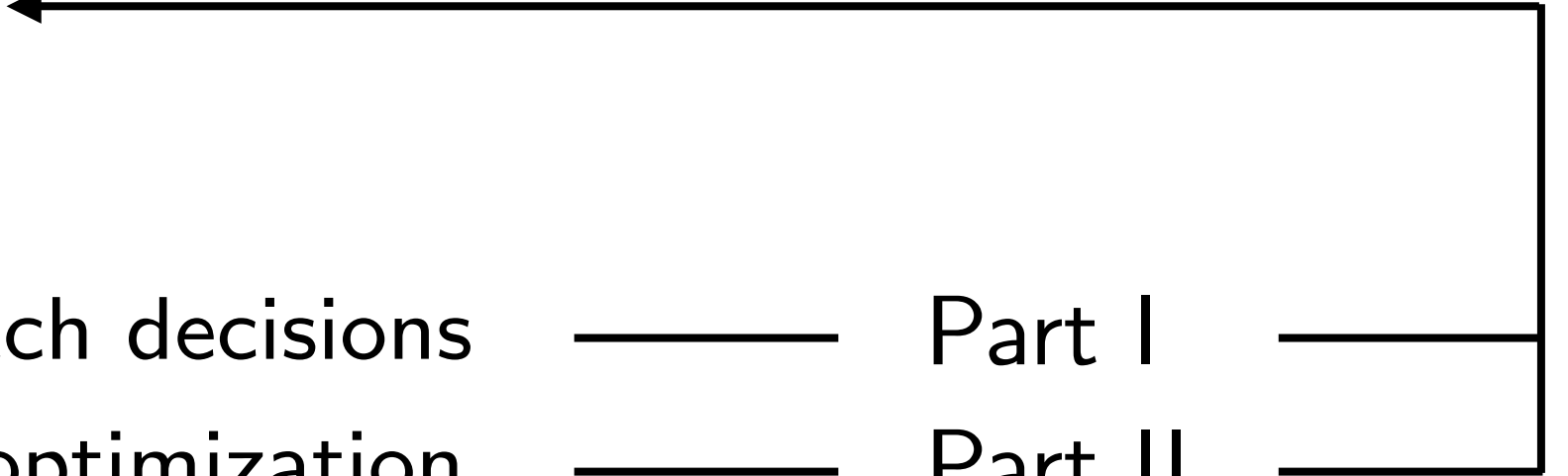
- ▶ Significant speed up and reported efficiency gains
  - ▶ Assisting decisions on ultrafast time scale
  - ▶ Approximates the efficiency of stochastic market design
- ▶ Imperfect predictions often lead to decision errors
  - ▶ Minimizing the impact of ML errors on pricing and dispatch decisions
  - ▶ Achieving equilibrium among ML models applied to grid optimization



## Typical grid optimization problem:

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- ▶ Significant speed up and reported efficiency gains
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    - ▶ Minimizing the impact of ML errors on pricing and dispatch decisions ——— Part I ———
    - ▶ Achieving equilibrium among ML models applied to grid optimization ——— Part II ———
- 

**Introduction**

**Controlling the impact of ML errors on electricity pricing**

**Nash equilibrium of ML models in electricity markets**

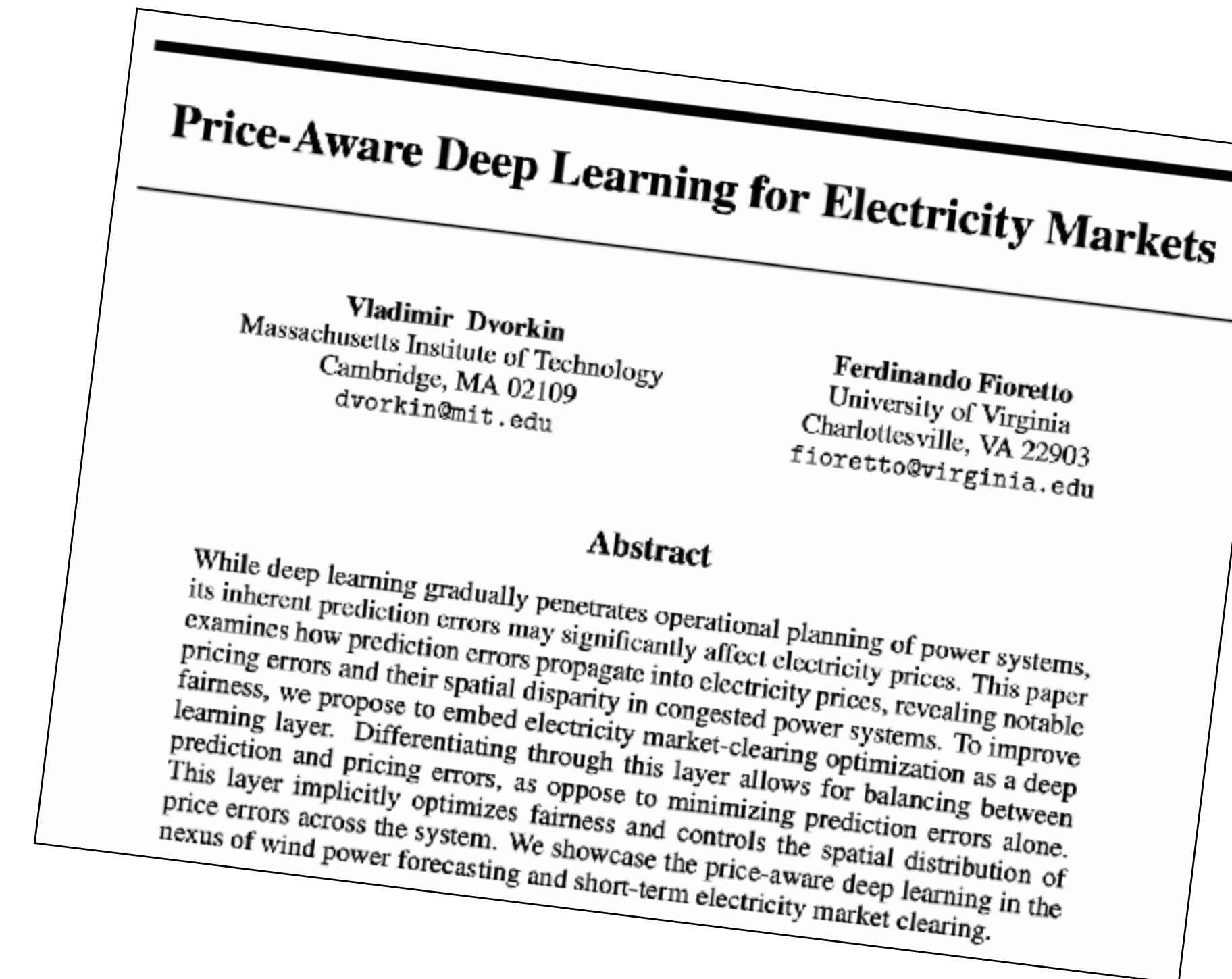
**Concluding remarks**

Introduction

Controlling the impact of ML errors on electricity pricing

Nash equilibrium of ML models in electricity markets

Concluding remarks



## DC optimal power flow:

$$\begin{aligned} \min_{\underline{p} \leq p \leq \bar{p}} \quad & \mathbf{p}^\top \mathbf{C} \mathbf{p} + \mathbf{c}^\top \mathbf{p} \\ \text{s.to} \quad & \mathbf{1}^\top (\mathbf{p} + \hat{\mathbf{w}} - \mathbf{d}) = \mathbf{0} : \lambda_b \\ & |\mathbf{F}(\mathbf{p} + \hat{\mathbf{w}} - \mathbf{d})| \leq \bar{\mathbf{f}} : \lambda_{\bar{f}}, \lambda_{\underline{f}} \end{aligned}$$

- ▶ Electricity market clearing based on DC-OPF
- ▶ Relies on the *forecast*  $\hat{\mathbf{w}}$  of wind power generation
- ▶ Forecast errors  $\rightarrow$  pricing errors via market optimization
- ▶ May not be a dominant generation resource, yet still exposes the entire electricity trading to errors

## Locational marginal prices:

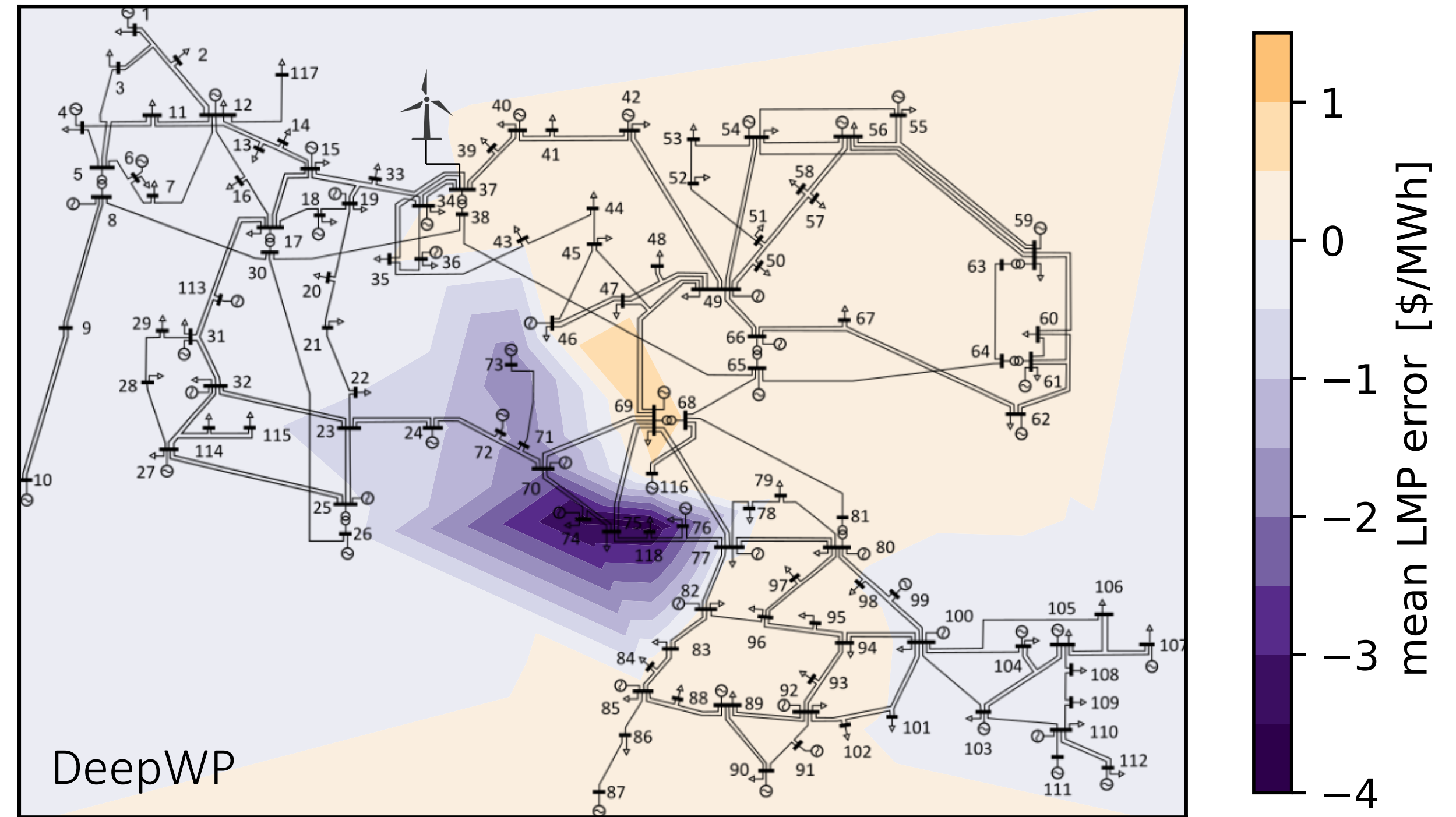
$$\lambda(\hat{\mathbf{w}}) = \underbrace{\mathbf{1} \cdot \lambda_b(\hat{\mathbf{w}})}_{\text{uniform part}} - \underbrace{\mathbf{F}^\top (\lambda_{\bar{f}}(\hat{\mathbf{w}}) - \lambda_{\underline{f}}(\hat{\mathbf{w}}))}_{\text{congestion part}}$$

DC optimal power flow:

$$\begin{aligned} \min_{\underline{p} \leq p \leq \bar{p}} \quad & p^\top C p + c^\top p \\ \text{s.to} \quad & \mathbf{1}^\top (p + \hat{w} - d) = 0 : \lambda_b \\ & |F(p + \hat{w} - d)| \leq \bar{f} : \lambda_{\bar{f}}, \lambda_{\underline{f}} \end{aligned}$$

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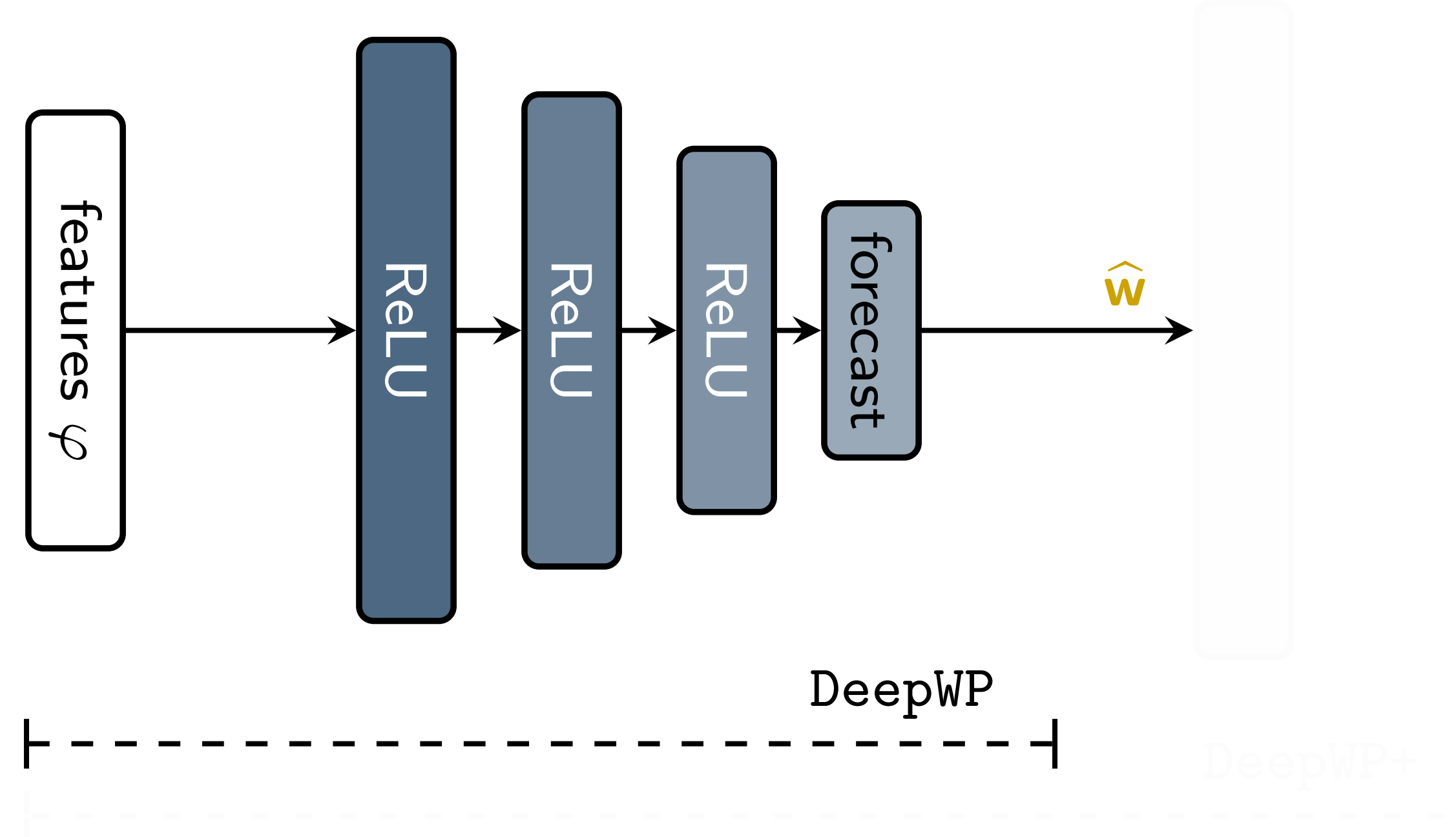
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Forecast errors from a single wind power plant propagate into locational marginal price (LMP) errors across the IEEE 118-Bus RTS. Electricity at certain buses is systematically over- or under-priced [DF23].

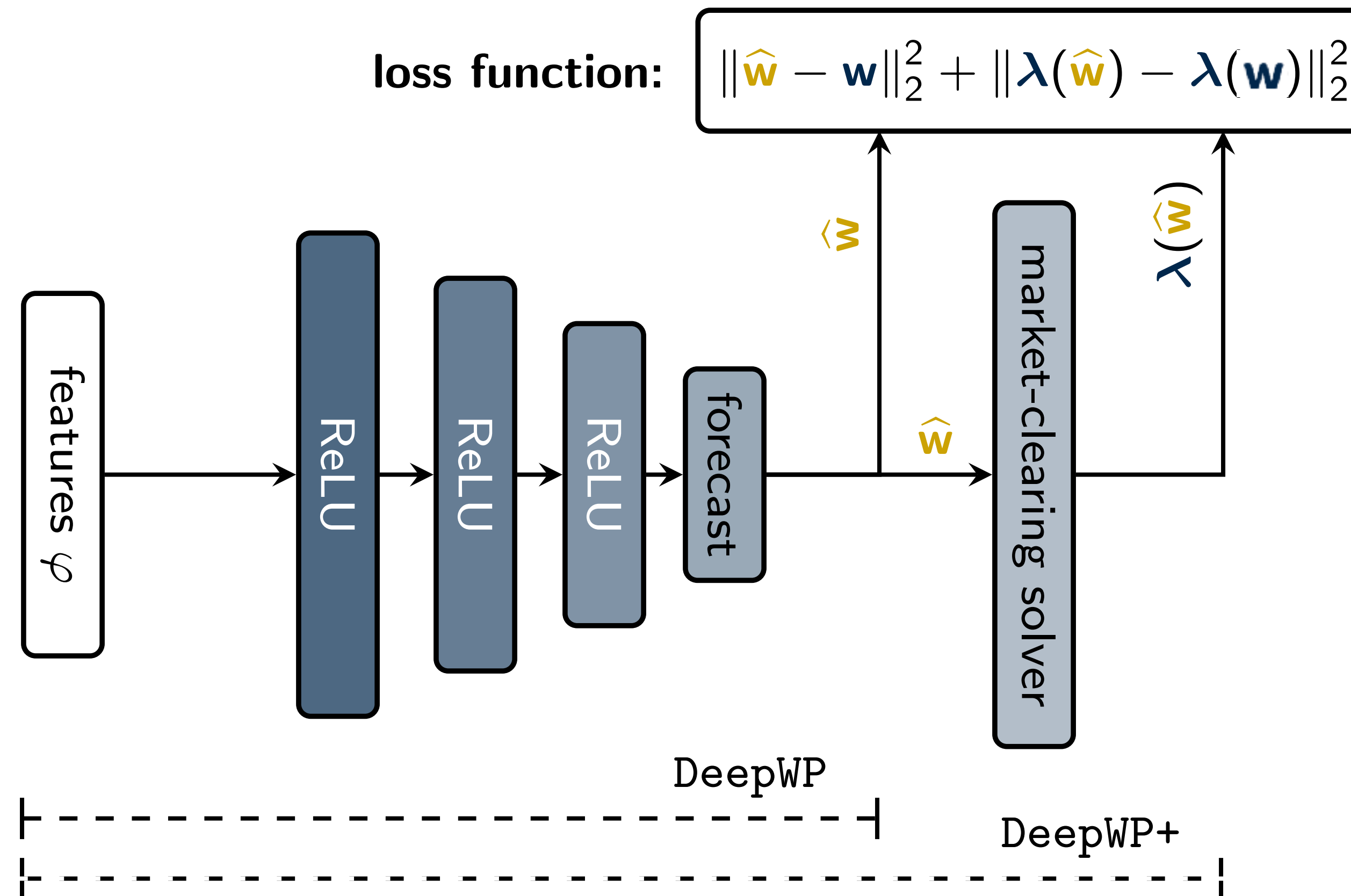
- ▶ Dataset  $\{(\varphi_1, \mathbf{w}_1), \dots, (\varphi_m, \mathbf{w}_m)\}$  of wind power records, with features  $\varphi$  and measurements  $\mathbf{w}$
- ▶ Two deep learning architectures **DeepWP** and **DeepWP+** for wind power forecasting:

loss function:  $\|\hat{\mathbf{w}} - \mathbf{w}\|_2^2$



▶ DeepWP+ incorporates market clearing as an optimization layer [AK17], which informs on pricing errors

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- ▶ DeepWP+ incorporates market clearing as an optimization layer [AK17], which informs on pricing errors

- ▶ Market-clearing optimization layer significantly slows down the training
- ▶ Thousands of market-clearing problems are solved at each training epoch
- ▶ We use QP duality and fast distributed algorithms to speed up the process

## Market-clearing optimization

$$\begin{aligned} \min_{\underline{\mathbf{p}} \leq \mathbf{p} \leq \bar{\mathbf{p}}} \quad & \mathbf{p}^\top \mathbf{C} \mathbf{p} + \mathbf{c}^\top \mathbf{p} \\ \text{s.to} \quad & \mathbf{1}^\top (\mathbf{p} + \hat{\mathbf{w}} - \mathbf{d}) = 0 \\ & |\mathbf{F}(\mathbf{p} + \hat{\mathbf{w}} - \mathbf{d})| \leq \bar{\mathbf{f}} \end{aligned}$$

large constrained optimization



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Market-clearing optimization  $\implies$  Equivalent primal form

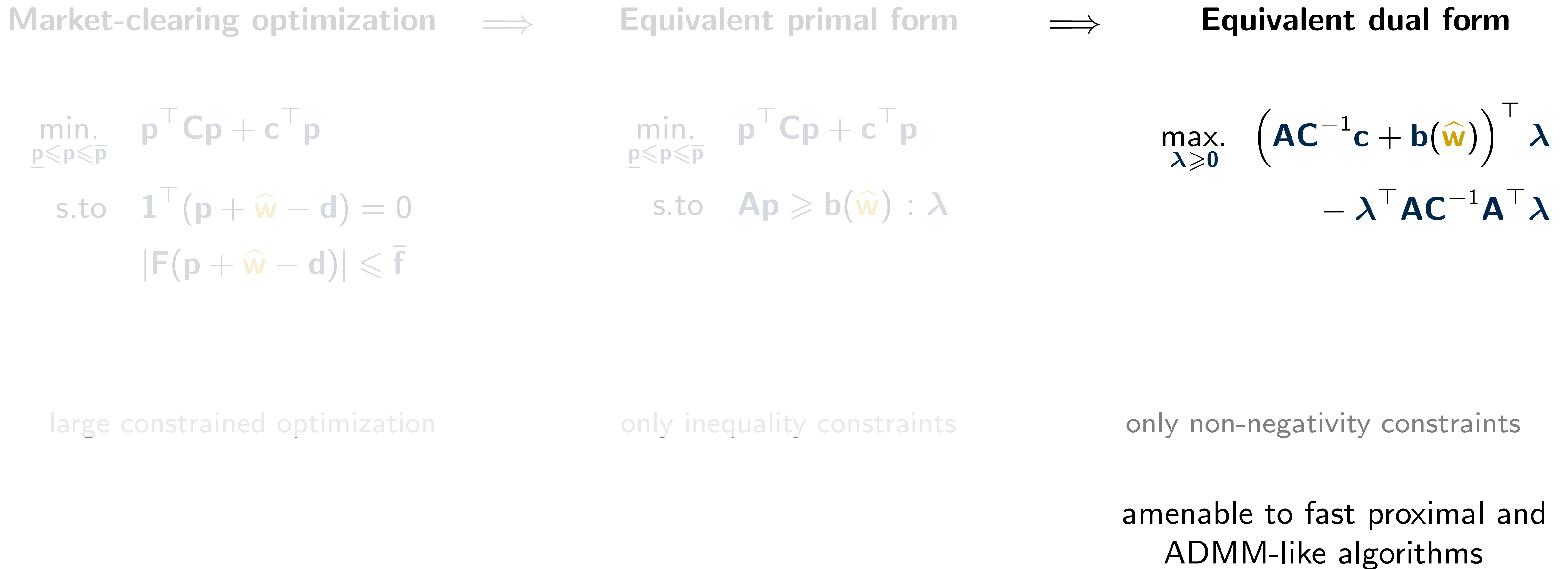
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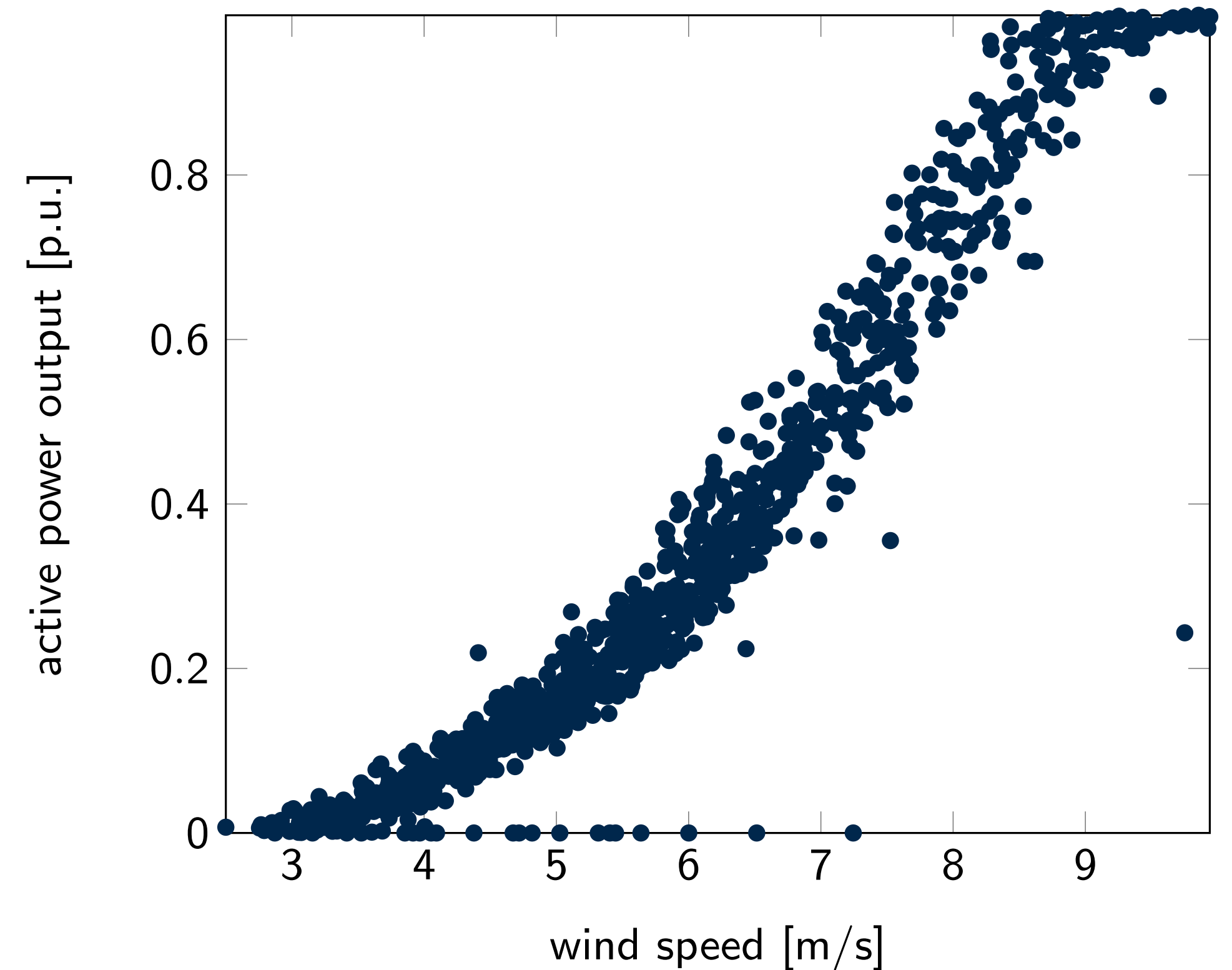
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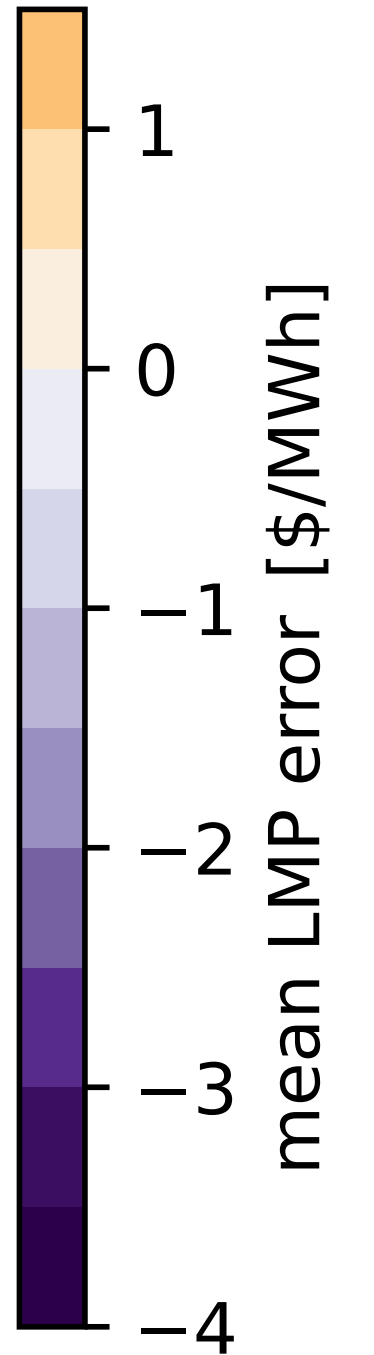
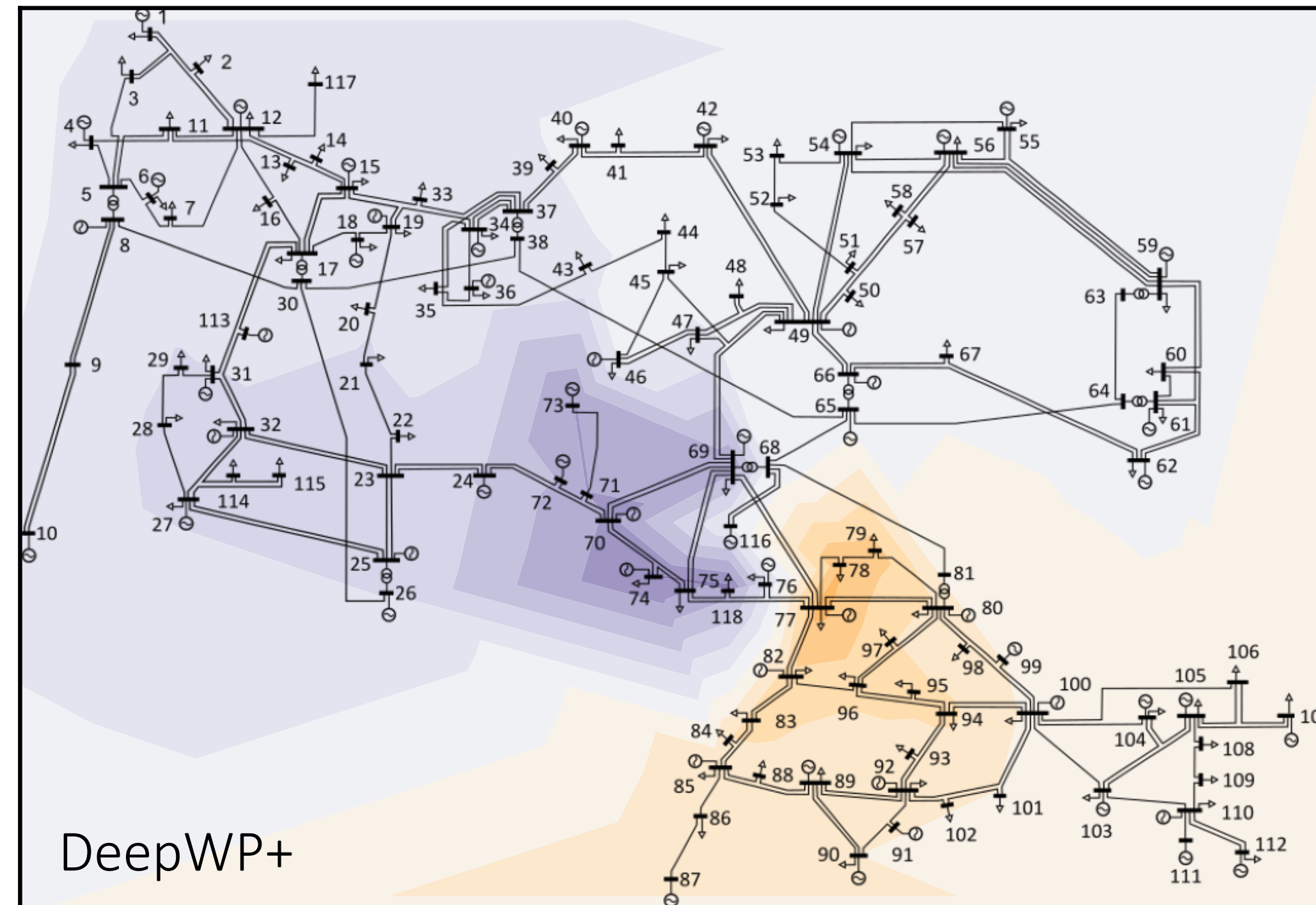
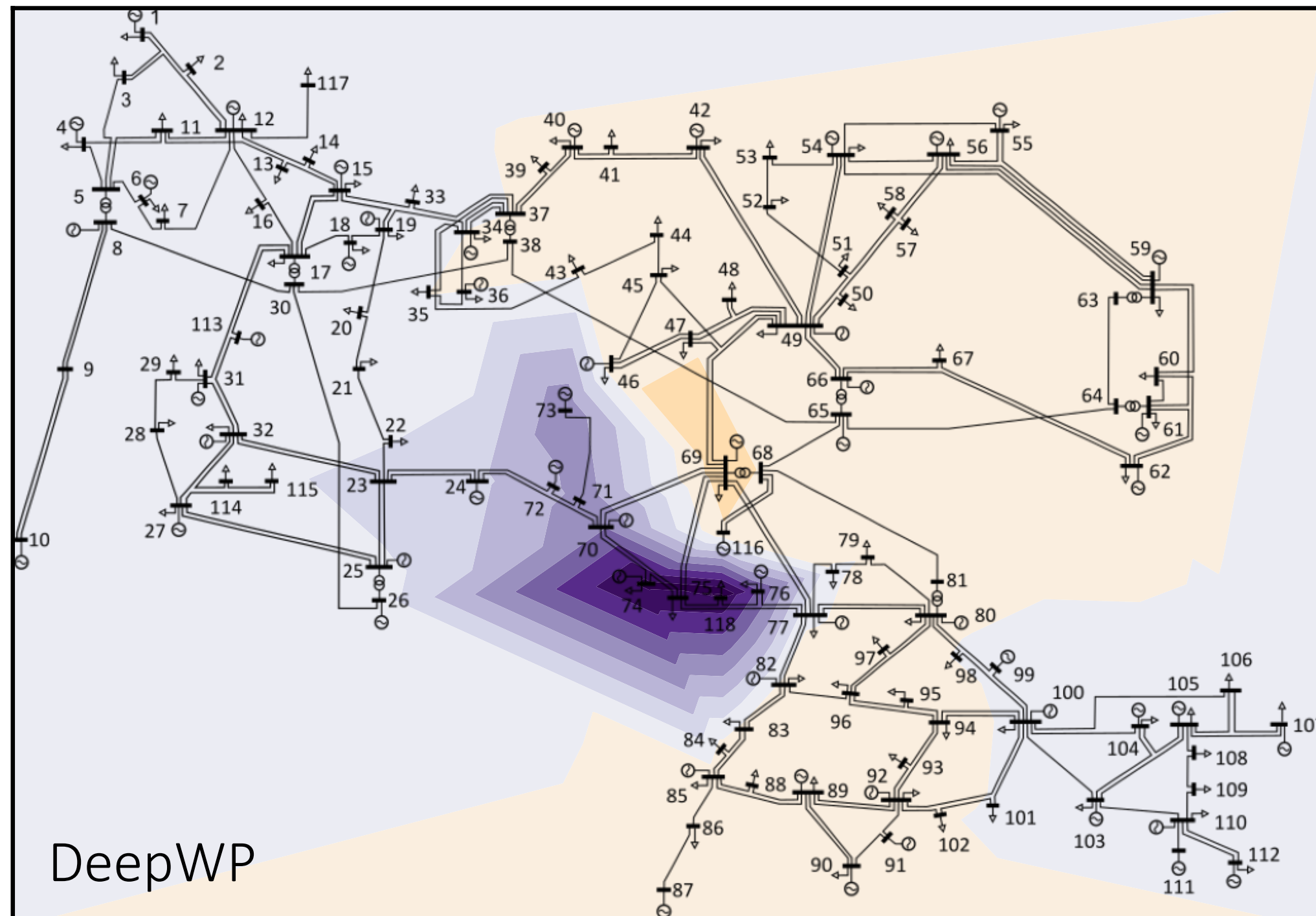
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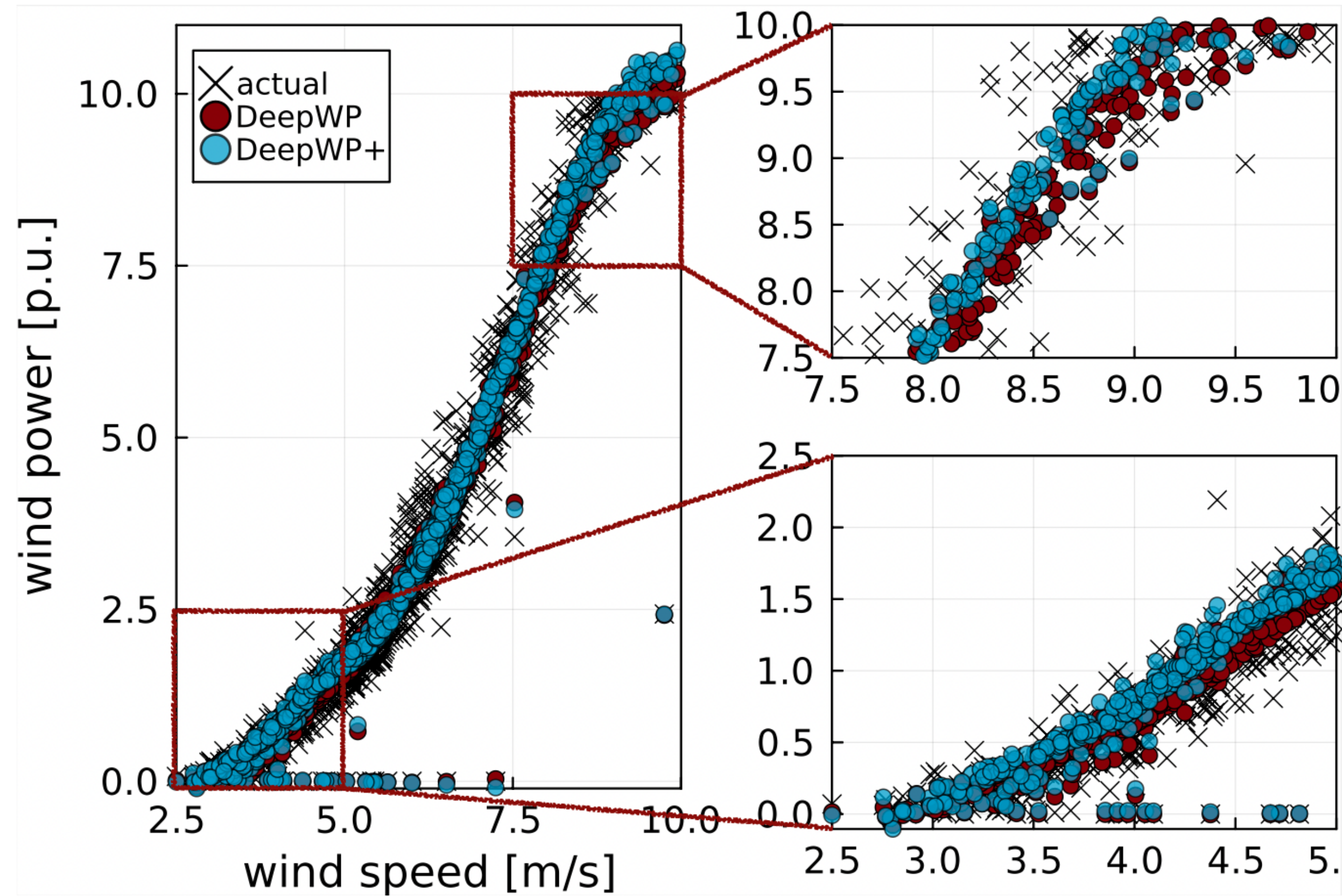
- ▶ 1,000 wind power records from a real wind power turbine:
  - ▶ Active power output
  - ▶ Wind speed and direction
  - ▶ Blade pitch angle
- ▶ DeepWP has 4 hidden layers with 30 neurons each. DeepWP+ additionally includes an opt. layer
- ▶ ADAM optimizer with varying learning rate





**DeepWP**: Forecast error objective – LMP errors  $[-4, 1]$  \$/MWh

**DeepWP+**: LMP error objective – LMP errors  $[-1, 1]$  \$/MWh



**DeepWP**: Minimizes the average forecast deviation

**DeepWP+**: Intentionally over-predicts in certain range of wind speeds

case	DeepWP			DeepWP+					
	RMSE( $\hat{w}$ )	RMSE( $\hat{\lambda}$ )	CVaR( $\hat{\lambda}$ )	RMSE( $\hat{w}$ )		RMSE( $\hat{\lambda}$ )		CVaR( $\hat{\lambda}$ )	
	MWh	\$/MWh	\$/MWh	MWh	gain	\$/MWh	gain	\$/MWh	gain
14_ieee	0.35	0.62	1.52	0.35	+0.6%	0.61	-0.6%	1.50	-0.8%
57_ieee	2.31	11.03	34.64	2.60	+11.2%	10.72	-2.9%	33.59	-3.1%
24_ieee	4.08	8.62	37.70	4.51	+9.6%	8.33	-3.5%	36.35	-3.7%
39_epri	5.94	11.15	31.21	6.43	+7.6%	10.19	-9.4%	28.02	-11.4%
73_ieee	4.02	5.12	16.21	5.51	+26.9%	4.24	-20.8%	13.41	-20.9%
118_ieee	2.29	3.59	11.32	2.60	+12.1%	2.88	-24.7%	9.06	-25.0%

- ▶ Price errors reduction comes at the expense of forecast error
- ▶ Price error reduction is more significant in larger networks

For more results, including **price fairness**: <https://arxiv.org/pdf/2308.01436>

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14_ieee	0.35	0.62	1.52	0.35	+0.6%	0.61	-0.6%	1.50	-0.8%
57_ieee	2.31	11.03	34.64	2.60	+11.2%	10.72	-2.9%	33.59	-3.1%
24_ieee	4.08	8.62	37.70	4.51	+9.6%	8.33	-3.5%	36.35	-3.7%
39_epri	5.94	11.15	31.21	6.43	+7.6%	10.19	-9.4%	28.02	-11.4%
73_ieee	4.02	5.12	16.21	5.51	+26.9%	4.24	-20.8%	13.41	-20.9%
118_ieee	2.29	3.59	11.32	2.60	+12.1%	2.88	-24.7%	9.06	-25.0%

- ▶ Price errors reduction comes at the expense of forecast error
- ▶ Price error reduction is more significant in larger networks

For more results, including **price fairness**: <https://arxiv.org/pdf/2308.01436>

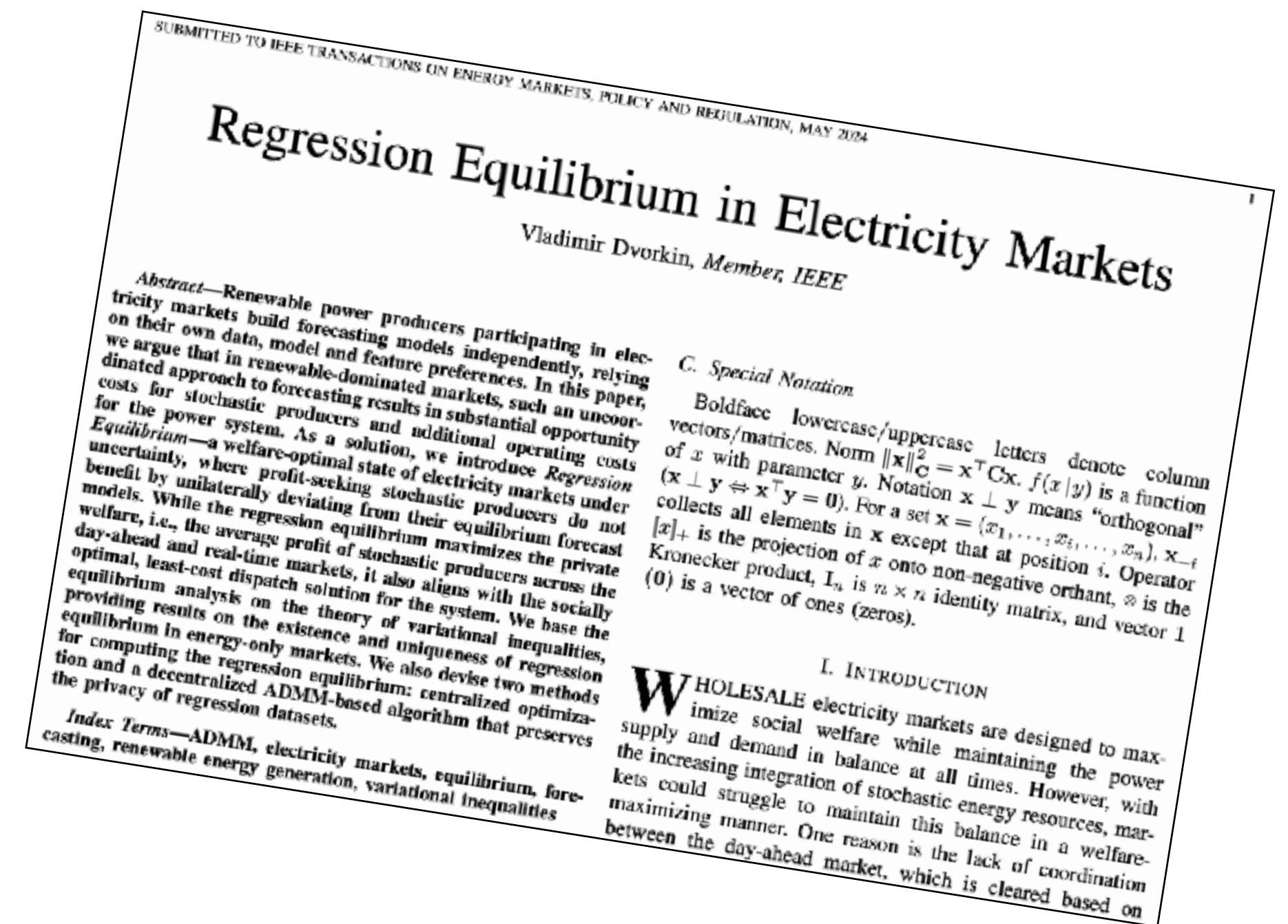


Introduction

Controlling the impact of ML errors on electricity pricing

Nash equilibrium of ML models in electricity markets

Concluding remarks



## Baseline approach to wind power forecasting:

- ▶ Collect a training dataset  $\mathcal{D} = \{(\varphi_1, \mathbf{w}_1), \dots, (\varphi_n, \mathbf{w}_n)\}$
- ▶ Machine learning model  $\mathbb{W}_\theta : \mathcal{F} \mapsto \mathcal{W}$  with parameter  $\theta$
- ▶ Learn optimal parameter  $\theta^*$  by minimizing a prediction loss

$$\min_{\|\theta\|_1 \leq \tau} \mathcal{L}(\theta | \mathcal{D}) = \frac{1}{n} \sum_{i=1}^n \|\mathbb{W}_\theta(\varphi_i) - \mathbf{w}_i\|_2^2$$

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## Revenue-optimal forecasting [PCK07, CK19, WSC23]:

1. Day-ahead stage: LMP  $\lambda_1$  pricing the forecast of wind power
2. Real-time stage: LMP  $\lambda_2$  pricing any forecast deviation

$$\max_{\|\theta\|_1 \leq \tau} \mathcal{R}^W(\theta | \mathcal{D}, \lambda_1, \lambda_2) = \frac{1}{n} \sum_{i=1}^n \left( \underbrace{\lambda_{1i} \mathbb{W}_\theta(\varphi_i)}_{\text{day-ahead revenue}} + \underbrace{\lambda_{2i} (\mathbf{w}_i - \mathbb{W}_\theta(\varphi_i))}_{\text{real-time revenue}} \right)$$

## Optimization of wind power producers

$$\max_{\|\boldsymbol{\theta}_j\| \leq \tau_j} \mathcal{R}^W(\boldsymbol{\theta}_j | \mathcal{D}_j, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) - \gamma \cdot \mathcal{L}(\boldsymbol{\theta}_j | \mathcal{D}_j)$$

for all producers  $j \in 1, \dots, b$

## Optimization of wind power producers

$$\max_{\|\theta_j\| \leq \tau_j} \mathcal{R}^W(\theta_j | \mathcal{D}_j, \lambda_1, \lambda_2) - \gamma \cdot \mathcal{L}(\theta_j | \mathcal{D}_j)$$

for all producers  $j \in 1, \dots, b$

feature selection

regularization  
by prediction loss

expected revenue

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## Optimization of controllable generators

$$\max_{\mathbf{p}_i, \mathbf{r}_i \in \mathcal{G}} \mathcal{R}^G(\mathbf{p}_i, \mathbf{r}_i | \boldsymbol{\lambda}_{1i}, \boldsymbol{\lambda}_{2i}) - c(\mathbf{p}_i, \mathbf{r}_i)$$

for all training samples  $i \in \mathcal{D}_{1:b}$

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for all producers  $j \in 1, \dots, b$

revenue from generation and regulation

## Optimization of controllable generators

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generation and regulation cost

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Others



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for all training samples  $i \in \mathcal{D}_{1:b}$

Others

## Market-clearing conditions at the 1<sup>st</sup> and 2<sup>nd</sup> stages

$$\mathbf{0} \leq \boldsymbol{\lambda}_{1i} \perp \mathbf{p}_i + \sum_{j=1}^b \mathbb{W}_{\boldsymbol{\theta}_j}(\boldsymbol{\varphi}_i) - \mathbf{d} \geq \mathbf{0}, \quad \mathbf{0} \leq \boldsymbol{\lambda}_{2i} \perp \mathbf{r}_i + \sum_{j=1}^b \mathbf{w}_{ji} - \sum_{j=1}^b \mathbb{W}_{\boldsymbol{\theta}_j}(\boldsymbol{\varphi}_i) \geq \mathbf{0}$$

for all training samples  $i \in \mathcal{D}_{1:b}$

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$$\max_{\|\boldsymbol{\theta}_j\| \leq \tau_j} \mathcal{R}^W(\boldsymbol{\theta}_j | \mathcal{D}_j, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) - \gamma \cdot \mathcal{L}(\boldsymbol{\theta}_j | \mathcal{D}_j)$$

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for all training samples  $i \in \mathcal{D}_{1:b}$

## Assumptions:

- ▶ Class of ML models  $\mathbb{W}_{\boldsymbol{\theta}}$  is **convex** in  $\boldsymbol{\theta}$ , e.g., kernel regression
- ▶ Training datasets are such that  $n \gg \text{card}[\boldsymbol{\varphi}]$  (unique regression solution)
- ▶ The intersection of private feasible regions is compact (at least one feasible dispatch  $\forall i \in \mathcal{D}_{1:b}$ )

**Main result:** regression equilibrium exists and is unique!

## Optimization of wind power producers

$$\max_{\|\theta_j\| \leq \tau_j} \mathcal{R}^W(\theta_j | \mathcal{D}_j, \lambda_1, \lambda_2) - \gamma \cdot \mathcal{L}(\theta_j | \mathcal{D}_j)$$

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for all training samples  $i \in \mathcal{D}_{1:b}$

**Equilibrium regression profile**  $\Theta^* = (\theta_1^*, \dots, \theta_b^*)$ , such that:

- ▶ Feasible operation of the power grid and markets
- ▶ Maximized wind power profits, with no incentives to deviate
- ▶ Minimized expected dispatch costs across the two markets

## Optimization of wind power producers

$$\max_{\|\theta_j\| \leq \tau_j} \mathcal{R}^W(\theta_j | \mathcal{D}_j, \lambda_1, \lambda_2) - \gamma \cdot \mathcal{L}(\theta_j | \mathcal{D}_j)$$

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Others

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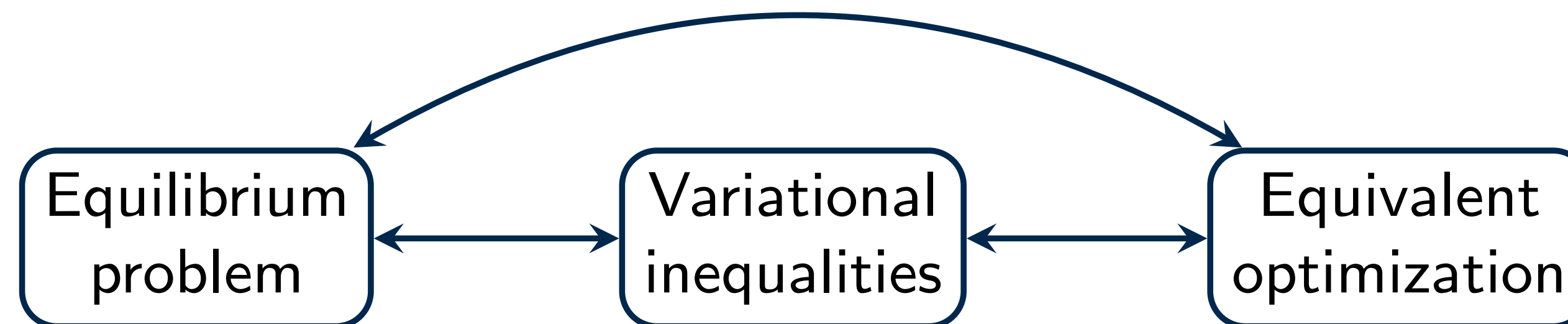
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**Equilibrium regression profile  $\Theta^* = (\theta_1^*, \dots, \theta_b^*)$ , such that:**

- ▶ Feasible operation of the power grid and markets
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**How to compute equilibrium regression  $\Theta^*$ ?**



- ▶ Equilibrium problem: stacks many private optimization problems
- ▶ Variational inequalities (VI): analyzes the interaction between private optimization problems
- ▶ In some special cases (like ours), VI connects equilibrium to a centralized optimization

For more details visit Appenix A in: <https://arxiv.org/pdf/2405.17753>

- ▶ By the Symmetry Principle Theorem [FP03], there exists an equivalent opt solving the equilibrium
- ▶ ... which happens to minimize the expected generation and regulation costs
- ▶ ... thus enhancing the temporal coordination of day-ahead and real-time markets

$$\begin{aligned}
 \min_{\Theta, \mathbf{p}, \mathbf{r}} \quad & \frac{1}{n} \sum_{i=1}^n c(\mathbf{p}_i, \mathbf{r}_i) + \gamma \|\Theta \boldsymbol{\varphi}_i - \mathbf{w}_i\|_2^2 && \text{Day-ahead + real-time cost} \\
 \text{s.to} \quad & \mathbf{1}^\top (\mathbf{p}_i + \Theta \boldsymbol{\varphi}_i - \mathbf{d}) = 0, && \text{regularized expected cost} \\
 & \mathbf{1}^\top (\mathbf{r}_i - \Theta \boldsymbol{\varphi}_i + \mathbf{w}_i) = 0, && \text{day-ahead power balance} \\
 & |\mathbf{F}(\mathbf{p}_i + \Theta \boldsymbol{\varphi}_i - \mathbf{d})| \leq \bar{\mathbf{f}}, && \text{real-time power balance} \\
 & |\mathbf{F}(\mathbf{p}_i + \Theta \boldsymbol{\varphi}_i - \mathbf{d}) \\
 & \quad + \mathbf{F}(\mathbf{r}_i - \Theta \boldsymbol{\varphi}_i + \mathbf{w}_i)| \leq \bar{\mathbf{f}}, && \text{day-ahead power flow limit} \\
 & \underline{\mathbf{p}} \leq \mathbf{p}_i + \mathbf{r}_i \leq \bar{\mathbf{p}}, && \text{real-time power flow limit} \\
 & |\mathbf{r}_i| \leq \bar{\mathbf{r}}, \quad \forall i = 1, \dots, n, && \text{generation limit} \\
 & |\Theta| \leq \tau && \text{regulation limit} \\
 & && \text{equilibrium feature selection}
 \end{aligned}$$

- ▶ For more details visit Appendix A in: <https://arxiv.org/pdf/2405.17753>

Step 1 Primal update of each wind power producer:

$$\boldsymbol{\theta}^k \leftarrow \max_{\|\boldsymbol{\theta}\| \leq \tau} \mathcal{R}^W(\boldsymbol{\theta} | \mathcal{D}, \boldsymbol{\lambda}_1^k, \boldsymbol{\lambda}_2^k) - \gamma \cdot \mathcal{L}(\boldsymbol{\theta} | \mathcal{D})$$

Step 2 Primal update of each conventional generator:

$$\mathbf{p}^k, \mathbf{r}^k \leftarrow \max_{\mathbf{p}, \mathbf{r} \in \mathcal{G}} \mathcal{R}^G(\mathbf{p}, \mathbf{r} | \boldsymbol{\lambda}_1^k, \boldsymbol{\lambda}_2^k) - c(\mathbf{p}, \mathbf{r})$$

Step 3 Electricity price updates:

$$\boldsymbol{\lambda}_1^{k+1} \leftarrow \left[ \boldsymbol{\lambda}_1^k - \varrho \left( \mathbf{p}^k + \mathbb{W}_{\boldsymbol{\theta}^k}(\boldsymbol{\varphi}) - \mathbf{d} \right) \right]_+$$

$$\boldsymbol{\lambda}_2^{k+1} \leftarrow \left[ \boldsymbol{\lambda}_2^k - \varrho \left( \mathbf{r}^k + \mathbf{w} - \mathbb{W}_{\boldsymbol{\theta}^k}(\boldsymbol{\varphi}) \right) \right]_+$$

- ▶ Resembles Walrasian auction: Equilibrium is computed via price exchange
- ▶ Proprietary training datasets are localized and not exchanged

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$$\theta^k \leftarrow \max_{\|\theta\| \leq \tau} \mathcal{R}^W(\theta | \mathcal{D}, \lambda_1^k, \lambda_2^k) - \gamma \cdot \mathcal{L}(\theta | \mathcal{D}) + \underbrace{\frac{\rho}{2} \left\| \mathbf{p}^{k-1} + \mathbb{W}_{\theta}(\varphi) - \mathbf{d} \right\|_2^2 + \frac{\rho}{2} \left\| \mathbf{r}^{k-1} + \mathbf{w} - \mathbb{W}_{\theta}(\varphi) \right\|_2^2}_{\text{ADMM feasibility terms}}$$

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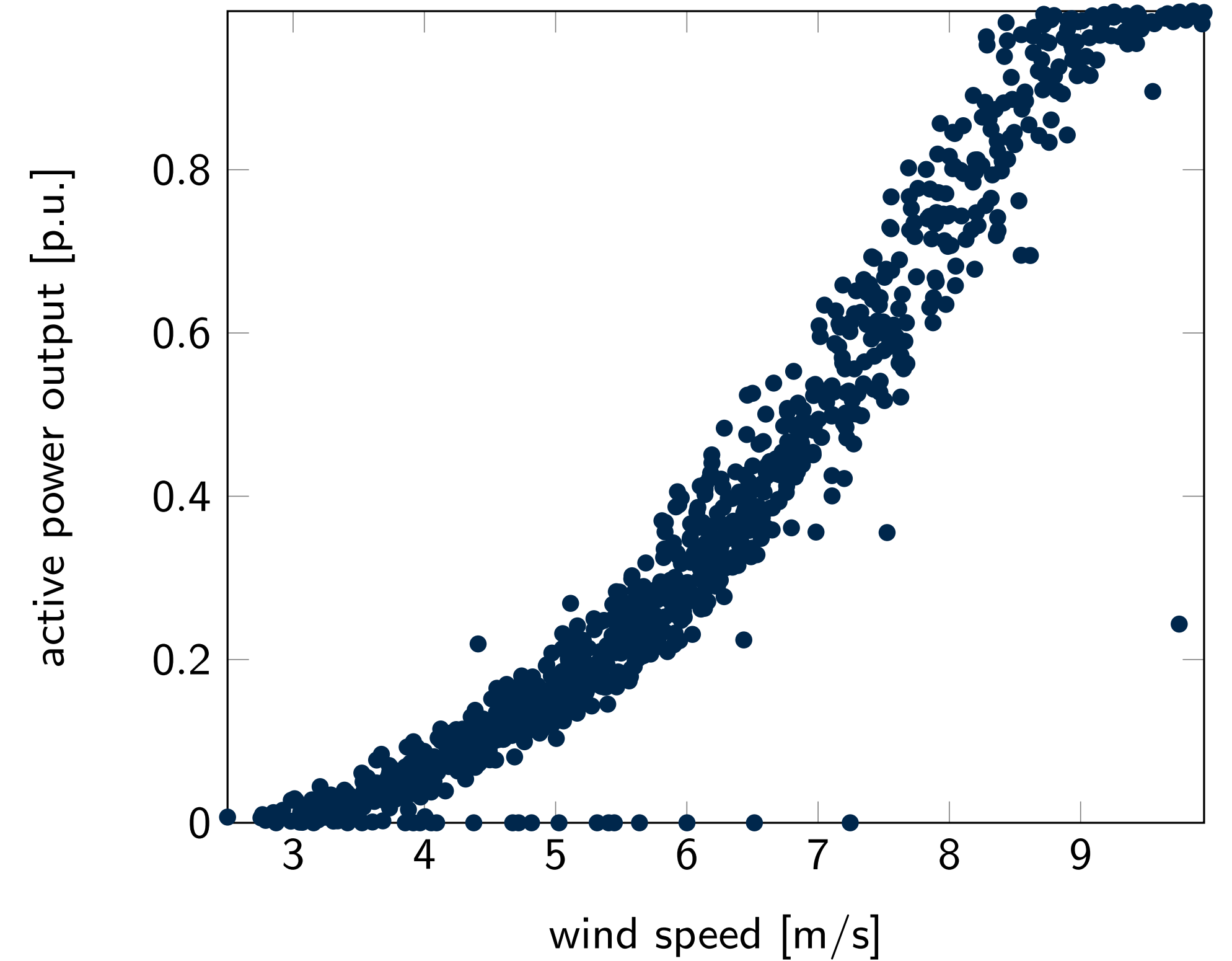
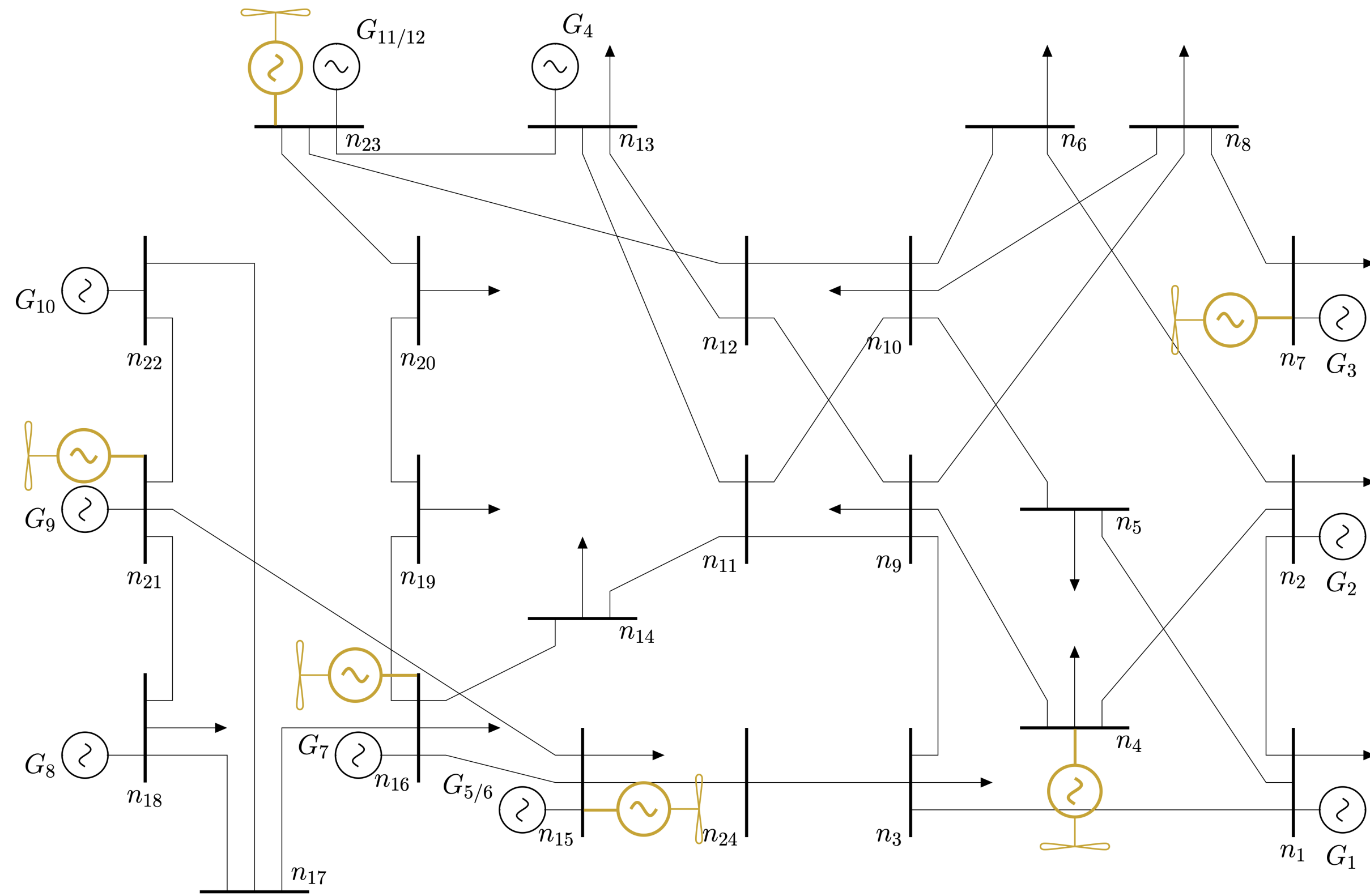
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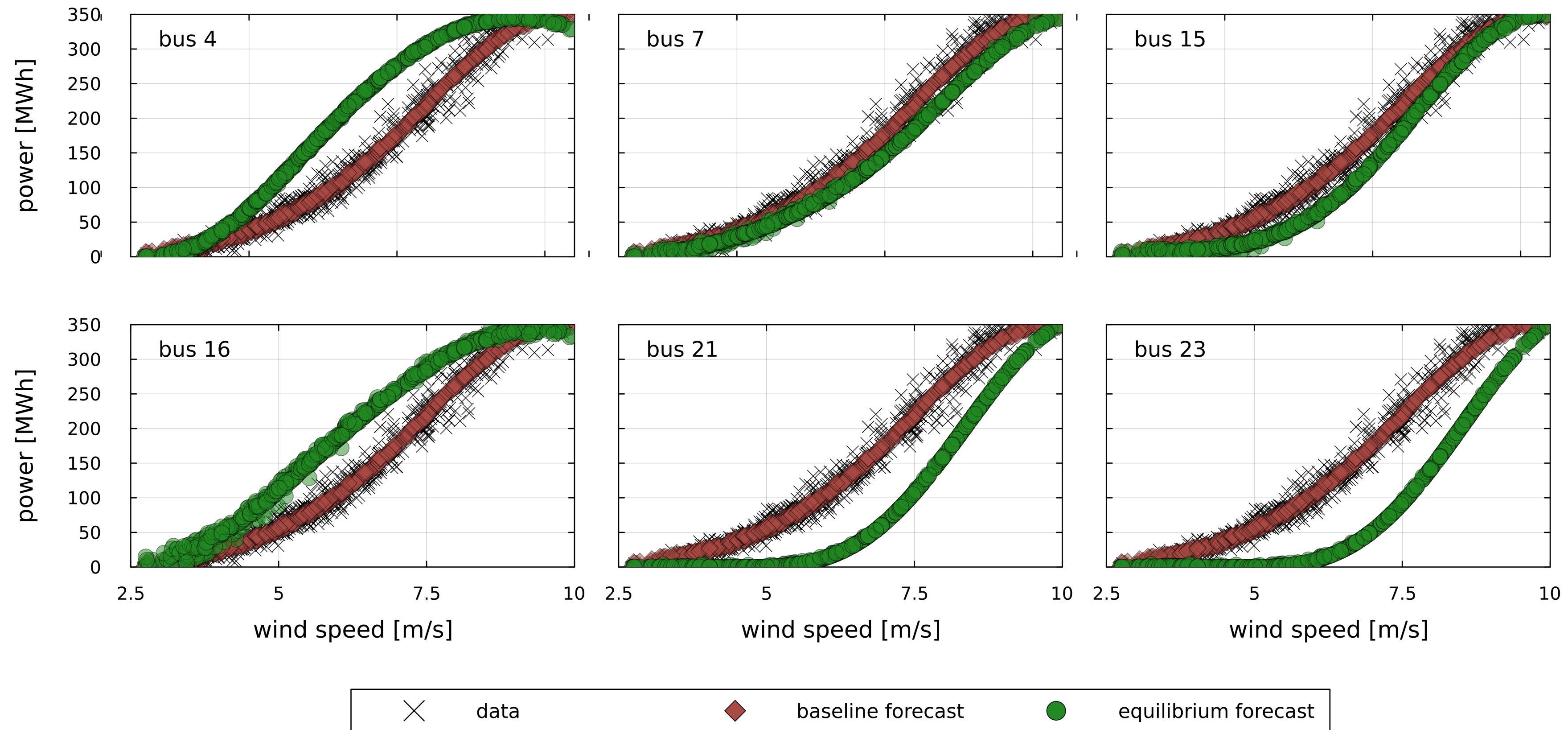
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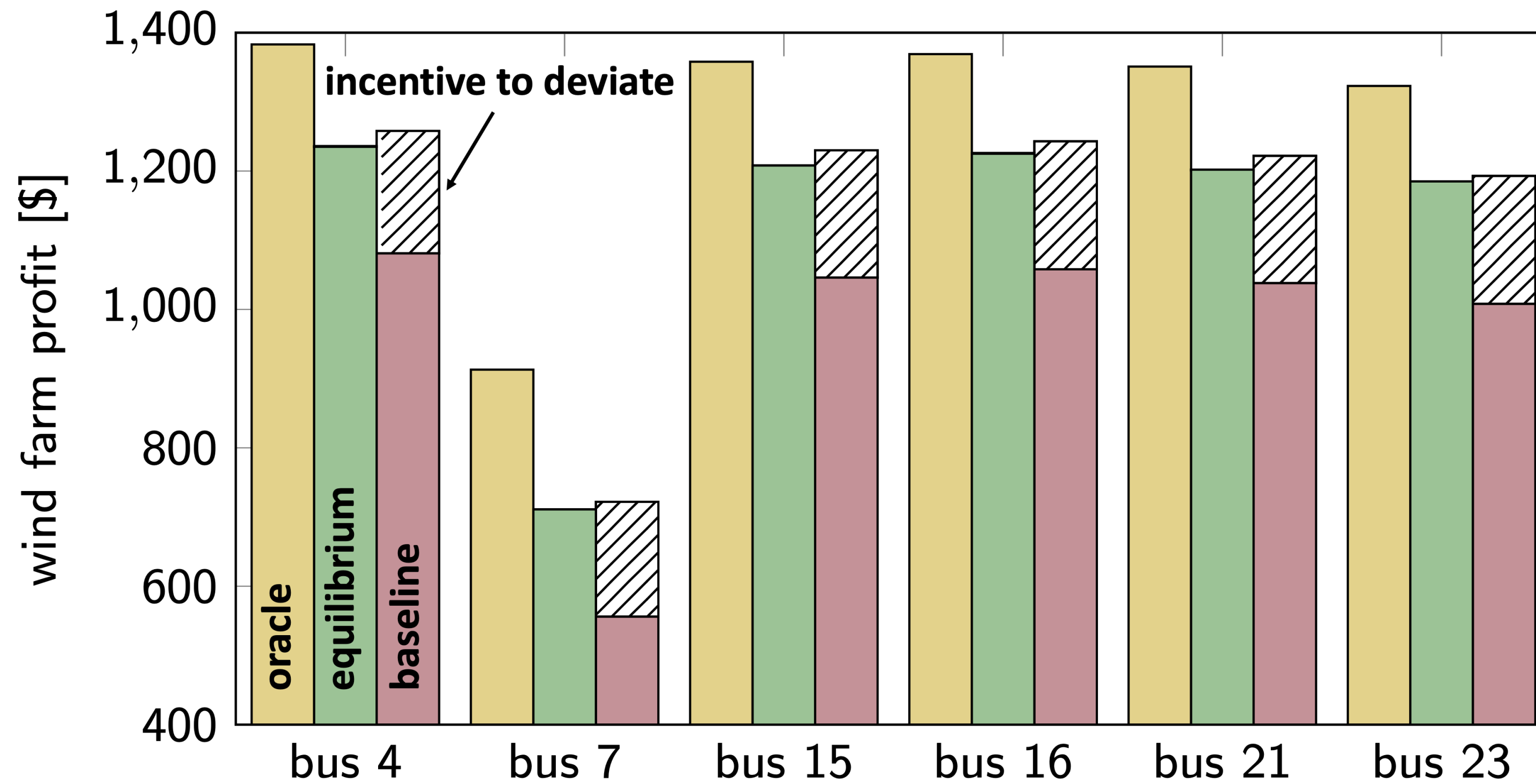


- ▶ 6 with farms with identical data and features
- ▶ Cover 38.4% of load at peak generation
- ▶ Kernel regression with 30 transformed features
- ▶ 5,000 training and 10,000 testing samples
- ▶ Although data is the same, how do equilibrium forecasts depend on the wind farm location in the grid?
- ▶ What are the equilibrium benefits in terms profits (any incentives to deviate?) and cost of electricity?



- ▶ **Baseline:** minimizes a prediction error
- ▶ **Equilibrium:** maximizes wind farm profits

**Systematic over- or under-prediction depending on the wind farm's location in the grid**



- ▶ Equilibrium regression yields larger profits for all wind farms
- ▶ There are large profit incentives to unilaterally deviate from the baseline regression
- ▶ And (almost) no incentives to deviate from the equilibrium regression

Regression	RMSE, MWh	Average dispatch cost, \$			Total dispatch cost error, \$	
		total	day-ahead	real-time	average	CVaR <sub>10%</sub>
Oracle	—	37,246	37,246	—	—	—
Baseline	88	39,223	37,459	1,764	1,977	8,626
Equilibrium	395	38,326	38,154	172	1,080	3,555

- ▶ Baseline regression: minimal forecast error, yet results in large real-time cost
- ▶ Equilibrium regression: large forecast errors, withholds cheap generation from the day-ahead market; yet, results in very cheap real-time re-dispatch
- ▶ Saving of 2.4% on average, and 13.6% on average across 10% of the worst-case scenarios

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## Part I:

- ▶ Erroneous ML models have a significant impact on pricing and dispatch decisions in electricity markets
- ▶ We integrate market-clearing optimization into training to inform them on specific decision objective

## Part II:

- ▶ Network coupling of private ML models (ripple effect on the entire electricity market)
- ▶ Nash regression equilibrium syncs private models and yields maximum profits
- ▶ It implicitly minimizes the cost across day-ahead and real-time markets ...
- ▶ ...thus delivering some benefits of stochastic market design in the existing deterministic markets



Part 1

Thank you for your attention!



Part 2



# ECE 598: Computational Power Systems

Term: Winter 2025

Credit Hours: 3 credits

Time: Fridays 10:30-13:30

Format : Lecture (75 min) + Break + Tutorial (75 min)

Instructor: Vladimir (Vlad) Dvorkin

E-mail: [dvorkin@umich.edu](mailto:dvorkin@umich.edu)

## Course Description

The growing digitization of power systems and the rapid integration of renewable energy resources call for new computational algorithms to support power system operations and electricity markets. In this course, students will learn the core computational problems in power systems and modern algorithms to solve those problems, while managing the trade-offs between performance, speed and data requirements.

In the first part of the course, students will familiarize themselves with optimization problems in power systems, including economic dispatch and market clearing for transmission grids, as well as voltage control and peer-to-peer markets for distribution grids. They will also learn how machine learning (ML) aids in solving these optimization problems. In the second part, the focus will be on decentralized/distributed decision-making in high-voltage and distribution grids, and how agents can autonomously solve dispatch, control, and learning problems using decentralized/distributed algorithms, such as dual decomposition, ADMM, and their variants. In the third part, students will acquire prescriptive analytics skills: it will introduce algorithms for decision-focused learning in the context of renewable power forecasting and other relevant analytical tasks. Students will work on final projects (to be agreed upon with the instructor) and present the results to their peers. Possible project topics include:

- Decentralized electricity market designs
- Carbon-constrained electric power dispatch and pricing
- Optimization algorithms for voltage control in distribution grids
- Power grid coordination with adjacent infrastructures (e.g., with data centers)

Each weekly session will consist of a lecture and a follow-up tutorial on the lecture materials.

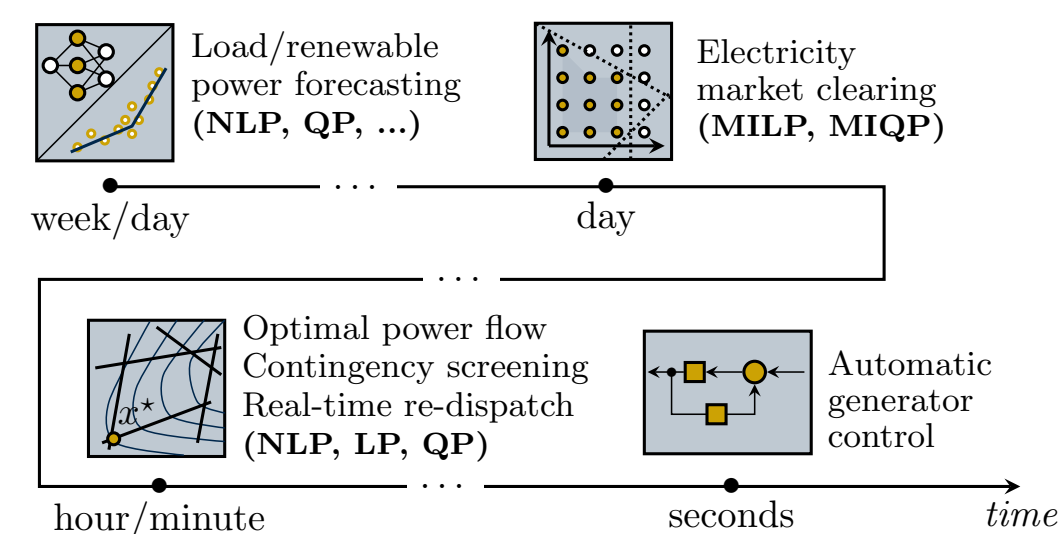





















Figure 1: Computational power systems timeline

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