# **Formal Privacy Guarantees** for Optimization Datasets in Power Systems

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## Power systems of the future as a collection of optimization problems



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### Power resource allocation

**Data:** cost, tech limits, topology Type: LP, QP, MIQP, NLP **Result:** cost-optimal and feas. allocation

Renewable power forecasting

**Data:** historical records, weather forecast **Type:** QP, convex or NLP **Result:** forecast w/ varying leading times

### Demand response

**Data:** loads and tech limits **Type:** LP, QP, MIQP, etc. **Result:** load timing and geo allocation

energy storage

PV gen











## When optimization spills secrets - abstraction

### **Resource** allocation

## **Regression** analysis



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Classification













## When optimization spills secrets - case of power systems

### **High-voltage systems**



Electricity prices [\$/MWh] at New York ISO, August 28, 2018

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 $\downarrow$  data leakage













## Power systems as a stroll in the fog

- Power systems are critical infrastructures with most of data being classified
- We have only a limited observably, e.g., MISO data disclosure portal
- Market participants hence act on a limited set of system data





Hedgehog in the Fog Yuri Norstein (1975)

**Example:** market clearing in the (small) IEEE 118-Bus system

- 1079 rows of element-specific data
- Each generator owns only 2 rows
- The rest of the system remains unknown









## In today's talk





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- 1. Introduction
- 2. Differential privacy basics
- 3. Privacy-preserving optimization via stochastic programming
- 4. Privacy-preserving synthetic dataset generation
- 5. Outlook





## 1. Introduction

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# Formalizing differential privacy (DP)



- ► Wind power records  $y, y', y'', ... \in [0, 1]$
- For given  $\alpha > 0$ , records y and y' are  $\alpha$ -adjacent if  $||y y'|| \leq \alpha$
- Let Lap $(\alpha/\varepsilon)$  be a zero-mean random Laplacian noise For some parameter  $\varepsilon > 0$ , the release is  $\varepsilon$ -DP if  $\frac{\Pr[y + \operatorname{Lap}(\alpha/\varepsilon) \in \widehat{y}]}{\Pr[y' + \operatorname{Lap}(\alpha/\varepsilon) \in \widehat{y}]} \leq \exp(\varepsilon)$

for any  $\alpha$ -adjacent pair (y, y') and any outcome  $\widehat{y}$ >>> V. Dvorkin









## **DP** basics: Laplace mechanism

Function  $f : \mathbb{D} \mapsto \mathbb{R}$  mapping datasets from data universe  $\mathbb{D}$  to reals

### Laplace mechanism of DP

Perturbed function  $\tilde{f}(\cdot) = f(\cdot) + Lap$  $\frac{\Pr\left[\tilde{f}(\mathcal{D})\right]}{\Pr\left[\tilde{f}(\mathcal{D}')\right]}$ 

for any pair  $\mathcal{D}, \mathcal{D}' \in \mathbb{D}$  and any outcome

• **Composition**: a series of  $\tilde{f}_1(\mathcal{D}), \ldots, \tilde{f}_k(\mathcal{D})$  of  $\varepsilon$ -DP computations ensures  $k\varepsilon$ -DP • Immunity to post-processing: if  $\tilde{f}(\mathcal{D})$  is  $\varepsilon$ -DP, then  $g \circ \tilde{f}(\mathcal{D})$  is also  $\varepsilon$ -DP

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• Worst-case sensitivity  $\Delta f$  of function f to datasets, i.e.,  $\Delta f = \max_{\mathcal{D} \sim \alpha \mathcal{D}'} \|f(\mathcal{D}) - f(\mathcal{D}')\|_1$ 

$$\left(\mu = 0, b = \frac{\Delta f}{\varepsilon}\right)$$
 is  $\varepsilon$ -DP for datasets, i.e.,  
 $\left(\hat{y} \in \widehat{y}\right]$   
 $\left(\hat{y} \in \widehat{y}\right] \leq \exp(\varepsilon)$   
ome  $\widehat{y}$ .





## **DP** basics: Report noisy max

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• Worst-case sensitivity  $\Delta = \max_{i} \Delta f_{i}$  of functions to datasets

### **Report noisy max**

What function takes the maximum value on a private dataset  $\mathcal{D}$ ?

for i = 1, ... $| \tilde{f}_i(\mathcal{D}) =$ end

Use case: what is the worst-case optimization model for a given (private) dataset?

Finite population of functions  $f_1, \ldots, f_k : \mathbb{D} \mapsto \mathbb{R}$  mapping datasets from universe  $\mathbb{D}$  to reals

$$., k$$
  
 $f_i(\mathcal{D}) + Lap(\Delta/arepsilon)$ 

- **return**:  $i^* \in \operatorname{argmax}_i \tilde{f}_i(\mathcal{D})$
- Releasing index  $i^*$  satisfies  $\varepsilon$ -DP (despite k computations on private data!)











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## Limits of DP applications to convex optimization



### Input perturbation

**1** Optimization dataset perturbation

$$\tilde{\mathcal{D}} = \mathcal{D} + \zeta, \quad \zeta \sim \mathsf{Lap}(\alpha/\varepsilon)$$

**2** Optimization on perturbed data  $x^*(\tilde{\mathcal{D}})$ 

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Conic optimization program • Optimization dataset  $\mathcal{D} = \{c, b, A\}$ • Optimal solution  $x^*$  is dataset-specific • Often,  $x^*(\mathcal{D}) \neq x^*(\mathcal{D}')$  for different datasets  $\mathcal{D}$  and  $\mathcal{D}'$ 

## **Output perturbation**













 $\blacktriangleright$  *l* and *l'* must be made indistinguishable

input and output perturbation strategies are equivalent and yield infeasible results













## Stochastic programming for private optimization queries

For a deterministic conic program, we device a chance-constrained (stochastic) counterpart

Solution vector  $x(\mathcal{D})$  is modeled as a **linear decision rule** of the form:

$$\widetilde{x}(\mathcal{D}) = \overline{x}(\mathcal{D}) + X(\mathcal{D})\boldsymbol{\zeta}$$

Identity query  $(\mathcal{X} = \{X | X = I\})$ :

$$ilde{x}(\mathcal{D}) = \overline{x}^*(\mathcal{D}) + X^*(\mathcal{D})\boldsymbol{\zeta} = \overline{x}^*(\mathcal{D}) + \boldsymbol{\zeta}$$

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 $\overline{x}$  – nominal solution vector

$$\zeta$$
 – zero-mean perturbation

Sum query  $(X = \{X | 1^{\top}X = 1\})$ :

 $1^{\top} \tilde{x}(\mathcal{D}) = 1^{\top} \overline{x}^{\star}(\mathcal{D}) + 1^{\top} X^{\star}(\mathcal{D}) \boldsymbol{\zeta} = 1^{\top} \overline{x}^{\star}(\mathcal{D}) + \boldsymbol{\zeta}$ 













# minimize $\mathbb{E}\left[c\cdot(\overline{x}+\zeta)\right]$ subject to $\Pr[\ell \leq \overline{x} + \zeta \leq u] \ge 1 - \eta$ ,

- ▶ Perturb. of  $\overline{x}^*$  is feasible with a high prob.
- Sub-optimal due to feasibility requirement

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 $\implies$ 

### **Deterministic program**

 $\begin{array}{ll} \underset{x}{\text{minimize}} & c^{\top}x\\ \text{subject to} & b - Ax \in \mathcal{K} \end{array}$ 

### Differential privacy of identity optimization queries

 $\implies \zeta \sim \mathsf{Lap}(\mathbf{\Delta}_{lpha} / arepsilon) \implies$ 

- dataset adjacency  $\alpha$
- solution sensitivity  $\Delta_{\alpha}$
- privacy budget  $\varepsilon$



$$\frac{\Pr[\overline{x}^{\star}(\mathcal{D}) + X^{\star}(\mathcal{D})\zeta = \widehat{x}]}{\Pr[\overline{x}^{\star}(\mathcal{D}') + X^{\star}(\mathcal{D}')\zeta = \widehat{x}]} \leqslant \epsilon$$

▶ for any optimization outcome x
 ▶ and α-adjacent dataset pair (D, D')



 $\exp(arepsilon)$  $\widehat{x}$  $(\mathcal{D},\mathcal{D}')$ 



## Private optimal power flow (OPF) problem

$$\begin{array}{l} \displaystyle \min_{\overline{x}, X \in \mathcal{X}} \quad \mathbb{E}[c^{\top}(\overline{x} + X\boldsymbol{\zeta})] \\ \text{s.t.} \quad 1^{\top}(\overline{x} + X\boldsymbol{\zeta} - \boldsymbol{d}) = 0 \\ \\ \displaystyle \mathsf{Pr} \begin{bmatrix} |F(\overline{x} + X\boldsymbol{\zeta} - \boldsymbol{d})| \leqslant f^{\max} \\ |x^{\min} \leqslant \overline{x} + X\boldsymbol{\zeta} \leqslant x^{\max} \end{bmatrix} \geqslant 1 - \eta \end{array}$$

Load vector d (in MWh) is private information

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• Must be stat. similar to any  $\alpha$ -adjacent load d'

perturbation strategy	OPF infeasibility (%)			OPF sub-optimality (%)		
	$\alpha = 1$	$\alpha = 3$	lpha= 10	$\alpha = 1$	$\alpha = 3$	lpha= 10
input	51.5	49.9	50.3	0.0	0.1	0.0
output	52.7	51.5	48.8	0.0	0.0	0.1
program	0.1	0.1	0.1	1.7	5.1	17.1



perturbed power balance

stochastic network limits

Queries in electricity markets

- System costs (objective function)
- Generation by a particular technology

1-DP system cost query on the IEEE 24-Bus RTS





## Private optimal power flow (OPF) problem

$$\begin{split} \min_{\overline{x}, X \in \mathcal{X}} & \mathbb{E}[c^{\top}(\overline{x} + X\boldsymbol{\zeta})] \\ \text{s.t.} & \mathbf{1}^{\top}(\overline{x} + X\boldsymbol{\zeta} - \boldsymbol{d}) = 0 \\ & \mathsf{Pr} \begin{bmatrix} |F(\overline{x} + X\boldsymbol{\zeta} - \boldsymbol{d})| \leqslant f^{\mathsf{max}} \\ |x^{\mathsf{min}} \leqslant \overline{x} + X\boldsymbol{\zeta} \leqslant x^{\mathsf{max}} \end{bmatrix} \geqslant 1 - \eta \end{split}$$

Load vector d (in MWh) is private information • Must be stat. similar to any  $\alpha$ -adjacent load d'



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expected generation cost

perturbed power balance

stochastic network limits

Queries in electricity markets

- System costs (objective function)
- Generation by a particular technology







## **Private monotone curve fitting**

$$\min_{\beta} \mathbb{E} \left[ \sum_{i=1}^{n} \left( \underbrace{y_{i} - \varphi(x_{i})^{\top} \beta}_{\text{business as usual}} \underbrace{-\varphi(x_{i})^{\top} \zeta}_{\text{perturbation}} \right)^{2} \right]$$
s.t.  $\mathbb{P} \left[ C(\beta + \zeta) \ge 0 \right] \ge 1 - \eta,$ 

### output perturbation strategy



-0.00/

- Dataset  $\{(y_1, x_1), \ldots, (y_n, x_n)\}$
- Minimize regression loss function
- ▶ By finding optimal weights  $\beta^*$  ...
- ... of basis functions in vector  $\varphi(x)$

### 60 $h(x) = 0.49\varphi_1(x) + 1.482\varphi_2(x)$ 30 y(x) 0 • -30 -60O 2 6 4 8 Х

### program perturbation strategy











## **Private monotone curve fitting**

$$\begin{split} \min_{\beta} & \mathbb{E} \left[ \sum_{i=1}^{n} \left( \underbrace{y_{i} - \varphi(x_{i})^{\top} \beta}_{\text{business as usual}} \underbrace{-\varphi(x_{i})^{\top} \zeta}_{\text{perturbation}} \right)^{2} \right] \\ \text{s.t.} & \mathbb{P} \left[ C(\beta + \zeta) \ge 0 \right] \ge 1 - \eta, \end{split}$$



Alstom.Eco.80



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- Dataset  $\{(y_1, x_1), \ldots, (y_n, x_n)\}$
- Minimize regression loss function
- ▶ By finding optimal weights  $\beta^*$  ...
- ... of basis functions in vector  $\varphi(x)$









## Private support vector machine (SVM) for classification

Dataset (x<sub>1</sub>, y<sub>1</sub>), ..., (x<sub>m</sub>, y<sub>m</sub>)
 Feature x<sub>i</sub> ∈ ℝ<sup>n</sup>, label y<sub>i</sub> ∈ {−1, 1}

$$\min_{\tilde{b}(\boldsymbol{\zeta}), \tilde{w}(\boldsymbol{\zeta}), z} \mathbb{E} \left[ \lambda \| \overline{w} \|^2 + \frac{1}{m} \mathbf{1}^\top z + \lambda \| W \right]$$
  
s.t. 
$$\Pr \left[ \begin{array}{c} y_i (\overline{w}^\top x_i - \overline{b}) \ge 1 - z_i - z_i \\ z_i \ge 0, \quad \forall i = 1, \dots, m \end{array} \right]$$

- Quering hyperplane parameters
- Deterministic hyperplane is very sensitive to perturbation
- Stochastic hyperplane, in contrast, is very robust to perturbation

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mean accuracy 51.2%

mean accuracy 97.6%





## Private support vector machine (SVM) for classification

► Dataset  $(x_1, y_1), ..., (x_m, y_m)$ Feature  $x_i \in \mathbb{R}^n$ , label  $y_i \in \{-1, 1\}$ 

$$\min_{\tilde{b}(\boldsymbol{\zeta}), \tilde{w}(\boldsymbol{\zeta}), z} \mathbb{E} \left[ \lambda \| \overline{w} \|^2 + \frac{1}{m} \mathbf{1}^\top z + \lambda \| W \right]$$
  
s.t. 
$$\Pr \left[ \begin{array}{c} y_i (\overline{w}^\top x_i - \overline{b}) \ge 1 - z_i - z_i \\ z_i \ge 0, \quad \forall i = 1, \dots, m \end{array} \right]$$

- Load data to classify OPF feasibility
- Output perturbation (OP) accuracy is small
- Program perturbation (PP) accuracy high and improves with a smaller constraint violation prob.  $\eta$

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**Computes a hyperplane**  $w^{\top}x_i - b$ ► Classification rule sign[ $w^{*\top}\hat{x} - b^{*}$ ]  $\| \boldsymbol{\zeta} \|^2$  $\begin{array}{c|c} -y_i((W\zeta)^\top x_i - B\zeta), \\ \eta \end{array} \geqslant 1 - \eta, \quad \begin{bmatrix} W \\ B \end{bmatrix} = I \end{array}$ OPF feasibility classification on IEEE 24-Bus RTS 90 80 ccuracy (%) 70 |-60 50 non-private OP PP (η = 5%) PP ( $\eta = 1\%$ ) PP ( $\eta = 0.01\%$ )







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## Synthetic datasets are not new to power systems



**Texas A&M University Grid Datasets** 

### Why these datasets may not satisfy our needs?

- "[...] data bears no relation to the actual grid [...] except that generation and load profiles are similar, based on public data"
- "This test case represents a synthetic (fictitious) transmission"
- "This case is synthetic and does not model the actual grid"

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**PyPSA-Eur: European synthetic data** 

Australian synthetic market data









# Wind power obfuscation (WPO) algorithm



# Wind power obfuscation (WPO) algorithm (Part I)

real dataset: 
$$D = \{(y_1, x_1), ..., (y_n, x_n)\}$$

synthetic dataset:  $\tilde{\mathcal{D}} = \{ (\tilde{y}_1, x_1), \dots, (\tilde{y}_n, x_n) \}$ 

- Regression on synthetic data  $\tilde{y}$  must match the regression on real data y We use regression loss and weights as a measure of accuracy
- Private estimation of regression parameters:

$$\mathsf{loss}: \quad \overline{\ell} = \ell(y) + \mathsf{Lap}\left(\frac{\delta_{\ell}}{\varepsilon}\right), \quad \mathsf{weights}: \quad \overline{\beta} = \beta(y) + \mathsf{Lap}\left(\frac{\delta_{\beta}}{\varepsilon}\right)$$

where  $\delta_{(.)}$  is the sensitivity of (.) to data  $\alpha$ -adjacent datasets Lemma (global sensitivity bounds):

$$\delta_{\ell} \leq \underset{i=1,\ldots,n}{\operatorname{maximum}} \left\| (X(X^{\top}X + \lambda I)^{-1}X^{\top} - I)(e_i \circ \alpha) \right\| \qquad \delta_{\beta} \leq \left\| (X^{\top}X + \lambda I)^{-1}X^{\top} \right\|_1 \alpha$$

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# Wind power obfuscation (WPO) algorithm (Part II)

**Step 1** Synthetic wind power measurements:

**Step 2** Private regression parameters estimation:

 $\overline{\ell} = \ell(y) + Lap(y)$ 

**Step 3** Synthetic dataset post-processing:

 $\widetilde{y} \in \underset{\widetilde{y}}{\operatorname{argmin}} \quad \bigcup_{\widetilde{\ell}}$ los

s.t. **0** ≤

 $\beta($ 

**Theorem:**  $\varepsilon_1 = \varepsilon/2$  and  $\varepsilon_2 = \varepsilon/4$  renders WPO  $\varepsilon$ -DP for  $\alpha$ -adjacent wind power datasets.



 $\tilde{y}^0 = y + \text{Lap}(\alpha/\varepsilon_1)$ 

$$\left(\delta_{\ell}/\varepsilon_{2}
ight) \quad \overline{eta} = eta(y) + \operatorname{Lap}\left(\delta_{\beta}/\varepsilon_{2}
ight)$$

$$\begin{split} \overline{\xi} &= \ell(\widetilde{y}) \| + \gamma_{\beta} \underbrace{\|\overline{\beta} - \beta(\widetilde{y})\|}_{\text{weight accuracy}} + \gamma_{y} \underbrace{\|\widetilde{y}^{0} - \widetilde{y}\|}_{\text{regularization}} \\ \leqslant \widetilde{y} \leqslant \mathbf{1} \\ \widetilde{y}), \ell(\widetilde{y}) \in \operatorname*{argmin}_{\beta} \underbrace{\|X\beta - \widetilde{y}\|}_{\ell} + \lambda \|\beta\| \end{split}$$











## Laplace Mechanism

Accuracy of the WPO Algorithm remains high with a growing privacy requirement  $\alpha$ 

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# Transmission capacity obfuscation (TCO) algorithm



## **Differentially private release of network parameters**

## **Optimal Power Flow (OPF) problem**

$$egin{aligned} \mathcal{C}(\overline{f}) &= \min_{p \in \mathcal{P}} & c^ op p & dispatch \ ext{s.t.} & \mathbf{1}^ op (p-d) &= 0 & power b \ ert F(p-d) ert \leqslant \overline{f} & power flow \end{aligned}$$

Laplace mechanism:

Laplace + Bilevel optimization:



Feasible and cost-consistent with respect to a **single** OPF model

 $\overline{\varphi}^{0} = \overline{f} + \operatorname{Lap}(\alpha/\varepsilon)$ 

Almost never feasible

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h costs

balance

w limit



How to release vector of transmission capacities f privately?

## Laplace & Exponential mechanisms + **Bilevel optimization**:

- ► LM for obfuscation
- EM for worst-case OPF models
- Bilevel opt. on worst-case models

Feasible and cost-consistent with respect to a **population** of OPF models





## **Differentially private transmission capacity obfuscation (TCO) algorithm**

**Step 1** Initialize synthetic data using LM:



**Theorem:**  $\varepsilon_1 = \varepsilon/2$  and  $\varepsilon_2 = \varepsilon/(4T)$  achieve  $\varepsilon$ —differential privacy

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$$\overline{\varphi}^{0} = \overline{f} + \mathsf{Lap}(\alpha/\varepsilon_{1})$$

**Step 2** Find the worst-case OPF model using EM:

$$\overline{f}) - \mathcal{C}_i^R(\overline{\varphi}^{t-1}) \Big\|_1 + \operatorname{Lap}(\overline{c}\alpha/\varepsilon_2), \forall i = 1, \dots, m$$

return index  $k^t$  of the worst-case model **Step 3** Compute the worst-case cost using LM:

$$\overline{\mathcal{C}}_{t} = \mathcal{C}_{k^{t}}(\overline{f}) + \operatorname{Lap}(\overline{c}\alpha/\varepsilon_{2})$$

**Step 4** Post-processing bilevel optimization:

in 
$$\sum_{\tau=1}^{t} \left\| \overline{\mathcal{C}}_{\tau} - \mathcal{C}_{k^{\tau}}(\overline{\varphi}) \right\| + \left\| \overline{\varphi} - \overline{\varphi}^{t-1} \right\|$$





## Laplace mechanism



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## **TCO Algorithm**











# IEEE 73-RTS benchmark: TCO feasibility and sup-optimality





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### more privacy (more noise)







## IEEE 73-RTS benchmark: TCO robustness bias



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after 5 iterations

after 10 iterations











**Concluding remarks** 

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What we used to say about synthetic datasets:

- "[...] data bears no relation to the actual grid [...]"
- "This test case represents [...] fictitious transmission"
- "This case is synthetic and does not model the actual grid"

## What we will say about synthetic datasets:

- "This synthetic dataset is produced based on the data from a real-world power grid"
- "It is not possible to infer the real data from this synthetic dataset"
- "Computational results on this data are consistent with the real data"







 $\blacktriangleright$  Rigorous privacy quantification  $\Longrightarrow$  legal compliance











## What is next?



### What has been done so far

- Noise addition to obfuscate private data
- Post-processing optimization to improve utility

### Diffusion models are the next step

- Privacy-preserving perturbation in the forward process
- Optimization in the reverse process to ensure utility

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## Thank you for your attention!

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### Let's stay in touch:







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